Exact solutions of Einstein and Einstein-Maxwell equations

0.1 Topics

- Exact solutions in General Relativity
- Cosmology and Astrophysics
- Quantum Fields
- Fundamental Relativity
- Alternative theories

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0.4 Research activity

0.4.1 Exact solutions in General Relativity

• It was found the new way of derivation of the Kerr and Kerr-Newman solution by adding to the Schwarzschild and Reissner-Nordstrom black holes a solitonic whirls. The main problem here were to integrate the Lax pair equations both for the Schwarzschild and Reissner-Nordstrom backgrounds in order to find the corresponding spectral matrices. These calculations were performed and with this approach we can have a new interpretation of the distribution of energy of the Kerr and Kerr-Newman black holes between their rotational and rest mass components. The first part of this work (Kerr) have been published [3], the second paper (Kerr-Newman) is in preparation.

• Simple exact solutions have been found which describe the universes which spatial geometries are asymptotically homogeneous and isotropic near the initial singularity, but which evolution goes under the influence of primordial magnetic fields. In all these "deformed" Friedmann models (spatially flat, open or closed), the initial magnetic fields are concentrated near some axis of symmetry and their lines are the circles – the lines of the azimuthal angle coordinate. Caused by the expansion of the universe, the time dependence of a magnetic field induces (in accordance with the Faraday law) the emergence of source-free electric fields. In comparison with the Friedmann models, the cosmological expansion goes with acceleration in spatial directions across the magnetic field, and with deceleration along the magnetic lines, so that in flat and open models, in fluid comoving coordinates, the lengths of the azimuthal circles of large enough radius or for late enough times decrease and vanish for $t \to \infty$. This means that in flat and open models, we have a partial dynamical closure of space-time at large distances from the axis, i.e. from the regions where the electromagnetic fields in our solutions are concentrated. To get simple exact solutions of the Einstein-Maxwell and perfect fluid equations, we assume for the perfect fluid (which supports the isotropic and homogeneous "background" Friedmann geometries) rather exotic, stiff matter equation of state. However, it seems reasonableto to expect that similar effects might take place in the mutual dynamicsof geometry and strong electromagnetic fields in the universes with more realistic matter equations of state [1].

• The work of G. Alekseev on the monodromy transform method for construction exact solution for the supergravity and pure gravity for any space-time dimensions was made in 2012 and its arXiv version have been included into the ICRANet 2012 report. Here we are indicating that slightly updated version of this this work have been published in 2013 in Physical Review D [2].

0.4.2 Cosmology and Astrophysics

Cosmology

• The work on the book "Cosmological Singularity" (V.Belinski) has been continued. The project is in progress under the official agreement with Cambridge University Press .

• It was discovered the new type of the *generic* cosmological singularity which is in line with the observed structure of our Universe. The short outline of this result is as following. Observations show that the early Universe was isotropic, homogeneous, and thermally balanced. Consequently the initial cosmological singularity should also be in conformity with these properties. In other words, the singularity should be an isotropic singularity of Friedmann type. But it is well known that the singularity in an exact Friedmann model is unstable in the presence of conventional types of matter. This instability is due to the sharp anisotropy which develops unavoidably near the generic cosmological singularity. Then for the standard models of matter, the spacetime cannot start expandin isotropically at the singularity unless there is an artificial fine-tuning of unknown origin. However, an intuitive understanding suggests that anisotropy can be damped by shear viscosity, which might result in the existence of a generic solution with an isotropic singularity.

To search for an analytical realization of such a possibility, it would be inappropriate to use just the Eckart or Landau-Lifshitz approaches to the relativistic hydrodynamics with dissipative processes. These theories are valid provided the characteristic times of the macroscopic motions of the matter are much bigger than the time of relaxation of the medium to the equilibrium state. Evidently it is not so near the cosmological singularity since all characteristic macroscopic times in this region tend to zero, in which case one needs a theory which takes into account Maxwell's relaxation times on the same footing as all other transport coefficients. In a literal sense such a theory does not exist; however, it can be constructed in an approximate form for the cases when a medium does not deviate too much from equilibrium and relaxation times do not noticeably exceed the characteristic macroscopic times. It is reasonable to expect that these conditions will be satisfied automatically for a generic solution (if it exists) near an isotropic singularity describing the beginning of the thermally balanced Friedmann Universe accompanied by the arbitrary infinitesimally small corrections.

