Exact solutions of Einstein and Einstein-Maxwell equations

0.1 Topics

- Exact solutions in General Relativity and Supergravity
- Cosmology and Astrophysics
- Quantum Fields
- Fundamental Relativity

0.2 Participants

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0.4 Research activity

0.4.1 Exact solutions in General Relativity and Supergravity

• In 2014 ICRANet started the new program "Exact solutions in the super-symmetric General Relativity" in collaboration with the group of Prof. H. Nicolai at Albert Einstein Institute at Potsdam (Germany). This new direction is now is in the list of the thematic of the ICRANet sector "Exact Solutions of the Einstein and Einstein-Maxwell equations". During 2014-2016 the work have been dedicated to the extension of the generating technique known as the Inverse Scattering Method (ISM) to the super-gravity (V. Belinski). Here we have two main problems: first to formulate the super-symmetric version of ISM for the two-dimensional integrable models in super-gravity and then to find a way to construct exact super-solitonic solutions. During 2016 i) both

of these problems was solved for the 2-dimensional extended N=16 supergravity, and ii) there was constructed the general (not necessary supersymmetric) extension of the pure gravity ISM to the case when fermionic fields are presented. The corresponding paper appeared in the arXiv and will be submitted to Phisical Review D. Reference (1).

• An exact analytical model of the process of collision and nonlinear interaction of gravitational and/or electromagnetic soliton waves and strong nonsoliton electromagnetic traveling waves of arbitrary profile propagating in the expanding universe was constructed. Based on exact solutions of the Einstein-Maxwell equations, the model demonstrates a series of nonlinear phenomena, such as (a) creation of gravitational waves in the collision of two electromagnetic waves, (b) creation of electromagnetic soliton waves in the collision of a gravitational soliton with traveling electromagnetic waves, (c) scattering of a part of a soliton wave in the direction of propagation of a traveling electromagnetic wave, and (d) quasiperiodic oscillating character of fields in the wave interaction region and multiple mutual transformations of gravitational and electromagnetic waves in this region. Part of this work already appeared in the 2015 ICRANET report but the complete updated version was published in Physical Review D only in 2016. Reference (2).

• A class of exact conformally static solutions of the Einstein-Maxwell field equations was found in which the gravitational and electromagnetic potentials are completely determined by a harmonic function. The equations of motion for neutral and charged particles in a spacetime background characterized by this class of solutions have been derived. As an example, focus made on the analysis of a particular harmonic function, which generates a singularity-free and asymptotically flat spacetime that describes the gravitational field of a punctual mass endowed with a magnetic field. In this particular case, it was investigated the main physical properties of equatorial circular orbits and it was shown that due to the electromagnetic interaction, it is possible to have charged test particles which stay at rest with respect to a static observer located at infinity. Additionally, it was obtained an analytic expression for the perihelion advance of test particles and the corresponding explicit value in the case of a punctual magnetic mass. The analytical expressions obtained from our analysis are sufficient for being confronted with observations in order to establish whether such objects can exist in nature. Reference (1).

0.4.2 Cosmology and Astrophysics

Cosmology

• The year 2016 have been dedicated to complete the book "The cosmological Singularity" by V.Belinski and M.Henneaux. The book (8 chapters and 4 technical Appendices) finelly is finished and will be send to the Cambridge University Press in January 2017.

• Homogeneous cosmological models with perfect-fluid sources in the framework of the Horava-Lifshitz model for quantum gravity have been investigated. It was shown that the Hamiltonian constraint of such spacetimes can be rewritten as the Cardy formula for the entropy in conformal field theory. The Cardy entropy is shown to depend explicitly on the value of the Horava parameter, so that it can be interpreted as determining the entropy and the gravitational interaction of the theory. Moreover, Verlinde's Pythagorean representation of the Hamiltonian constraint is also valid in the case of homogeneous Horava-Lifshitz spacetimes. These results can be interpreted as a further indication of a deep relationship between gravity, thermodynamics and holography in the quantum regime. Reference (1).

