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Supermassive Black Holes in Galactic Bulges

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ABSTRACT

Growing evidence indicate supermassive black holes (SMBHs) in a mass range of $M_{\rm BH}\sim 10^6-10^{10}M_\odot$ lurking in central stellar bulges of galaxies. Extensive observations reveal fairly tight power laws of $M_{\rm BH}$ versus the mean stellar velocity dispersion σ of the host stellar bulge. Together with evidence for correlations between $M_{\rm BH}$ and other properties of host bulges, the dynamic evolution of a bulge and the formation of a central SMBH should be linked. In this Letter, we reproduce the empirical $M_{\rm BH}-\sigma$ power laws based on our recent theoretical analyses (Lou & Wang; Wang & Lou; Lou, Jiang & Jin) for a self-similar general polytropic quasi-static dynamic evolution of bulges with self-gravity and spherical symmetry and present a sensible criterion of forming a central SMBH. The key result is $M_{\rm BH}=\mathcal{L}\ \sigma^{1/(1-n)}$ where $2/3<n<1$ and $\mathcal{L}$ is a proportional coefficient characteristic of different classes of host bulges. By fitting and comparing several empirical $M_{\rm BH}-\sigma$ power laws, we conclude that SMBHs and galactic bulges grow and evolve in a coeval manner and most likely there exist several classes of galactic bulge systems in quasi-static self-similar evolution and that to mix them together can lead to an unrealistic fitting. Based on our bulge-SMBH model, we provide explanations for intrinsic scatter in the relation and a unified scenario for the formation and evolution of SMBHs in different classes of host bulges.

Key words: black hole physics — galaxies: bulges — galaxies: evolution — galaxies: nuclei — hydrodynamics — quasars: general

1 INTRODUCTION

It is now widely accepted that supermassive black holes (SMBHs) within a mass range of $10^6\sim 10^{10}M_\odot$ ($M_\odot=2\times 10^{33}$g is the solar mass) form at the centres of spiral and elliptical galaxies (e.g., Lynden-Bell 1969; Kormendy & Richstone 1995; Kormendy 2004). Observationally, SMBH masses $M_{\rm BH}$ correlate with various properties of spiral galaxy bulges or elliptical galaxies, including bulge luminosities (e.g., Kormendy & Richstone 1995; Magorrian et al. 1998; Marconi & Hunt 2003), stellar bulge masses $M_{\rm bulge}$ (e.g., Magorrian et al. 1998; Marconi & Hunt 2003; Häring & Rix 2004), $M_{\rm BH}$ versus $M_{\rm bulge}$ relations in active (AGN) and inactive galaxies (e.g., Wandel 1999, 2002; McLure & Dunlop 2002), galaxy light concentrations (e.g., Graham et al. 2001), the Sérsic index (Sérsic 1968) of surface brightness profile (e.g., Graham & Driver 2007), inner core radii (e.g., Lauer et al. 2007), spiral arm pitch angles (e.g., Seigar et al. 2008), bulge gravitational binding energies (e.g., Aller & Richstone 2007) and mean stellar velocity dispersions σ (e.g., Ferrarese & Merritt 2000; Gebhardt et al. 2000; Tremaine et al. 2002; Ferrarese & Ford 2005; Hu 2008). These empirical correlations strongly suggest a physical link between SMBHs and their host bulges (e.g., Springel, Matteo & Hernquist 2005; Li, Haiman & MacLow 2007).

Among these empirical relations, $M_{\rm BH}$ and σ correlate tightly in a power law with an intrinsic scatter of $\lesssim 0.3$ dex (e.g., Novak et al. 2006). This relation was explored theoretically (e.g., Silk & Rees 1998; Fabian 1999; Blandford 1999) before observations (e.g., Ferrarese & Merritt 2000; Gebhardt et al. 2000) and the emphasis was on outflow effects for galaxies. The idea was further elaborated by King (2003). A model of singular isothermal sphere with rotation (Adams, Graff & Richstone 2001) was proposed for the $M_{\rm BH}-\sigma$ relation. This relation was also studied in a semi-analytic model (Kauffmann & Haehnelt 2000) with starbursts while SMBHs being formed and fueled during major mergers. Accretion of collisional dark matter onto SMBHs may also give the $M_{\rm BH}-\sigma$ relation (e.g., Ostriker 2000; see Haehnelt 2004 for a review). There are also numerical simulations to model feedbacks from SMBHs and stars on host galaxies.

