Theoretical Astroparticle Physics

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2 Participants

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3 Brief description

Astroparticle physics is a new field of research emerging at the intersection of particle physics, astrophysics and cosmology. Theoretical development in these fields is mainly triggered by the growing amount of experimental data of unprecedented accuracy, coming both from the ground based laboratories and from the dedicated space missions.

3.1 Relativistic plasma

Electron-positron plasma is of interest in many fields of physics and astrophysics, e.g. in the early universe, active galactic nuclei, the center of our Galaxy, compact astrophysical objects such as hypothetical quark stars, neutron stars and gamma-ray bursts sources. It is also relevant for the physics of ultraintense lasers and thermonuclear reactions. We study physical properties of dense and hot electron-positron plasmas. In particular, we are interested in the issues of its creation and relaxation, its kinetic properties and hydrodynamic description, baryon loading and radiation from such plasmas.

Two different states exist for electron-positron plasma: optically thin and optically thick. Optically thin pair plasma may exist in active galactic nuclei and in X-ray binaries. The theory of relativistic optically thin nonmagnetic plasma and especially its equilibrium configurations was established in the 80s by Svensson, Lightman, Gould, Haug and others. It was shown that relaxation of the plasma to some equilibrium state is determined by a dominant reaction, e.g. Compton scattering or bremsstrahlung.

Developments in the theory of gamma ray bursts from one side, and observational data from the other side, unambiguously point out on existence of optically thick pair dominated non-steady phase in the beginning of formation of GRBs. The spectrum of radiation from optically thick plasma is usually assumed to be thermal.

Experiments with high intensity laser beams interacting with each other as well as with solid targets aim at creation of relativistic plasmas and their study in laboratory conditions. The goal of such experiments is reproduction of astrophysical plasmas in controlled environment.

In a series of publications we consider kinetic, electrodynamic, hydrodynamic and observational properties of relativistic plasma.

3.1.1 Numerical scheme for evaluating the collision integrals for triple interactions in relativistic plasma

The most general description of relativistic plasma dynamics is given in terms of distribution function, where particle collisions are described by the integrals of differential cross-section (or a matrix element) over the phase space (Cercignani and Kremer, 2012; Groot et al., 1980; Vereshchagin and Aksenov, 2017). The binary interactions between photons and electrons are the subject of classical textbooks in QED. There are some analytic expressions for reaction rates in thermal equilibrium of binary interactions. Most numerical kinetic codes account for binary interactions.

The triple interactions are less represented in the literature. Several papers present analytic expressions for reaction rates in equilibrium in asymptotic cases, such as for low energy photons or at nonrelativistic temperatures. Non-equilibrium rates in relativistic regime can be computed only numerically. There are few numerical codes dealing with triple interactions in approximate way. Specifically, efficient codes were developed which describe only binary collisions without induced emission (Pe'er and Waxman, 2005), binary collisions with induced emission (Prakapenia et al., 2018), or binary and triple collisions not far from thermal equilibrium (Chluba and Sunyaev, 2012).

We are developing a new numerical scheme for computation of collision integrals in relativistic plasmas. It was first introduced in the work (Aksenov et al., 2004) and then applied to the study of thermalization in relativistic plasma of Boltzmann particles (Aksenov et al., 2007, 2009a,b, 2010a), for the computation of relaxation timescales (Aksenov et al., 2010b), and description of electron-positron plasma creation in strong electric fields (Benedetti et al., 2013). Recently plasma degeneracy has been taken into account in the computational scheme (Prakapenia et al., 2018) which allowed to follow plasma thermalization for degenerate plasmas as well (Prakapenia et al., 2019).

This year we focused on generalization of our method for calculation of collision integrals specifically treating triple interactions in relativistic plasma.

