F. Becattini
Universita' di Firenze

L'adronizzazione come fenomeno statistico

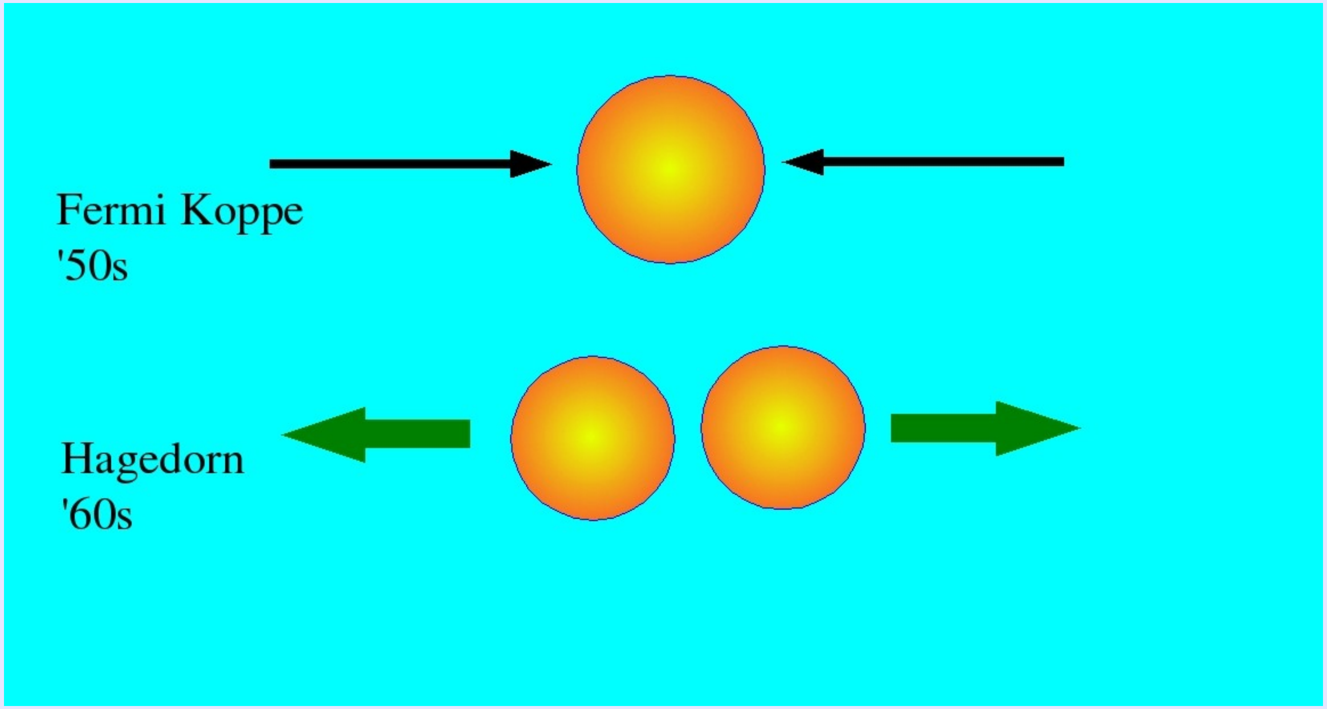
SOMMARIO

- Introduzione
- Il modello statistico di adronizzazione
- Risultati
- Interpretazioni

See F. B., arXiv:0901.3643

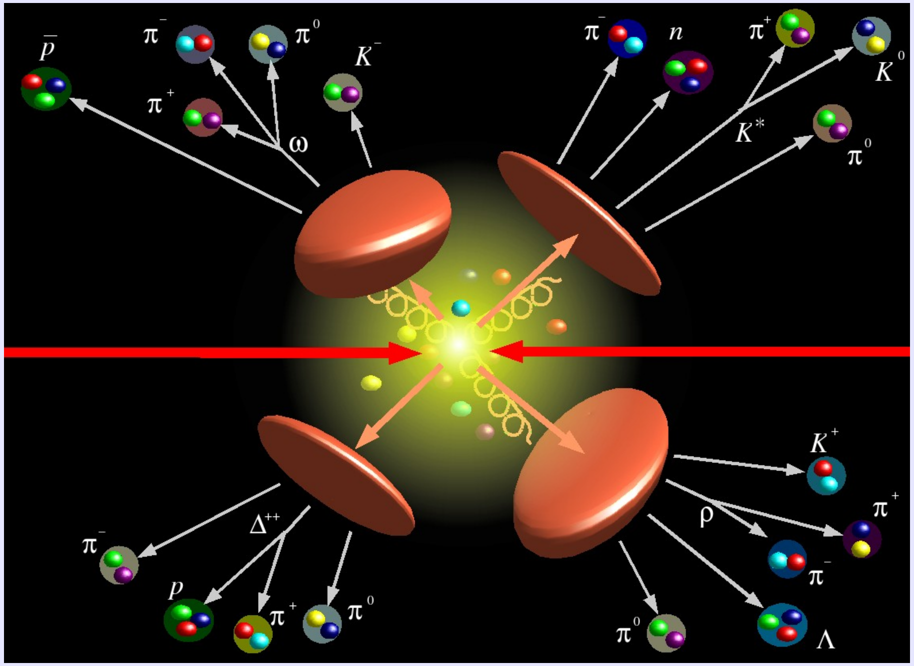
Roma La Sapienza
29 Aprile 2009

Historical prologue

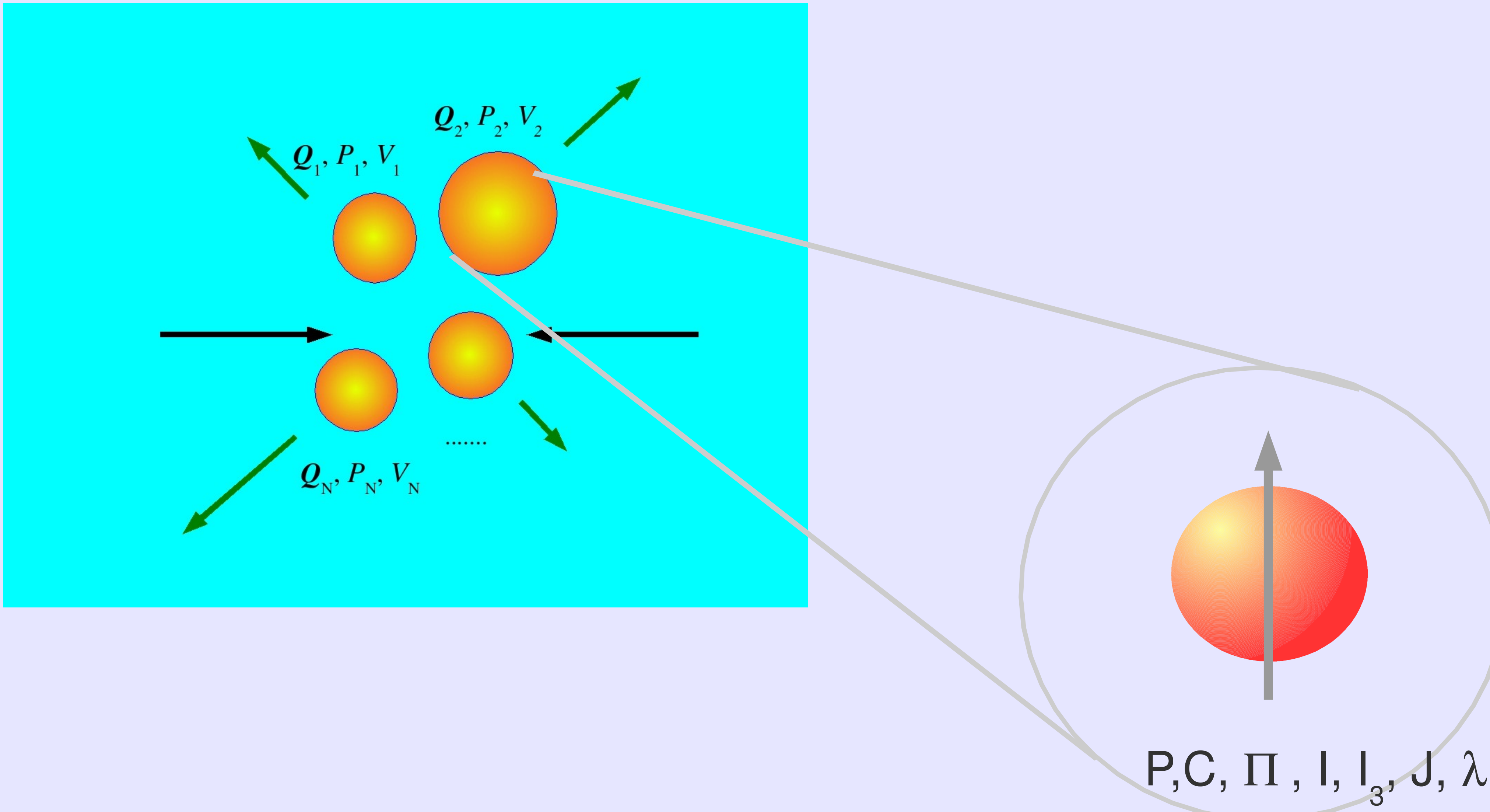


Multiple hadron production proceeds from highly excited regions (clusters or fireballs) emitting hadrons according to a pure statistical law

In modern view, the statistical model is a model of hadronization, describing the process of hadron formation at the scale where QCD is no longer perturbative



The fundamental brick of the SHM is the CLUSTER or FIREBALL: a massive *extended* relativistic object with inner charges



Extension is the key property

Cluster ~ bag of the MIT model (relativistic extended massive object)

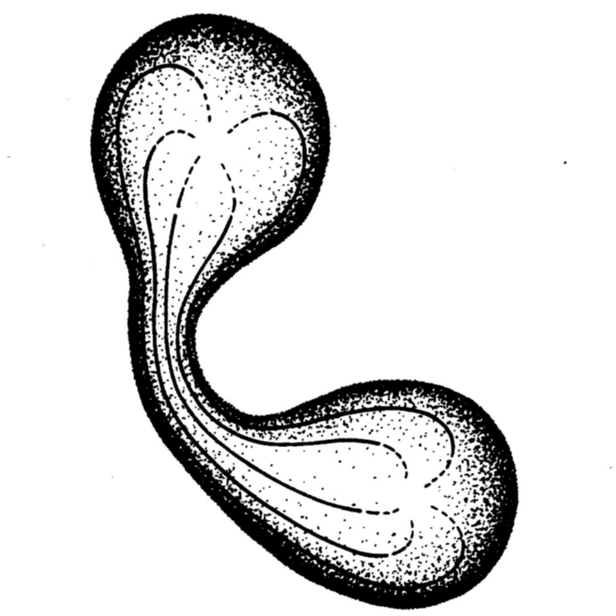


FIG. 1. A color-singlet bag attempting to fission into two bags which are not color singlets. The flux lines of the colored gluon field are shown explicitly.

The statistical hadronization model can be seen as an effective model for the "formidable task" of calculating bag decays

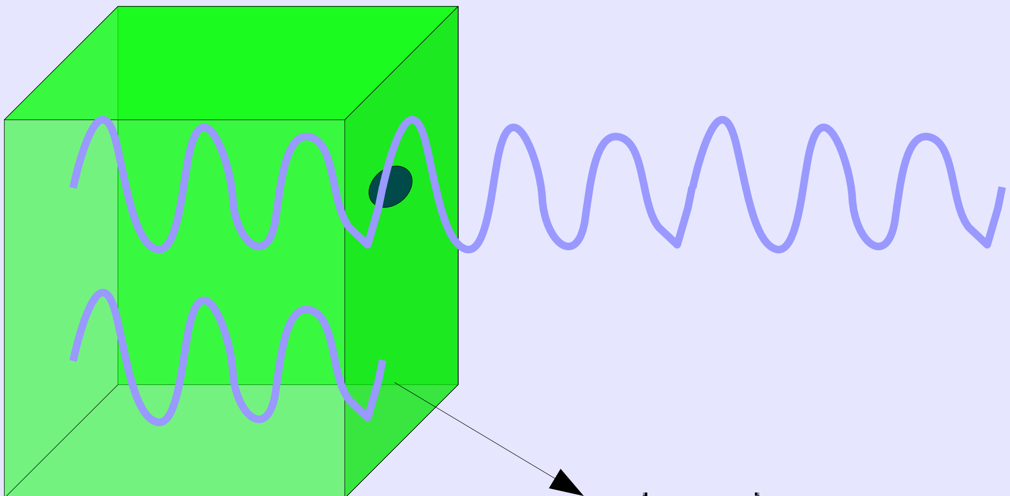
A. Chodos et al., Phys. Rev. D 12 (3471) 1974

The SHM's *urprinzip*

Every localized multihadronic state within the cluster compatible with conservation laws is equally likely

The word “localized” gives finite extension a crucial role

Localized vs asymptotic states



$$|f\rangle = |p_1, \sigma_1; p_2, \sigma_2, \ldots\rangle$$

$$|h_V\rangle$$

described by the occupation numbers of the field in the box

The distinction is unimportant if the volume is sufficiently larger than (cubic root of) Compton wavelenghts, but it is crucial if they are comparable

Example: one particle in non-relativistic Quantum Mechanics

$$|h_V\rangle = |\mathbf{k}\rangle = \begin{cases} \frac{1}{\sqrt{V}} \exp(i\mathbf{k}\cdot\mathbf{x}) & \text{if } \mathbf{x} \in V \\ 0 & \text{if } \mathbf{x} \notin V \end{cases} \qquad \mathbf{k} = \pi n_x/L_x \pi \hat{\mathbf{i}} + n_y/L_y \hat{\mathbf{j}} + \pi n_z/L_z \hat{\mathbf{k}}$$

$$\langle \mathbf{k}|\mathbf{p}\rangle = \frac{1}{\sqrt{(2\pi)^3 V}} \int_V d^3x \; \mathrm{e}^{\mathrm{i}(\mathbf{p}-\mathbf{k})\cdot\mathbf{x}}$$

Quantum Field Theory: localized and asymptotic states differ also for the *numbers of quanta*

$$|N\rangle_V = \alpha_{0,N}|0\rangle + \alpha_{1,N}|1\rangle + \ldots + \alpha_{N,N}|N\rangle + \ldots$$

$$|0_V\rangle \neq |0\rangle$$

Casimir effect

One can write non-bijective (Bogoliubov) relations connecting creation operators for the localized and full-space field problems

$$a_{\mathbf{k}} = \int d^3p \, F(\mathbf{k},\mathbf{p}) \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{p}}}{2\sqrt{\varepsilon_{\mathbf{k}}\varepsilon_{\mathbf{p}}}} \, a_{\mathbf{p}} + F(\mathbf{k},-\mathbf{p}) \frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{p}}}{2\sqrt{\varepsilon_{\mathbf{k}}\varepsilon_{\mathbf{p}}}} \, a^{\dagger}_{\mathbf{p}}$$

Translating the postulate into formulae

F.B., J. Phys. Conf. Ser. 5, 175 (2005), hep-ph 0410403

The cluster is described as a *mixture of states*

$$\hat{\rho} \propto \sum_{h_V} \mathsf{P}_i |h_V\rangle\langle h_V| \mathsf{P}_i \equiv \mathsf{P}_i \mathsf{P}_V \mathsf{P}_i$$

P_i is the projector onto initial cluster's quantum numbers

$$\mathsf{P}_i = \mathsf{P}_{P,J,\lambda,\Pi} \mathsf{P}_\chi \mathsf{P}_{I,I_3} \mathsf{P}_{\mathbf{Q}}$$

P 4-momentum
J spin
λ helicity
π parity
χ C-parity
Q abelian charges
I, I_3 isospin

P_i can be further factorized if worked out into the rest frame where *P = (M,**0**)*

$$\mathsf{P}_{P,J,\lambda,\pi} = \delta^4(P - \hat{P}) \mathsf{P}_{J,\lambda} \frac{\mathsf{I} + \pi\hat{\Pi}}{2}$$

Can define a probability of observing an asymptotic multi-particle state $|f\rangle$

$$p_f \propto \langle f|\mathsf{P}_i \mathsf{P}_V \mathsf{P}_i|f\rangle$$

This is positive definite and fulfills symmetry requirements.