Observationally, there are two empirical types of bulges: classical bulges (spiral galaxies with classical bulges or ellip-

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tical galaxies) and pseudobulges (e.g., Kormendy et al. 2004; Drory & Fisher 2007). While SMBHs in classical bulges are formed after major mergers, pseudobulges do not show obvious merger signatures. Interestingly, pseudobulges also manifest a $M_{\rm BH}-\sigma$ power law yet with a different exponent (e.g., Kormendy & Gebhardt 2001; Hu 2008).

Self-similar dynamics of a conventional polytropic gas sphere has been studied earlier (e.g., Yahil 1983; Suto & Silk 1988; Lou & Wang 2006 (LW06) and references therein). LW06 obtained novel self-similar quasi-static dynamic solutions which approach singular polytropic spheres (SPS) after a long time. Such polytropic dynamic solutions have been further generalized and applied to study protostar formation, "champagne flows" in H II regions, stellar core collapse, rebound shocks and the formation of compact stellar objects in a single fluid model (LW06; Lou & Wang 2007; Wang & Lou 2007, 2008; Hu & Lou 2008) as well as galaxy clusters in a two-fluid model (Lou, Jiang & Jin 2008; LJJ hereafter). Here, we take such quasi-static solutions in a general polytropic fluid to describe long-time evolution of host stellar bulges and the formation of central SMBHs and to establish $M_{\rm BH}-\sigma$ power laws.

2 A POLYTROPIC SELF-SIMILAR DYNAMIC MODEL FOR $M_{\rm BH}-\sigma$ POWER LAWS

For the dynamic evolution of a stellar bulge in a galaxy, we adopt a few simplifying assumptions. First, we treat the stellar bulge as a spherical polytropic fluid as the typical age $\sim 10^9$ yr of galactic bulges is long (e.g., Frogel 1998; Gnedin, Norman & Ostriker 1999) that they are continuously adjusted and relaxed. Stellar velocity dispersions produce an effective pressure P against the self-gravity as in the Jeans equation (e.g., Binney & Tremaine 1987). Secondly, the total mass of the interstellar medium in a galaxy is $\sim 10^7-10^8 M_\odot$ (e.g., Gnedin et al. 1999), only $10^{-2}\sim 10^{-3}$ of the total bulge mass. Although gas densities in broad and narrow line regions of AGNs are high, the filling factor is usually small ($\sim 10^{-3}$; Osterbrock & Ferland 2006) and the gas breaks into clumpy clouds. Thus gas is merged into our stellar fluid as an approximation. Thirdly, the diameter of broad line regions of AGNs is only ~ 0.1 pc (Osterbrock & Ferland 2006) and the disc around a SMBH is even smaller while a galactic bulge size is $\gtrsim 1$ kpc. We thus ignore small-scale structures around the central SMBH of a spherical bulge. Finally, as rotation curves of galaxies show (e.g., Binney & Tremaine 1987), the effect of dark matter halo in the innermost region (around several kpcs) of a galaxy may be neglected. So the dark matter is not included as we study the bulge dynamics.

Hydrodynamic equations of a general polytropic bulge model with spherical symmetry are mass conservation

$$\frac{\partial M}{\partial t}+u\frac{\partial M}{\partial r}=0 \qquad \text{and} \qquad \frac{\partial M}{\partial r}=4\pi r^2\rho\ , \tag{1}$$

or equivalently

$$\frac{\partial \rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho u)=0\ , \tag{2}$$

radial momentum conservation (LW06; Wang & Lou 2008)

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}-\frac{GM}{r^2}\ , \tag{3}$$

and 'specific entropy' conservation along streamlines (LJJ)

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)\left(\frac{P}{\rho^\gamma}\right)=0\ , \tag{4}$$

where r is radius and t is time; $M(r,t)$ is the enclosed mass and $u(r,t)$ is the bulk radial flow speed; $P(r,t)$ is the effective pressure and $\rho(r,t)$ is the mass density; γ is the polytropic index for the stellar bulge fluid; G is the gravity constant.