The main target of the efforts of many authors up to the final formulation of the generalized relativistic theory by Israel and Stewart was to bring the theory into line with relativistic causality, that is, to eliminate the supraluminal propagation of the thermal and viscous excitations. The existence of such supraluminal effects was the main stumbling block for Eckart's and Landau and Lifshitz's descriptions of dissipative fluids. As usual, in the vicinity to

the singularity where the energy density ε diverges, the coefficient of viscosity η can be approximated by the power law asymptotics $\eta \sim \varepsilon^{\nu}$ with some exponent ν. Beforehand the value of this exponent is unknown and we have to investigate its entire range from minus infinity to plus infinity. As for the relaxation time τ of the shear stresess, the choice is more definite. It is well known that $\eta/\varepsilon\tau$ represents a measure of velocity of propagation of the shear excitations. Then we can model this ratio by a positive constant f, that is the relation between relaxation time and viscosity coefficient we can choose as $\eta = f\varepsilon\tau$. It turns out that from the entire range $-\infty < \nu < \infty$ of exponent ν only one point $\nu = 1/2$ exists for which the Friedmann singularity becomes stable and at the same time no supraluminal signals exist in its vicinity.

Our results show that the viscoelastic material with shear viscosity coefficient $\eta \sim \sqrt{\varepsilon}$ can stabilize the Friedmann singularity, leading to the existence of a *generic* solution of the Einstein equations possessing an isotropic initial or final singularity. Depending on the free parameters of the theory such solution can be either of smooth power law asymptotics near the singularity or it can have the character of damping oscillations. The last possibility reveals itself as a weak trace of the chaotic oscillatory regime which is characteristic for the most general asymptotics near the cosmological singularity and which cannot be described in closed analytical form The present work shows that the shear viscosity can smooth such chaotic behavior up to the quiet oscillations which have simple asymptotic expressions in terms of the elementary functions [4].

• One can use cosmography to present constraints on the kinematics of the Universe, without postulating any underlying theoretical model. To this end, it is reasonable to use a Monte Carlo Markov chain analysis to perform comparisons to the supernova Ia Union 2 compilation, combined with the Hubble Space Telescope measurements of the Hubble constant, and the Hubble parameter data sets. The authors introduce a sixth order cosmographic parameter and show that it does not enlarge considerably the posterior distribution when comparing to the fifth order results, and also propose a way to construct viable parameter variables to be used as alternatives of the redshift z. These can overcome both the problems of divergence and lack of accuracy associated with the use of z. Moreover, it was shown that it is possible to improve the numerical fits by reparametrizing the cosmological distances. In addition, it is necessary to constrain the equation of state of the Universe as a whole by the use of cosmography. Thus, the expressions can be derived which can be directly used to fit the equation of state and the pressure derivatives up to fourth order. To this end, it is necessary to depart from a pure cosmographic analysis and to assume the Friedmann equations as valid. All results are consistent with the Lambda CDM model, although alternative fluid models, with nearly constant pressure and no cosmological constant, match the results accurately as well [17].

• The formalism of Geometrothermodynamics it was used to derive a se-

ries of fundamental equations for thermodynamic systems. It is shown that all these fundamental equations can be of help in the context of relativistic cosmology to derive diverse scenarios which include the standard cosmological model, a unified model for dark energy and dark matter, and an effective inflationary model [18].

Astrophysics

• It was improved the effective-one-body (EOB) description of nonspinning coalescing black hole binaries by incorporating several recent analytical advances, notably: (i) logarithmic contributions to the conservative dynamics; (ii) resummed horizon-absorption contribution to the orbital angular momentum loss; and (iii) a specific radial component of the radiation reaction force implied by consistency with the azimuthal one. Then it was completed this analytically improved EOB model by comparing it to accurate numerical relativity (NR) simulations performed by the Caltech-Cornell-CITA group for different mass ratios. In particular, the comparison to NR data allows us to determine with high-accuracy the value of the main EOB radial potential. It was introduced a new technique for extracting from NR data an intrinsic measure of the phase evolution. Aligning the NR-completed EOB quadrupolar waveform and the NR one at low frequencies, it was found that they keep agreeing (in phase and amplitude) within the NR uncertainties throughout the evolution for all mass ratios considered. It was also found good agreement for several subdominant multipoles without having to introduce and tune any extra parameters [5].

