Theoretical Astroparticle Physics

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  - Pauli blocking effects on pair creation in strong electric field
- Strong fields in astrophysics
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2 Participants

2.1 ICRANet participants

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2.2 Ongoing collaborations

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3 Brief description

Astroparticle physics is a new field of research emerging at the intersection of particle physics, astrophysics and cosmology. Theoretical development in these fields is mainly triggered by the growing amount of experimental data of unprecedented accuracy, coming both from the ground based laboratories and from the dedicated space missions.

3.1 Relativistic plasma

Electron-positron plasma is of interest in many fields of physics and astrophysics, e.g. in the early universe, active galactic nuclei, the center of our Galaxy, compact astrophysical objects such as hypothetical quark stars, neutron stars and gamma-ray bursts sources. It is also relevant for the physics of ultraintense lasers and thermonuclear reactions. We study physical properties of dense and hot electron-positron plasmas. In particular, we are interested in the issues of its creation and relaxation, its kinetic properties and hydrodynamic description, baryon loading and radiation from such plasmas.

Two different states exist for electron-positron plasma: optically thin and optically thick. Optically thin pair plasma may exist in active galactic nuclei and in X-ray binaries. The theory of relativistic optically thin nonmagnetic plasma and especially its equilibrium configurations was established in the 80s by Svensson, Lightman, Gould, Haug and others. It was shown that relaxation of the plasma to some equilibrium state is determined by a dominant reaction, e.g. Compton scattering or bremsstrahlung.

Developments in the theory of gamma ray bursts from one side, and observational data from the other side, unambiguously point out on existence of optically thick pair dominated non-steady phase in the beginning of formation of GRBs. The spectrum of radiation from optically thick plasma is usually assumed to be thermal.

Experiments with high intensity laser beams interacting with each other as well as with solid targets aim at creation of relativistic plasmas and their study in laboratory conditions. The goal of such experiments is reproduction of astrophysical plasmas in controlled environment.

In a series of publications we consider kinetic, electrodynamic, hydrodynamic and observational properties of relativistic plasma. In 2023 we focused on electrodynamic part and pair creation, accounting for quantum statistics of particles.

3.1.1 Pauli blocking effects on pair creation in strong electric field

Quantum electrodynamics predicts Schwinger (1951) the creation of electron-positron pairs in strong electric field out of vacuum as a nonperturbative process with the field strength exceeding the critical value $E_c = m_e^2 c^3/\hbar e \simeq 1.3 \times 10^{16}$ V/cm, where m_e is the electron mass, e is its charge, c is the speed of light and $\hbar$ is reduced Planck constant.

In this work Prakapenia and Vereshchagin (2023b) we use classical kinetic equations with the source term due to the Schwinger process, as well as Maxwell equations to describe electron-positron pair plasma and electric field evolution. Previously it was shown that such approach gives accurate results in comparison to the full quantum treatment Kluger et al. (1991, 1992). Such approach was used previously in order to study kinetic evolution of electron-positron-photon plasma emerging from overcritical electric field Benedetti et al. (2013). There starting from pure vacuum and external overcritical electric field it was shown that kinetic evolution proceeds in the following time sequence: first pair creation starts which generates electric currents back reacting on external electric field inducing collective oscillations. Simultaneously electron-positron pairs are accelerated in external field back and forth reaching ultrarelativistic velocities: their kinetic energy oscillates as well. Subsequent creation of more pairs in this oscillating electric field leads to tranformation of kinetic energy of pairs into their internal energy. At this point pair distribution function is highly anisotropic. Finally, pairs start to annihilate into photons, which leads to isotropization of their distribution in momentum space.

The goal of the work Prakapenia and Vereshchagin (2023b) is to study kinetic evolution of pair plasma with initially non-vacuum states. We assume that in addition to overcritical electric field electron-positron pairs are present from the beginning and their distribution function f is given by the Fermi-

Dirac function with some temperature T and chemical potential μ. As we are interested in the effects of quantum degeneracy, we explore the influence of particle distribution in the phase space on dynamics of pairs and electric field. Electron-positron pairs in initial state are assumed to obey the Fermi-Dirac statistics

$$f = \left[1 + e^{\left(\sqrt{p^2+1}-\mu\right)/kT}\right]^{-1}. \tag{3.1.1}$$

Note that equilibrium distribution with relativistic temperature and $\mu = 0$ corresponds to $f < 1/2$ for small momenta, while fully degenerate distribution with $T = 0$ corresponds to $f > 1/2$ for small momenta. For initial conditions with $\mu = 0$ we find that when electric energy dominates over pair energetically pair production is as efficient, as in the vacuum case. Conversely, when pairs energy dominates over electric field pair production is suppressed. This is expected, because in the pair dominated region the Schwinger process is suppressed and plasma keeps oscillating with relativistic plasma frequency.