The microcanonical partition function is recovered summing over all final states

$$\sum_f p_f \propto \sum_f \langle f|\mathsf{P}_i \mathsf{P}_V \mathsf{P}_i|f\rangle \propto \mathrm{tr} \mathsf{P}_V \mathsf{P}_i = \sum_{h_V} \langle h_V|\mathsf{P}_i|h_V\rangle \equiv \Omega$$

Mixture vs pure state

Proper quantum description of a cluster as a *pure state* superposition of localized states

$$|\psi\rangle = \sum_{h_V} c_{h_V} \mathsf{P}_i |h_V\rangle$$

From the postulate: $|c_{h_V}|^2 \quad \text{independent of} \;\; h_V$

$$\begin{aligned} |\langle f|\psi\rangle|^2 &= |\sum_{h_V} \langle f|\mathsf{P}_i|h_V\rangle c_{h_V}|^2 = \text{const} \sum_{h_V} |\langle f|\mathsf{P}_i|h_V\rangle|^2 \\ &+ \sum_{h_V \neq h'_V} \langle f|\mathsf{P}_i|h_V\rangle\langle h'_V|\mathsf{P}_i|f\rangle c_{h_V} c^*_{h'_V} \end{aligned}$$

same p_f as before

If coefficients c_{hV} have random phases, the interference term vanishes and an effective mixture description is recovered

General formulae for a multi-particle channel

F. B., L. Ferroni: Eur. Phys. J. C 35, 243 (2004); Eur. Phys. J. C 51, 259 (2007)

Take $\mathsf{P}_i = \delta^4(P_0 - \hat{P})\delta_{Q_0,\hat{Q}}$

Without quantum statistics

$$\Omega_N = \frac{V^N}{(2\pi)^{3N}} \left(\prod_j \frac{(2S_j+1)^{N_j}}{N_j!} \right) \int d^3p_1 \ldots \int d^3p_N \; \delta^4(P_0 - \sum_i p_i) \langle 0 | \mathsf{P}_V | 0 \rangle$$

With quantum statistics

$$\Omega_{\{N_j\}} = \int d^3\mathrm{p}_1 \ldots d^3\mathrm{p}_N \; \delta^4(P_0 - P_f) \prod_j \sum_{\{h_{n_j}\}} \frac{(\mp 1)^{N_j + H_j} (2J+1)^{H_j}}{\prod_{n_j=1}^{N_j} n_j^{h_{n_j}} h_{n_j}!} \prod_{l_j=1}^{H_j} F_{n_{l_j}} \langle 0 | \mathsf{P}_V | 0 \rangle$$

partitions of Nj

$$\sum_{n_j=1}^{N_j} n_j h_{n_j} = N_j \qquad \sum_{n_j=1}^{N_j} h_{n_j} = H_j \qquad \sum_j N_j = N \qquad F_{n_l} = \prod_{i_l=1}^{n_l} \frac{1}{(2\pi)^3} \int_V d^3\mathrm{x} \; \mathrm{e}^{i\mathbf{x}\cdot(\mathbf{p}_{c_l(i_l)} - \mathbf{p}_{i_l})}$$

Cluster expansion of the microcanonical partition function

Relativistic invariance and dynamical content

$$\Gamma_N \propto \frac{V^N}{(2\pi)^{3N}} \left(\prod_j \frac{(2S_j+1)^{N_j}}{N_j!} \right) \int d^3p_1 \ldots \int d^3p_N \, \delta^4(P_0 - \sum_i p_i)$$

$$= \frac{1}{(2\pi)^{3N}} \left(\prod_j \frac{(2S_j+1)^{N_j}}{N_j!} \right) \left[\prod_{i=1}^N \int d^4p_i \; \Upsilon \cdot p_i \delta(p_i^2 - m_i^2) \theta(p_i^0) \right] \delta^4(P_0 - \sum_i p_i)$$

Four-volume $\Upsilon = (\gamma V, \gamma \mathbf{v} V)$ M. Chaichian, R. Hagedorn, M. Hayashi, Nucl. Phys. B92 (1975) 445

Comparing with the well-known formula

$$\Gamma_N \propto \sum_{\sigma_1,\ldots,\sigma_N} \frac{1}{(2\pi)^{3N}} \left(\prod_j \frac{1}{N_j!} \right) \int \frac{d^3p_1}{2\varepsilon_1} \ldots \int \frac{d^3p_N}{2\varepsilon_N} \, |M_{fi}|^2 \delta^4(P_0 - \sum_i p_i)$$

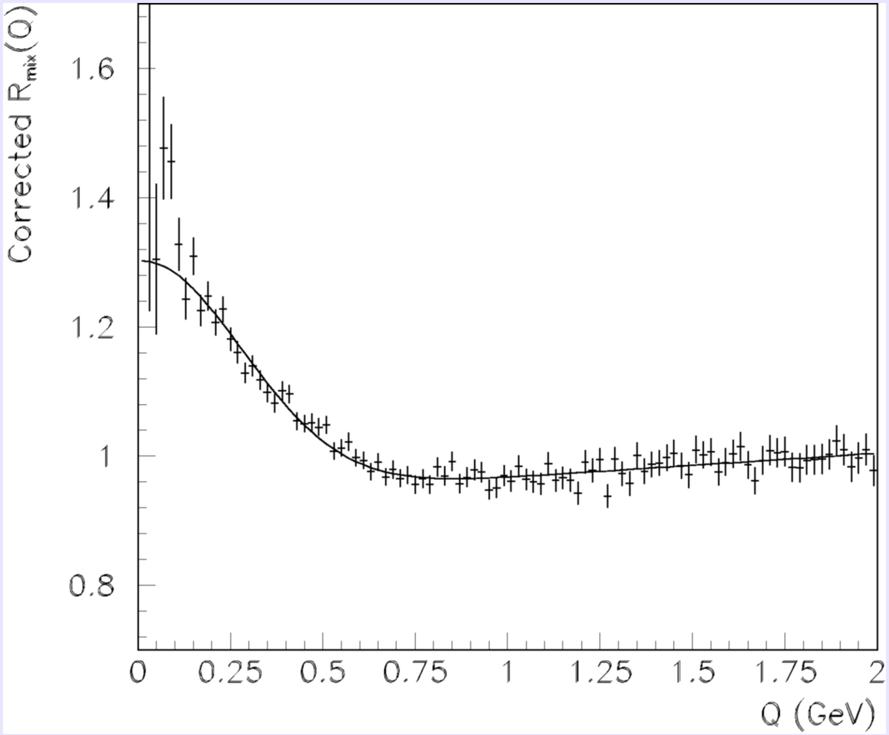
$$|M_{fi}|^2 = \prod_{i=1}^N \Upsilon \cdot p_i = \left(\frac{1}{\rho} \right)^N \prod_{i=1}^N P \cdot p_i$$

ρ energy density

Finite volume and quantum correlations

$$\Gamma_{\{N_j\}} \propto \int d^3\mathrm{p}_1 \ldots d^3\mathrm{p}_N \; \delta^4(P_0 - P_f) \prod_j \sum_{\{h_{n_j}\}} \frac{(\mp 1)^{N_j+H_j} (2J+1)^{H_j}}{\prod_{n_j=1}^{N_j} n_j^{h_{n_j}} h_{n_j}!} \prod_{l_j=1}^{H_j} F_{n_{l_j}}$$

Terms beyond leading in the cluster expansion account for BE and FD correlations.
Vanish for $V \longrightarrow \infty$



These observations support the idea of a finite volume in high energy collisions

e+e- collisions at $\sqrt{s}$ = 91.2 GeV
ALEPH coll., Phys. Rep. 294 (1998) 1

Interactions: hadron-resonance gas

While asymptotic states can only be strongly stable hadrons the energy-momentum projector must include interactions among them

A very nice and powerful theorem established by Dashen-Ma-Bernstein states

R. Dashen, S. K. Ma, H. Bernstein, Phys. Rev. 187 (1969) 345

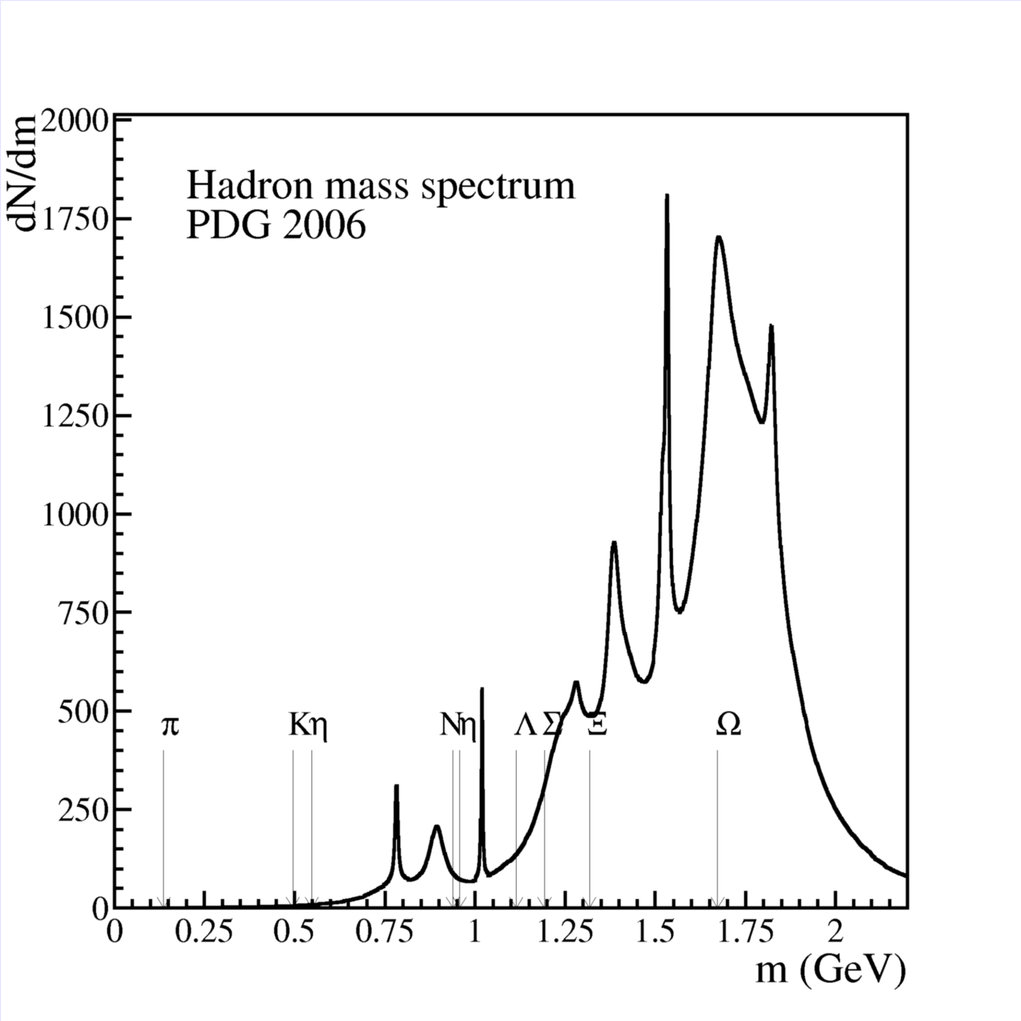
$$\mathrm{tr}\delta^4(P-\hat{P}) = \mathrm{tr}\delta^4(P-\hat{P}_0) + \frac{1}{4\pi i}\mathrm{tr}\left[\delta^4(P-\hat{P}_0)\hat{\mathcal{S}}^{-1}\frac{\overleftrightarrow{\partial}}{\partial E}\hat{\mathcal{S}}\right]$$

$\hat{\mathcal{S}}$ is the reduced scattering matrix on the energy-momentum P shell

CAVEAT: Such a theorem requires the thermodynamic limit

It can be proved that retaining only the resonant part of the interaction (and neglecting resonance interference) the microcanonical partition function reduces to that of a gas of free hadrons and resonances with distributed mass

hadron-resonance gas



The hadron gas is “the” system where this method applies owing to the very large number of resonances

Energy density (temperature) should be large enough to excite most resonances

The theorem states that the trace can be decomposed into two simple terms but it has never been proved whether it applies to single trace terms

$$\mathrm{tr}\,\delta^4(P-\hat{P}) = \mathrm{tr}\,\delta^4(P-\hat{P}_0) + \frac{1}{4\pi i}\mathrm{tr}\left[\delta^4(P-\hat{P}_0)\hat{\mathcal{S}}^{-1}\frac{\overleftrightarrow{\partial}}{\partial E}\hat{\mathcal{S}}\right]$$

It is assumed

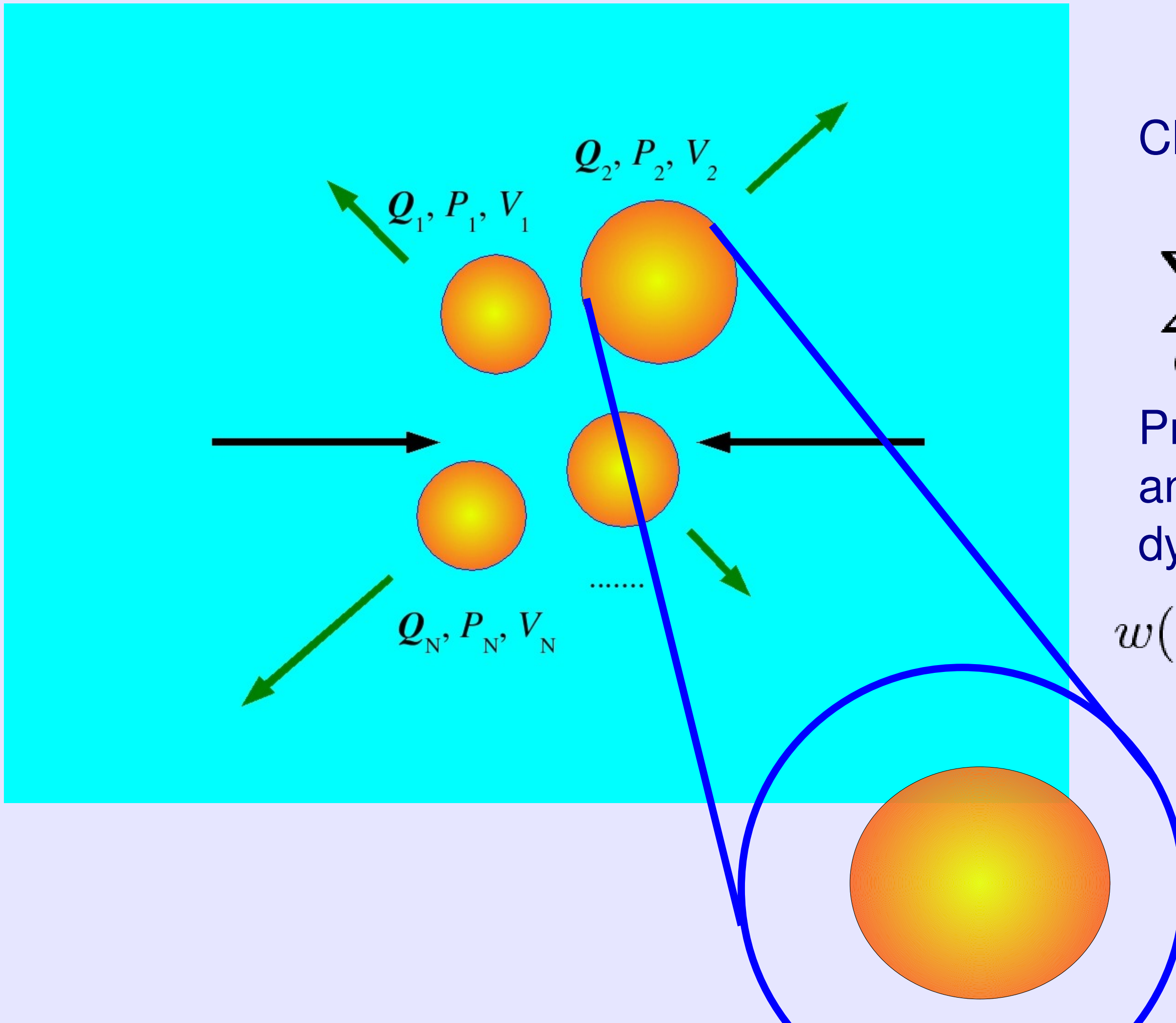
$$\mathrm{tr}_{\{N_j\}}\delta^4(P-\hat{P}) = \mathrm{tr}_{\{N_j\}}\delta^4(P-\hat{P}_0) + \frac{1}{4\pi i}\mathrm{tr}_{\{N_j\}}\left[\delta^4(P-\hat{P}_0)\hat{\mathcal{S}}^{-1}\frac{\overleftrightarrow{\partial}}{\partial E}\hat{\mathcal{S}}\right]$$

In the resonating approximation all above terms are positive

SUMMARY OF HYPOTHESES FOR HadResGas

- Thermodynamic limit
- Overall vanishing contribution from a whole class of non-symmetric diagrams
- Validity of DMB theorem for single channels

High energy collisions: multiple cluster production



Clusters are charge-entangled

$$\sum_{\mathbf{Q}_i} c(\mathbf{Q}_1, \mathbf{Q}_2, \ldots)|\mathbf{Q}_1, \mathbf{Q}_2, \ldots\rangle$$