• A new analytical approach to the motion and radiation of (comparable mass) binary systems has been introduced in 1999 under the name of Effective One Body (EOB) formalism. Now it was reviewed the basic elements of this formalism, and discuss some of its recent developments. Several recent comparisons between EOB predictions and Numerical Relativity (NR) simulations have shown the aptitude of the EOB formalism to provide accurate descriptions of the dynamics and radiation of various binary systems (comprising black holes or neutron stars) in regimes that are inaccessible to other analytical approaches (such as the last orbits and the merger of comparable mass black holes). In synergy with NR simulations, post-Newtonian theory and Gravitational Self-Force computations, the EOB formalism is likely to provide an efficient way of computing the very many accurate template waveforms that are needed for Gravitational Wave data analysis purposes [6].

• It was completed the analytical determination, at the 4th post-Newtonian approximation, of the main radial potential describing the gravitational interaction of two bodies within the effective one-body formalism. The (non logarithmic) coefficient measuring this 4th post-Newtonian interaction potential is found to be linear in the symmetric mass ratio. Its main part which

is idependent on the mass ratio is obtained by an analytical gravitational self-force calculation that unambiguously resolves the formal infrared divergencies which currently impede its direct post-Newtonian calculation. Its linear part is deduced from recent results of Jaranowski and Schafer, and is found to be significantly negative [7].

• It was studied to what extent the effective-one-body description of the dynamical state of a nonspinning, coalescing binary black hole (considered either at merger, or after ringdown) agrees with numerical relativity results. This comparison uses estimates of the integrated losses of energy and angular momentum during ringdown, inferred from recent numerical-relativity data. We find that the values, predicted by the effective-one-body formalism, of the energy and angular momentum of the system agree at the per mil level with their numerical-relativity counterparts, both at merger and in the final state. This gives a new confirmation of the ability of effective-one-body theory to accurately describe the dynamics of binary black holes even in the strong-gravitational-field regime. Our work also provides predictions (and analytical fits) for the final mass and the final spin of coalescing black holes for all mass ratios [8].

• The Numerical-Relativity-Analytical-Relativity (NRAR) collaboration is a joint effort between members of the numerical relativity, analytical relativity and gravitational-wave data analysis communities. The goal of the NRAR collaboration is to produce numerical-relativity simulations of compact binaries and use them to develop accurate analytical templates for the LIGO/Virgo Collaboration to use in detecting gravitational-wave signals and extracting astrophysical information from them. It was described the results of the first stage of the NRAR project, which focused on producing an initial set of numerical waveforms from binary black holes with moderate mass ratios and spins, as well as one non-spinning binary configuration which has a mass ratio of 10. All of the numerical waveforms are analysed in a uniform and consistent manner, with numerical errors evaluated using an analysis code created by members of the NRAR collaboration. We compare previously-calibrated, non-precessing analytical waveforms, notably the effective-one-body (EOB) and phenomenological template families, to the newly-produced numerical waveforms. We find that when the binary's total mass is around 100-200 solar masses, current EOB and phenomenological models of spinning, non-precessing binary waveforms have overlaps above 99% (for advanced LIGO) with all of the non-precessing-binary numerical waveforms with mass ratios less or equal 4, when maximizing over binary parameters. This implies that the loss of event rate due to modelling error is below 3%. Moreover, the non-spinning EOB waveforms previously calibrated to five non-spinning waveforms with mass ratio smaller than 6 have overlaps above 99.7% with the numerical waveform with a mass ratio of 10, without even maximizing on the binary parameters [9].

• It was presented a relativistic model describing a thin disk surrounded by

a halo in the presence of an electromagnetic field. The model is obtained by solving the Einstein-Maxwell equations on a particular conformastatic spacetime background and by using the distributional approach for the energy-momentum tensor. A class of solutions is obtained in which the gravitational and electromagnetic potentials are completely determined by a harmonic function only. A particular solution is given that is asymptotically flat and singularity free, and satisfies all the energy conditions [19].

• It was studied the circular motion of charged test particles in the gravitational field of a charged mass described by the Reissner-Nordstrom spacetime, focusing on the physical differences between black holes and naked singularities. It have been performed the most complete classification of circular orbits for different sets of the main physical parameters, and study numerically the behavior of the angular momentum and energy of the charged test particle. This analysis shows in an alternative manner that the behavior of circular orbits can be used to distinguish between black holes and naked singularities [20].

• It was carried out the detailed analysis of the orbital circular motion of electrically neutral test particles on the equatorial plane of the Kerr-Newman spacetime. Many details of the motion in the cases of black hole and naked singularity sources was pointed out. It was identified four different types of orbital regions, which depend on the properties of the orbital angular momentum, and defined four different kinds of naked singularities, according to the values of the charge-to-mass ratio of the source. It was shown that the presence of a particular type of counter-rotating test particle is sufficient to uniquely identify naked singularities. It was pointed out that the structure of the stability regions can be used to differentiate between black holes and naked singularities [21].

