Electron-positron pairs in physics and astrophysics

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- Nonlinear electrodynamics and rate of pair creation
- Pair production and annihilation in QED
- Semi-classical description of pair production in a general electric field
- Phenomenology of electron-positron pair creation and annihilation
- The Breit-Wheeler cutoff in high-energy γ-rays
- The extraction of blackholic energy from a black hole by vacuum polarization processes
- Thermalization of the mildly relativistic pair plasma
- Plasma oscillations in uniform electric fields
- The hydrodynamical expansion of electron-positron-photon plasma (Dyadophere formation) in gravitational collapse
- Plasma oscillations and radiation in nonuniform electric fields
- Fractional Effective Action at strong electromagnetic fields
- Einstein-Euler-Heisenberg theory and charged black holes
- Electron and positron pair production in gravitational collapse
- Gravitational and electric energies in gravitational collapse

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3. Brief description

3.1. Abstract

Due to the interaction of physics and astrophysics we are witnessing in these years a splendid synthesis of theoretical, experimental and observational results originating from three fundamental physical processes. They were originally proposed by Dirac, by Breit and Wheeler and by Sauter, Heisenberg, Euler and Schwinger. For almost seventy years they have all three been followed by a continued effort of experimental verification on Earth-based experiments. The Dirac process, $e^+e^- \to 2\gamma$, has been by far the most successful. It has obtained extremely accurate experimental verification and has led as well to an enormous number of new physics in possibly one of the most fruitful experimental avenues by introduction of storage rings in Frascati and followed by the largest accelerators worldwide: DESY, SLAC etc. The Breit–Wheeler process, $2\gamma \to e^+e^-$, although conceptually simple, being the inverse process of the Dirac one, has been by far one of the most difficult to be verified experimentally. Only recently, through the technology based on free electron X-ray laser and its numerous applications in Earth-based experiments, some first indications of its possible verification have been reached. The vacuum polarization process in strong electromagnetic field, pioneered by Sauter, Heisenberg, Euler and Schwinger, introduced the concept of critical electric field $E_c = m_e^2 c^3/(e\hbar)$. It has been searched without success for more than forty years by heavy-ion collisions in many of the leading particle accelerators worldwide.

The novel situation today is that these same processes can be studied on a much more grandiose scale during the gravitational collapse leading to the formation of a black hole being observed in Gamma Ray Bursts (GRBs). This report is dedicated to the scientific race. The theoretical and experimental work developed in Earth-based laboratories is confronted with the theoretical interpretation of space-based observations of phenomena originating on cosmological scales. What has become clear in the last ten years is that all the three above mentioned processes, duly extended in the general relativistic framework, are necessary for the understanding of the physics of the gravitational collapse to a black hole. Vice versa, the natural arena where these processes can be observed in mutual interaction and on an unprecedented scale, is indeed the realm of relativistic astrophysics.

We systematically analyze the conceptual developments which have fol-

lowed the basic work of Dirac and Breit–Wheeler. We also recall how the seminal work of Born and Infeld inspired the work by Sauter, Heisenberg and Euler on effective Lagrangian leading to the estimate of the rate for the process of electron–positron production in a constant electric field. In addition of reviewing the intuitive semi-classical treatment of quantum mechanical tunneling for describing the process of electron–positron production, we recall the calculations in *Quantum Electro-Dynamics* of the Schwinger rate and effective Lagrangian for constant electromagnetic fields. We also review the electron–positron production in both time-alternating electromagnetic fields, studied by Brezin, Itzykson, Popov, Nikishov and Narozhny, and the corresponding processes relevant for pair production at the focus of coherent laser beams as well as electron beam–laser collision. We finally report some current developments based on the general JWKB approach which allows to compute the Schwinger rate in spatially varying and time varying electromagnetic fields.

We also recall the pioneering work of Landau and Lifshitz, and Racah on the collision of charged particles as well as experimental success of AdA and ADONE in the production of electron–positron pairs.

We then turn to the possible experimental verification of these phenomena. We review: (A) the experimental verification of the $e^+e^- \rightarrow 2\gamma$ process studied by Dirac. We also briefly recall the very successful experiments of e^+e^- annihilation to hadronic channels, in addition to the Dirac electromagnetic channel; (B) ongoing Earth based experiments to detect electron–positron production in strong fields by focusing coherent laser beams and by electron beam–laser collisions; and (C) the multiyear attempts to detect electron–positron production in Coulomb fields for a large atomic number $Z > 137$ in heavy ion collisions. These attempts follow the classical theoretical work of Popov and Zeldovich, and Greiner and their schools.

We then turn to astrophysics. We first review the basic work on the energetics and electrodynamical properties of an electromagnetic black hole and the application of the Schwinger formula around Kerr–Newman black holes as pioneered by Damour and Ruffini. We only focus on black hole masses larger than the critical mass of neutron stars, for convenience assumed to coincide with the Rhoades and Ruffini upper limit of 3.2 $M_\odot$. In this case the electron Compton wavelength is much smaller than the spacetime curvature and all previous results invariantly expressed can be applied following well established rules of the equivalence principle. We derive the corresponding rate of electron–positron pair production and introduce the concept of dyadosphere. We review recent progress in describing the evolution of optically thick electron–positron plasma in presence of supercritical electric field, which is relevant both in astrophysics as well as ongoing laser beam experiments. In particular we review recent progress based on the Vlasov-Boltzmann-Maxwell equations to study the feedback of the created electron–positron pairs on the original constant electric field. We evidence the exis-

tence of plasma oscillations and its interaction with photons leading to energy and number equipartition of photons, electrons and positrons. We finally review the recent progress obtained by using the Boltzmann equations to study the evolution of an electron–positron-photon plasma towards thermal equilibrium and determination of its characteristic timescales. The crucial difference introduced by the correct evaluation of the role of two and three body collisions, direct and inverse, is especially evidenced. We then present some general conclusions.