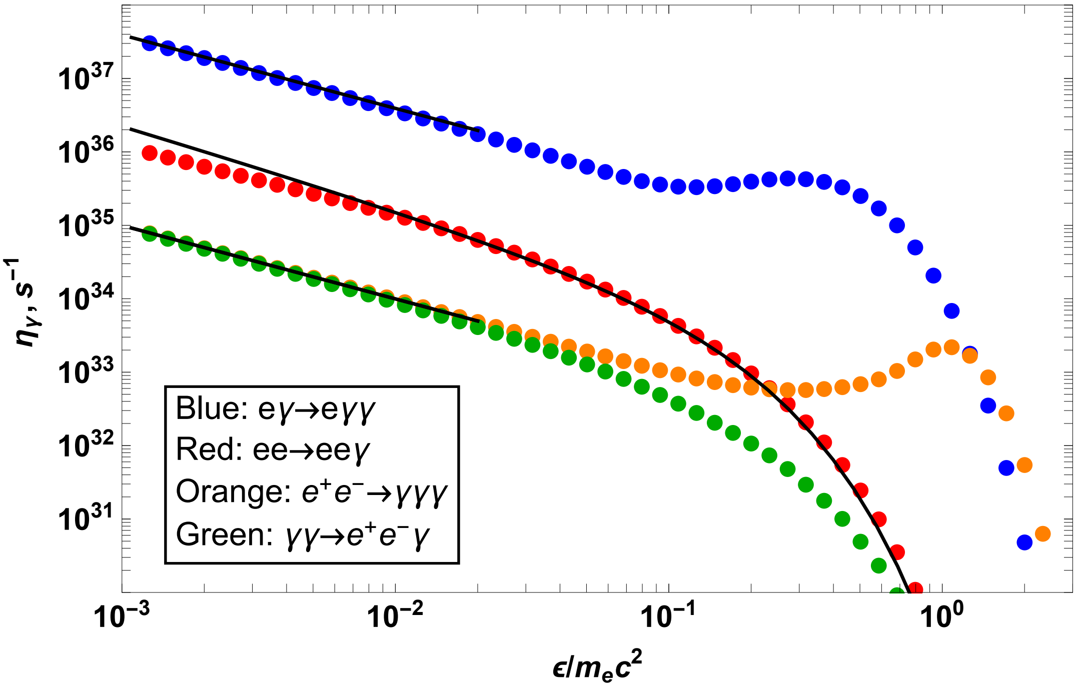

Figure 3.1: Thermal photon emissivity η_γ as a function of photon energy. Solid curves represent Svensson (Svensson, 1984) and Haug (Haug, 1975) analytical formulas, see text for details. From top to bottom: double Compton scattering, relativistic bremsstrahlung, three-photon annihilation, radiative pair production. The number of nodes is: 60 for energy, 32 for ϕ angle and 16 for μ angle. The distribution functions are taken at equilibrium with $\theta = 0.1$ and chemical potential being zero.

In addition, a new kinematic approach, which improve the scheme performance, has been developed for triple interactions.

In particular we propose the method to compute Uehling-Uhlenbeck collision integrals for all binary and triple interactions in relativistic plasma with drastic improvement in computation time with respect to existing methods. Plasma is assumed isotropic in momentum space. The set of binary reactions consists of: Moeller and Bhabha scattering, Compton scattering, two-photon pair annihilation, and two-photon pair production. The set of triple interactions consists of: electron-electron bremsstrahlung, electron-positron bremsstrahlung, double Compton scattering, three photon annihilation radiative pair production and their inverse processes. All these interactions are described by QED matrix elements, so that collision integrals are computed from the first principles by direct intergation over momenta of interacting particles. In our method exact energy and particle number conservation laws are satisfied. Reaction rates are compared, where possible, with the corresponding analytical expressions and convergence of numerical rates is demonstrated.

Figure 3.1 shows numerical thermal photon emissivity coefficient compared with analytic formulas (black curves). All solid curves besides bremsstrahlung are plotted up to the photon energy $e = 0.02$, as they are valid only in the soft photon limit. There is a good agreement with Svensson formula (A10) in Svensson (1984) for the double Compton scattering (blue dots). Bremsstrahlung emissivity (red dots) show deviations from Haug formula (2.10) in Haug (1975) for soft photons; this deviation decreases with increasing resolution in angles. Emissivities for three-photon annihilation (orange dots) and radiative pair production (green dots) are compared with eq. (A18) and (A20) in Svensson (1984), respectively, differ by a factor of particle density in the case of non-zero chemical potential and become identical in the case of zero chemical potential. The latter case is presented in Figure 3.1. Indeed, thermal emissivities of both processes coincide for soft photons, and become different with increasing photon energy. Overall, our numerical results show a good agreement with the corresponding non-relativistic formulas. This work is supported within the joint BRFFR-ICRANet-2018 funding programme within the Grant No. F19ICR-001.