Probabilities are unknown to the SHM and are determined by previous dynamics

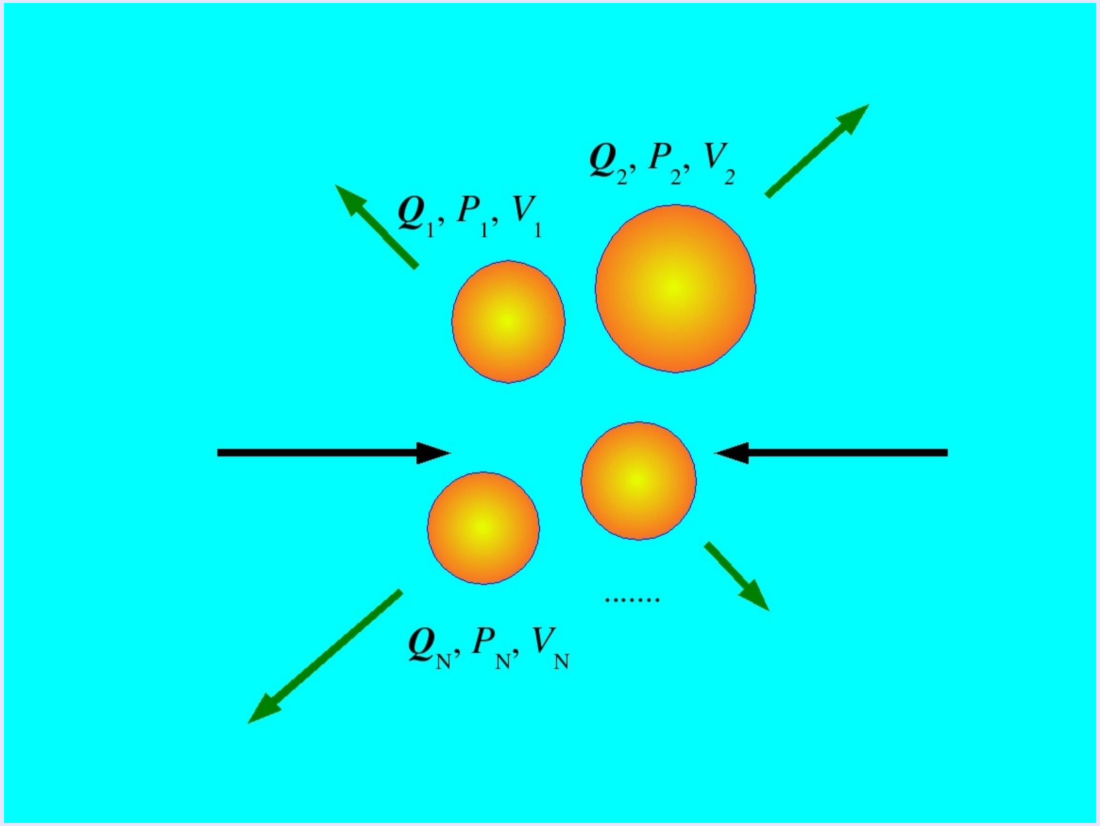
$$w(\mathbf{Q}_1, \mathbf{Q}_2, \ldots) = |c(\mathbf{Q}_1, \mathbf{Q}_2, \ldots)|^2 = ?$$

To be hadronized micro-canonically with

$$\Gamma_{\{N_j\}} \propto \frac{V^N}{(2\pi)^{3N}} \left(\prod_j \frac{1}{N_j!} \right) \int d^3p_1 \ldots \int d^3p_N \; \delta^4(P_0 - \sum_i p_i)$$

We need to know the charge-momentum distribution probabilities

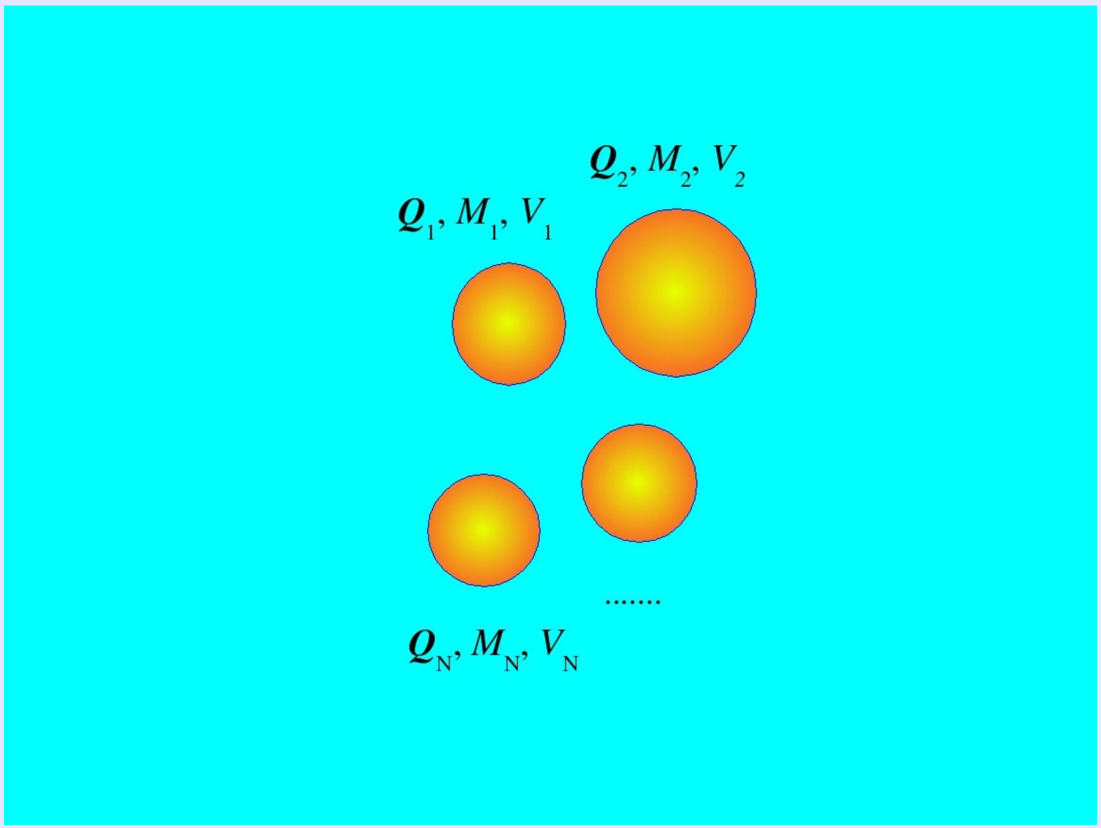
$$w(P_1, \mathbf{Q}_1; P_2, \mathbf{Q}_2, \ldots)$$



We need to know the charge-momentum distribution probabilities $w(P_1, \mathbf{Q}_1; P_2, \mathbf{Q}_2, \ldots)$

However, for Lorentz scalars, such as multiplicities, only mass distribution matters

$$w(P_1^2, \mathbf{Q}_1; P_2^2, \mathbf{Q}_2, \ldots)$$



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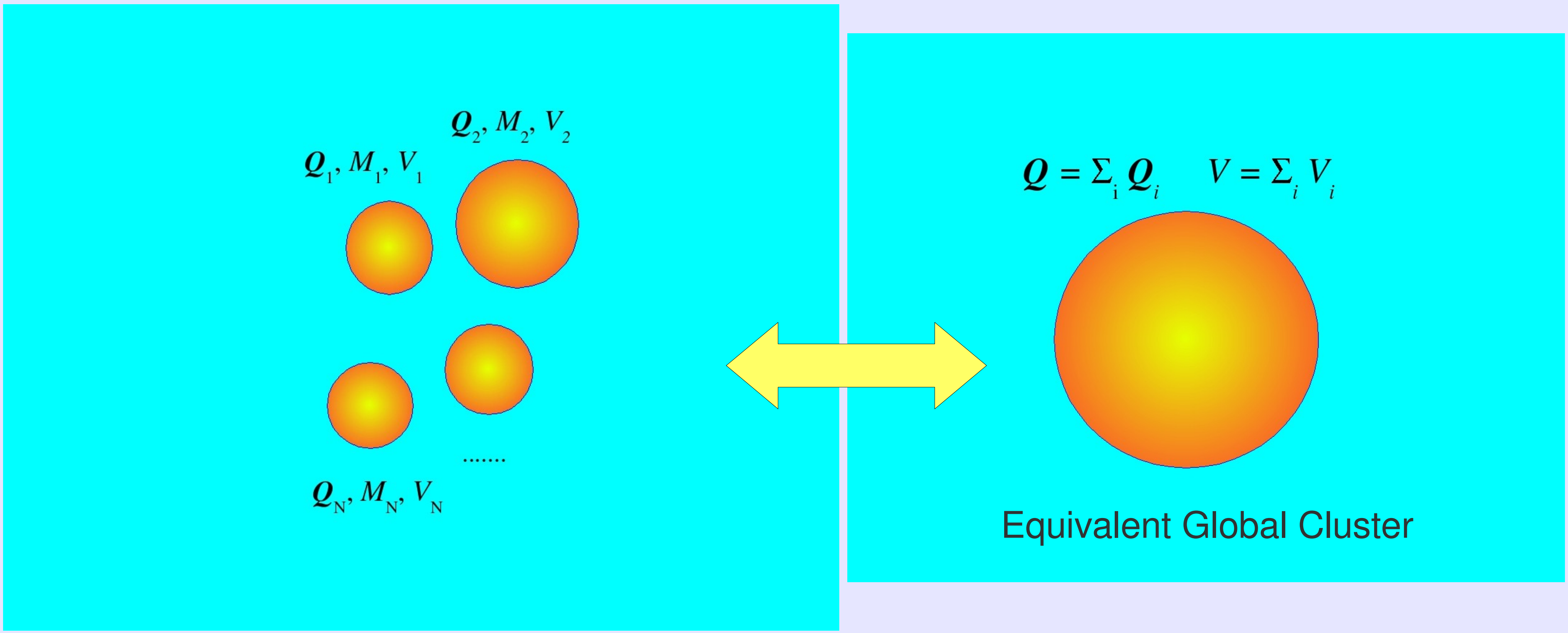
$$w(P_1^2, \mathbf{Q}_1; P_2^2, \mathbf{Q}_2, \ldots)$$



The equivalence applies provided that the probabilities *w* are “statistical”, i.e. they are obtained from maximizing the splitting entropy

For a detailed derivation see **F. B., G. Passaleva, Eur. Phys. J. C 23 (2002) 551**

If the EGC is large enough, one can go from a micro-canonical to a canonical calculation where (M,V) is replaced by (T,V)

Primary multiplicities in Boltzmann approx.

$$\langle n_j \rangle = \frac{(2S+1)V}{(2\pi)^3} \gamma_S{}^{N_s} \int d^3p \, \mathrm{e}^{-\sqrt{p^2+m_j^2}/T} \, \frac{Z(\mathbf{Q}-\mathbf{q}_j)}{Z(\mathbf{Q})}$$

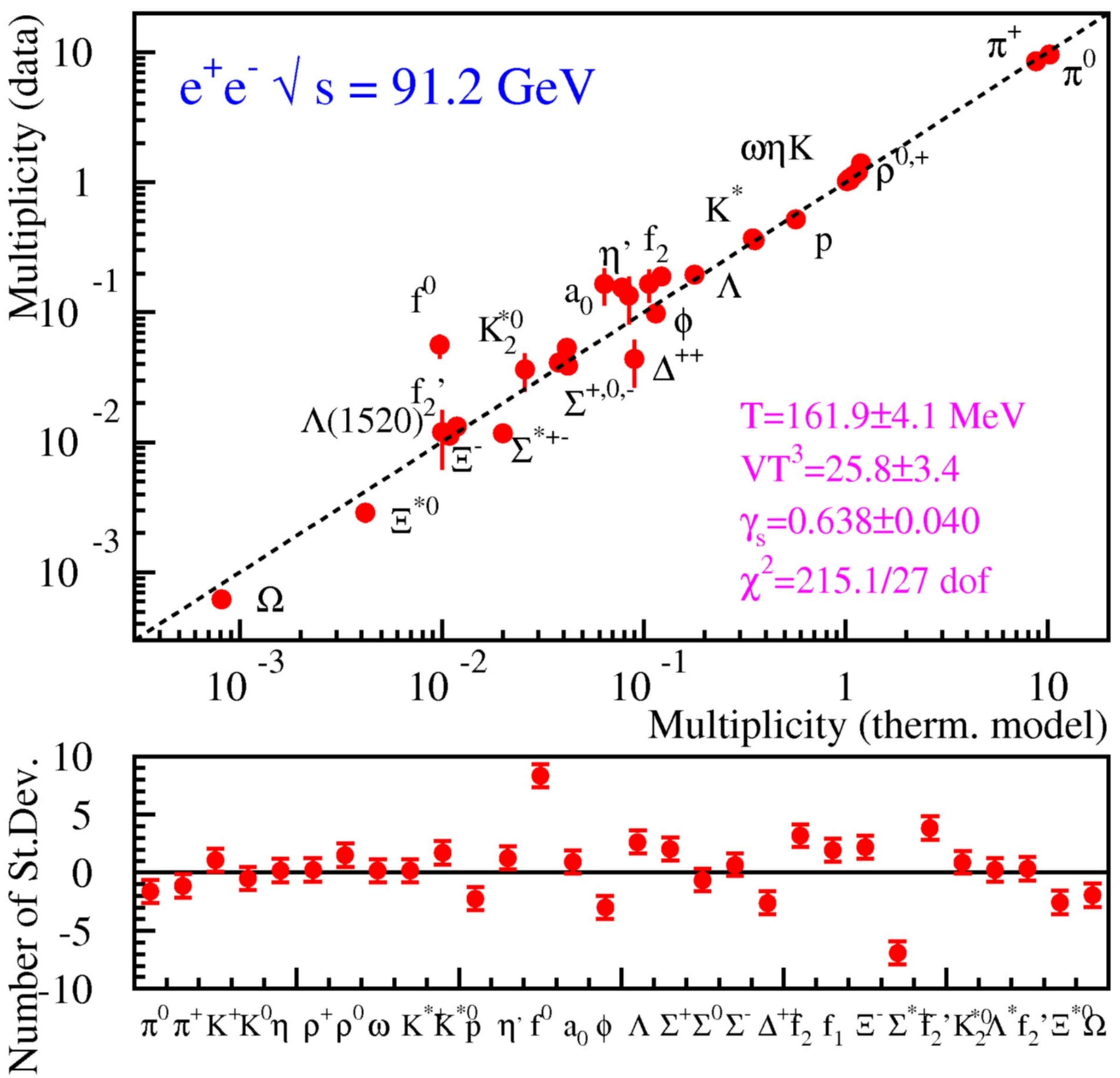
extra strangeness suppression

$$\mathbf{Q} = (Q, B, S, ..)$$
$$\mathbf{q}_j = (Q_j, B_j, S_j, ...)$$

REMARKS

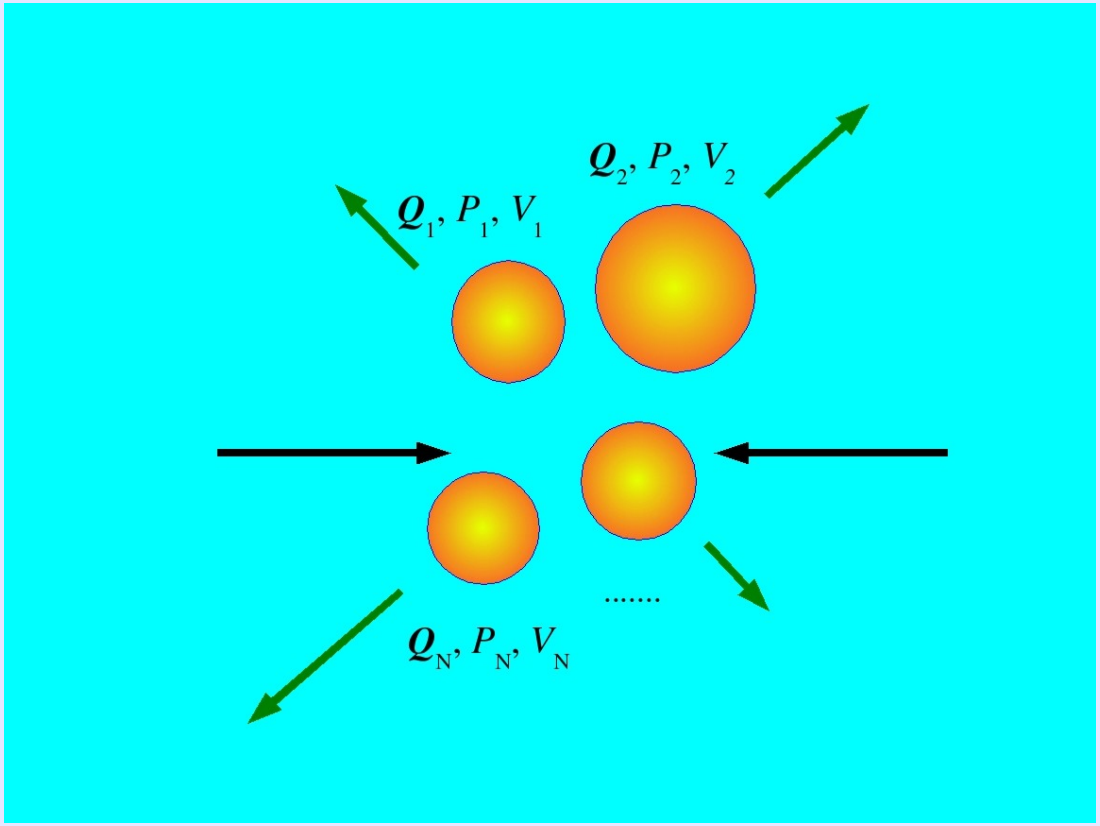
- Note that M as well as V of the EGC can fluctuate and so can T
- In general, for individual hadronizing clusters only mass and volume are physical; temperature cannot be defined for single clusters, only for the EGC.
- Therefore, in elementary collisions a local T does not exist
- Individual clusters in heavy ions are likely so large that they can be handled canonically (possibly grand-canonically): a local temperature may be well defined.