For initial conditions with $f > 1/2$ the statistical factor present in kinetic and Maxwell equations become negative, which implies particle annihilation in external electric field. Naively one could expect that pair annihilation would result in amplification of electric field. However, this does not happen. Diminishing of the number of pairs (and hence their rest mass energy) leads not to increase of the energy density of electric field, but to increase of internal energy of pairs. This is because electric field accelerates particles redistributing them in momentum space. There is no possibility to use inverse Schwinger process to enhance electric field.

As an example, we present in Fig. 3.1 the distribution function $f(p_\perp, p_{||}, t)$ as function of momentum parallel to electric field $p_{||}$ and orthogonal to it $p_\perp$..It is evident that pair annihilation leads to depletion of the distribution function only for small orthogonal momenta $p_\perp$, where the source term is significant. For larger $p_\perp$ the source term is negligible.

Main result in this work is the demonstration how quantum exclusion principle suppresses pair creation in overcritical uniform electric field, which in turn modifies the back reaction dynamics. We studied electron-positron pair creation and oscillations with initial vacuum state as well as with electron-positron plasma initially present. Two cases can be distinguished. 1) When the energy in electric field dominates that in pairs oscillations are induced, which leads to opening up of the phase space and consequent prolific pair

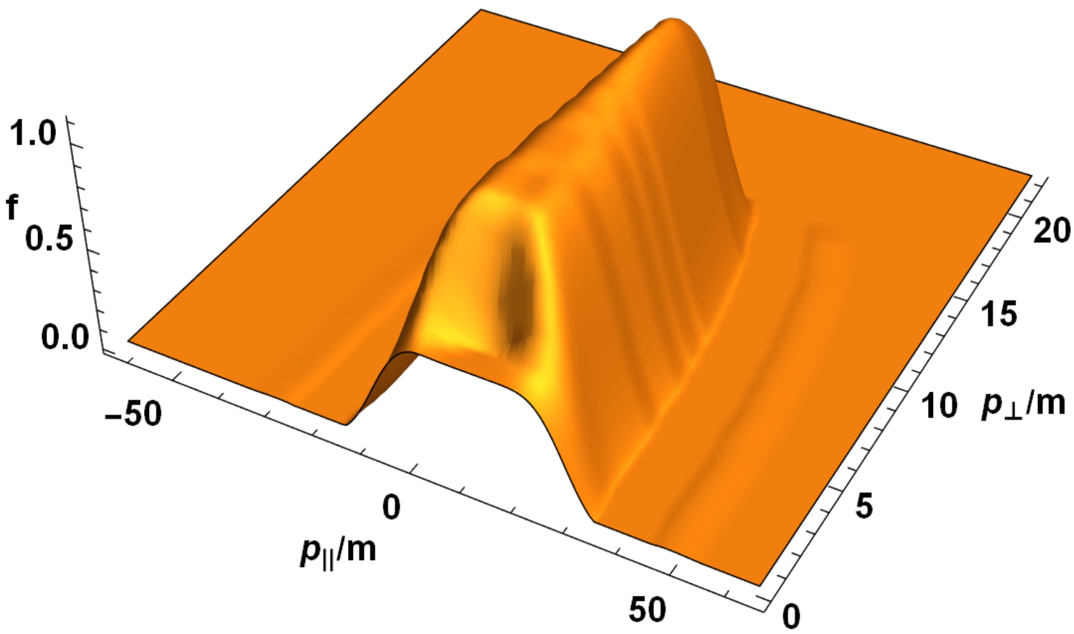


Figure 3.1: Distribution function of electron-positron pairs after 16 oscillations with $E = 4E_c$ and initial distribution (3.1.1) with $T = 0$ and $\mu = 16m_ec^2$.

creation. 2) In the opposite case, when pairs dominate energetically over electric field, plasma oscillations do occur with much higher frequency, since electric field is unable to displace them significantly in momentum space: as a consequence pair creation remains strongly suppressed.