Results of this work were reported at the 4th Zeldovich virtual meeting in September 2020. These results are published in Physics of Plasmas, 2020.

3.1.2 Pauli blocking effects in thermalization of relativistic plasma

Pauli exclusion principle is a fundamental principle of quantum mechanics and it manifests in many branches of physics: condensed matter, chemistry, molecular biology, and other fields. In particular Pauli blocking effect is relevant for relativistic plasma. The degree of plasma degeneracy is characterized

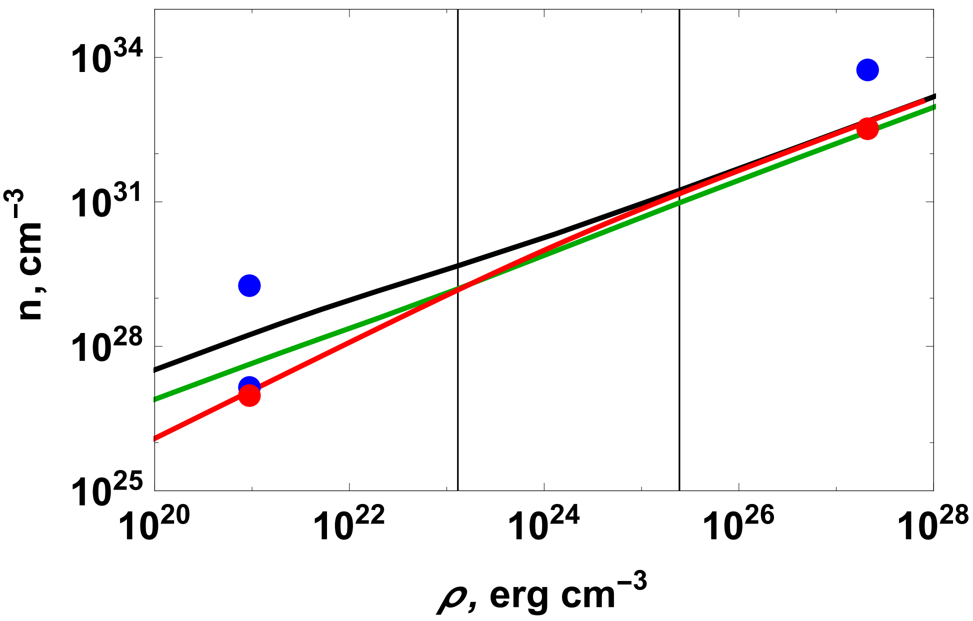

Figure 3.2: Number density - energy density diagram of a photon-electron-positron plasma. Green curve corresponds to thermal equilibrium state. Black curve shows the transition from nondegenerate $D > 1$ to degenerate $D < 1$ plasma, where D is defined by Eq. (3.1.1). Red curve corresponds to fully degenerate pair state defined by Eq. (3.1.2). Vertical line on the left corresponds to the transition from nonrelativistic to relativistic pair plasma ($\theta_{fin} = 0.3$). Vertical line on the right corresponds to relativistic pair plasma with $\theta_{fin} = 1$. (Initial conditions for calculations are denoted by dots for pairs (red) and photons (blue).

by the parameter (Groot et al., 1980)

$$D = \frac{1}{n\lambda_{th}^3}, \tag{3.1.1}$$

where n is number density of particles in plasma, $\lambda_{th} = \frac{c\hbar}{kT}$ is the thermal wave-length, k is Boltzmann constant, T is temperature, $\hbar = h/(2\pi)$, h is Planck constant.

