Exact solutions of Einstein and Einstein-Maxwell equations

1 Topics

- Exact solutions in General Relativity and Supergravity
- Cosmology and Astrophysics
- Quantum Fields
- Fundamental Relativity

2 Participants

- G.A.Alekseev
- V.A.Belinski
- T.Damour
- M.Novello
- H.Quevedo

3 PhD and Post Graduate Students

- Alessandro Bravetti (Italy)
- Eduardo Bittencourt (Brazil)
- Christine Gruber (Austria)
- Philipp Fleig (Germany)
- David Garcia (Mexico)
- Diego Tapias (Mexico)
- Saken Toktarbay (Kazakhstan)

4 Research activity

4.1 Exact solutions in General Relativity and Supergravity

• In July of this year ICRANet started the new program "Exact solutions in the supersymmetric General Relativity" in collaboration with the group of Prof. Hermann Nicolai at Albert Einstein Institute at Potsdam (Germany). This new direction is added now to the list of the thematics of the ICRANet sector "Exact Solutions of the Einstein and Einstein-Maxwell equations". The foremost target is construction of the exact solutions for supergravitational solitons. During July-November the work have been dedicated to the extension of the generating technique known as the Inverse Scattering Method (ISM) to the supergravity. Here we have two main problems: first to formulate the supersymmetric version of ISM for the two-dimensional integrable models in supergravity and then to find a way how we can use such ansatz to find the integrable physical models in four-dimensional space-time. The first part was solved and the paper is in preparation. The second part is much more sophisticated and the work is still in progress [1].

• Some classes of the so called "travelling wave" solutions of Einstein and Einstein - Maxwell equations in General Relativity and dynamical equations for massless bosonic fields in string gravity in four and higher dimensions was described. Similarly to the well known pp-waves, these travelling wave solutions may depend on arbitrary functions of a null coordinate which determine the arbitrary profiles and polarizations of the waves. However, in contrast with pp-waves, these waves do not admit the null Killing vector fields and can exist in some curved (expanding and spatially homogeneous) background space-times, where these waves propagate in certain directions without any scattering. Mathematically, some of these classes of solutions arise as the fixed points of Kramer-Neugebauer transformations for hyperbolic integrable reductions of the mentioned above field equations or in the other cases – after imposing of the ansatz that these waves do not change the part of spatial metric transversal to the direction of wave propagation. It is worth to note that striking simple forms of all presented solutions make possible a consideration of nonlinear interaction of these waves with the background curvature and singularities as well as a collision of sandwiches of such waves with solitons or with each others in the backgrounds where such travelling waves may exist [2].

• The monodromy transform approach, developed originally for solution of integrable reductions of vacuum Einstein equations and electrovacuum Einstein - Maxwell equations in General Relativity, was shown to be applicable to solution of the field equations which govern the bosonic dynamics of string gravity in four and higher dimensions and 5D minimal supergravity for space-times with the Abelian isometry group of codimension 2. In the present work we discuss a choice of (matrix-valued for these cases) monodromy data for construction of solutions which satisfy physically reasonable conditions (e.g., regularity of the axis of symmetry). We describe also a convenient "canonical" form of the matrix monodromy data and some discrete non-gauge symmetries of the spectral problem which can be used to restore the generic data from these "canonical" ones [3].

• Applications of the monodromy transform approach to construction of exact solutions of electrovacuum Einstein - Maxwell field equations are considered. Examples of new solutions are given. In this work the three new static solutions of Einstein – Maxwell equations were presented: (a) the solution for two Reissner-Nordstrom sorces (black holes or singularities) with proportional charges and masses $e_1/m_1 = e_2/m_2$ (this solution was known before, but here it was presented in a new, striking simple form), (b) the solution for two extremely charged massive sources which charges have opposite signs $e_1 = m_1$, $e_2 = -m_2$, (c) a very simple solution for a static magnetic dipole. In the cases (a) and (b) the conical singularities are present on the axis; the parameters e_1, m_1, e_2, m_2 are not physical charges and masses but these are only the mass and charge parameters. In the case (c) the source possess rather complicate structure and may consist of a pair of magnetic monopoles with opposite equal magnetic charges [4].

4.2 Cosmology and Astrophysics

4.2.1 Cosmology

• The work on the book "Cosmological Singularity" (V.Belinski) has been continued. The project is in progress under the official agreement with Cambridge University Press .

• It was written and published the review on the cosmological singularity containing both the old results and some new detailes (particularly on the asymptotic behavior near singularity of the general non-diagonal Bianchi IX model). This material will be used as a chapter for the book "Cosmological Singularity" [5].

• A review on the influence of the shear viscosity on the character of the cosmological singularity has been written. The results are very interesting because they elucidate first time that there exists some types of viscous matter which are able to provide the stable isotropic cosmological singularity. This

means that the Universe can start by the Friedmann Big Bang without any fine tunning [6].