As the bulk flow of stellar fluid is slow, we invoke the novel self-similar quasi-static solutions (LW06; LJJ) to model the bulge evolution under spherical symmetry. We use a self-similar transformation (LW06; LJJ) to solve the general polytropic fluid equations (1) − (4), namely

$$r=K^{1/2}t^n x\ ,\quad \rho=\frac{\alpha(x)}{4\pi G t^2}\ ,\quad u=K^{1/2}t^{n-1}v(x)\ ,$$
$$P=\frac{Kt^{2n-4}}{4\pi G}\beta(x)\ ,\qquad M=\frac{K^{3/2}t^{3n-2}m(x)}{(3n-2)G}\ . \tag{5}$$

Here, x is the independent dimensionless similarity variable while K and n are two scaling indices;[1] $\alpha(x)$ is the reduced mass density and $v(x)$ is the reduced radial flow speed; $\beta(x)$ is the reduced pressure and $m(x)$ is the reduced enclosed mass; reduced variables α, β, v and m are functions of x only. We require $n>2/3$ for a positive mass.

By transformation (5), we readily construct self-similar quasi-static dynamic solutions from the general polytropic fluid equations (1)−(4) that approach the SPS as the leading term for small x. Properties of such asymptotic solutions to the leading order (LW06; Wang & Lou 2008; LJJ) are summarized below. Both initial ($t\rightarrow 0^+$) and final ($t\rightarrow +\infty$) mass density profiles scale as $\sim r^{-2/n}$; accordingly, the bulge enclosed mass profile is $M\propto r^{3-2/n}$. As $r\rightarrow 0^+$ or $t\rightarrow+\infty$, the density and enclosed mass profiles are

$$M=\frac{nAK^{1/n}}{(3n-2)G}r^{(3n-2)/n}\ ,\qquad \rho=\frac{A}{4\pi G}K^{1/n}r^{-2/n}\ , \tag{6}$$

where $A\equiv\{n^{2-q}/[2(2-n)(3n-2)]\}^{1/(q+\gamma-2)}$ and $q\equiv 2(n+\gamma-2)/(3n-2)$. For either $x\rightarrow 0^+$ or $x\rightarrow+\infty$, the reduced velocity $v\rightarrow 0$, which means at a time t, for either $r\rightarrow 0^+$ or $r\rightarrow+\infty$ the flow speed $u\rightarrow 0$, or at a radius r, when t is short or long enough, the radial flow speed $u\rightarrow 0$. Our model describes a self-similar bulge evolution towards a nearly static configuration after a long time lapse, appropriate for galactic bulges at the present epoch.

As the effective pressure P results from stellar bulge velocity dispersion, we readily derive the mean velocity dispersion σ in a bulge. By specific entropy conservation along streamlines, we relate P with ρ and M (LJJ) and derive the P profile from our quasi-static solutions. We then take the local stellar velocity dispersion as $\sigma_L(r,t)=(\gamma P/\rho)^{1/2}$. The asymptotic expression when $t\rightarrow+\infty$ for the local stellar velocity dispersion in our model is therefore

$$\sigma_L(r)=\gamma^{1/2}K^{1/(2n)}n^{q/2}A^{(q+\gamma-1)/2}r^{(n-1)/n}\ . \tag{7}$$

To compare with observations, we derive the spatially averaged stellar velocity dispersion σ within the bulge. The bulge boundary is taken as the radius $r_{\rm c}$ where ρ drops to a value ρ_c indistinguishable from the environment. Practically, the mean velocity dispersion σ is usually estimated within

[1] Here, n is not the Sérsic index of surface brightness profile.