Regarding fermions only (electrons and positrons without photons) it is important to note that both fully degenerate and thermal states have comparable number density of particles for ultrarelativistic average energy per particle, specifically the number density for fully degenerate state is

$$n_{cr} = \frac{8\pi}{3h^3c^3}\varepsilon_F^3, \tag{3.1.2}$$

and the number density in ultrarelativistic limit for thermal state is

$$n_{th} = \frac{12\pi\zeta(3)}{h^3c^3}(kT)^3, \tag{3.1.3}$$

where ε_F is the Fermi energy, which plays a role of an upper particle energy boundary.

The number density – energy density diagram for relativistic electron-positron plasma is presented in Fig. 3.2. The black line corresponds to $D = 1$; below this curve $D > 1$ and plasma is nondegenerate, while above this curve $D < 1$ and plasma is degenerate. Green curve corresponds to thermal equilibrium state. Red curve shows the maximum number density for electron-positron pairs (fully degenerate state). Note that thermal equilibrium state is very near to the border $D = 1$.

Consider the case of electron-photon plasma. The positive charge, needed to compensate for electron charge, is assumed to be present, in the form of protons or ions. In this work it is assumed that positively charged particles are cold, and their presence does not affect electron degeneracy. The probability of creation of electron-positron pairs from photons at nonrelativistic temperature is exponentially suppressed. Then an initial fully degenerate electron state can be preserved for a time larger compared to the characteristic pair annihilation time. As a result, Pauli blocking becomes important for thermalization process.

In Fig. 3.3 we show the result of the simulation for the case of nonrelativistic photon-electron plasma with superdegenerate initial state (solid curves) and analogous simulation with nondegenerate initial electron state (dashed curves). Electron number is constant and photon number is changing due to imbalance in Double Compton scattering and Bremsstrahlung processes. There is a sharp difference between degenerate (solid curve) and nondegenerate (dashed curve) cases. For fully degenerate initial conditions Pauli blocking significantly reduces reaction rates. As a consequence kinetic evolution in degenerate case starts much later, only at $t \sim 10^{-15}$ sec with decrease of thermodynamic quantities. Then electron distribution quickly establishes