F. B., P. Castorina, J. Manninen, H. Satz, Eur. Phys. J. C 56 (2008) 493



The latest update of the early analysis: F. B., Z. Phys. C 69, 485 (1996)

Heavy flavoured hadron production



In $e^+e^- \to Q\bar{Q}$ where Q is a heavy quark, enforce two clusters with non-vanishing numbers of heavy quarks

Primary multiplicity of the jth heavy flavoured species is predicted to be

$$\langle n_j \rangle = \gamma_s^{N_{Sj}} z_j \frac{\sum_i \gamma_s^{N_{si}} z_i \zeta(\mathbf{Q} - \mathbf{q}_j - \mathbf{q}_i)}{\sum_{i,k} \gamma_s^{N_{si}} \gamma_s^{N_{sk}} z_i z_k \zeta(\mathbf{Q} - \mathbf{q}_i - \mathbf{q}_k)},$$

$$z_j = \frac{(2S_j + 1)V}{(2\pi)^3} \int d^3p \, \mathrm{e}^{-\sqrt{p^2+m_j^2}/T}$$

Heavy flavoured hadron yields measured at √s = 91.2 GeV

Particle		Experiment (E)	Model (M)	Residual	$(M-E)/E$ [%]
D^0	[28]	0.559 ± 0.022	0.5406	-0.83	-3.2
D^+	[28]	0.238 ± 0.024	0.2235	-0.60	-6.1
D^{*+}	[28–30]	0.2377± 0.0098	0.2279	-1.00	-4.1
D^{*0}	[31]	0.218 ± 0.071	0.2311	0.18	6.0
D_1^0	[32, 33]	0.0173 ± 0.0039	0.01830	0.26	5.8
D_2^{*0}	[32, 33]	0.0484 ± 0.0080	0.02489	-2.94	-48.6
D_s	[28]	0.116 ± 0.036	0.1162	0.006	0.19
D_s^*	[28]	0.069 ± 0.026	0.0674	-0.06	-2.4
D_{s1}	[33, 34]	0.0106 ± 0.0025	0.00575	-1.94	-45.7
D_{s2}^*	[34]	0.0140 ± 0.0062	0.00778	-1.00	-44.5
Λ_c	[28]	0.079 ± 0.022	0.0966	0.80	22.2
$(B^0+B^+)/2$	[35]	0.399 ± 0.011	0.3971	-0.18	-0.49
B_s	[35]	0.098 ± 0.012	0.1084	0.87	10.6
$B^*/B(\mathrm{uds})$	[36–39]	0.749 ± 0.040	0.6943	-1.37	-7.3
$B^{**}\times BR(B^{(*)}\pi)$	[40–42]	0.180 ± 0.025	0.1319	-1.92	-26.7
$(B_2^*+B_1)\times BR(B^{(*)}\pi)$	[41]	0.090 ± 0.018	0.0800	-0.57	-11.4
$B_{s2}^*\times BR(BK)$	[41]	0.0093 ± 0.0024	0.00631	-1.24	-32.1
b-baryon	[35]	0.103 ± 0.018	0.09751	-0.30	-5.3
Ξ_b^-	[35]	0.011 ± 0.006	0.00944	-0.26	-14.2

χ^2/dof = 24.6/19 *WITHOUT ANY NEW FREE PARAMETER*

F. B., P. Castorina, J. Manninen, H. Satz, Eur. Phys. J. C 56 (2008) 493

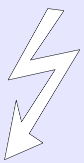
What if using incorrect framework?

If this result was accidental, one would expect random χ^2 fluctuations

FIT TO LONG-LIVED PARTICLES AT 91.2 GeV

- **No resonances (with Γ > 9 MeV)**

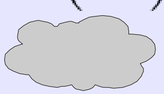
χ^2 = 391/12



- **Resonances, but no exact conservation of B, S, Q**

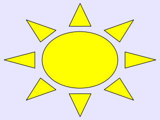
$$\langle n_j \rangle = \frac{(2S_j+1)V}{(2\pi)^3} \gamma_S{}^{N_s} \int d^3p \, \mathrm{e}^{-\sqrt{p^2+m_j^2}/T} \, \cancel{\frac{Z(\mathbf{Q}-\mathbf{q}_j)}{Z(\mathbf{Q})}}$$

χ^2 = 103/12

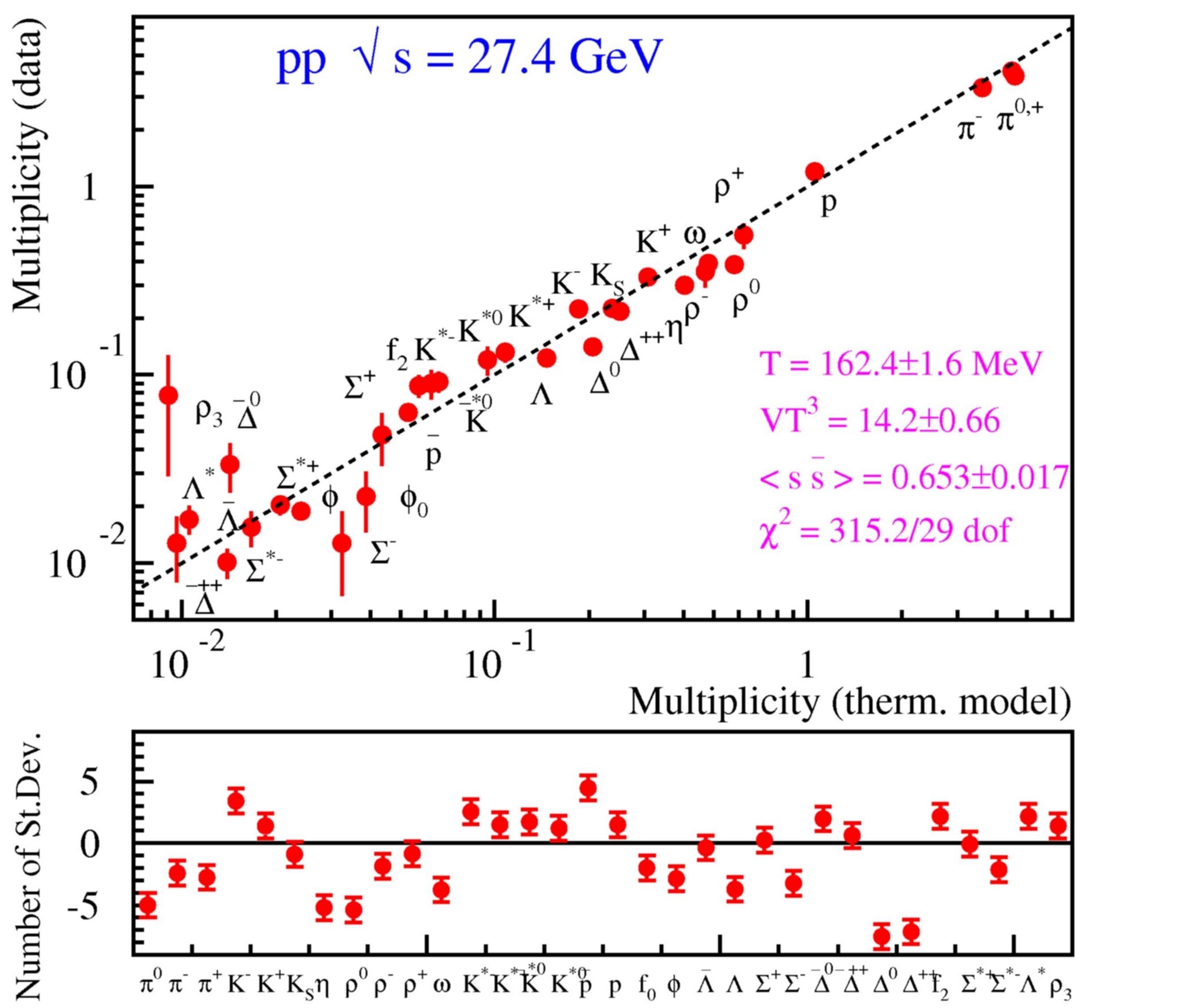


- **Correct implementation**

χ^2 = 39.3/12

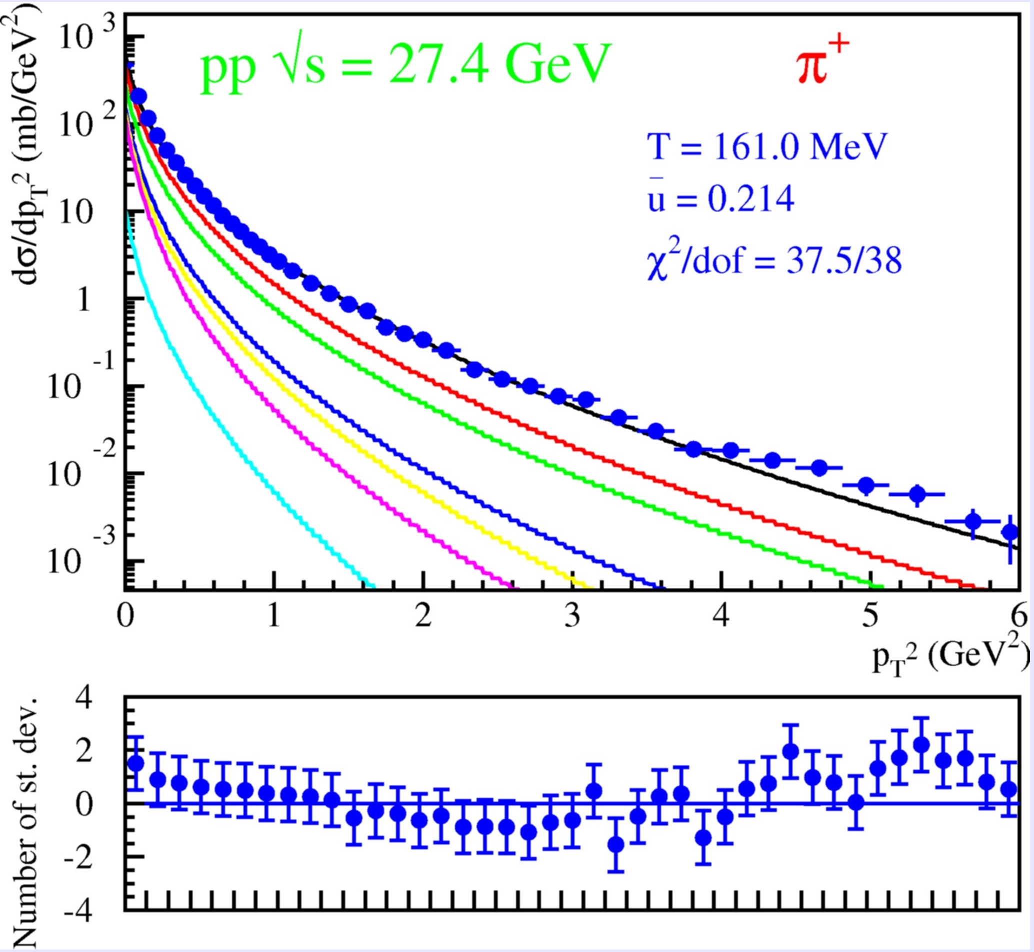


F. B., G. Passaleva, Eur. Phys. J. C 23 (2002) 551

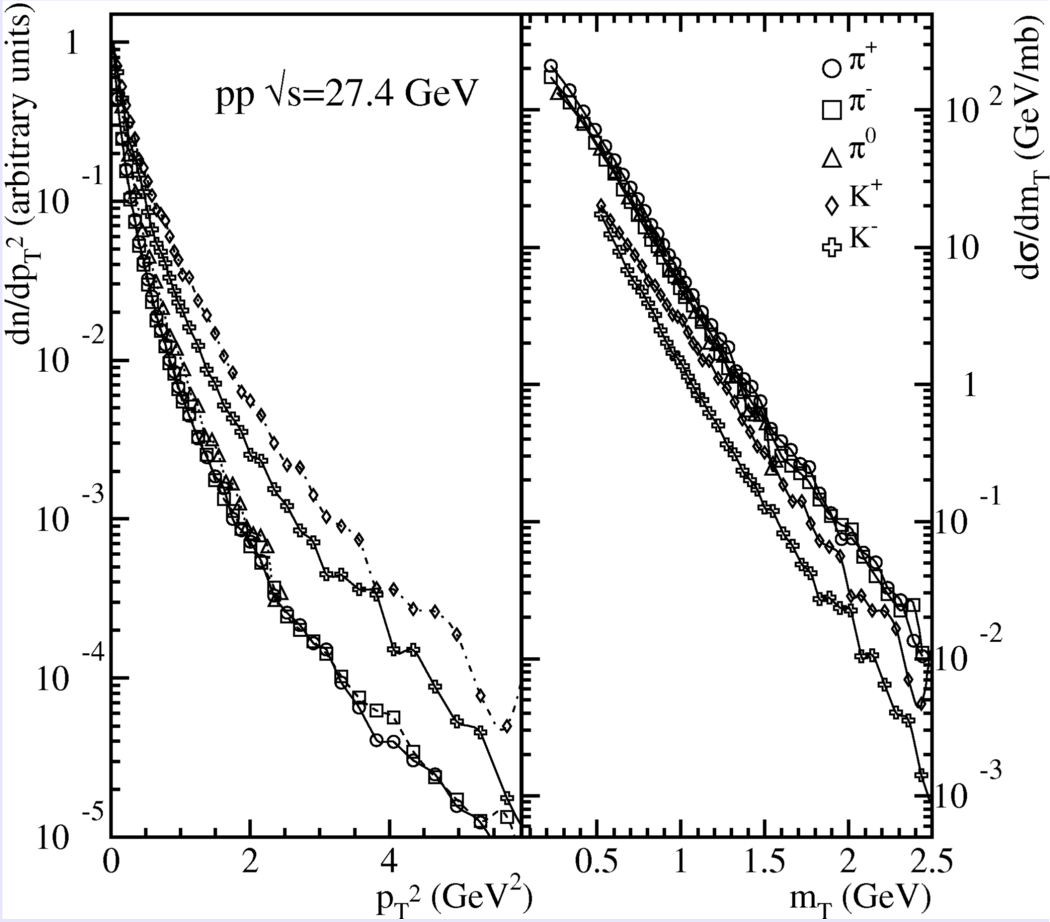


NOTE: no systematic errors were quoted in the paper where most data comes from

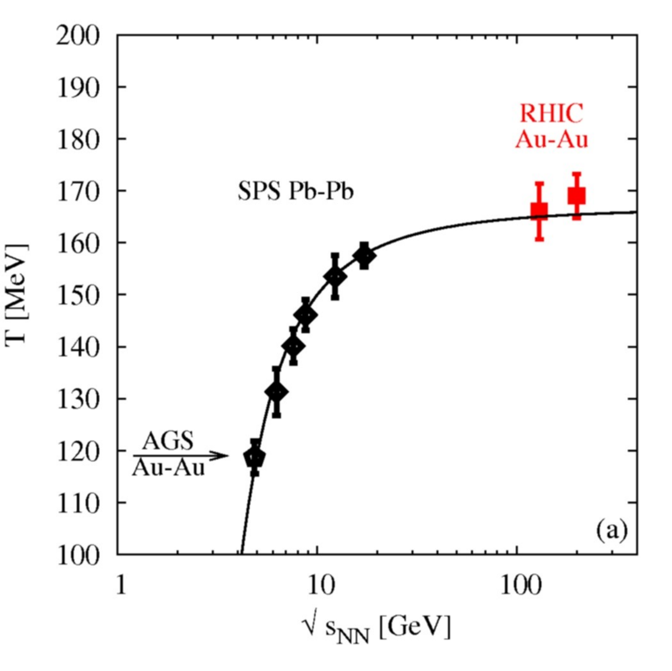
Transverse momentum spectra in hadronic collisions