This work is supported within the joint BRFFR-ICRANet-2023 funding programme within the Grant **No. F23ICR-001**.

Results of this work were reported at the 5th Zeldovich meeting held in Yerevan, Armenia on June 12-17, 2023. These results are published in Physical Review D, see Prakapenia and Vereshchagin (2023b). This work has been selected as Top-10 results from 2023 at the National Academy of Sciences of Belarus.

3.2 Strong fields in astrophysics

Despite strong efforts the Schwinger process is not yet reachable in laboratory conditions. However, one may look for this process in some extreme astrophysical environments, for reviews see Ruffini et al. (2010); Vereshchagin and Prakapenia (2022). Various kinetic effects in strong electromagnetic fields are discussed in Vereshchagin and Aksenov (2017). We focus on physical processes in strong electric field such as pair production and their evolution in external fields, which may be probed by astrophysical observations.

3.2.1 Pair production in hot electrospheres of compact astrophysical objects

In this work we revisited pair production in compact astrophysical objects endowed with strong electric field on their surface. The region with overcritical $E > E_c$ electric field in these objects is called *electrosphere*. Electrospheres are predicted for such hypothetical objects as superheavy nuclei Migdal et al. (1976) and quark nuggets Forbes et al. (2010). The magnitude of electric field in electrosphere depends on the sharpness of the boundary of positively charged component Mishustin et al. (2010). Usov in his seminal paper Usov (1998) proposed that hot quark stars may be a source of pair winds, potentially observable at cosmological distances. The main parameter determining the wind is the temperature on the quark star surface T_S. Based on this work detailed study of particle interactions was performed by Aksenov et al. Aksenov et al. (2004, 2005) predicting observed properties of hot quark stars. Usov's rate is used in many publications since then.

In our paper Prakapenia and Vereshchagin (2023a) we reconsider Usov's mechanism of pair creation in electrosphere of compact objects. First, we show that the reasoning under Usov's results contain some flaws. Then, we provide new arguments how pair creation can operate, and derive the rate of pair creation together with pair luminosity for electrosphere of a compact astrophysical object. Finally we perform self-consistent simulations for electron-positron pair creation and electric field evolution in electrosphere using Vlasov-Boltrzmann equations, describing distribution functions of electrons and positrons, and Maxwell equations describing time and space evolution of electric field. Our results indicate that hot electrosphere indeed is a source of strong pair wind.

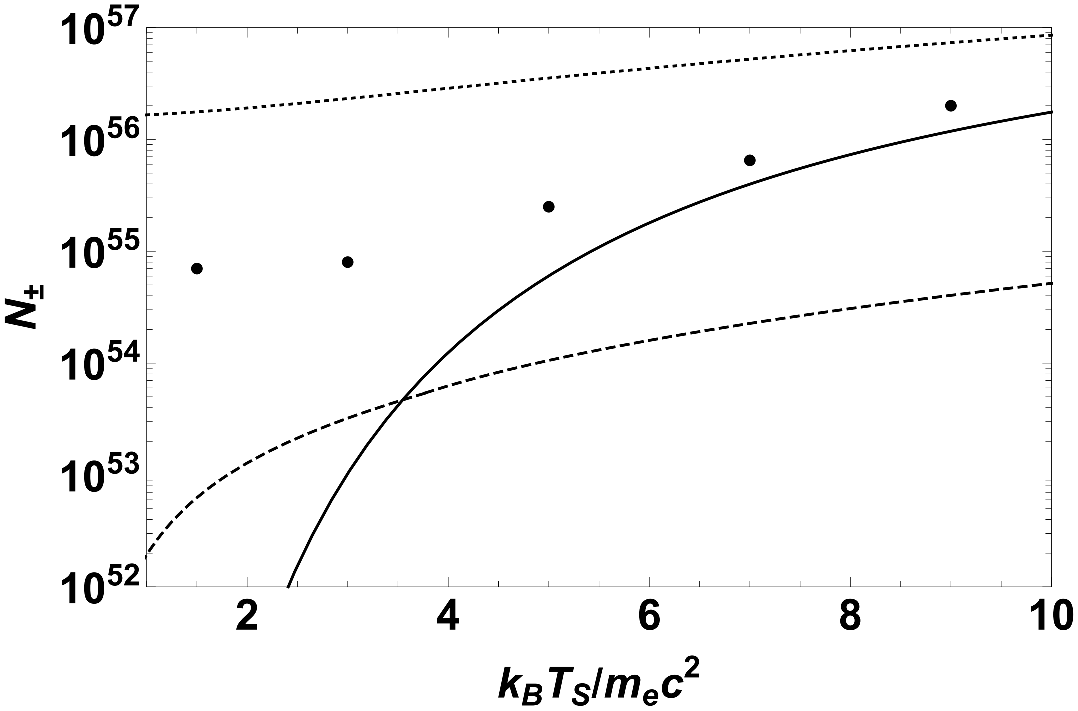
In particular, our kinetic simulation reveals two physical effects in hot electrosphere, which were ignored in previous analyses. The first effect is the inflation of electrosphere due thermal evaporation of electrons, leading to its spatial extension to distances much larger than the electrostatic solution implies. The second effect is enhancement of the rate of pair creation due to pair simultaneous acceleration by the electric field, first established in Benedetti et al. (2011). The latter effect can operate at electric fields values up to $E \lesssim 127E_c$. Both effects are crucial for estimation of pair creation rate, especially at low temperatures with strongly degenerate electrons, where analytical formulas fail to reproduce numerical rates, see Fig. 3.2.