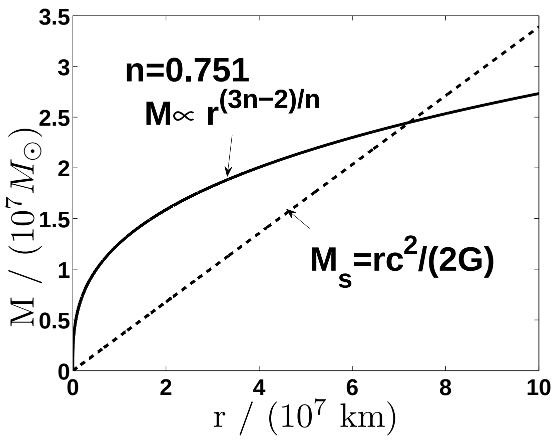

Figure 1. The criterion of forming a central SMBH in a self-similar quasi-static bulge evolution in a general polytropic formulation (LW06; LJJ). The enclosed mass power law is $M \propto r^{0.337}$ with $n = 0.751$ (solid curve). Meanwhile, we draw a straight dashed line $M_{\rm s} = frc^2/(2G)$ with $f = 1$ for the mass of the central SMBH versus the Schwarzschild radius r. Here at $r_{\rm s} = 7.2 \times 10^7$ km, the straight line intersects the enclosed mass power-law curve for a $2.45 \times 10^7 M_\odot$ SMBH. The region within $r_s = 7.2 \times 10^7$ km is the SMBH in this example. A self-similar general polytropic quasi-static solution of $n < 1$ can thus form a SMBH at the stellar bulge centre by this criterion.

the half-light radius which is less than the outer bulge photometric radius. As σ_L decreases with increasing r by equation (7), the mean σ would be higher within the half-light radius. In principle, the environmental density for different bulges is not the same and it is difficult to give the density exactly. Here, we take a reasonable critical density ρ_c as the criterion to define the boundary of a bulge, which means when the density of a bulge drops to the critical density ρ_c, we define the domain within this radius r_c as the bulge. For a class of bulges with the same n, ρ_c is regarded as a constant. So the difference in ρ_c reflects different environments of bulges with different parameter n. One can readily show that within r_c

$$\begin{aligned} \sigma &= \frac{3}{4\pi r_c^3}\int_0^{r_c} \sigma_L(r) 4\pi r^2 dr \\ &= \Big[3n^{1+q/2}\gamma^{1/2}/(4n-1)\Big](4\pi G\rho_c)^{(1-n)/2} A^{3nq/4} K^{1/2} \\ &\equiv \mathcal{Q} K^{1/2} \ . \end{aligned} \tag{8}$$

This relation is particularly satisfying on the intuitive ground. A SMBH forms at the centre of a galactic bulge that evolves in a self-similar quasi-static manner. Such a SMBH was formed by the core collapse of collections of stars and gas towards the bulge centre and grows rapidly by matter accretion at an earlier phase (e.g., Lynden-Bell 1969; Hu et al. 2006; see Haehnelt 2004 and references therein for a review of the joint formation of SMBHs and galaxies). As the growth timescale for SMBHs is only $\sim 10^5$ yr, our quasi-static solutions describe the relatively quiescent phase of galactic bulges after the formation of SMBHs as a longer history of a bulge evolution. The stellar fluid made up of stars and condensed gas clouds has a slow bulk flow speed towards the central SMBH, sustaining a reservoir of mass accretion for the circumnuclear torus and/or disc on smaller scales. Besides, there have been observational evidences to show that stars torn up by the tidal force of the SMBH may account for the observed X-rays at the centre of bulges (e.g., Zhao, Haehnelt & Rees 2002; Komossa 2004; Komossa et al. 2004; Komossa et al. 2008), which presents a scenario of how our stellar fluid being accreted by a central SMBH.

We now introduce the criterion of forming a SMBH in a spherical stellar bulge. A SMBH mass $M_{\rm BH}$ and its Schwarzschild radius $r_{\rm s}$ are related by $M_{\rm BH} = fr_{\rm s}c^2/(2G)$ where c is the speed of light and f is an adjustable factor of order unity. According to equation (6), the mass enclosed within the radius[2] r is $M \propto r^{3-2/n}$. At a certain radius $r = r_{\rm s} = [(3n-2)fc^2/(2nAK^{1/n})]^{n/(2n-2)}$ where it happens[3] $M = r_{\rm s}c^2/(2G)$, a SMBH forms. Only those quasi-static bulges with $n < 1$ can form central SMBHs as shown in Figure 1; we derive $M_{\rm BH} = [fc^2/(2G)][(3n-2)fc^2/(2nA)]^{n/(2n-2)}K^{1/(2-2n)}$ and the power law below

$$\begin{aligned} M_{\rm BH} = &\left(\frac{nA}{3n-2}\right)^{n/(2-2n)}\left(\frac{2}{fc^2}\right)^{(3n-2)/(2-2n)} \\ &\times \frac{\mathcal{Q}^{1/(n-1)}}{G}\ \sigma^{\ 1/(1-n)} \equiv \mathcal{L}\ \sigma^{\ 1/(1-n)}\ , \end{aligned} \tag{9}$$

where the coefficient $\mathcal{L}$ depends on fc^2, G, n, γ, ρ_c, and the exponent $1/(1-n) > 3$ because of the requirement $2/3 < n < 1$. Other researchers have their own criteria, which differ from ours, to define the black holes in their models to give a $M_{\rm BH} - \sigma$ relation (e.g., Adams et al. 2001).

