# Exact solutions of Einstein and Einstein-Maxwell equations

## 0.1 Topic:

Mathematical physics in application to General Relativity.

## 0.2 Participants:

- V.A.Belinski
- H.Quevedo

## 0.3 Publications (having ICRA or ICRANet affiliation):

**• V. A. Belinski "Fermi coordinates and static observer in Schwarzschild spacetime", Phys. Rev. D, 102, 064044 (2020).**

In this paper we construct the Fermi coordinates along any arbitrary line in simple analytical way without use orthogonal frames and their transport. In this manner we extend the Eddington approach to the construction of the Fermi metric in terms of the Riemann tensor. In the second part of the present article we show how the proposed approach works practically by applying it for deriving the Fermi coordinates for the static observer in the Schwarzschild spacetime.

**• S.V.Serdio and H.Quevedo "Singularity theorems in Schwarzschild spacetimes", European Physical Journal Plus, 135, 636 (2020).**

We study the conceptual details and the physical interpretation of the two prominent singularity theorems due to Penrose and Hawking. Their usage is discussed in detail for the Schwarzschild spacetime with positive and negative mass. First, we present a detailed mathematical proof to formally guarantee the existence of a singularity of geodesic incompleteness for the case of positive mass. Second, we discuss the applicability of the mathematical tools used by the theorems in the negative mass case. The physical implications of the validity or inconsistency of the hypotheses of such theorems on the latter case are also exhibited. As far as this analysis is concerned, some clues are produced regarding future research that could result in general properties for the naked singularities.

• **R.Giambò, O.Luongo, H.Quevedo "Repulsive regions in Lemaître–Tolman–Bondi gravitational collapse", Physics of the Dark Universe, 30, 100721 (2020).**

We show that in the inhomogeneous Lemaitre-Tolman-Bondi space-time there are specific regions in which repulsive gravity exists. To find these regions, we use an invariant definition of repulsive gravity based upon the behavior of the curvature eigenvalues. In addition, we analyze the effects of repulsive gravity on the dynamics of the gravitational collapse. In particular, we investigate the collapse in case of the parabolic solution for the effective scale factor of the Lemaitre-Tolman-Bondi metric, corresponding to the marginally bound case. Exploring the corresponding cut-offs at which gravity becomes repulsive, we notice that black holes with dominant repulsive effects are not excluded a priori. Indeed, we demonstrate that the collapse leads, in general, to the formation of a central naked singularity, however, for particular values of the free parameters entering the model, black holes with dominant repulsive gravity can exist. We show that the expected physical process is not modified as the marginally bound condition is dropped out. Moreover, we show that this is true independently of the hypothesis that at the energy-momentum tensor is built up in terms of pressureless matter. Further, we demonstrate that geodesic deviations can depend on the sign of the curvature eigenvalues. Finally, we give an astrophysical interpretation of black holes with dominant repulsive gravity. Indeed, we argue that compact objects with dominant repulsive gravity could be interpreted as progenitors of Gamma Ray Bursts.

• **V. Pineda-Reyes, L. F. Escamilla-Herrera, C. Gruber, F. Nettel and H. Quevedo "Modeling reparametrizations in thermodynamic phase space", Physica A: Statistical Mechanics and its Applications, 563, 125464 (2020).**

We investigate the consequences of reparametrizations in the geometric description of thermodynamics analyzing the effects on the thermodynamic phase space. It is known that the contact and Riemannian structures of the thermodynamic phase space are related to thermodynamic equilibrium and statistical fluctuations in the Boltzmann-Gibbs statistical mechanics. The physical motivation for this analysis rests upon the possibility of having, instead of a direct control of the intensive parameters determining the state of the corresponding physical reservoirs, the control of a set ofdifferentiable functions of the original variables. Likewise, we consider a set of differentiable functions of the extensive variables accounting for the possibility of not having direct access to the original variables. We find that different geometric

structures in the thermodynamic phase space can be used to describe its contact and Riemannian structures, while preserving the metric structure on the thermodynamic space of equilibrium states, if we restrict ourselves to a particular set of reparametrizations. We also single out a rank-two tensor that geometrically comprises the information about such reparametrizations in the thermodynamic phase space.

• **H.Quevedo, M.N. Quevedo and A.Sánchez "Geometrothermodynamics of Black Hole Binary Systems", International Journal of Modern Physics D, 29, 2050053 (2020).**

We study a stationary and axisymmetric binary system composed of two identical Kerr black holes, whose physical parameters satisfy the Smarr thermodynamic formula. Then, we use the formalism of geometrothermodynamics to show that the spatial distance between the black holes must be considered as a thermodynamic variable. We investigate the main thermodynamic properties of the system by using the contact structure of the phase space, which generates the first law of thermodynamics and the equilibrium conditions. The phase transition structure of the system is investigated through the curvature singularities of the equilibrium space. It is shown that the thermodynamic and stability properties and the phase transition structure of the binary system strongly depend on the distance between the black holes.