Numerical simulations show that, as expected, the Schwinger process operates for nonzero temperature T_S. Positrons are accelerated in the Coulomb barrier and move outward. The total outward flux is approximately neutral due ability of electrons to overcome the Coulomb barrier. The distribution of the electric field and electron density is quasi static, and pair creation does not back react on the electrosphere. As a result electrons do not occupy all empty states and the process operates continuously.

Our results are summarized in Fig. 3.2. Usov's original prediction is shown by dashed curve. The rate obtained from the high field approximation $E \gg E_c$ to the Schwinger formula neglecting Pauli blocking factors is shown by dotted curve. The rate obtained from the Schwinger formula with Pauli blocking factors is shown by solid curve. Dots represent numerical results from solution of the Vlasov-Maxwell equations.

Initial conditions are chosen from electrostatic solution to the Maxwell-Vlasov system, neglecting pair creation. We recall that electrostatic solution

Figure 3.2: Pair creation rate in the electrosphere $\dot{N}_{\pm}$ according to Usov (dashed curve), ignoring Pauli blocking (dotted curve) and taking into account Pauli blocking but neglecting pair acceleration (solid curve). Dots represent numerical results.



of the the full Maxwell equations exists only in the case $T_S = 0$, that is for fully degenerate electrons. Only in this case the chemical potential $\mu = e\varphi$ equals Fermi energy, which means that there are no electrons with energies exceeding the Coulomb barrier. However, when the surface temperature is non-zero $T_S > 0$ there is a small part of electrons with energies larger than the Coulomb barrier. These electrons move outward the surface increasing the electric field outside the surface of the compact object, leading to electrosphere inflation.

The initial distribution function of electrons is assumed to be the Fermi-Dirac one, see eq. 3.1.1, where the chemical potential $\mu = e\varphi$ and the temperature T_S are obtained from the Poisson equation.

As an example we present the final spatial distribution of electric field and plasma density in Fig. 3.3 for $T_S = 3m_e c^2$. Electric field and electron