F. B., G. Passaleva, Eur. Phys. J. C 23 (2002) 551

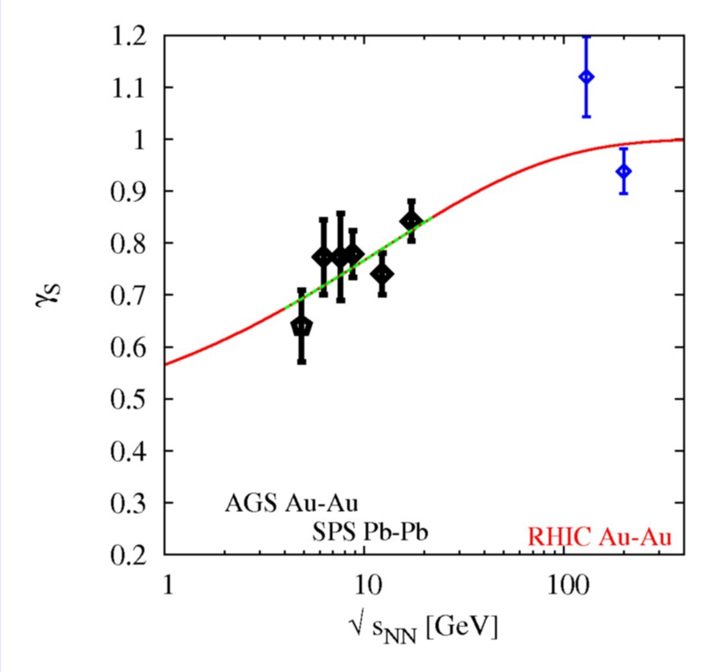


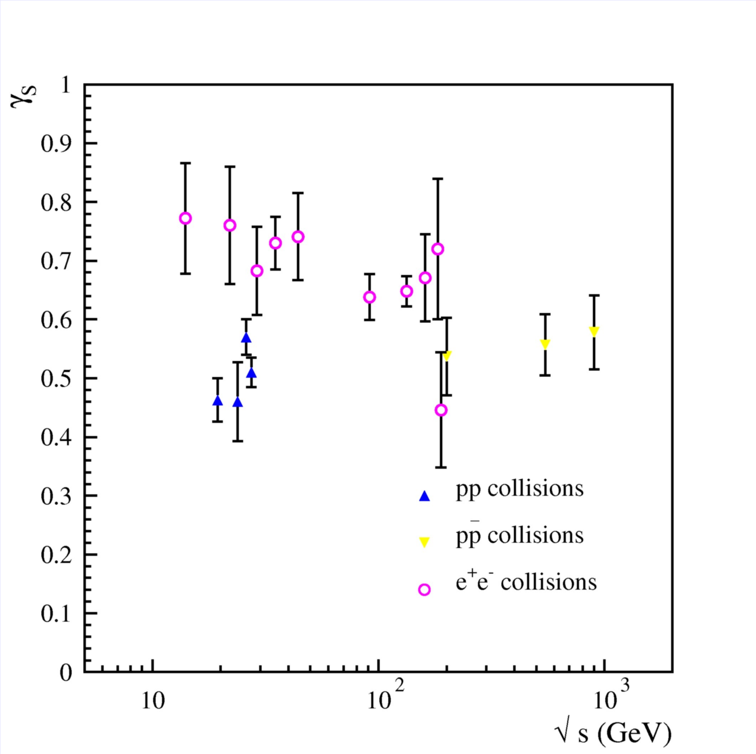
Heavy ions

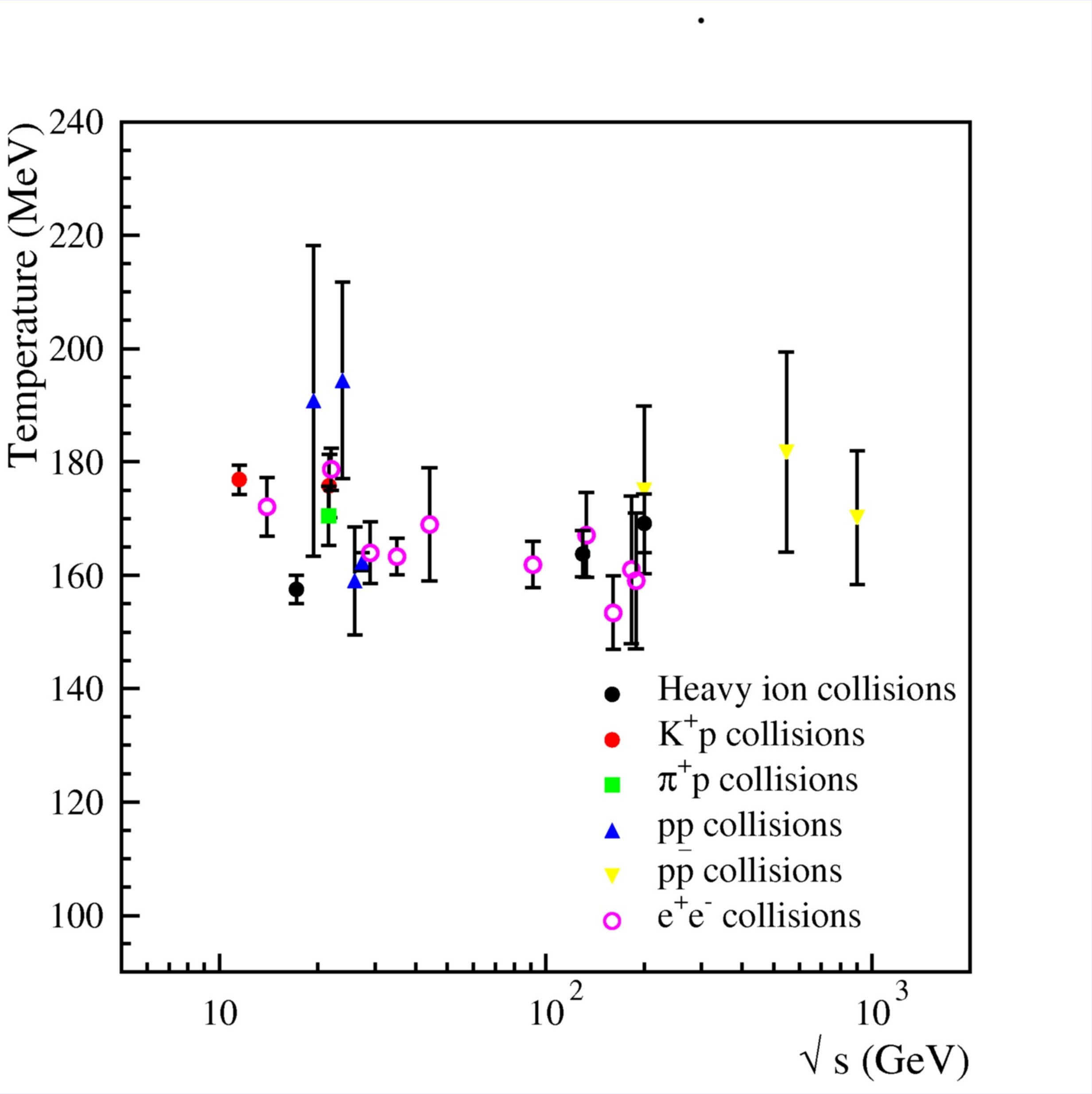


Fits to the statistical model are equally good in heavy ion collisions.
Temperatures have similar asymptotic values

The only clear difference is that extra strangeness suppression parameter is larger than in elementary collisions and approaches 1 as energy increases







Critical energy density at the hadronization ?

Why do we observe such thermal features?

L. McLerran, "Lectures on RHIC physics", hep-ph 0311028

This would appear to be a compelling case for the production of a Quark Gluon Plasma. The problem is that the fits done for heavy ions to particle abundances work even better in e^+e^- collisions. One definitely expects no Quark Gluon Plasma in e^+e^- collisions. There is a deep theoretical question to be understood here: How can thermal models work so well for non-thermal systems? Is there some simple saturation of phase space?

"Phase space dominance": J.Hormuzdiar, S. D. H. Hsu, G.Mahlon, Int. J. Mod. Phys. E 12, 649 (2003)

Can we derive / justify this behaviour within QCD?

In the statistical model there is essentially one scale, which is the energy density at which hadronization takes place: all the rest is conservation laws (except for γ_s, which is probably associated to the m_s).

Some very recent proposals...

- AdS-CFT correspondance

N. Evans, A. Tedder, *A holographic model of hadronization*,
Phys. Rev. Lett. 100 (2008) 162003

(see also Hatta et al.)

- Hadron radiation as a QCD counterpart of the Hawking-Unruh thermal radiation

P. Castorina, D. Kharzeev, H. Satz, Eur. Phys. J. C52 (2007) 187

Quantum thermalization of classically chaotic systems

M. Srednicki, Phys. Rev. E 50 (1994) 888

Based on the validity of Berry's conjecture: the high-lying energy eigenfunctions of a classically chaotic and ergodic system are Gaussian random numbers

Srednicki considers a classical gas of hard spheres (= classically chaotic, ergodic system) and shows that, provided Berry's conjecture is true, the bounded quantum gas of hard spheres must exhibit a thermal single-particle spectrum in a PURE QUANTUM STATE

Even more intriguingly, he shows that, starting from a wave function which does not have thermal spectrum, the system will become thermal within a time of the order of $1/\Delta$ where Δ is the spread in initial energy

A. Krzywicki (hep-ph 0204411) advocates this as the underlying mechanism for the apparent thermal features in hadroproduction in high energy collisions

Berry's conjecture (1)

The high-lying eigenfunctions of a quantum system whose classical counterpart is chaotic and ergodic appear to be random Gaussian numbers $\psi_\alpha(\mathbf{x})$ in configuration space, with a two-point correlation function given by

$$\langle \psi_\alpha(\mathbf{x}+\frac{\mathbf{s}}{2})\psi^*_\beta(\mathbf{x}-\frac{\mathbf{s}}{2})\rangle_{EE} = \delta_{\alpha\beta}\frac{\int d^3p\, \mathrm{e}^{i\mathbf{p}\cdot\mathbf{s}}\ \delta(E_\alpha - H(\mathbf{x},\mathbf{p}))}{\int d^3p\, d^3x\ \delta(E_\alpha - H(\mathbf{x},\mathbf{p}))}$$

EE stands for eigenstate ensemble. A fictitious ensemble whose meaning is: the typical eigenfunction behaves as if it was drawn at random from the eigenstate ensemble

The best intuitive example of such a behaviour is the superposition of many plane waves with random phases: because of the central limit theorem, the amplitudes $\psi_\alpha(\mathbf{x})$ are normally distributed

Berry's conjecture (2)

Berry's conjecture has been found to be valid numerically for many systems and it has even been suggested as a good definition of Quantum Chaos

$$\langle W_\alpha(\mathbf{x},\mathbf{p})\rangle_{EE} = \langle \frac{1}{(2\pi)^3} \int d^3s\, \mathrm{e}^{-i\mathbf{p}\cdot\mathbf{s}}\, \psi_\alpha(\mathbf{x}+\frac{\mathbf{s}}{2})\psi^*_\alpha(\mathbf{x}-\frac{\mathbf{s}}{2})\rangle_{EE} = \frac{\delta(E_\alpha - H(\mathbf{x},\mathbf{p}))}{\int d^3p\, d^3x\ \delta(E_\alpha - H(\mathbf{x},\mathbf{p}))}$$

The “typical” Wigner function becomes the statistical microcanonical distribution

It can be easily shown that, given the assumed two-point correlation function and if the volume is large, the probability of a momentum state is microcanonical:

$$\langle \rho(\mathbf{p})\rangle_{EE} = \frac{\int d^3x\ \delta(E_\alpha - H(\mathbf{x},\mathbf{p}))}{\int d^3p\, d^3x\ \delta(E_\alpha - H(\mathbf{x},\mathbf{p}))}$$

QUESTIONS

Is classical QCD chaotic?
(YES: T. Biro, C. Gong, B. Muller, H. Markum, R. Pullirsch)

Does Berry's conjecture apply to quantum fields?

Can we extend Srednicki's reasoning to dynamical hadron production?

Recent and possible future developments

- Hadron radiation as a QCD counterpart of the Hawking-Unruh thermal radiation
P. Castorina, D. Kharzeev, H. Satz, Eur. Phys. J. C52 (2007) 187

- Implementation of the (broken) symmetry $SU(3)_f$ replacing the exact U(1)'s
(F.B., J. Manninen in progress)

The aim of these studies is to explain the extra strangeness suppression parameter γ_S

Statistical model in a MonteCarlo code: MCSTHAR++ project

C. Bignamini (U. Pavia, Mcnet Karlsruhe) , F. Piccinini (U. and INFN Pavia)

Collaborating: F.B., L. Ferroni (LBNL), S. Gieseke (Karlsruhe)

Hadronization module currently being interfaced with HERWIG and HERWIG++

- Parton shower
- Cluster formation
- Replace HERWIG default hadronization procedure with microcanonical decay of each HERWIG cluster

Complete tuning to LEP data to be started hopefully soon

Molteplicità per le specie adorniche			
Adroni	Misura	Cariche libere	Cariche fissate
π^0	9.61 ± 0.29	10.27 ± 0.01	10.42 ± 0.04
$\pi^\pm$	8.50 ± 0.10	9.08 ± 0.01	8.81 ± 0.03
$K^\pm$	1.127 ± 0.026	1.098 ± 0.003	1.16 ± 0.01
K_S^0	1.0376 ± 0.0096	0.999 ± 0.003	1.09 ± 0.01
η	1.059 ± 0.086	0.993 ± 0.003	1.14 ± 0.01
ρ^0	1.40 ± 0.13	1.220 ± 0.004	1.25 ± 0.01
$\rho^\pm$	1.20 ± 0.22	1.145 ± 0.003	1.06 ± 0.01
ω	1.024 ± 0.059	0.993 ± 0.003	0.97 ± 0.01
K^{*0}	0.357 ± 0.022	0.35 ± 0.002	0.352 ± 0.006
$K^{*\pm}$	0.370 ± 0.012	0.359 ± 0.002	0.0356 ± 0.006
η'	0.166 ± 0.047	0.095 ± 0.001	0.107 ± 0.003
ϕ	0.0977 ± 0.0058	0.108 ± 0.001	0.205 ± 0.005
$f_2(1270)$	0.188 ± 0.020	0.124 ± 0.001	0.127 ± 0.003
K_2^*	0.036 ± 0.011	0.0217 ± 0.0005	0.020 ± 0.001
$f_2'(1525)$	0.0120 ± 0.0058	0.0088 ± 0.0003	0.019 ± 0.001
p	0.519 ± 0.018	0.543 ± 0.002	0.506 ± 0.006
$a_0(980)$	0.135 ± 0.054	0.0890 ± 0.0009	0.071 ± 0.03
f_0	0.1555 ± 0.0085	0.0663 ± 0.0008	0.067 ± 0.03
Λ	0.1943 ± 0.0038	0.196 ± 0.001	0.177 ± 0.005
Σ^+	0.0535 ± 0.0052	0.0473 ± 0.0007	0.037 ± 0.002
Σ^0	0.0389 ± 0.0041	0.0473 ± 0.0007	0.043 ± 0.002
Σ^-	0.0410 ± 0.0037	0.0436 ± 0.0004	0.046 ± 0.002
$\Sigma^\pm$	0.0868 ± 0.0087	0.091 ± 0.001	0.082 ± 0.004
Δ^{++}	0.044 ± 0.016	0.0846 ± 0.0007	0.059 ± 0.002
Ξ^-	0.01319 ± 0.00052	0.0123 ± 0.0003	0.009 ± 0.001
Σ^{*+}	0.0118 ± 0.0011	0.0206 ± 0.0005	0.015 ± 0.001
$\Lambda(1520)$	0.0112 ± 0.0014	0.0109 ± 0.0003	0.010 ± 0.001
Ξ^{*0}	0.00289 ± 0.00050	0.0041 ± 0.0002	0.0028 ± 0.0005
Ω^-	0.00062 ± 0.00012	0.00088 ± 0.00009	0.0010 ± 0.0003

T. Gabbriellini, Master thesis, University of Florence 2004

Same quality of the agreement data-model

CONCLUSIONS

The statistical-thermal features observed in elementary as well as heavy ion collisions cannot be accidental