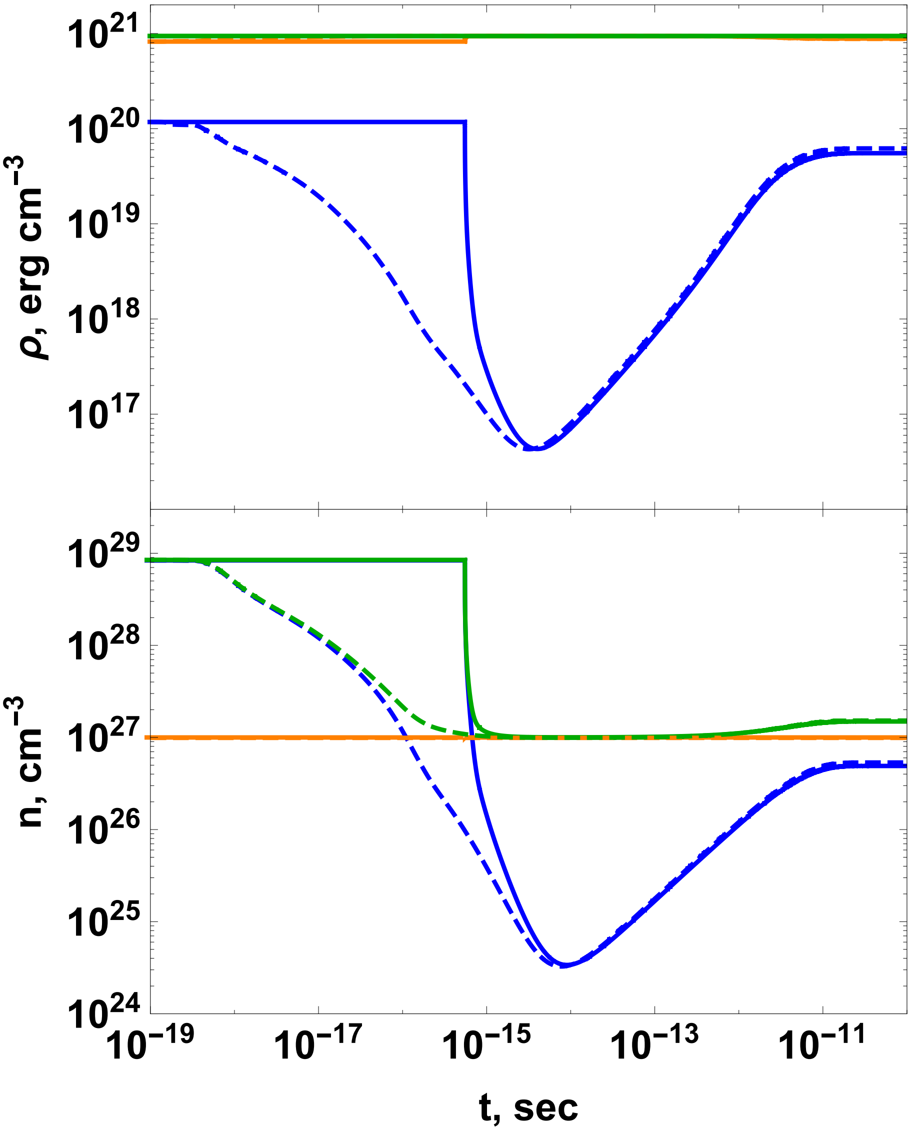

Figure 3.3: Time evolution of energy density (top) and particle number density (bottom) for nonrelativistic photon-electron plasma with degenerate initial pair state (solid) and nondegenerate initial electron state (dashed). Final equilibrium temperature is $\theta_{fin} = 0.05$.

the Fermi-Dirac form due to fast Coulomb scattering process. We note that photon state at that moment is not described yet by the Bose-Einstein distribution, because Compton scattering timescale is longer than Coulomb scattering one. The simulation shows that balance in Compton scattering process sets at the time moment $t \sim 6 \times 10^{-12}$ sec and thermal equilibrium sets at $t \sim 10^{-11}$ sec. Thus, both binary and triple processes are unbalanced until final thermal equilibrium is established. This work is supported within the joint BRFFR-ICRANet-2018 funding programme within the Grant No. F19ICR-001. These results are published in Physics Letters A, 2020.

3.1.3 Diffusive photospheres in gamma-ray bursts

Thermal components are detected in time resolved spectra during the early afterglow in a number of GRBs (Page et al., 2011; Starling et al., 2012; Sparre and Starling, 2012; Friis and Watson, 2013; Valan et al., 2018; Izzo et al., 2019). So far several mechanisms to generate such emission are proposed. They include a shock breakout from a progenitor star or a stellar wind (Campana et al., 2006) and a hot cocoon formed when the relativistic jet emerges from the stellar surface (Pe'er et al., 2006; Nakar and Piran, 2017). Some authors argue that shock breakouts are not energetic enough and do not last long enough to explain observed thermal emission (Valan et al., 2018), leaving cocoons as a favourite model. In addition, there is an alternative proposal of a cloud or a clump with small mass, accelerated by the GRB outflow (Ruffini et al., 2017).