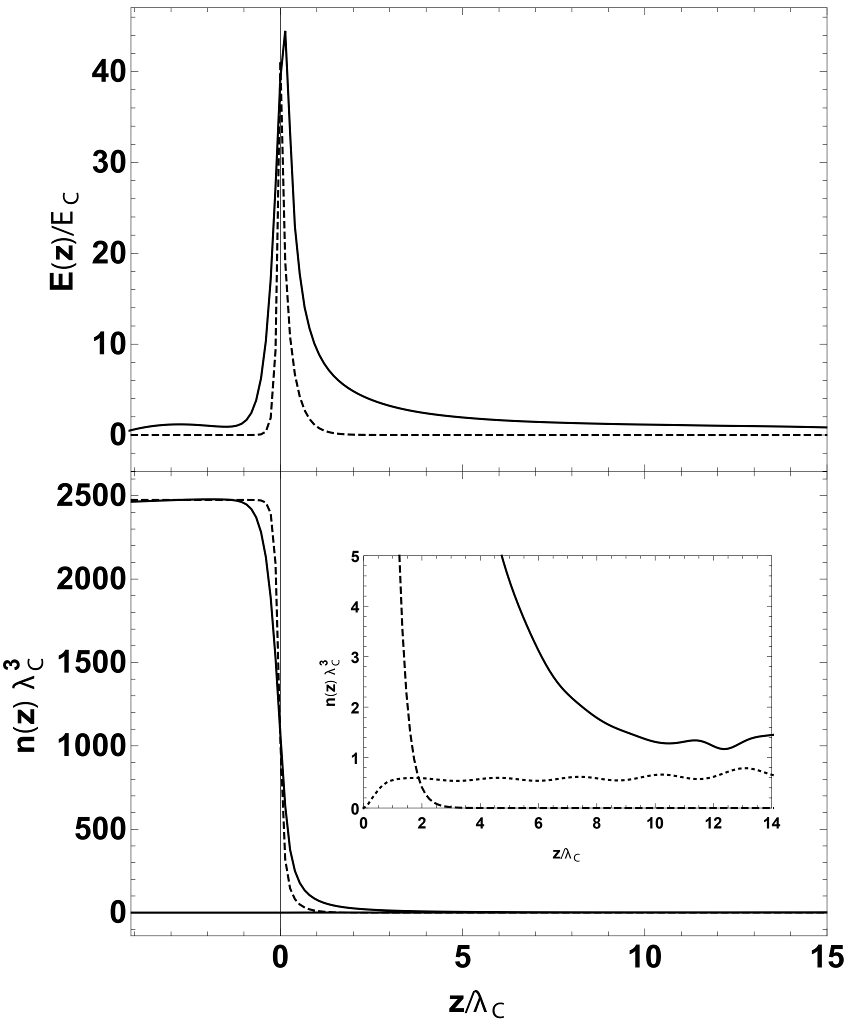


Figure 3.3: Electric field (top) and electron number density (bottom) as function of distance from the surface of the compact object. Dashed curves represent electrostatic solution. Solid curves represents inflated electrosphere. Inset shows electron (solid) and positron (dotted) spatial distribution. Here $T_S = 3m_e$.

number density as function of distance from the surface of the compact object are shown. Electrostatic solutions, used as initial conditions for simulations, are represented by dashed curves. Solutions of dynamical equations are shown by solid curves. Electrosphere inflation due to evaporation of electrons, whose energy exceeds the Coulomb barrier leads to extension of the region of overcritical electric field from $z \sim \lambda_C$ up to $z \sim 10\lambda_C$. Electron-positron pairs are mostly produced near the surface at $z = 0$, where electric field is the largest. The combined flux of electrons and positrons is clearly visible in the insets in Fig. 3.3 at $z > 10\lambda_C$.The main conclusion is that the rate of pair creation in electrosphere is largely underestimated in the literature. Moreover, the luminosity in pairs is determined not only by the temperature, but by the acceleration provided by the electric field. We find that the luminosity in pairs can be as large as

$$L_{\pm} \simeq 1.3 \times 10^{52}\ \mathrm{erg/s} \left(\frac{E}{5 \times 10^{17} \mathrm{V/cm}} \right)^3 .$$

In this estimate the typical value of electric field $E = 30E_c$ obtained from electrostatic configurations is used.

This work is supported within the joint BRFFR-ICRANet-2023 funding programme within the Grant **No. F23ICR-001**. It is submitted for publication in the Astrophysical Journal.

3.3 Charged particle motion near black holes

This year we continued the project dedicated to the motion of charged particles near black holes, supported by the joint ICRANet-BRFFR program. The purpose of the work is determination of electromagnetic field of a test charge moving in the vicinity of a black hole, as well as determination of its observational characteristics and application of obtained results to astrophysical problems of radiation in the vicinity of black holes. It is proposed to use the general covariant approach to calculate the retarded potentials of the electromagnetic field of a particle moving in the vicinity of a black hole.

3.3.1 Electromagnetic field of a charge asymptotically approaching a spherically symmetric black hole

The problem of finding the electromagnetic field of a charge in a gravitational field within general relativity has a long history. While the basic results for the total radiated energy and the spectrum of the radiation of a charged particle in the vicinity of a black hole have been established, the question remains as to the fate of the electromagnetic field of the charged particle as it approaches the horizon of the black hole. In the literature this question was approached only within analysis of the electromagnetic field of a charge at rest in the gravitational field of a black hole Richard Hanni and Remo Ruffini (1973); Linet (1976); K. S. Thorne, R. H. Price, D. A. Macdonald (1986); Ruffini (1973).