• It was proposed a method for constructing the specific heat for the universe of spatially flat homogeneous and isotropic spacetime. The cosmography to represent the specific heat in terms of measurable quantities have been used and it was shown shown that a negative specific heat at constant volume and a zero specific heat at constant pressure are compatible with observational data. The most general cosmological model which is compatible with the values obtained for the specific heat of the universe was derived and it was shown that it alleviates the fine-tuning and the coincidence problems of the Lambda CDM model [7].

• The problem of the cosmic acceleration is revisited by using the fact that the adiabatic speed of sound can be assumed to be negligible small. Within the context of general relativity, the total energy budget is recovered under the hypothesis of a vanishing speed of sound by assuming the existence of one fluid only. It was found a cosmological model which reproduces the main results of the Lambda CDM paradigm at late-times, showing an emergent cosmological constant, which is not at all related with the vacuum energy term. As a consequence, the model presented behaves as a unified dark energy model [8].

• Geometrothermodynamics is a mathematical formalism that intends to describe the properties of thermodynamic systems in terms of concepts of differential geometry. It was shown that it is possible to consider thermodynamic systems as extremal surfaces embedded in the thermodynamic phase space. Any extremal surface is determined by a relationship that can be interpreted as a fundamental equation from which all physical properties of the corresponding thermodynamic system can be derived. The particular examples have been considered from which it was derived the thermodynamics of several cosmological models and it was shown that they describe different phases of the evolution of the Universe, including inflation [9].

4.2.2 Astrophysics

• Tidal interactions have a significant influence on the late dynamics of compact binary systems, which constitute the prime targets of the upcoming network of gravitational-wave detectors. It was refined the theoretical description of tidal interactions (hitherto known only to the second post-Newtonian level) by extending our recently developed analytic self-force formalism, for extreme mass-ratio binary systems, to the computation of several tidal invariants. Specifically, it was computed, to linear order in the mass ratio and to the 7.5 th post-Newtonian order, the following tidal invariants: the square and the cube of the gravitoelectric quadrupolar tidal tensor, the square of the gravitomagnetic quadrupolar tidal tensor, and the square of the gravito-

electric octupolar tidal tensor. By combining analytical and numerical results it was possible to provide simple, accurate analytic representations of the global strong-field behavior of the gravitoelectric quadrupolar tidal factor. A striking finding is that the linear-in-mass-ratio piece in the latter tidal factor changes sign in the strong-field domain, to become negative (while its previously known second post-Newtonian approximant was always positive) [10].

• A new, tunable effective-one-body (EOB) model of the motion and radiation of coalescing black hole binaries with arbitrary mass ratio and aligned spins was presented. The most novel feature is the introduction, and systematic use, of the (gauge-invariant) concept of the centrifugal radius r_c. In the spinning small mass-ratio limit, the main radial potential expressed in terms of r_c differs very little (and only multiplicatively so) from the usual Schwarzschild potential $1 - 2M/r_c$. The disagreement between numerical and theoretical results are remarkably small over the entire spin range [11].

• It was proposed a new way of analyzing the ringdown part of the gravitational wave signal emitted by coalescing black hole binaries. By contrast with the usual linear decomposition of the multipolar complex waveform h(t) in a sum of quasi-normal modes, our procedure relies on a multiplicative decomposition of h(t) as the product of the fundamental quasi-normal mode with a remaining time-dependent complex factor whose amplitude and phase are separately fitted. As an illustrative example, we apply our analysis and fitting procedure to the ringdown part of a sample of sixteen equal-mass, spinning, nonprecessing, numerical waveforms computed with the SPEC code, now publicly available in the SXS catalogue. This approach yields an efficient and accurate way to represent the ringdown waveform, thereby offering a new way to complete the analytical effective-one-body inspiral-plus-plunge waveform [12].

• It was computed analytically, to linear order in the mass-ratio, the "geodetic" spin precession frequency of a small spinning body orbiting a large (nonspinning) body to the eight-and-a-half post-Newtonian order, thereby extending previous analytical knowledge which was limited to the third post-Newtonian level. These results are obtained applying analytical gravitational self-force theory to the first-derivative level generalization of Detweiler's gauge-invariant redshift variable. We compare our analytic results with strong-field numerical data recently obtained by S.R. Dolan et al. [PRD, 89, 064011 (2014)]. Our new, high-post-Newtonian-order results capture the strong-field features exhibited by the numerical data. We argue that the spin-precession will diverge as the light-ring is approached. We transcribe our kinematical spin-precession results into a corresponding improved analytic knowledge of one of the two (gauge-invariant) effective gyro-gravitomagnetic ratios characterizing spin-orbit couplings within the effective-one-body formalism. We provide simple, accurate analytic fits both for spin-precession and the effective gyro-gravitomagnetic ratio. The latter fit predicts that the linear-in-

mass-ratio correction to the gyro-gravitomagnetic ratio changes sign before reaching the light-ring. This strong-field prediction might be important for improving the analytic modeling of coalescing spinning binaries [13].