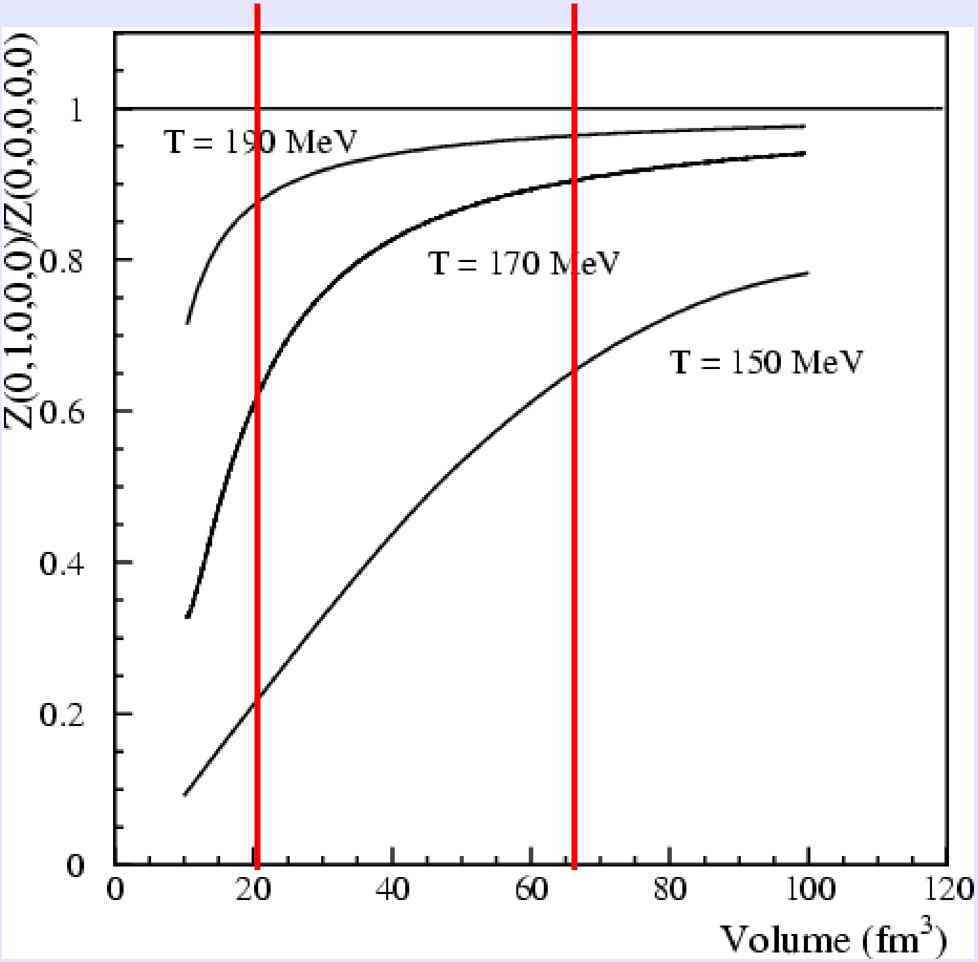
I believe that the reason of this behaviour is common to all collisions and must be quantum mechanical in its origin: *“hadrons are born at equilibrium”* (R. Hagedorn, 1970)

- Quantum chaos?
- Hawking-Unruh radiation?
- Recovered from AdS-CFT?
- ...

SPARE SLIDES

Canonical suppression

Example: neutron chemical factor in a completely neutral cluster



In GC should be:

$$\langle n_j \rangle = \frac{V(2J_j+1)}{(2\pi)^3} \int d^3p \, \exp\left(-\sqrt{p^2+m_j^2}/T\right)$$

Whilst in C

$$\langle n_j \rangle = \frac{V(2J_j+1)}{(2\pi)^3} \int d^3p \, \exp\left(-\sqrt{p^2+m_j^2}/T\right) \frac{Z(-q_j)}{Z(0)}$$

For $V \to \infty$ $\langle n \rangle_C = \langle n \rangle_{GC}$

$\langle n \rangle_C < \langle n \rangle_{GC}$

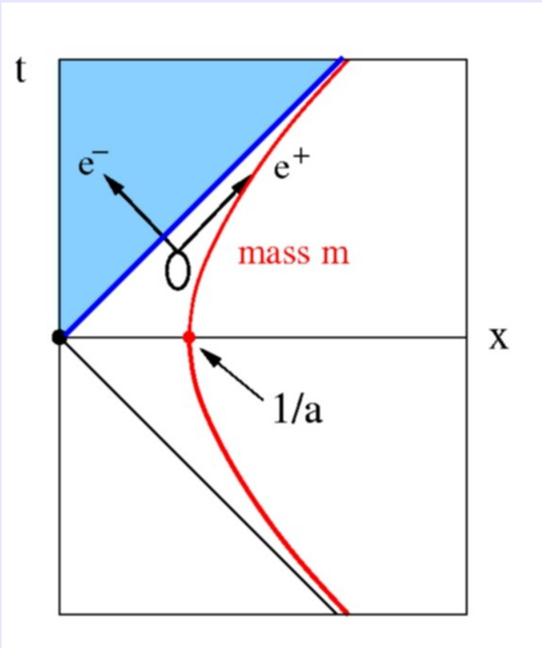
Canonical suppression

Hadronization as Hawking-Unruh radiation

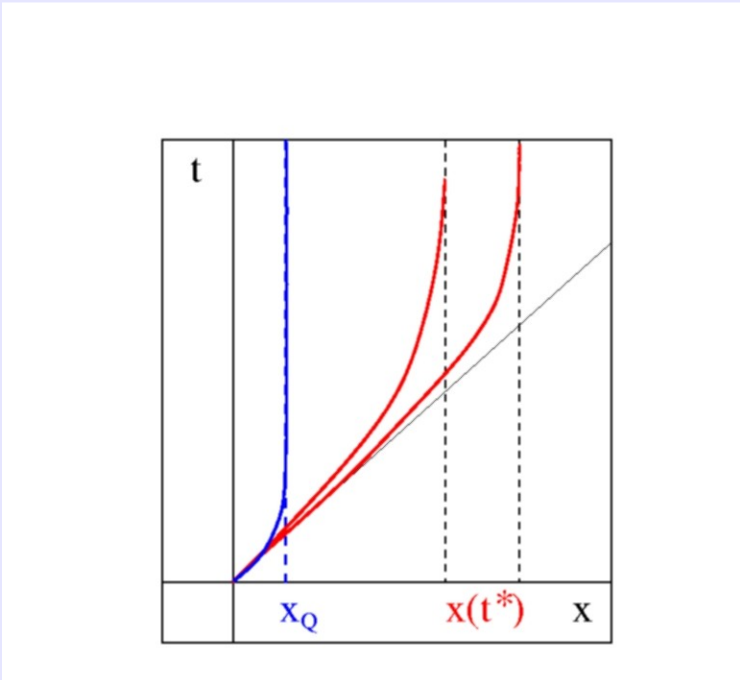
P. Castorina, D. Kharzeev, H. Satz, Eur. Phys. J. C52 (2007) 187

IDEA: analogy between black holes and QCD confinement, which is effectively seen as the formation of an event horizon for coloured signals

Hawking radiation from event horizon can only be thermal because it cannot convey any information



Unruh radiation: $T = \hbar a / 2\pi c$



Event horizon in qq production

Possible deep relation between pair production in a strong field (QED = Schwinger effect) and Unruh radiation

$$R_{pair} \sim \exp[-\pi m^2/eE] = \exp[-m/T_{Unruh}]$$

$$T_{Unruh} = \frac{a}{2\pi} = \frac{2eE}{m}$$

New paradigm: the acceleration involved in pair-produced particles in the colour field determines the temperature of the emitted radiation.

For the colour field, inspiring of the above formula:

$$a_q = \frac{\sigma}{\sqrt{P^2}/2} = \frac{\sigma}{\sqrt{m_q^2 + k_q^2}} = \frac{\sigma}{\sqrt{m_q^2 + \frac{\sigma^2}{4m_q^2+2\pi\sigma}}} \equiv \frac{\sigma}{w_q}$$

Massless quarks

$$T = \frac{a}{2\pi} = \sqrt{\frac{\sigma}{2\pi}}$$

Massive quarks in a hadron:
averaging the accelerations

$$\bar{a} = \frac{w_1 a_1 + w_2 a_2}{w_1 + w_2}$$

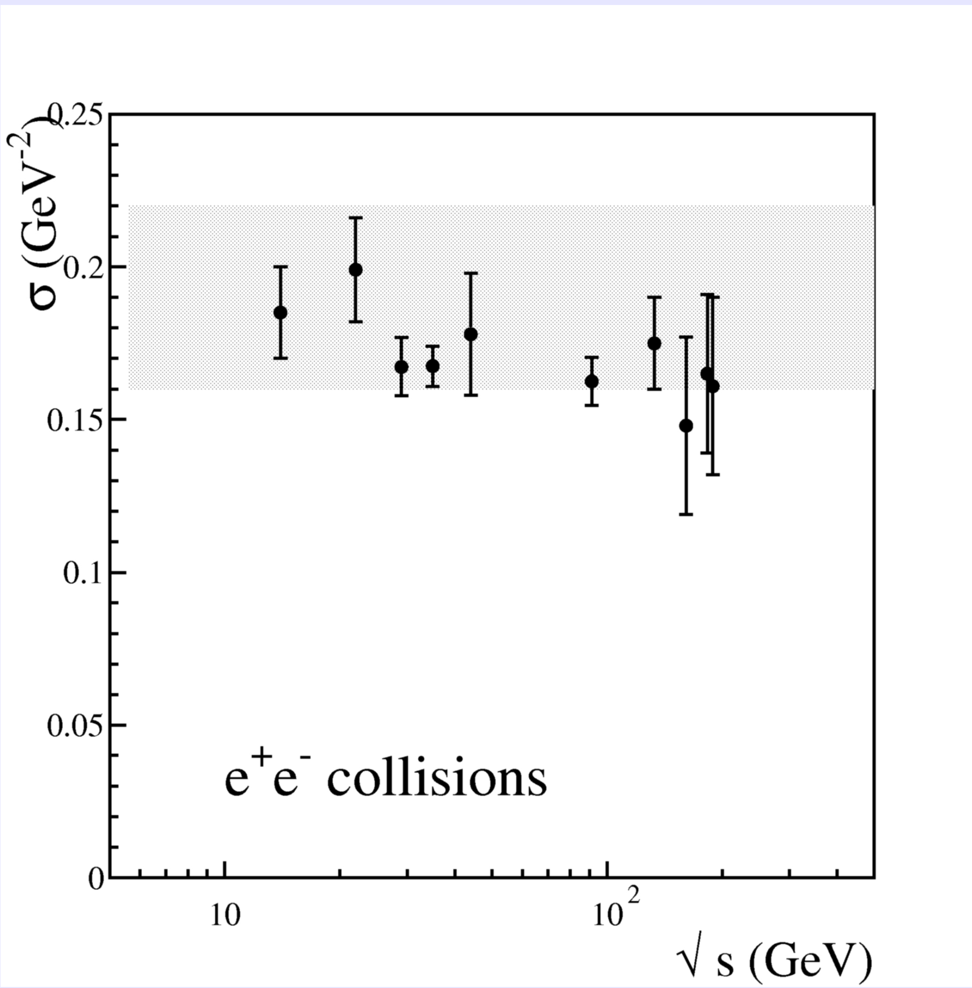
Temperature of hadron radiation becomes dependent on the quark masses

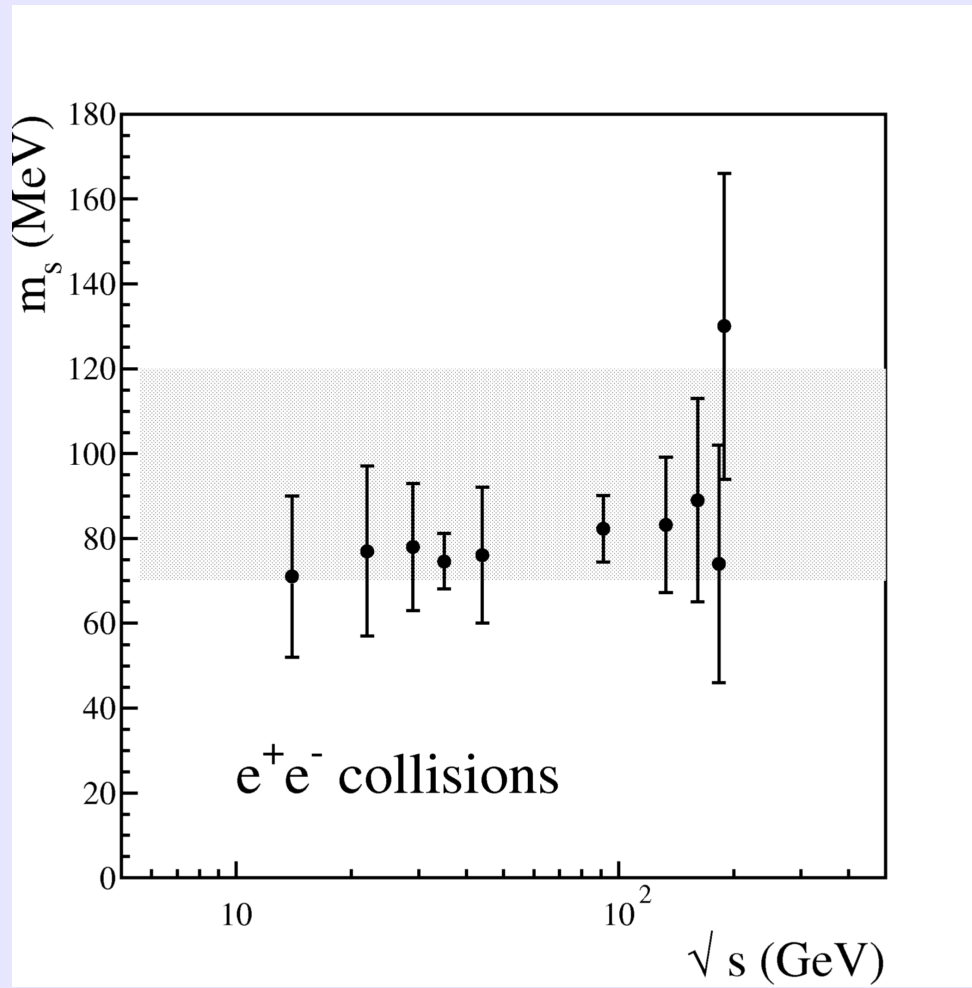
$\sigma = 0.2\ \mathrm{GeV}^2$

T	$m_s = 0.075$	$m_s = 0.100$	$m_s = 0.125$
$T(00)$	0.178	0.178	0.178
$T(0s)$	0.172	0.167	0.162
$T(ss)$	0.166	0.157	0.148
$T(000)$	0.178	0.178	0.178
$T(00s)$	0.174	0.171	0.167
$T(0ss)$	0.170	0.164	0.157
$T(sss)$	0.166	0.157	0.148

Results

Grey bands: present world best estimates





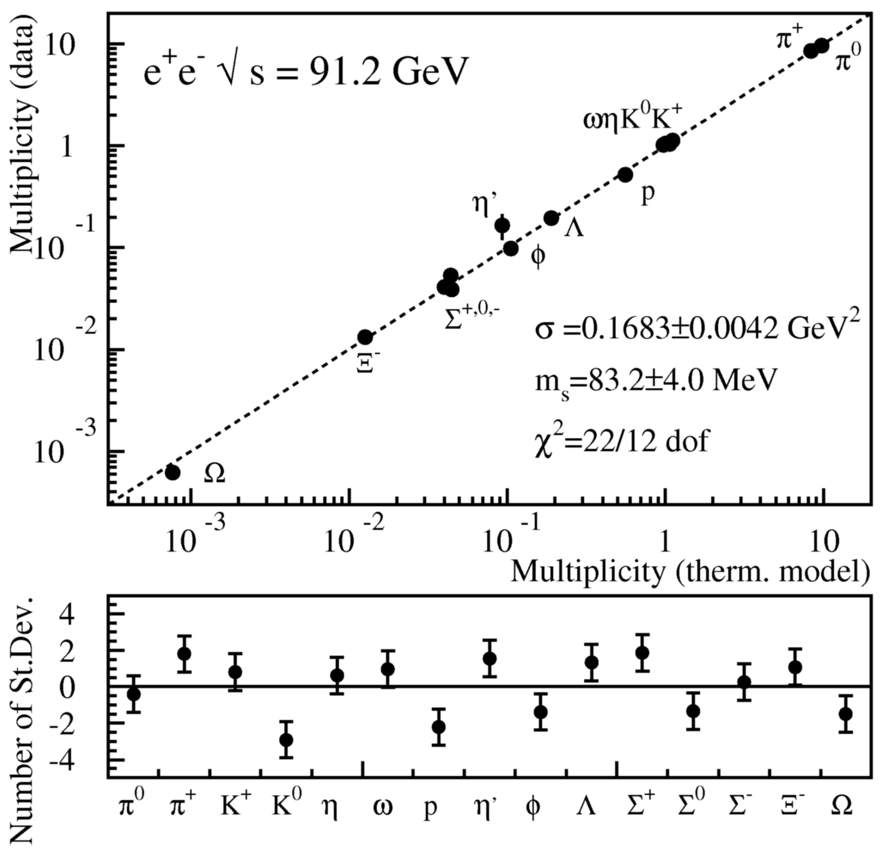
Analysis of the whole set of available data in e+e- collisions