Astrophysics

• The fourth post-Newtonian (4PN) two-body dynamics has been recently tackled by several different approaches: effective field theory, ADM Hamiltonian, action-angle-Delaunay averaging, effective-one-body, gravitational self-force, first law of dynamics, and Fokker action. We review the achievements of these approaches and discuss the complementarity of their results. Our main conclusions are: (i) the results of the first complete derivation of the 4PN dynamics [T. Damour, P. Jaranowski, and G. Schäfer, Phys. Rev. D, **89**, 064058 (2014)] have been, piecewise, fully confirmed by several subsequent works; (ii) the results of the Delaunay-averaging technique [T. Damour, P. Jaranowski, and G. Schäfer, Phys. Rev. D, **91**, 084024 (2015)] have been confirmed by several independent works; and (iii) several claims in a recent harmonic-coordinates Fokker-action computation [L. Bernard et al.,arXiv:1512.02876] are incorrect, but can be corrected by the addition of a couple of ambiguity parameters linked to subtleties in the regularization of infrared and ultraviolet divergences. Reference (2).

• It was raised the analytical knowledge of the eccentricity expansion of the Detweiler-Barack-Sago redshift invariant in a Schwarzschild spacetime up to the 9.5th post-Newtonian order (included) for the e2 and e4 contributions, and up to the 4th post-Newtonian order for the higher eccentricity contributions through e20. This information was convert into an analytical

knowledge of the effective-one-body radial potentials through the 9.5th post-Newtonian order. These analytical results are compatible with current corresponding numerical self-force data. Reference (3).

• The first analytic computation of the Detweiler-Barack-Sago gauge-invariant redshift function for a small mass in eccentric equatorial orbit around a spinning black hole have been made. The results give the redshift contributions that mix eccentricity and spin effects, through second order in eccentricity, second order in spin parameter, and the eight-and-a-half post-Newtonian order. Reference (4).

• It was analized the applications of general relativity in relativistic astrophysics in order to solve the problem of describing the geometric and physical properties of the interior and exterior gravitational and electromagnetic fields of compact objects. The focus on the interpretation of exact solutions of Einstein's equations in terms of their multipole moments structure was made. In view of the lack of physical interior solutions, it was proposed an alternative approach in which higher multipoles should be taken into account. Reference (5).

• The equilibrium configurations of uniformly rotating white dwarfs, using Chandrasekhar and Salpeter equations of state, in the framework of Newtonian physics have been investigated. The Hartle formalism was applied to integrate the field equation together with the hydrostatic equilibrium condition. It was considered the equations of structure up to the second order in the angular velocity, and compute all basic parameters of rotating white dwarfs to test the so-called moment of inertia, rotational Love number and quadrupole moment (I-Love-Q) relations. It was found that the I-Love-Q relations are also valid for white dwarfs regardless of the equation of state and nuclear composition. In addition, it was shown that the moment of inertia, quadrupole moment and eccentricity (I-Q-e) relations are valid as well. Reference (6).

0.4.3 Fundamental Relativity

• It was developed a simple model to study classical fields on the background of a fluctuating spacetime volume. The model was applied to formulate the stochastic Einstein equations with a perfect-fluid source. The particular case of a stochastic Friedmann-Lemaitre-Robertson-Walker cosmology was considered, and it was shown that the resulting field equations can lead to solutions which avoid the initial big bang singularity. By interpreting the fluctuations as the result of the presence of a quantum spacetime, the conclusion is that classical singularities can be avoided even within a stochastic model that include quantum effects in a very simple manner. Reference (7).

• It was shown that to investigate the thermodynamic properties of charged phantom spherically symmetric anti-de Sitter black holes, it is necessary to consider the cosmological constant as a thermodynamic variable so that the corresponding fundamental equation is a homogeneous function defined on an extended equilibrium space. All thermodynamic properties of this class of black holes was explored by using the classical physical approach, based upon the analysis of the fundamental equation, and the alternative mathematical approach as proposed in geometrothermodynamics. Both approaches are compatible and lead to equivalent results. Reference (8).