• It was computed (analytically), to the eight-and-a-half post-Newtonian order, and to linear order in the mass ratio, the radial potential describing (within the effective one-body formalism) the gravitational interaction of two bodies, thereby extending previous analytic results. These results are obtained by applying analytical gravitational self-force theory (for a particle in circular orbit around a Schwarzschild black hole) to Detweiler's gauge-invariant redshift variable. The calculations show the increase of the numbers entering the post-Newtonian expansion coefficients as the order increases. Also it was studied the convergence of the post-Newtonian expansion as the expansion parameter $u = GM/(2cr)$ leaves the weak-field domain $u \ll 1$ to enter the strong field $u = O(1)$ domain [14].

• It was probed the gravitational interaction of two black holes in the strong-field regime by computing the scattering angle of hyperbolic-like, close binary-black-hole encounters as a function of the impact parameter. The fully general-relativistic result from numerical relativity is compared to two analytic approximations: post-Newtonian theory and the effective-one-body formalism. As the impact parameter decreases, so that black holes pass within a few times their Schwarzschild radii, it was found that the post-Newtonian prediction becomes quite inaccurate, while the effective-one-body one keeps showing a good agreement with numerical results. Because a regime explored is very different from the one considered so far with binaries in quasi-circular orbits, the results open a new avenue to improve analytic representations of the general-relativistic two-body Hamiltonian [15].

• The authors complete the analytical determination, at the 4th post-Newtonian (4PN) approximation, of the conservative dynamics of gravitationally interacting two-point-mass systems. This completion is obtained by resolving the infra-red ambiguity which had blocked a previous 4PN calculation [P.Jaranowski and G.Schäfer, Phys. Rev. D 87, 081503(R) (2013)] by taking into account the 4PN breakdown of the usual near-zone expansion due to infinite-range tail-transported temporal correlations found long ago [L.Blanchet and T.Damour, Phys. Rev. D 37, 1410 (1988)]. This leads to a Poincare-invariant 4PN-accurate effective action for two masses, which mixes instantaneous interaction terms (described by a usual Hamiltonian) with a (time-symmetric) nonlocal-in-time interaction [16].

• A stationary generalization of the static q-metric have been found which is the simplest generalization of the Schwarzschild solution that contains a quadrupole parameter. It possesses three independent parameters that are related to the mass, quadrupole moment and angular momentum. The geometric and physical properties of this exact solution of Einstein's vacuum equations was investigated, and it was shown that it can be used to describe the exterior gravitational field of rotating, axially symmetric, compact objects

[17].

• It was investigated in detail the circular motion of test particles on the equatorial plane of the ergoregion in the Kerr spacetime. It was found all the regions inside the ergoregion where circular motion is allowed, and analyzed their stability properties and the energy and angular momentum of the test particles. The structure of the stability regions has definite features that make it possible to distinguish between black holes and naked singularities. The naked singularity case presents a very structured non-connected set of regions of orbital stability. The properties of the circular orbits turn out to be so distinctive that they allow the introduction of a complete classification of Kerr spacetimes, each class of which is characterized by different physical effects that could be of particular relevance in observational astrophysics. The presence of counterrotating particles and zero angular momentum particles inside the ergoregion of a specific class of naked singularities is interpreted as due to the presence of a repulsive field generated by the central source of gravity [18].

4.3 Quantum Fields

• The quantum dynamics of a supersymmetric squashed three-sphere by dimensionally reducing to one timelike dimension the action of D=4 simple supergravity for a Bianchi IX cosmological model was studied. After imposition of the diffeomorphism constraints, the wave function of the Universe becomes a spinor of Spin(8,4) depending on the three squashing parameters, which satisfies Dirac, and Klein-Gordon-like, wave equations describing the propagation of a quantum spinning particle reflecting off spin-dependent potential walls. The algebra of the SUSY constraints and of the Hamiltonian one is found to close. One finds that the quantum Hamiltonian is built from operators that generate a 64-dimensional representation of the maximally compact sub-algebra of the rank 3 hyperbolic Kac-Moody algebra AE3. The (quartic-in-fermions) squared-mass term entering the Klein-Gordon-like equation has several remarkable properties: 1) it commutes with all the other (Kac-Moody related) building blocks of the Hamiltonian; 2) it is a quadratic function of the fermion number NF; 3) it is negative in most of the Hilbert space. The latter property leads to a possible quantum avoidance of the singularity ("cosmological bounce"), and suggests imposing the boundary condition that the wavefunction of the Universe vanish when the volume of space tends to zero. The space of solutions is a mixture of "discrete-spectrum states" (explicitely given) and of continuous-spectrum states (parametrized by arbitrary functions entering some initial-value problem). The predominantly negative values of the squared-mass term lead to a "bottle effect" between small and large volume-Universes and to a possible reduction of the continuous spectrum to a discrete spectrum of quantum states looking like

excited versions of the Planckian-size Universes described by the discrete states at fermionic NF = 0 and 1 levels [19].