F. B., P. Castorina, J. Manninen, H. Satz, arXiv:0805.0964 Eur. Phys. J. C in press

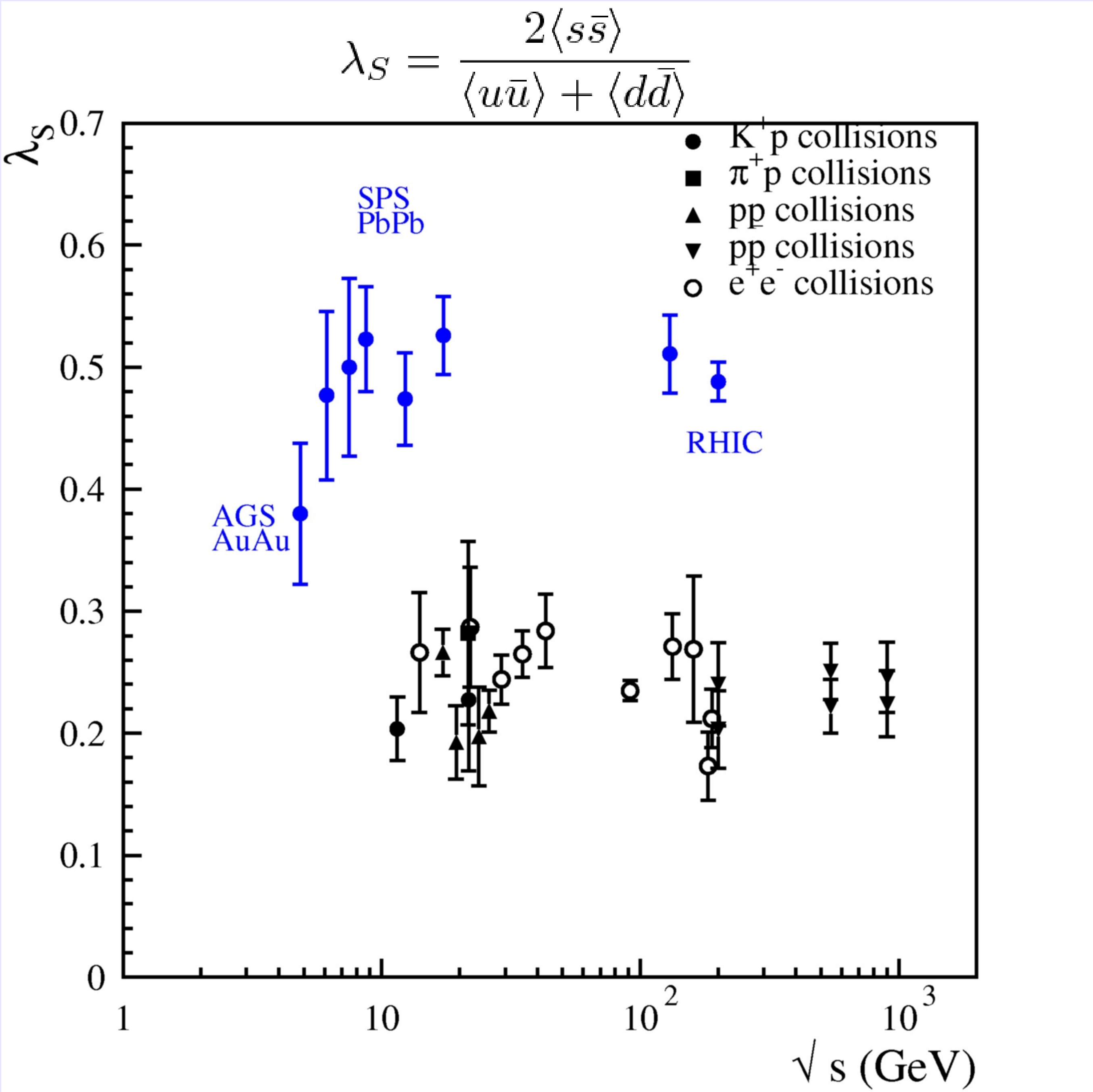
Primary mult's in Boltzmann approx.

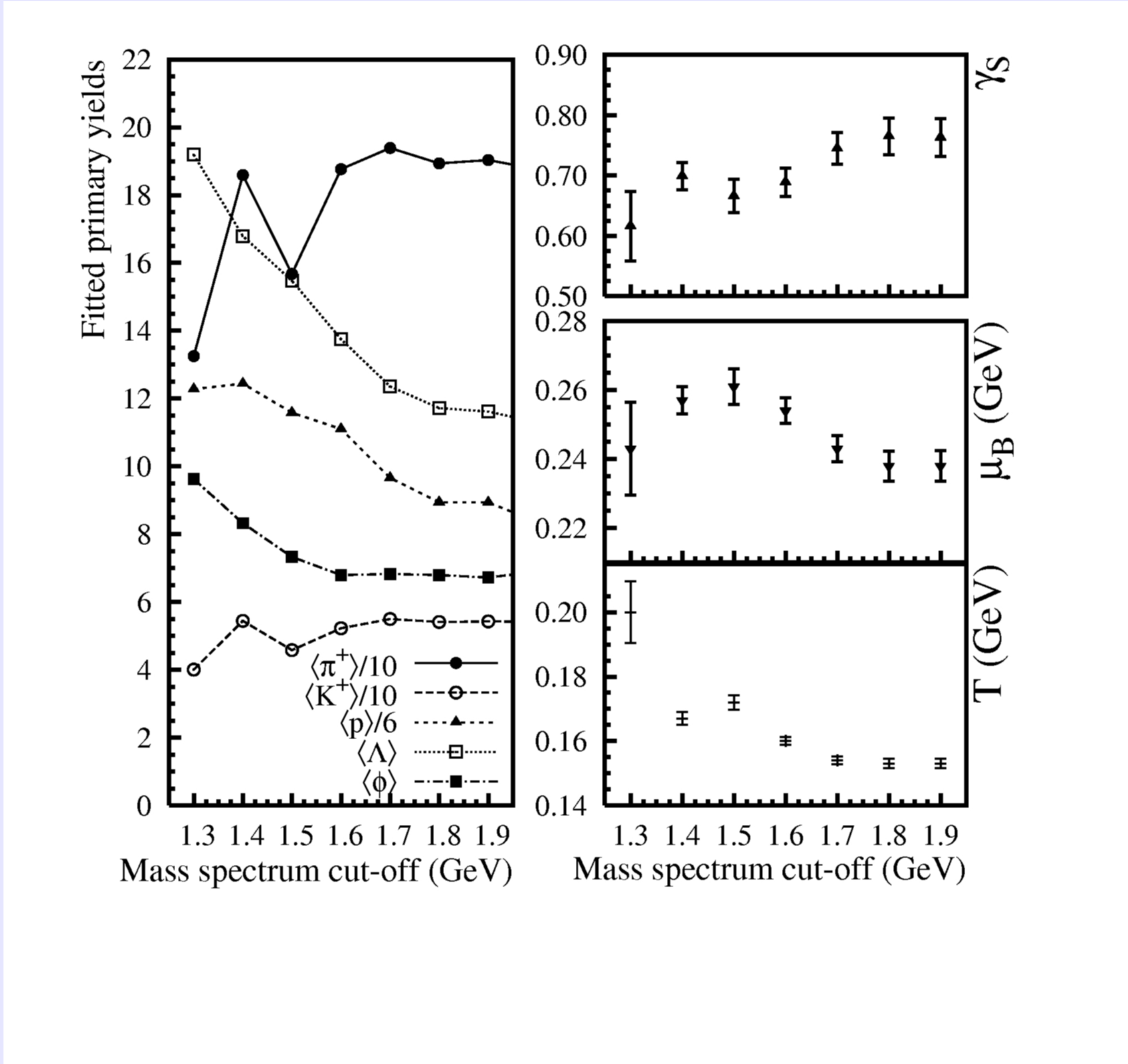
$$\langle n_j \rangle = \frac{(2S+1)V}{(2\pi)^3} \int d^3p \, e^{-\sqrt{p^2+m_j^2}/T(j)} \frac{Z(\mathbf{Q}-\mathbf{q}_j)}{Z(\mathbf{Q})} \qquad \begin{array}{l} \mathbf{Q} = (Q, B, S, ..) \\ \mathbf{q}_j = (Q_j, B_j, S_j, ...) \end{array}$$

$\sqrt{s}$	$R_u + R_d$	R_s	R_c	R_b
14	0.46	0.09	0.37	0.08
22	0.46	0.09	0.36	0.09
29	0.46	0.09	0.36	0.09
35	0.46	0.09	0.36	0.09
43	0.46	0.09	0.36	0.09
91.25	0.39	0.22	0.17	0.22
133	0.41	0.18	0.23	0.18
161	0.42	0.17	0.24	0.17
183	0.42	0.16	0.26	0.16
189	0.42	0.16	0.26	0.16



Wroblewski ratio: current status





F. B., M. Gazdzicki, A. Keranen, J. Manninen, R. Stock, Phys. Rev. C 69 (2004) 024905

Comparison with other models

From **DELPHI coll., *"The next round of....identified particles"*, hep-ex 9511011**

	JETSET 7.3 PS	JETSET 7.4 PS	ARIADNE 4.06	JETSET 7.4 ME	HERWIG 5.8 C	LEP[44],[46],[47],[49]	Stat. mod.
Charged Particles							
$< N_{ch} >$	20.87	20.81	20.80	20.86	20.94	20.95 $\pm$ 0.21	
Pseudoscalar Mesons							
$\pi^{\pm}$	17.19	17.09	17.13	17.36	17.66	17.1 $\pm$ 0.4	16.93
π^0	9.85	9.83	9.82	10.03	9.81	9.9 $\pm$ 0.08	9.88
$K^{\pm}$	2.20	2.23	2.19	2.15	2.11	2.42 $\pm$ 0.13	2.29
K^0	2.13	2.17	2.12	2.10	2.08	2.12 $\pm$ 0.06	2.20
η	1.07	1.10	1.09	1.16	1.02	0.73 $\pm$ 0.07	1.00
η'(958)	0.10	0.09	0.10	0.10	0.14	0.17 $\pm$ 0.05	0.104
D^+	0.19	0.20	0.20	0.20	0.24	0.20 $\pm$ 0.03	
D^0	0.46	0.49	0.48	0.49	0.53	0.40 $\pm$ 0.06	
$B^{\pm}, B^0$	0.36	0.36	0.36	0.36	0.36	0.34 $\pm$ 0.06	
Scalar Mesons							0.069
$f_0(980)$	0.17	0.16	0.17	0.16		0.14 $\pm$ 0.06	
Vector Mesons							
$\rho^{\circ}(770)$	1.29	1.27	1.26	1.29	1.43	1.40 $\pm$ 0.1	1.10
$K^{*\pm}(892)$	0.78	0.77	0.79	0.77	0.74	0.78 $\pm$ 0.08	0.706
$K^{*0}(892)$	0.80	0.77	0.81	0.78	0.74	0.77 $\pm$ 0.09	0.691
$\phi(1020)$	0.109	0.107	0.107	0.102	0.099	0.086 $\pm$ 0.018	0.127
$D^{*\pm}(2010)$	0.18	0.22	0.19	0.22	0.22	0.17 $\pm$ 0.02	
$D^{*0}(2007)$	0.20	0.22	0.20	0.22	0.23	.	
Tensor Mesons							
$f_2(1270)$	0.29	0.29	0.29	0.30	0.26	0.31 $\pm$ 0.12	0.0972
Baryons							
p	0.97	0.97	0.96	0.90	0.78	0.92 $\pm$ 0.11	0.898
Λ^0	0.361	0.349	0.365	0.309	0.368	0.348 $\pm$ 0.013	0.308
Ξ^-	0.0288	0.0300	0.0300	0.0256	0.0493	0.0238 $\pm$ 0.0024	0.0227
$\Delta^{++}(1232)$	0.158	0.160	0.136	0.158	0.154	0.077 $\pm$ 0.018	0.135
$\Sigma^{\pm}(1385)$	0.037	0.036	0.032	0.033	0.065	0.0380 $\pm$ 0.0062	0.0667
$\Xi^0(1530)$	0.0073	0.0069	0.0063	0.0060	0.0249	0.0063 $\pm$ 0.0014	0.00768
Ω^-	0.0013	0.0019	0.0021	0.0010	0.0077	0.0051 $\pm$ 0.0013	0.00167
Λ_b^0	0.032	0.033	0.032	0.029	0.007	0.031 $\pm$ 0.016	

Table 8: The Production Rates for the different Generators compared to LEP data

χ^2 = 71.8/5

χ^2 = 90/16

Free parameters in PYTHIA

Specific for light-flavoured abundances

Parameter	Name	Default	Range gen.	Fit Result Val.	stat.	sys.
λ_{QCD}	PARJ(81)	0.4	0.25 - 0.35	0.297	± 0.004	$^{+0.007}_{-0.008}$
Q_0	PARJ(82)	1.0	1.0 - 2.0	1.56	± 0.11	$^{+0.21}_{-0.15}$
a	PARJ(41)	0.5	0.1 - 0.5	0.417	± 0.022	$^{+0.011}_{-0.015}$
b	PARJ(42)	0.9	0.850	optimized		
σ_q	PARJ(21)	0.35	0.36 - 0.44	0.408	± 0.005	$^{+0.004}_{-0.004}$
$P(^1S_0)_{ud}$	-	0.5	0.3 - 0.5	0.297	± 0.021	$^{+0.102}_{-0.011}$
$P(^3S_1)_{ud}$	-	0.5	0.2 - 0.4	0.289	± 0.038	$^{+0.004}_{-0.026}$
$P(^1P_1)_{ud}$	-	0.	see text	0.096		
$P(oth.P - states)_{ud}$	-	0.	see text	0.318		
γ_s	PARJ(2)	0.30	0.27 - 0.31	0.308	± 0.007	$^{+0.004}_{-0.036}$
$P(^1S_0)_s$	-	0.4	0.3 - 0.5	0.410	± 0.038	$^{+0.026}_{-0.013}$
$P(^3S_1)_s$	-	0.6	0.2 - 0.4	0.297	± 0.021	$^{+0.020}_{-0.004}$
$P(P - states)_s$	-	0.	see text	0.293		
ϵ_c	PARJ(54)	-	variable	-0.0372	± 0.0007	$^{+0.0011}_{-0.0012}$
$P(^1S_0)_c$	-	0.25	0.26			
$P(^3S_1)_c$	-	0.75	0.44	adj. to data		
$P(P - states)_c$	-	0.	0.3			
ϵ_b	PARJ(55)	-	variable	-0.00284	± 0.00005	$^{+0.00012}_{-0.00010}$
$P(^1S_0)_b$	-	0.25	0.175			
$P(^3S_1)_b$	-	0.75	0.525	adj. to data		
$P(P - states)_b$	-	0.	0.3			
$P(qq)/P(q)$	PARJ(1)	0.1	0.08 - 0.11	0.099	± 0.001	$^{+0.005}_{-0.002}$
$P(us)/P(ud)/\gamma_s$	PARJ(3)	0.4	0.65	adj. to data		
$P(ud1)/P(ud0)$	PARJ(4)	0.05	0.07	adj. to data		
extra baryon supp.	PARJ(19)	0.	0.5	adj. to data only uds		
extra η supp.	PARJ(25)	1.0	0.65	0.65 ± 0.06		
extra η' supp.	PARJ(26)	1.0	0.23	0.23 ± 0.05		

Table 49: Parameter setting and fit results for JETSET 7.4 PS with default decays

From **DELPHI coll.**, ***"The next round of....identified particles"***, **hep-ex 9511011**

The meaning of χ^2

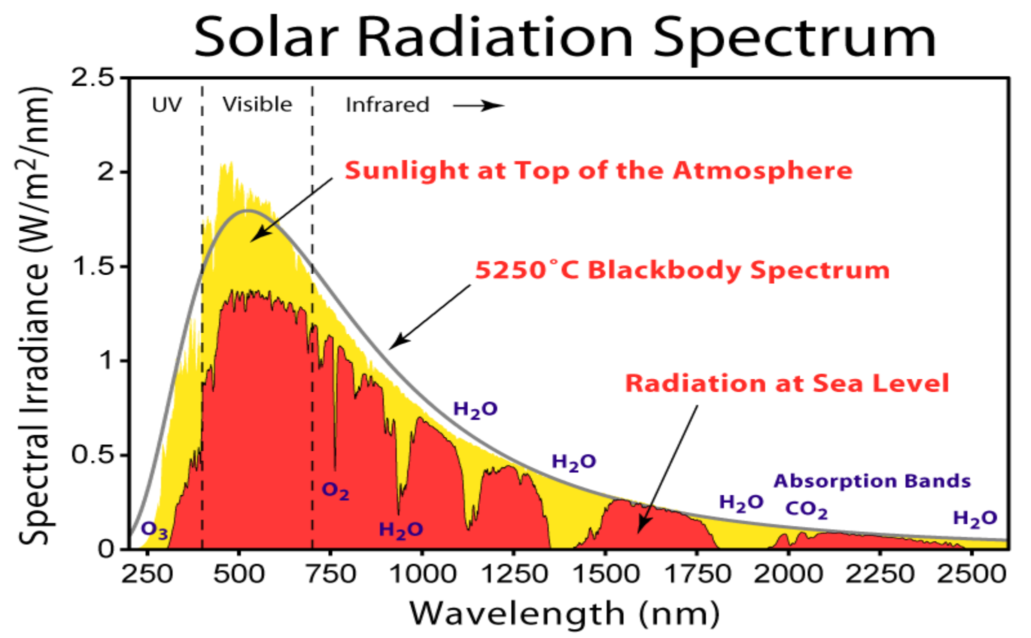
In general: χ^2 test is used to reject a hypothesis.
This hypothesis is not "the model is good" but "the formula we are using is good"

EXAMPLE: if we were to fit electroweak measurements with tree-level Standard Model formulae, we would get a very bad χ^2 but this would not mean Standard Model is not good. Fit quality is excellent if we include radiative corrections (this is how top mass was predicted)

2nd EXAMPLE: temperature of the sun surface is inferred from a black-body fit to the light spectrum measured on top atmosphere. The fit is horrible, because the sun is not a perfect blackbody (absorption lines in the corona etc.).