• It was studied a time-independent, spherically symmetric, self-gravitating systems minimally coupled to a scalar field with U(1) gauge symmetry: charged boson stars. Numerical solutions to the Einstein-Maxwell equations coupled to the relativistic Klein-Gordon equation was found. It was shown that bound stable configurations exist only for values of the coupling constant less than or equal to a certain critical value. The metric coefficients and the relevant physical quantities, such as the total mass and charge, turn out to be, in general, bound functions of the radial coordinate, reaching their maximum values at a critical value of the scalar field at the origin. The stability problem from both the quantitative and qualitative point of view was discussed. It was taken into account the electromagnetic contribution to the total mass and investigated the stability issue considering the binding energy per particle. It was verified the existence of configurations with positive binding energy in which objects that are apparently bound can be unstable against small perturbations, in full analogy with the effect observed in the mass-radius relation of neutron stars [22].

• A relativistic model describing a thin disk system composed of two flu-

ids was presented. The system is surrounded by a halo in the presence of a non-trivial electromagnetic field. It was shown that the model is compatible with the variational multi-fluid thermodynamics formalism, allowing to determine all the thermodynamic variables associated with the matter content of the disk. The asymptotic behaviour of these quantities indicates that the single fluid interpretation should be abandoned in favour of a two-fluid model [29].

0.4.3 Quantum Fields

• It was studied the quantum dynamics of a supersymmetric squashed three-sphere by dimensionally reducing (to one timelike dimension) the action of D=4 simple supergravity for an SO(3)-homogeneous (Bianchi IX) cosmological model. The quantization of the homogeneous gravitino field leads to a 64-dimensional fermionic Hilbert space. The algebra of the supersymmetry constraints and of the Hamiltonian one is found to close. One finds that the quantum Hamiltonian is built from operators that generate a 64-dimensional representation of the (infinite-dimensional) maximally compact sub-algebra of the rank-3 hyperbolic Kac-Moody algebra AE3. Some exponentials of these operators generate a spinorial extension of the Weyl group of AE3 which describe (in the small wavelength limit) the chaotic quantum evolution of the universe near the cosmological singularity [10].

0.4.4 Fundamental Relativity

• It was presented a thorough analysis on the invariance of the most widely used metrics in the Geometrothermodynamics programme with centrering the attention in the invariance of the curvature of the space of equilibrium states under a change of fundamental representation. Assuming that the systems under consideration can be described by a fundamental relation which is a homogeneous function of a definite order, it was demonstrated that such invariance is only compatible with total Legendre transformations in the present form of the programme. The explicit form of a metric which is invariant under total Legendre transformations and whose induced metric produces a curvature which is independent of the fundamental representation it was given. Also study a generic system with two degrees of freedom and whose fundamental relation is homogeneous of order one was studied[23].

• The method of topological quantization to obtain the bosonic string topological spectrum propagating on a flat background have been applied. It was defined the classical configuration of the system, and construct the corresponding principal fiber bundle (pfb) that uniquely represents it. The topological spectrum is defined through the characteristic class of the pfb. It was found the explicit expressions for the topological spectrum for particular con-

figurations of the bosonic strings on a Minkowski background and showed that they lead to a discretization of the total energy of the system [24].

• It was used the formalism of Geometrothermodynamics to describe chemical reactions in the context of equilibrium thermodynamics. Any chemical reaction in a closed system is shown to be described by a geodesic in a two-dimensional manifold that can be interpreted as the equilibrium space of the reaction. First this was shown in the particular cases of a reaction with only two species corresponding to either two ideal gases or two van der Waals gases. Then it was considered the case of a reaction with an arbitrary number of species. The initial equilibrium state of the geodesic is determined by the initial conditions of the reaction. The final equilibrium state, which follows from a thermodynamic analysis of the reaction, is shown to correspond to a coordinate singularity of the thermodynamic metric which describes the equilibrium manifold [25].

• In this work the authors employ a recently devised metric within the Geometrothermodynamics program to study ordinary thermodynamic systems. The new feature of this metric is that, in addition to Legendre symmetry, it exhibits invariance under a change of representation. This metric was derived in a previous work by the authors while addressing the problem of the conformal structure of the thermodynamic metrics for different representations. Here, it was presented an thorough analysis for the ideal gas, the van der Waals fluid, the one dimensional Ising model and some other systems of cosmological interest [26].