• Sonoluminescence is a process in which a strong sound field is used to produce light in liquids. It is possible to explain sonoluminescence as a phase transition from ordinary fluorescence to a superradiant phase. It was considered a spin-boson model composed of a single bosonic mode and an ensemble of NN identical two-level atoms. It was assumed that the whole system is in thermal equilibrium with a reservoir at some temperature. It was shown that, in a ultrastrong-coupling regime, between the two-level atoms and the electromagnetic field it is possible to have a cooperative interaction of the molecules of the gas in the interior of the bubble with the field, generating sonoluminescence [20].

4.4 Fundamental Relativity

• The formalism of Geometrothermodynamics was used to describe chemical reactions in the context of equilibrium thermodynamics. Any chemical reaction in a closed system is shown to be described by a geodesic in a 2-dimensional manifold that can be interpreted as the equilibrium space of the reaction. First is shown this in the particular cases of a reaction with only two species corresponding to either two ideal gases or two van der Waals gases. Then it was considered the case of a reaction with an arbitrary number of species. The initial equilibrium state of the geodesic is determined by the initial conditions of the reaction. The final equilibrium state, which follows from a thermodynamic analysis of the reaction, is shown to correspond to a coordinate singularity of the thermodynamic metric which describes the equilibrium manifold [21].

• In this work is employed a recently devised metric within the Geometrothermodynamics program to study ordinary thermodynamic systems. The new feature of this metric is that, in addition to Legendre symmetry, it exhibits invariance under a change of representation. This metric was derived in a previous work by the authors while addressing the problem of the conformal structure of the thermodynamic metrics for different representations. Now it is presented a thorough analysis for the ideal gas, the van der Waals fluid, the one dimensional Ising model and some other systems of cosmological interest [22].

• Repulsive gravity has been investigated in several scenarios near compact objects by using different intuitive approaches. Here, we propose an invariant method to characterize regions of repulsive gravity, associated to black holes and naked singularities. Our method is based upon the behavior of the curvature tensor eigenvalues, and leads to an invariant definition of a repulsion radius. The repulsion radius determines a physical region, which can be interpreted as a repulsion sphere, where the effects due to repulsive

gravity naturally arise. Further, it was shown that the use of effective masses to characterize repulsion regions can lead to coordinate-dependent results whereas, in our approach, repulsion emerges as a consequence of the spacetime geometry in a completely invariant way. Our definition is tested in the spacetime of an electrically charged Kerr naked singularity and in all its limiting cases. A positive mass can generate repulsive gravity if it is equipped with an electric charge or an angular momentum. It was obtained the reasonable results for the spacetime regions contained inside the repulsion sphere whose size and shape depend on the value of the mass, charge and angular momentum. Consequently, one can define repulsive gravity as a classical relativistic effect by using the geometry of spacetime only [23].

• This work expands for spinor fields the recently developed Dynamical Bridge formalism which relates a linear dynamics in a curved space to a nonlinear dynamics in Minkowski space. Astonishingly, this leads to a new geometrical mechanism to generate a chiral symmetry breaking without mass, providing an alternative explanation for the undetected right-handed neutrinos. We consider a spinor field obeying the Dirac equation in an effective curved space constructed by its own currents. This way, both chiralities of the spinor field satisfy the same dynamics in the curved space. Subsequently, the dynamical equation is re-expressed in terms of the flat Minkowski space and then each chiral component behaves differently. The left-handed part of the spinor field satisfies the Dirac equation while the right-handed part is trapped by a Nambu-Jona-Lasinio (NJL) type potential [24].

• Propagation of light in nonlinear materials has been studied in the regime of the geometrical optics. It was shown that a spherically symmetric medium at rest with some specific dielectric properties can be used to produce an exact analogue model for a class of space-times which includes spherically symmetric and static black hole solutions. The optical model presented can be a useful tool to reproduce in laboratory the behavior of optical null geodesics near a compact object with an observable gravitational Schwarzschild radius [25].

5 Teaching activity

V. Belinski ”Shear viscosity effects in cosmology”, three lectures course for International Relativistic Astrophysics PhD Erasmus Mundus Program (Nice, 26 February-1 March, 2014).

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