Obviously, the particle cannot be at rest in the external gravitational field without an external force. Much more interesting from a physical point of view is the problem of the electromagnetic field produced by a free-falling charged particle in the external gravitational field of a Schwarzschild black hole. Finding this field is the aim of the present work Komarov et al. (2023).

The equations describing the electromagnetic field appeared to coincide with the Regge-Wheeler equations describing small perturbations of a black hole Regge and Wheeler (1957); Zerilli (1970). These equations were solved numerically and the spectrum of the first few multipoles together with the total radiated energy was found in J. Tiomno (1972); R. Ruffini (1972).

Consider the particle moving radially in the vicinity of Schwarzschild black hole. In Schwarzschild coordinates $\{t, r, \theta, \phi\}$ the metric of space-time has the following form

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \,. \tag{3.3.1}$$

We choose orientation of the coordinate system in such a way that trajectory of the particle satisfies the condition $\theta = 0$. Due to axial symmetry of the describing system, electromagnetic field does not depend on azimuthal angle ϕ. As it has been shown in Ruffini et al. (1972), in this case electromagnetic field of the particle can be represented as

$$\begin{cases} A_0 = \sum_{l=0}^{\infty} f_l(r,t) P_l(\cos\theta)\,, \\ A_1 = \sum_{l=0}^{\infty} h_l(r,t) P_l(\cos\theta)\,, \\ A_2 = \sum_{l=0}^{\infty} k_l(r,t) \dfrac{\mathrm{d}P_l(\cos\theta)}{\mathrm{d}\theta}\,,\ A_3 = 0\,. \end{cases}$$

Here $P_l(x)$ are Legendre polynomials with multipole index l.

Introduce multipole coefficients $b_l(r,t)$ by the following equations

$$b_l(r,t) = \frac{r^2}{l(l+1)} \left(\frac{\partial h_l}{\partial t} - \frac{\partial f_l}{\partial r} \right)\,, \text{ for } l > 0\,; \tag{3.3.2}$$

$$b_0(r,t) = r^2 \left(\frac{\partial h_0}{\partial t} - \frac{\partial f_0}{\partial r} \right)\,. \tag{3.3.3}$$

From (3.3.2) and (3.3.3) it follows that

$$F_{10} = \frac{1}{r^2} b_0 + \sum_{l=1}^{\infty} \frac{l(l+1)}{r^2} b_l(r,t) P_l(\cos\theta)\,.$$

It is convenient to introduce the tortoise coordinate r^*, which can be defined in the region outside the black hole as

$$r^*(r) = r + 2M \ln\left(r/2M - 1\right)\,; \qquad \frac{\mathrm{d}r}{\mathrm{d}r^*} = 1 - 2M/r\,.$$

It follows from the definition that when r changes from $2M$ to ∞ (in the region outside the black hole), r^* changes from $-\infty$ to ∞. Let $\tilde{b}_l(r^*,\omega)$ be the Fourier transform of $b_l(r(r^*),t)$. The following equation is obtained for Fourier components J. Tiomno (1972); R. Ruffini (1972); Ruffini et al. (1972)

$$\tilde{b}_l'' + \left[\omega^2 - U(r^*)\right] \tilde{b}_l = e^{i\omega T(r)} a_l(r^*)\,, \tag{3.3.4}$$

where

$$U(r^*) = \left(1 - \frac{2M}{r(r^*)}\right) \frac{l(l+1)}{r(r^*)^2}, \tag{3.3.5}$$

$$a_l(r^*) = \frac{q}{2\pi} \frac{2l+1}{l(l+1)} \left[T''^*) + i\omega(T'^*))^2 - i\omega\right], \tag{3.3.6}$$

$$a_0(r^*) = \frac{q}{2\pi} \left[T''^*) + i\omega(T'^*))^2 - i\omega\right]. \tag{3.3.7}$$

Here the prime denotes derivative with respect to tortoise coordinate r^*; $T(r^*) = t$ represents the equation of the worldline of the particle. For instance, when particle starts at the spatial infinity and without initial velocity, it has the form

$$T(r^*) = -\frac{2}{3\sqrt{2M}} r(r^*)^{3/2} - 2\sqrt{2Mr(r^*)} - 2M \ln\left(\frac{\sqrt{r(r^*)} - \sqrt{2M}}{\sqrt{r(r^*)} + \sqrt{2M}}\right). \tag{3.3.8}$$

Equation (3.3.4) is well-known Regge-Wheeler equation.