While the $M_{\rm BH}-\sigma$ power law is very tight with intrinsic scatter $\leqslant 0.3$ dex for SMBHs and host bulges (e.g., Novak & Faber 2006), such scatter is large enough to accommodate different exponents in the $M_{\rm BH} - \sigma$ relation (Ferrarese & Merritt 2000; Gebhardt et al. 2000; Tremaine et al. 2002). By relation (9), we have a natural interpretation for intrinsic scatters in the observed $M_{BH} - \sigma$ power law. In our model, all bulges with the same n lie on a straight line with the exponent $1/(1-n)$ as shown in Figure 2. For a fixed n, different bulges are represented by different K values in transformation (5), leading to different $M_{\rm BH}$ and σ. However, for bulges with different n values, they lie on different lines. For elliptical galaxies or bulges in spiral galaxies, they appear to eventually take the self-similar evolution described above with a certain n value. But pseudobulges may take on different n values. Observationally, we cannot determine a priori the specific n value for a bulge but simply attempt to fit all bulges with a single exponent, which then leads in part to intrinsic scatter in the observed $M_{\rm BH} - \sigma$ power law.

To show this, we fit three published $M_{\rm BH} - \sigma$ power laws in Figure 2. The first one is $M_{\rm BH} = 1.2 \times 10^8 M_\odot(\sigma/200\ {\rm km\ s^{-1}})^{3.75}$ given in Gebhardt et al. (2000) with our parameters $\{n,\ \gamma,\ \rho_c\}$ being $\{0.733,\ 1.327,\ 0.47\ M_\odot{\rm pc}^{-3}\}$; and the five points (asterisks $*$) correspond to $K = \{0.8,\ 1,\ 2,\ 3,\ 4\} \times 10^{23}$ cgs unit and $r_c = 0.73,\ 0.82,\ 1.15,\ 1.41,\ 1.63$ kpc. The second one is $\log(M_{\rm BH}/M_\odot) = 8.13 + 4.02\log(\sigma/200\ {\rm km\ s^{-1}})$ given in Tremaine et al. (2002) with our parameters

[2] In the domain of bulges and SMBHs, these asymptotic expressions represent a very good approximation of the exact solution.
[3] We may take the last stable orbit as the cutoff radius or a mildly relativistic case of $M_{\rm BH} = fr_{\rm s}c^2/(2G)$ with $0.2 \lesssim f \lesssim 3$ to form central SMBHs, and this would not alter our results significantly.

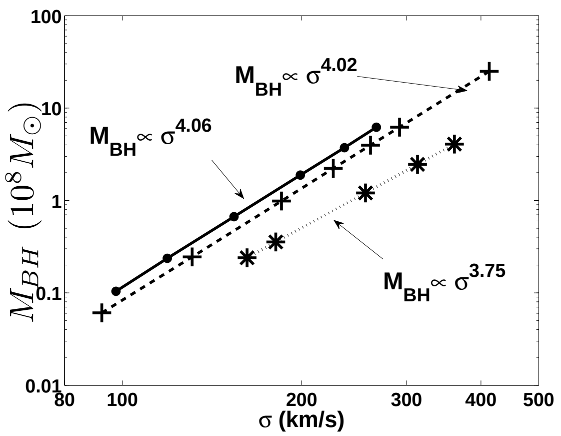

Figure 2. Power-law $M_{\rm BH}-\sigma$ relations in our general polytropic model for quasi-static self-similar bulge evolution and the formation of a central SMBH. Three indices $n = 0.7537$, 0.7512, 0.7333 are adopted for three different classes, namely, solid line (Hu 2008), dashed line (Tremaine et al. 2002), dotted line (Gebhardt et al. 2000), of $M_{\rm BH}-\sigma$ power law (9). Given n, we calculate the SMBH mass $M_{\rm BH}$ and the mean stellar velocity dispersion σ for a certain K value in self-similar transformation (5). A bulge has different mean velocity dispersion σ for different K values. Each bulge-SMBH system is represented by a point in this plot. All such systems of same n lie on a straight line, while systems of different n values correspond to different lines in our model.