• The Legendre-invariant formalism of geometrothermodynamics have been used to investigate the geometric properties of the equilibrium space of a spherically symmetric phantom black hole with electric charge and dilaton. It was found that at certain points of the equilibrium space the thermodynamic curvature is characterized by the presence of singularities that are interpreted as phase transitions. It was investigated the phase transition structure by using the standard approach of black hole thermodynamics based upon the analysis of the heat capacity and response functions. There is compatibility between the two approaches. In addition, a new type of phase transition is found, which is due to the presence of phantom energy and corresponds to a transition between black hole states with different stability properties. Reference (9).

• The Newman–Janis Ansatz was used first to obtain the stationary Kerr metric from the static Schwarzschild metric. Many works have been devoted to investigate the physical significance of this Ansatz, but no definite answer has been given so far. We show that this Ansatz can be applied in general to conformastatic vacuum metrics, and leads to stationary generalizations which, however, do not preserve the conformal symmetry. Investigation was made also for the particular case when the seed solution is given by the Schwarzschild spacetime and it was shown that the resulting rotating configuration does not correspond to a vacuum solution, even in the limiting case of slow rotation. In fact, it describes in general a relativistic fluid with anisotropic pressure and heat flux. This implies that the Newman–Janis Ansatz strongly depends on the choice of representation for the seed solution. Reference (10).

• A novel approach to the Effective One-Body description of gravitationally interacting two-body systems is introduced. This approach is based on the post-Minkowskian approximation scheme (perturbation theory in G, without assuming small velocities), and employs a new dictionary focussing on the functional dependence of the scattering angle on the total energy and the total angular momentum of the system. Using this approach, it was proved to all orders in v/c two results that were previously known to hold only to a limited post-Newtonian accuracy: (i) the relativistic gravitational

dynamics of a two-body system is equivalent, at first post-Minkowskian order, to the relativistic dynamics of an effective test particle moving in a Schwarzschild metric; and (ii) this equivalence requires the existence of an exactly quadratic map between the real (relativistic) two-body energy and the (relativistic) energy of the effective particle. The same energy map is also shown to apply to the effective one-body description of two masses interacting via tensor-scalar gravity. Reference (11).

• The problem of describing the gravitational field of compact stars in general relativity has been reviewed. The focus made on the deviations from spherical symmetry which are expected to be due to rotation and to the natural deformations of mass distributions. It was assumed that the relativistic quadrupole moment takes into account these deviations, and was considered the class of axisymmetric static and stationary quadrupolar metrics which satisfy Einstein's equations in empty space and in the presence of matter represented by a perfect fluid. It was formulated the physical conditions that must be satisfied for a particular spacetime metric to describe the gravitational field of compact stars. A brief review of the main static and axisymmetric exact solutions of Einstein's vacuum equations, satisfying all the physical conditions was pesented. It was discussed how to derive particular stationary and axisymmetric solutions with quadrupolar properties by using the solution generating techniques which correspond either to Lie symmetries and Bäcklund transformations of the Ernst equations or to the inverse scattering method applied to Einstein equations. As for interior solutions, it is necessary to apply alternative methods to obtain physically meaningful solutions, and review a method which allows us to generate interior perfect-fluid solutions. Reference (12).

• The Hartle formalism to study equilibrium configurations in the framework of Newtonian gravity has been applied. This approach allows one to study in a simple manner the properties of the interior gravitational field in the case of static as well as stationary rotating stars in hydrostatic equilibrium. It was shown that the gravitational equilibrium conditions reduce to a system of ordinary differential equations which can be integrated numerically. It was derived all relevant equations up to the second order in the angular velocity. Moreover, it was found explicitly the total mass, the moment of inertia, the quadrupole moment, the polar and equatorial radii, the eccentricity and the gravitational binding energy of the rotating body. The procedure to calculate the gravitational Love number also was developed. The formalism has been tested for the case of white dwarfs and its compatibility with the known results in the literature was shown. Reference (13).

•• *Finally it is worth to mention the five works which have been done in the past 2015 year (see ICRANet report 2015) but they appeared in the respectable refereed journals only in 2016 or just in December 2015. References (14).*

Bibliography

[1] V. A. Belinski ”On the integrable gravity coupled to fermions”, arXiv:1611.02924 (2016), will be send to PRD.