It is well known that this equation mathematically coincides with the stationary Schrodinger equation for a particle in a potential barrier. This barrier is represented in Fig. 3.4. It has a maximum for $r^* = r_0^* \equiv 3M + 2M \ln(1/2)$. The simplest way to obtain approximate analytic solution of this equation is to represent such a barrier with the Dirac delta function located in the maximum of the potential

$$U(r^*) = U_0 \delta(r^* - r_0^*). \tag{3.3.9}$$

In order to obtain analytic solution we approximate this barrier with a Dirac delta function, as well as with a rectangular barrier.

Our final result is

$$b_l(r(r^*), t) = \begin{cases} -\rho \dfrac{2l+1}{l(l+1)}, \\ \text{if } T(r^*) < t < r^* + T(r^*) - r_0^*; \\ -\rho \dfrac{2l+1}{l(l+1)} e^{\frac{U_0}{2}\omega(r^* + T(r_0^*) - r_0^* - t)}, \\ \text{if } t > r^* + T(r_0^*) - r_0^*. \end{cases}$$

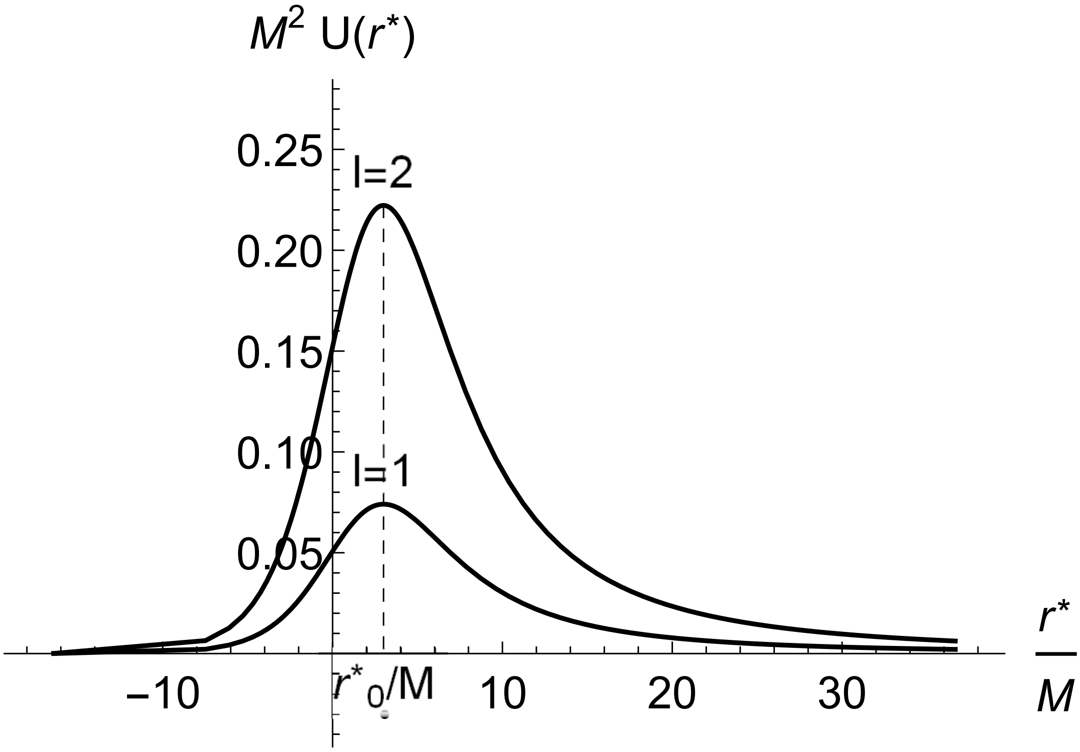


Figure 3.4: Potential barrier of the Regge-Wheeler equation for multipole moments $l = 1$ and $l = 2$.