$\{n,\ \gamma,\ \rho_c\}$ being $\{0.7512,\ 1.330,\ 0.0122\ M_\odot{\rm pc}^{-3}\}$; and the seven points (plus signs +) correspond to $K = \{1,\ 2,\ 4,\ 6,\ 8,\ 10,\ 20\}\times 10^{22}$ cgs unit and $r_c = 2.92,\ 4.13,\ 5.83,\ 7.14,\ 8.25,\ 9.22,\ 13.04$ kpc. The third one is $\log(M_{\rm BH}/M_\odot) = 8.28 + 4.06\log(\sigma/200\ {\rm km\ s^{-1}})$ given in Hu (2008) with our parameters $\{n,\ \gamma,\ \rho_c\}$ being $\{0.7537,\ 1.332,\ 0.00364\ M_\odot{\rm pc}^{-3}\}$; and the six points (solid dots) correspond to $K = \{0.6,\ 0.9,\ 1.5,\ 2.5,\ 3.5,\ 4.5\}\times 10^{23}$ cgs unit and $r_c = 5.74,\ 7.03,\ 9.08,\ 11.72,\ 13.87,\ 15.73$ kpc. Clearly, to fit all these points in Figure 2 with a single power law, we would get a different result with higher intrinsic scatter. In fact, there is yet another $M_{\rm BH}-\sigma$ relation given in Ferrarese & Ford (2005), namely $\log(M_{\rm BH}/M_\odot) = 8.22 + 4.86\log(\sigma/200\ {\rm km\ s^{-1}})$ (also Ferrarese & Merritt 2000). If we were to fit this relation according to our model (e.g., $n = 0.7942$, $\rho_c = 1.46\times 10^{-7}M_\odot{\rm pc}^{-3}$, $r_c = 3.25$ Mpc), the critical mass density ρ_c would be too small and the bulge size for a certain average velocity dispersion would be too large. So it seems that our model favours multiple power laws contained in the available data. In the three more sensible fitting examples above, bulge inflow speeds of stellar fluid are slow ($\sim 0.1-1$ km s^{-1}), an evolution feature of our self-similar quasi-static solutions. Near the SMBH boundary $r_{\rm s}$, the inflow rest mass-energy flux falls in the range of $10^{40}-10^{45}$ erg s^{-1} in these examples, sufficient to supply the observed X-ray luminosities (Komossa et al. 2008). There can be outgoing accretion shocks around a SMBH in these inflows. As the age of galactic bulges is so long ($\sim 10^9$ yr) (Frogel 1998) that such shocks should have already gone outside bulges and dispersed or merged into surroundings.

Besides the $M_{\rm BH}-\sigma$ relation, observations reveal $M_{\rm BH}\propto M_{\rm bulge}^{1.12}$ with $M_{\rm bulge}$ being the stellar bulge mass (e.g., Häring & Rix 2004 and references therein) and $M_{\rm BH}\propto E_{\rm g}^{0.6}$ with $E_{\rm g}$ being the absolute value of the bulge gravitational binding energy (e.g., Aller & Richstone 2007). Using our criterion of forming a SMBH and the bulge radius r_c, we derive a power law between $M_{\rm BH}$ and $M_{\rm bulge}$ as $M_{\rm BH}\propto M_{\rm bulge}^{1/(3-3n)}$ according to equation (6). For $n = 0.75$, our result leads to relations in Adams et al. (2001) but for a nonisothermal general SPS. The bulge gravitational binding energy, without contributions from dark matter halo and a disc as in Aller & Richstone (2007), is $E_{\rm g}\approx\int_0^{r_c} GM\rho 4\pi r dr$. For self-similar quasi-static dynamic solutions in general polytropic fluid model, we obtain $M_{\rm BH}\propto E_{\rm g}^{1/(5-5n)}$.