Most papers dealing with the photospheric emission, e.g. (Mészáros and Rees, 2000; Pe'er, 2008; Pe'er and Ryde, 2011; Beloborodov, 2011; Lundman et al., 2013; Santana et al., 2016; Bhattacharya et al., 2018), for a review see (Pe'er and Ryde, 2017), adopt the hydrodynamic model of a steady and infinite wind. However, finite duration of GRBs implies finite width of the wind. Winds of *finite duration* are classified as photon thin and photon thick (Bégué et al., 2013; Ruffini et al., 2013; Vereshchagin, 2014). Decoupling of photons from plasma in the latter case occurs simultaneously in the entire outflow, while in the former case photons are transported to the boundaries of the outflow by radiative diffusion, just like in nonrelativistic outflows, e.g. in supernova ejecta. Emission in this case originates not at the photospheric radius, but at smaller radii. The photon thick case, corresponding to the steady wind, appears to be justified for typical GRB parameters. Photon thin regime is not considered in the literature. Thermal components are also detected in time resolved spectra during the early afterglow in a number of GRBs (Page et al., 2011; Starling et al., 2012; Sparre and Starling, 2012; Friis and Watson, 2013; Valan et al., 2018; Izzo et al., 2019). So far several mechanisms to generate such emission are proposed. They include a shock breakout from a progenitor star or a stellar wind (Campana et al., 2006) and a hot cocoon formed when the relativistic jet emerges from the stellar surface (Pe'er et al., 2006; Nakar and Piran, 2017). Some authors argue that shock breakouts are not energetic enough and do not last long enough to explain observed thermal emission (Valan et al., 2018), leaving cocoons as a favourite model. In addition, there is an alternative proposal of a cloud or a clump with small mass, accelerated by the GRB outflow (Ruffini et al., 2017).

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From the theoretical viewpoint the equation determining the Lorentz factor η of the outflow is

$$\frac{\eta}{l} = \frac{\zeta^{3/2}\left(1+z\right)^3}{d_L^{1/2}} \left(\frac{\sigma Y F_{obs}}{m_p c^3 \mathcal{R}^3}\right)^{1/2}, \tag{3.1.4}$$

where z is cosmological redshift, d_L is the luminosity distance, ζ is a numerical factor of order unity, σ is the Thomson cross section, Y is the fraction of the total luminosity L and the energy emitted in X and γ-rays *in the prompt phase,* m_p is proton mass, c is the speed of light, and $\mathcal{R} \equiv \left(F_{obs}^{BB}/\sigma_{SB}T_{obs}^4\right)^{1/2}$. Here σ_{SB} is the Stefan-Boltzmann constant, F_{obs}^{BB} is observed flux, T_{obs} is observed temperature. Therefore, the Lorentz factor can be determined if outflow width l is known. In particular case $l = R_0$, where R_0 is the nozzle radius of the outflow, from (3.1.4) it follows the minimum Lorentz factor for which the photon thin case applies

$$\eta_{thin} = (1+z)\left(d_L \frac{YF_{obs}\sigma}{m_p c^3 \mathcal{R}}\right)^{1/2} \frac{4^{3/2}}{\zeta^{5/2}\xi^6}\left(\frac{F_{obs}^{BB}}{YF_{obs}}\right)^{3/2}, \qquad (3.1.5)$$

where ξ is a numerical factor of order unity.

All GRBs reported in Valan et al. (2018) with measured redshifts and thermal component detected in their early afterglows were considered, namely GRBs 060218, 090618, 101219B, 111123A, 111225A, 121211A, 131030A, 150727A, 151027A. The results indicate that in several cases (GRBs 060218, 111225A and 150727A) the inferred Lorentz factors are relatively small, $\Gamma < 10$, while in other cases (GRBs 090618, 131030A and 151027A) the inferred Lorentz factors are larger, $\Gamma > 10$. Such differences suggest two possible sources of the thermal component: mildly relativistic cocoons or highly relativistic jets. These results are the first indication that radiative diffusion may play an important role not only in nonrelativistic outflows, but also in ultra-relativistic outflows, represented by GRBs.

Results of this work were reported at the 14th International Conference on Gravitation, Astrophysics and Cosmology (ICGAC-14), virtual meeting, August 17 – 21, 2020. These results are published in MNRAS, 2020.

4 Publications

1. M. A. Prakapenia, I. A. Siutsou and G. V. Vereshchagin, "Numerical scheme for evaluating the collision integrals for triple interactions in relativistic plasma", Phys. Plasmas 27, 113302 (2020) pp. 1-10.