For the particle that is approaching asymptotically the black hole event horizon ($t \to +\infty$) it follows that b_l tends to zero exponentially. It implies that all components of electromagnetic field tend to zero exponentially when $t \to +\infty$. Note that this result is independent on the function $T(r^*)$ and valid for both our approximations. It is valid also when external electromagnetic field as well as radiation reaction are present.

This work is supported within the joint BRFFR-ICRANet-2023 funding programme within the Grant **No. F23ICR-003**.

Results of this work were reported at the 5th Zeldovich meeting held in Yerevan, Armenia on June 12-17, 2023. These results are published in Physical Review D, see Komarov et al. (2023).

4 Publications

1. M. A. Prakapenia and G. V. Vereshchagin, “Pauli blocking effects on pair creation in strong electric field”, Phys. Rev. D 108, 013002 (2023).

   The process of electron-positron pair creation and oscillation in a uniform electric field is studied, taking into account the Pauli exclusion principle. Generally, we find that pair creation is suppressed; hence, coherent oscillations occur on longer timescales. Considering pair creation in already existing electron-positron plasma, we find that the dynamics depends on pair distribution function. We considered Fermi-Dirac distribution of pairs and found that for small temperatures pair creation is suppressed, while for small chemical potentials it increases: heating leads to enhancement of pair creation.

2. M. A. Prakapenia and G. V. Vereshchagin, “Pair creation in hot electrosphere of compact astrophysical objects”, submitted to ApJ (2023).

   The mechanism of pair creation in electrosphere of compact astrophysical objects such as quark stars or neutron stars is revisited, paying attention to evaporation of electrons and acceleration of electrons and positrons, previously not addressed in the literature. We perform a series of numerical simulations using the Vlasov-Maxwell equations. The rate of pair creation strongly depends on electric field strength in the electrosphere. Despite Pauli blocking is explicitly taken into account, we find no exponential suppression of the pair creation rate at low temperatures. The luminosity in pairs increases with temperature and it may reach up to $L_\pm\sim 10^{52}$ erg/s, much larger than previously assumed.

3. Komarov, S. O. ; Gorbatsievich, A. K. ; Vereshchagin, G. V., ”Electromagnetic field of a charge asymptotically approaching a spherically symmetric black hole”, Phys. Rev. D 108 (2023) 104056.

   We consider a test charged particle falling onto a Schwarzschild black hole and evaluate its electromagnetic field. The Regge-Wheeler equation is solved analytically by approximating the potential barrier with Dirac delta function and rectangular barrier. We show that for asymptotically large times measured by a distant observer the electromagnetic field approaches the spherically symmetric electrostatic field. This implies that in the region accessible to a distant observer the initial state of separated charge and the electromagnetic field outside the event horizon of Schwarzschild black hole becomes asymptotically indistinguishable from the Reisnner-Nordström solution. The implications of this result for some astrophysical models of black holes accreting charged particles are discussed.

4. S. O. Komarov, A. K. Gorbatsievich, A. S. Garkun, and G. V. Vereshchagin, ”Electromagnetic Radiation and Electromagnetic Self-Force of a Point Charge in the Vicinity of the Schwarzschild Black Hole”, Nonlinear Phenomena in Complex Systems, 26 (2023), pp.77 - 82.

   A point charge, radially moving in the vicinity of a black hole is considered. Electromagnetic field in a wave zone and in the small neighbourhood of the charge is calculated. Numerical results of the calculation of the spectrum of electromagnetic radiation of the point charge are presented. Covariant approach for the calculation of the electromagnetic self-force is used for the case of the slowly moving charge. Numerical results for the self-force in the case of the slow motion of a particle are obtained and compared to the results in literature.

5. G. V. Vereshchagin, “On diffusive photospheres in Gamma-Ray Bursts”, in proceedings of the Sixteenth Marcel Grossmann Meeting, World Scientific, 2023, pp. 2989-3001.

6. G. V. Vereshchagin and D. Begue, “Summary of the parallel session GB3”, in proceedings of the Sixteenth Marcel Grossmann Meeting, World Scientific, 2023, pp. 3002-3008.

4.1 Invited talks at international conferences

1. “On direct and inverse Schwinger process in pair plasma”, The Fifth Zeldovich meeting, Yerevan, Armenia, June 12-16, 2023.

2. “Creazione di elettroni-positroni e principio di esclusione di Pauli”, European Researchers’ Night, ICRANet, Pescara, 29 September 2023.

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