• It was studied the thermodynamics and geometrothermodynamics of different black hole configurations in more than four spacetime dimensions. The approach used was the response functions to find the conditions under which second order phase transitions occur in higher-dimensional static Reissner-Nordstrom and stationary Kerr black holes. The results indicate that the equilibrium manifold of all these black hole configurations is in general curved and that curvature singularities appear exactly at those places where second order phase transitions occur [27].

• The thermodynamics of Maxwell-Dilaton black holes has been extensively studied. It has served as a fertile ground to test ideas about temperature through various definitions of surface gravity. It was performed an independent analysis of these black hole solutions in both, Einstein and Jordan, frames. It was explored a set of definitions for the surface gravity and observed the different predictions they make for the near extremal configuration of these black holes. Finally, motivated by the singularity structure in the interior of the event horizon, it was used a holographic argument to remove the micro-states from the disconnected region of this solution. In this manner, it was possible to construct a frame independent entropy from which one obtains a temperature which agrees with the standard results in the non-extremal regime, and has a desirable behaviour around the extremal configurations according to the third law of black hole mechanics [28].

• It was shown that the path of any accelerated body in an arbitrary spacetime with some background metric can be described as a geodesic in a new effective "dragged metric" that depends only on the background metric and on the motion of the body. Such procedure allows the interpretation of all kinds of non-gravitational force as modifications of the spacetime metric. This method of effective elimination of the forces by changing metric of the substratum can be understood as a generalization of the d'Alembert principle applied to all relativistic processes [13].

• Propagation of light in nonlinear materials was studied in the regime of the geometrical optics. It was shown that a spherically symmetric medium at rest with some specific dielectric properties can be used to produce an exact analogue model for a class of space-times which includes spherically symmetric and static black hole solutions. The optical model presented can be a useful tool to reproduce in laboratory the behavior of optical null geodesics near a compact object with an observable gravitational Schwarzschild radius [14].

• A new form of contribution for the anomalous magnetic moment of all particles have been investigated. This common origin is displayed in the framework of a recent treatment of electrodynamics that is based on the introduction of an electromagnetic metric which has no gravitational character. This effective metric constitutes a universal pure electromagnetic process perceived by all bodies, charged or not charged. As a consequence it yields a complementary explanation for the existence of anomalous magnetic moment for charged particles and even for non-charged particles like neutrino [15].

0.4.5 Alternative theories

• A comprehensive review (102 pages) of the equations of general relativity in the quasi-Maxwellian (QM) formalism introduced by Jordan, Ehlers and Kundt is made. The main interest concerns its applications to the analysis of the perturbation of standard cosmology in the Friedman framework. The major achievement of the QM scheme is its use of completely gauge independent quantities. It is shown that in the QM scheme we deal directly with observable quantities. This reveals its advantage over the old method introduced by Lifshitz et al. that deals with perturbations in the standard Einstein framework. For completeness, we compare the QM scheme to the gauge independent method of Bardeen, a procedure consisting on particular choices of the perturbed variables as a combination of gauge dependent quantities [11].

• It was investigated the dependence of thermodynamic properties of black holes on the choice of statistical ensemble for a particular class of Einstein-Maxwell-Gauss-Bonnet black holes with cosmological constant. The authors

used partial Legendre transformations in the thermodynamic limit in order to compare the results in different ensembles, and show that the phase transition structure depend on the choice of thermodynamic potential. This result implies that thermodynamic metrics which are partially Legendre invariant cannot be employed to describe black hole thermodynamics, and partly explains why a particular thermodynamic metric has been used so far in the framework of black hole geometrothermodynamics [16].

• A geometric scalar theory of gravity was constructed. The construction was described using the "background field method" introduced by Gupta, Feynman, Deser and others as a field theory formulation of general relativity. The previous criticisms against scalar gravity was analyzed and it was shown how the present proposal avoids these difficulties. This concerns not only the theoretical complaints but also those related to observations. In particular, it was claryfied that the widespread belief of the conjecture that the source of scalar gravity must be the trace of the energy-momentum tensor—which is one of the main difficulties to couple gravity with electromagnetic phenomenon in previous models—does not apply to our geometric scalar theory. From the very beginning this is not a special relativistic scalar gravity. The adjective "geometric" pinpoints its similarity with general relativity: this is a metric theory of gravity. Some consequences of this new scalar theory are explored [12].

0.5 Teaching activity

V. Belinski "Relativistic dissipative thermodynamics and its application to cosmology", three lectures course for International Relativistic Astrophysics PhD Erasmus Mundus Program (Nice, 11-13 September, 2013).

0.6 References

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