As another class of bulges, pseudobulges are thought to have formed without merging in contrast to classical bulges. Pseudobulges again follow a $M_{\rm BH}-\sigma$ power law (Hu 2008), i.e., $\log(M_{\rm BH}/M_\odot) = 7.5 + 4.5\log(\sigma/200\ {\rm km\ s^{-1}})$. Pseudobulges may take a different self-similar quasi-static evolution for a different n. For their different formation history, they show a different $M_{\rm BH}-\sigma$ power law as observed. For $\{n,\ \gamma,\ \rho_c\}$ being $\{0.7778,\ 1.34,\ 0.000426\ M_\odot{\rm pc}^{-3}\}$, we can fit the empirical power law for pseudobulges; e.g., with $K = 6\times 10^{22}$ cgs unit, we have $\sigma = 124$ km s^{-1} and $r_c = 5.742$ kpc for elliptical galaxies.

3 CONCLUSIONS AND DISCUSSION

On the basis of a self-similar quasi-static dynamic evolution of a general polytropic sphere (LW06, LJJ), we establish $M_{\rm BH} = \mathcal{L}\ \sigma^{\ 1/(1-n)}$ with $2/3 < n < 1$ by equation (9). Our first conclusion is $1/(1-n) > 3$ which appears consistent with observations so far. Secondly, uncertainties in the formation criterion of a SMBH [i.e., factor f in $M_{\rm BH} = fr_sc^2/(2G)$] and in the choice of r_c and thus ρ_c will not change the form of equation (9) and the n value but will only affect the value of $\mathcal{L}$. Thirdly, the tight $M_{\rm BH}-\sigma$ power laws and other relations among the SMBH mass $M_{\rm BH}$ and known properties of host stellar bulges strongly suggest coeval growths of SMBHs and galactic bulges (e.g., Page et al. 2001; Haehnelt 2004; Kauffmann & Haehnelt 2000; Hu et al. 2006). Fourthly in our model, while forming a SMBH at the bulge centre (e.g., by core collapse of gas and stars or by merging), the spherical general polytropic bulge evolves in a self-similar quasi-static phase for a long time. We then reproduce well-established empirical $M_{\rm BH}-\sigma$ power laws. Different energetic processes appear to give rise to different scaling index n values, which finally determines the slope of the $M_{\rm BH}-\sigma$ relations in a logarithmic presentation.

Besides classical bulges and pseudobulges, there are also 'core' elliptical galaxies (i.e., those with apparent 'cores' of relatively flat brightness; Lauer et al. 1995; Hu 2008), thought to have formed by 'dry' mergers (i.e., almost without gas). A steeper $M_{\rm BH}-\sigma$ relation exists in these galaxies as compared to that for classical bulges (e.g., Lauer et al. 1995; Laine et al. 2003; Lauer et al. 2007).

This can also be accommodated in our unified scenario that all hosts of SMBHs may finally evolve into self-similar quasi-static phase with different scaling parameters (i.e., different index n for the slope and different ρ_c for the normalization of the $M_{\rm BH}-\sigma$ relation). While these bulges can be of quite different kinds in galaxies, they have these similar tight relations and we thus provide a unified self-similar dynamic framework to model the relatively quiescent evolution phase of SMBH host bulges and the growth of SMBH masses. As the observed $M_{\rm BH}-\sigma$ relation for classical bulges is tight, the elliptical galaxies and spiral galaxies appear to take on close n values for merging processes.

In our model, n is a key scaling index to determine the exponent of the $M_{\rm BH}-\sigma$ power law. The smaller the value of n is, the steeper the density profile is and the smaller the index of the $M_{\rm BH}-\sigma$ relation is. If SMBHs are formed by collapse of stars and gas and a less steeper density distribution may provide a more effective mechanism to form SMBHs, then we conclude that for a certain value of velocity dispersions, [4] the smaller the mass of an initially formed SMBH is, the smaller the value of n is.

After a long lapse, our quasi-static solutions approach the static SPS solution as the leading term that is independent of the timescale. So all the described relations here are nearly independent of time, as long as the systems have evolved for a long enough time. It is not obvious to decide when the host bulges began to take the described self-similar evolution. But even if we take different times in our model, our results would remain largely the same. The only difference is that for an early time, we may have a chance to observe accretion shocks within the region of stellar bulges. Such shocks are characterized by a rapid inner density rise of several times and a rapid inner rise of stellar velocity dispersions depending on the strength of accretion shocks.

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[4] The mean velocity dispersion $\sigma = \mathcal{Q}K^{1/2}$ is not sensitive to n.