   Binary interactions in relativistic plasma, such as Coulomb and Compton scattering as well as pair creation and annihilation are well known and studied in details. Triple interactions, namely relativistic bremsstrahlung, double Compton scattering, radiative pair production, triple pair production/annihilation and their inverse processes, are usually considered as emission processes in astrophysical problems, as well as in laboratory plasmas. Their role in plasma kinetics is fundamental [1]. We present a new conservative scheme for computation of Uehling-Uhlenbeck collision integral for all triple interactions in relativistic plasma based on direct integration of exact QED matrix elements. Reaction rates for thermal distributions are compared, where possible, with the corresponding analytic expressions, showing good agreement. Our results are relevant for quantitative description of relativistic plasmas out of equilibrium, both in astrophysical and laboratory conditions.

2. M. A. Prakapenia and G. V. Vereshchagin, "Pauli blocking effects in thermalization of relativistic plasma", Phys. Lett. A, Vol. 384 (2020) 126679.

   We investigate the effects of Pauli blocking on thermalization process of relativistic plasma by solving relativistic Uehling-Uhlenbeck equations with QED collision integral for all binary and triple processes. With this purpose we consider nonequilibrium initial state of plasma to be strongly degenerate. We found that when electron-positron annihilation is efficient, initial plasma degeneracy is quickly destroyed. As a result in a wide range of final temperatures ranging from nonrelativistic to mildly relativistic $0.1 m_e c^2 \leq k_B T \leq 10 m_e c^2$ thermalization is not affected by Pauli blocking. Conversely, when electron-positron annihilation process is inefficient, thermalization process in such degenerate plasma is strongly affected by Pauli blocking. This is possible either in a nonrelativistic plasma, with equilibrium temperature $k_B T \leq 0.3 m_e c^2$, or in photon-electron plasma. In these cases all reaction rates are strongly suppressed by Pauli blocking and thermalization does not occur until electrons can populate energy states above the Fermi energy. Soon after this happens thermalization proceeds suddenly in an avalanche-like process. Such rapid thermalization can be a unique footprint of strongly degenerate plasma.

3. G. V. Vereshchagin and I. A. Siutsou, "Diffusive photospheres in gamma-ray bursts", MNRAS 494 (2020), pp. 1463-1469.

   Photospheric emission may originate from relativistic outflows in two qualitatively different regimes: last scattering of photons inside the outflow at the photospheric radius or radiative diffusion to the boundary of the outflow. In this work, the measurement of temperature and flux of the thermal component in the early afterglows of several gamma-ray bursts along with the total flux in the prompt phase is used to determine initial radii of the outflow as well as its Lorentz factors. Results indicate that in some cases the outflow has relatively low Lorentz factors ($\Gamma < 10$), favouring cocoon interpretation, while in other cases Lorentz factors are larger ($\Gamma > 10$), indicating diffusive photospheric origin of the thermal component, associated with an ultrarelativistic outflow.

4. M. A. Prakapenia and G.V. Vereshchagin, "Bose-Einstein condensation in relativistic plasma", European Physics Letters, 128 (2019) 50002.

   The phenomenon of Bose-Einstein condensation is traditionally associated with and experimentally verified for low temperatures: either of nano-Kelvin scale for alkali atoms [1-3] or room temperatures for quasi-particles [4,5] or photons in two dimensions [6]. Here we demonstrate out of first principles that for certain initial conditions non-equilibrium plasma at relativistic temperatures of billions of Kelvin undergoes condensation, predicted by Zeldovich and Levich in their seminal work [7]. We determine the necessary conditions for the onset of condensation and discuss the possibilities to observe such a phenomenon in laboratory and astrophysical conditions.

4.1 Invited talks at international conferences

1. "Diffusive photospheres and thermal emission in early afterglows of gamma-ray bursts", The 14th International Conference on Gravitation,

Astrophysics and Cosmology (ICGAC-14), virtual meeting, August 17 – 21, 2020.

2. “On Bose-condensation of photons in relativisticplasma”, The Forth Zeldovich virtual meeting, 7 – 11 September 2020.

4.2 Lectures

1. “Nobel prize in Physics 2020: laureates and their results”

   (G.V. Vereshchagin)

   Lecture at the virtual meeting of the Belarusian Physical Society, 23 October 2020.

2. “Nobel per la Fisica 2020”

   (G.V. Vereshchagin)

   La Notte dei Ricercatori 2020, ICRANet online meeting, 27 November 2020.

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