χ^2 value depends on the relative accuracy measurements, for an imperfect model or an approximate formulation of the model

χ^2 minimization is a useful tool to determine the best parameters even when the test is unsuccesful



Comparison with heavy ion collisions at RHIC 200

At face value, χ^2 is far worse in e+e- than in heavy ions at RHIC 200.
Does this mean that the statistical model does not work in e+e- whereas it does in heavy ion collisions?

Au-Au @ 200 GeV

χ^2 = 17.8/8 (fit to dN/dy's STAR midrap data)

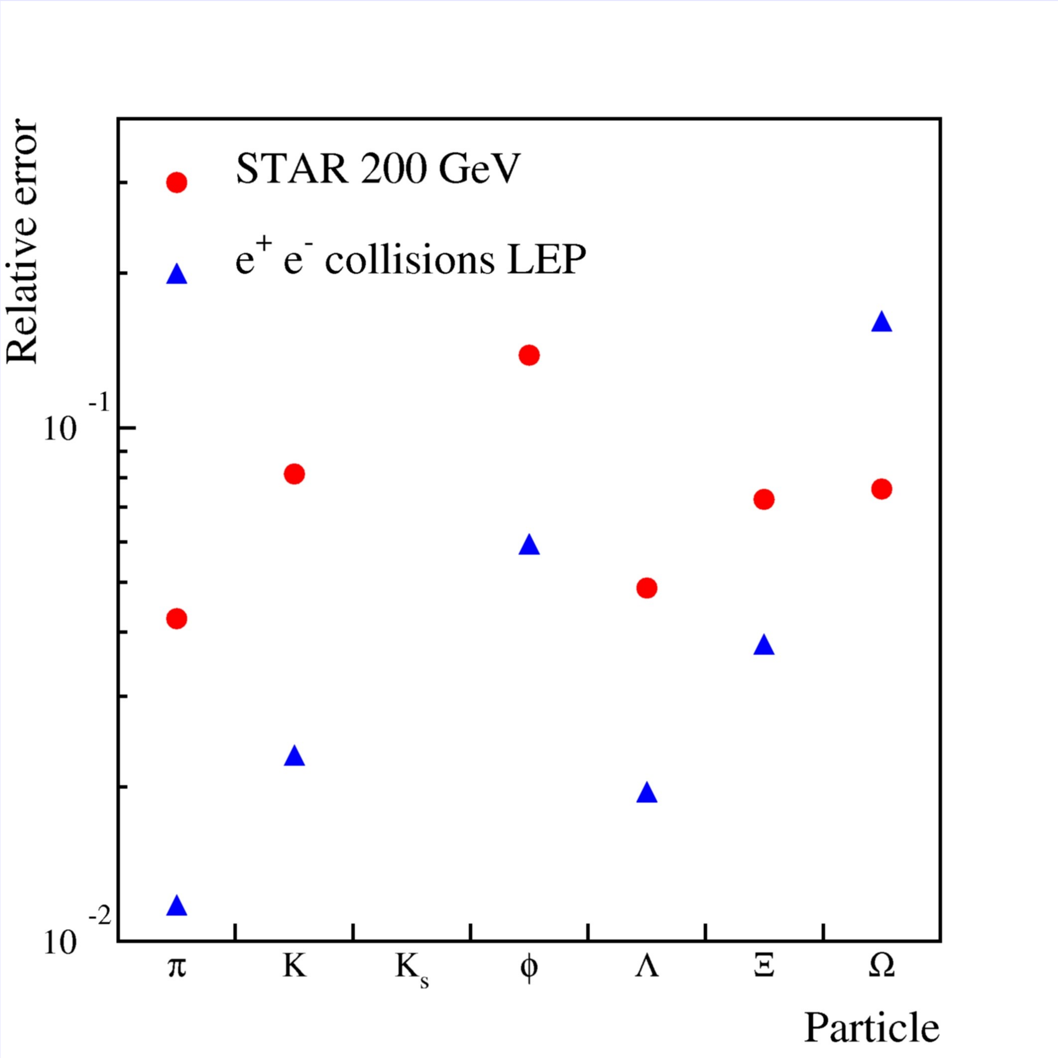
J. Manninen, F. Becattini, Phys. Rev. C 78 (2008) 054901

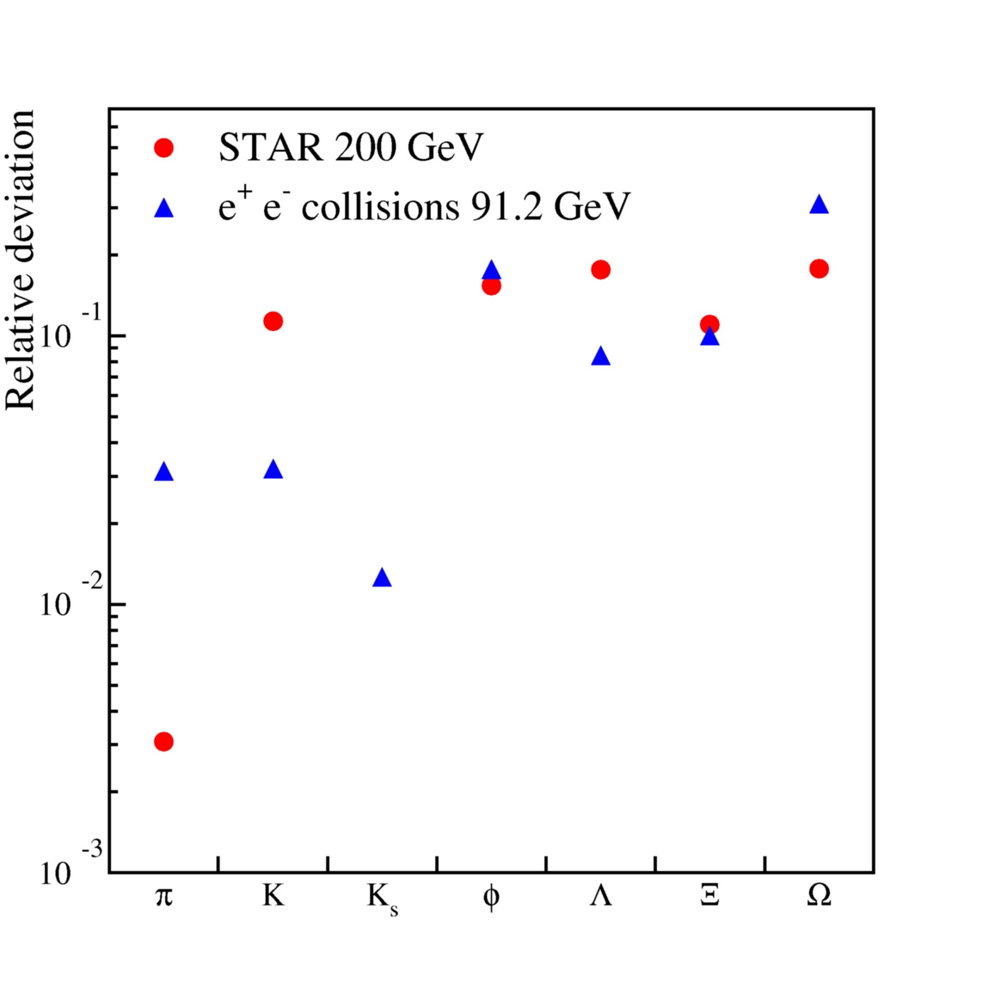
e+e- @ 91.25 GeV

χ^2 = 215/27 (fit to LEP compilation)

F. B., P. Castorina, J. Manninen, H. Satz, Eur. Phys. J. C 56 (2008) 493

Are heavy ion mult's really more “thermal” than e+e-?





Comparison should be fairly made

Comparison of fits to a set of <u>12 long-lived particles</u>

H I : π^+, π^-, K^+, K^-, ϕ, p, $\bar{p}$, Λ, $\bar{\Lambda}$, Ξ^-, $\bar{\Xi}^+$, $\Omega + \bar{\Omega}$

e+e-: π^+, π^0, K^+, K^0, ϕ, p, Λ, Σ^+, Σ^0, Σ^-, Ξ^-, Ω

HI : $\chi^2 = 17.9/8$ with 1 more free parameter

e+e- : $\chi^2 = 37/9$

Comparison of fits to a set 12 long-lived particles enforcing the <u>same relative errors</u>

HI : $\chi^2 = 14.7/8$ with 1 more free parameter

e+e- : $\chi^2 = 11.9/9$

$$\langle n_j \rangle = \frac{(2S+1)V}{(2\pi)^3} {\gamma_S}^{N_s} \int d^3p \, \mathrm{e}^{-\sqrt{p^2+m_j^2}/T} \, \frac{Z(\mathbf{Q}-\mathbf{q}_j)}{Z(\mathbf{Q})}$$

This formula used to fit multiplicities is a "tree-level" one relying on several simplifying assumptions:

- Maximal disorder in mass-charge distribution (=existence of EGC, needed to fit one T and one V)

- The use of the hadron-resonance gas model, neglecting non-resonant interaction

- Within the resonant approximation:
  - neglecting interference terms (non-symmetric diagrams in S-matrix decomposition)
  - assuming validity of DMB theorem at finite V
  - assuming validity of DMB formulae for specific channels besides full trace
  - assuming complete knowledge of the resonance spectrum up to a cut-off mass

Multiplicity fits

- Calculate (light-flavoured) primary hadron yields according to

(simplified Boltzmann formula)

$$\langle n_j \rangle = \frac{(2S_j+1)V}{(2\pi)^3} {\gamma_S}^{N_s} \int d^3p \, \mathrm{e}^{-\sqrt{p^2+m_j^2}/T} \, \frac{Z(\mathbf{Q}-\mathbf{q}_j)}{Z(\mathbf{Q})}$$

$$\mathbf{Q} = (Q, B, S, ..) \quad \mathbf{q}_j = (Q_j, B_j, S_j, ...)$$

- Perform hadronic decays until experimental definition of multiplicity is matched

$$\langle n_j \rangle = \langle n_j \rangle_{prim} + \sum_k \langle n_k \rangle \mathrm{BR}(k \to j)$$

- Optimize parameters

done in two steps: taking into account errors on masses, widths, BR's
iterating the fit (effective variance method)

The simplest example: again one particle in non-relativistic Quantum Mechanics with energy conservation

$$|h_V\rangle = |\mathbf{k}\rangle = \begin{cases} \frac{1}{\sqrt{V}} \exp(i\mathbf{k}\cdot\mathbf{x}) & \text{if } \mathbf{x} \in V \\ 0 & \text{if } \mathbf{x} \notin V \end{cases} \qquad \mathbf{k} = \pi n_x/L_x \pi \hat{\mathbf{i}} + n_y/L_y \hat{\mathbf{j}} + \pi n_z/L_z \hat{\mathbf{k}}$$

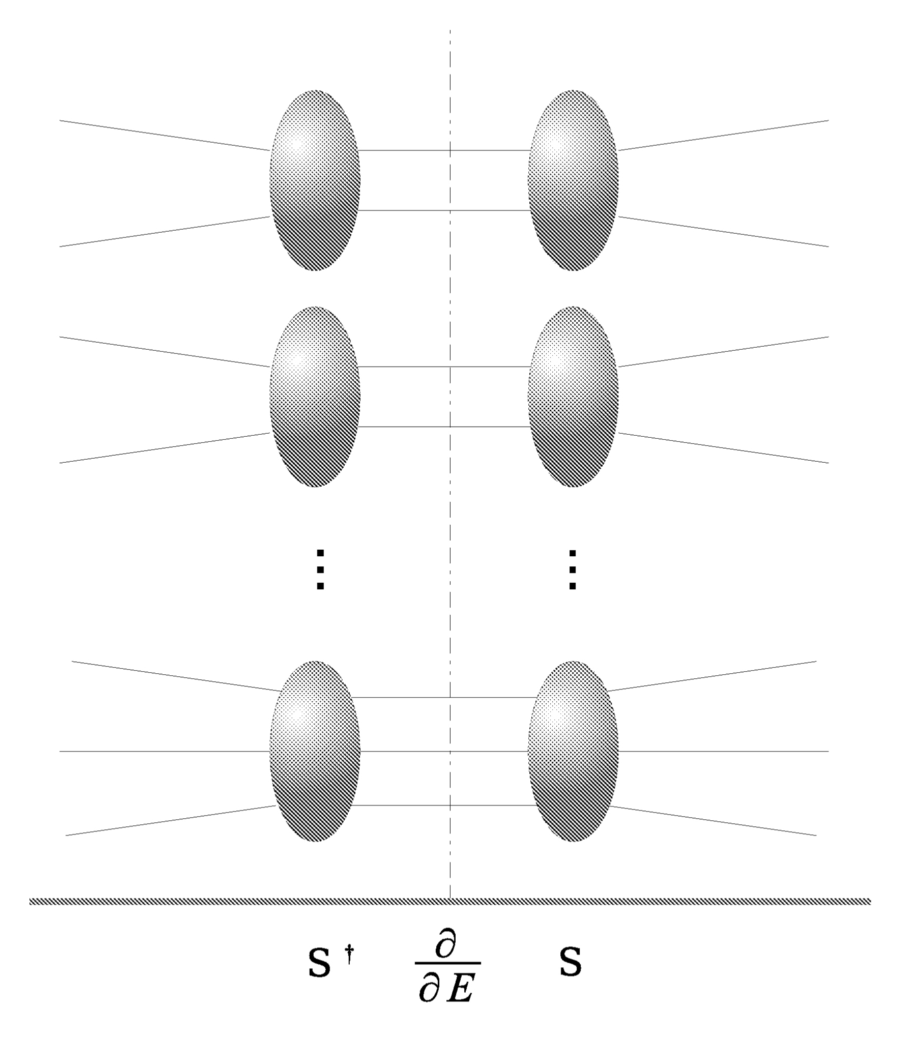
Therefore:

$$\Omega = \sum_{\mathbf{k}} \langle \mathbf{k}|\delta(E-\hat{H})|\mathbf{k}\rangle = \sum_{\mathbf{k}} \int d^3p\, |\langle \mathbf{k}|\mathbf{p}\rangle|^2 \delta\left(E - \frac{p^2}{2m}\right)$$

Because of the completeness relation: $\displaystyle\sum_{\mathbf{k}} \frac{1}{V} \exp[i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')] = \delta^3(\mathbf{x}-\mathbf{x}')$

$$\Rightarrow \quad \sum_{\mathbf{k}} |\langle \mathbf{k}|\mathbf{p}\rangle|^2 = \frac{V}{(2\pi)^3}$$

$$\Rightarrow \quad \Omega = \frac{V}{(2\pi)^3} \int d^3p\, \delta\left(E - \frac{p^2}{2m}\right)$$



The hadron-resonance gas only includes the contribution of symmetric diagrams from the cluster decomposition of the scattering matrix

Non-symmetric diagrams are neglected.
They depend on unknown complex phases (resonance interference parameters)
and might give an overall vanishing contribution