[2] G.A. Alekseev ”Collision of strong gravitational and electromagnetic waves in the expanding universe”
Phys. Rev. D **93**, 061501(R) (2016).

Bibliography

[1] A. Gutierrez-Pineres, A. Capistrano, H. Quevedo "Test particles in a magnetized conformastatic spacetime",
Phy. Rev. D, **93**, 124009 (2016).

Bibliography

[1] O. Luongo, G. B. Pisani, H. Quevedo "Cardy-Verlinde entropy in Horava-Lifshitz gravity", Phys. Rev. D, **93**, 064057 (2016).

[2] T. Damour, P. Jaranowski, and G. Schäfer "Conservative dynamics of two-body systems at the fourth post-Newtonian approximation of general relativity", Phys. Rev. D, **93**, 084014 (2016).

[3] D. Bini, T. Damour and A. Geralico "New gravitational self-force analytical results for eccentric orbits around a Schwarzschild black hole", Phys. Rev. D, **93**, 104017 (2016).

[4] D. Bini, T. Damour and Andrea Geralico "High post-Newtonian order gravitational self-force analytical results for eccentric equatorial orbits around a Kerr black hole", Phys. Rev. D, **93**, 124058 (2016).

[5] H. Quevedo "Multiple structure of compact objects", arXiv:1606.05985 (June 2016).

[6] K. Boshkayev, H. Quevedo and B. Zhami "I-Love-Q relations for white dwarf stars", Month. Not. Roy. Astron. Soc., DOI: 10.1093/mnras/stw2614, October 2016.

[7] V. Dzhunushaliev, H. Quevedo "Stochastic Einstein equations with fluctuating volume", arXiv:1603.00951 (March 2, 2016).

[8] H. Quevedo, M.N. Quevedo, A. Sanchez "Geometrothermodynamics of phantom ADS black holes", Eur. Phys. Journ. C, **76**, 110 (2016).

[9] H. Quevedo, M.N. Quevedo, A. Sanchez "Einstein-Maxwell dilaton phantom black holes: thermodynamics and geometrothermodynamics" Phys. Rev. D, **94**, 024057 (2016).

[10] A. C. Gutiérrez-Piñeres, H. Quevedo "Newman-Janis Ansatz in conformastatic spacetimes", GRG, **48**, 146 (2016).

[11] T. Damour "Gravitational scattering, post-Minkowskian approximation and Effective One-Body theory", arXiv:1609.00354 [gr-qc] (September 2016).

[12] H. Quevedo "Quadrupolar metrics", arXiv:1606.09361 [gr-qc] (June 2016).

[13] K. Boshkayev, H. Quevedo, Z. Kalymova and B. Zhami "Hartle formalism for rotating Newtonian configurations", Eur. Journ. Phys., **37,** 065602 (2016).

[14] 1) K. Boshkayev, E. Gasperín, A. C. Gutiérrez-Piñeres, H. Quevedo, and S. Toktarbay "Motion of test particles in the field of a naked singularity", Phys. Rev. D, **93**, 024024 (2016).

2) K. A. Boshkayev, H. Quevedo, M. S. Abutalip, Zh. A. Kalymova, Sh. S. Suleymanova "Geodesics in the field of a rotating deformed gravitational source", IJMP(A), **31**, Issue 02n03 (2016).

3) S. Balmelli and T. Damour "New effective-one-body Hamiltonian with next-to-leading order spin-spin coupling", Phys. Rev. D, **92**, 124022 (December 2015).

4) D. Bini, T. Damour, and A. Geralico "Spin-dependent two-body interactions from gravitational self-force computations", Phys. Rev. D, **92**, 124058 (December 2015); Erratum Phys. Rev. D, **93**, 109902 (2016)

5) A. Nagar, T. Damour, C. Reisswig, and D. Pollney "Energetics and phasing of non-precessing spinning coalescing black hole binaries", Phys. Rev. D, **93**, 044046 (2016).