The results reviewed in this report are going to be submitted to decisive tests in the forthcoming years both in physics and astrophysics. To mention only a few of the fundamental steps in testing in physics we recall the starting of experimental facilities at the National Ignition Facility at the Lawrence Livermore National Laboratory as well as corresponding French Laser the Mega Joule project. In astrophysics these results will be tested in galactic and extragalactic black holes observed in binary X-ray sources, active galactic nuclei, microquasars and in the process of gravitational collapse to a neutron star and also of two neutron stars to a black hole giving origin to GRBs. The astrophysical description of the stellar precursors and the initial physical conditions leading to a gravitational collapse process will be the subject of a forthcoming report. As of today no theoretical description has yet been found to explain either the emission of the remnant for supernova or the formation of a charged black hole for GRBs. Important current progress toward the understanding of such phenomena as well as of the electrodynamical structure of neutron stars, the supernova explosion and the theories of GRBs will be discussed in the above mentioned forthcoming report. What is important to recall at this stage is only that both the supernovae and GRBs processes are among the most energetic and transient phenomena ever observed in the Universe: a supernova can reach energy of $\sim 10^{54}$ ergs on a time scale of a few months and GRBs can have emission of up to $\sim 10^{54}$ ergs in a time scale as short as of a few seconds. The central role of neutron stars in the description of supernovae, as well as of black holes and the electron–positron plasma, in the description of GRBs, pioneered by one of us (RR) in 1975, are widely recognized. Only the theoretical basis to address these topics are discussed in the present report.

3.2. The three fundamental contributions to the electron-positron pair creation and annihilation and the concept of critical electric field

The annihilation of electron–positron pair into two photons, and its inverse process – the production of electron–positron pair by the collision of two photons were first studied in the framework of quantum mechanics by P.A.M. Dirac and by G. Breit and J.A. Wheeler in the 1930s (Dirac (1930); Breit and Wheeler (1934)).

A third fundamental process was pioneered by the work of Fritz Sauter and Oscar Klein, pointing to the possibility of creating an electron–positron pair from the vacuum in a constant electromagnetic field. This became known as the 'Klein paradox' and such a process named as *vacuum polarization*. It would occur for an electric field stronger than the critical value

$$E_c \equiv \frac{m_e^2 c^3}{e\hbar} \simeq 1.3 \cdot 10^{16}\,\mathrm{V/cm}. \tag{3.2.1}$$

where m_e, e, c and $\hbar$ are respectively the electron mass and charge, the speed of light and the Planck's constant.

The experimental difficulties to verify the existence of such three processes became immediately clear. While the process studied by Dirac was almost immediately observed Klemperer (1934) and the electron–positron collisions became possibly the best tested and prolific phenomenon ever observed in physics. The Breit–Wheeler process, on the contrary, is still today waiting a direct observational verification. Similarly the vacuum polarization process defied dedicated attempts for almost fifty years in experiments in nuclear physics laboratories and accelerators all over the world, see Section 7 in the following article.

From the theoretical point of view the conceptual changes implied by these processes became immediately clear. They were by vastness and depth only comparable to the modifications of the linear gravitational theory of Newton introduced by the nonlinear general relativistic equations of Einstein. In the work of Euler, Oppenheimer and Debye, Born and his school it became clear that the existence of the Breit–Wheeler process was conceptually modifying the linearity of the Maxwell theory. In fact the creation of the electron–positron pair out of the two photons modifies the concept of superposition of the linear electromagnetic Maxwell equations and impose the necessity to transit to a nonlinear theory of electrodynamics. In a certain sense the Breit–Wheeler process was having for electrodynamics the same fundamental role of Gedankenexperiment that the equivalence principle had for gravitation. Two different attempts to study these nonlinearities in the electrodynamics were made: one by Born and Infeld Born (1933, 1934); Born and Infeld (1934) and one by Euler and Heisenberg Heisenberg and Euler (1936). These works prepared the even greater revolution of Quantum Electro-Dynamics by Tomonaga Tomonaga (1946), Feynman Feynman (1948, 1949b,a), Schwinger Schwinger (1948, 1949a,b) and Dyson Dyson (1949a,b).

In Section 3 in the following article we review the fundamental contributions to the electron–positron pair creation and annihilation and to the concept of the critical electric field. In Section 3.1 of the following article we review the Dirac derivation Dirac (1930) of the electron–positron annihilation process obtained within the perturbation theory in the framework of relativistic quantum mechanics and his derivation of the classical formula for the cross-section $\sigma^{\rm lab}_{e^+e^-}$ in the rest frame of the electron

$$\sigma^{\rm lab}_{e^+e^-} = \pi\left(\frac{\alpha\hbar}{m_e\, c}\right)^2 (\hat\gamma-1)^{-1}\left\{\frac{\hat\gamma^2+4\hat\gamma+1}{\hat\gamma^2-1}\ln[\hat\gamma+(\hat\gamma^2-1)^{1/2}]-\frac{\hat\gamma+3}{(\hat\gamma^2-1)^{1/2}}\right\},$$

where $\hat\gamma \equiv \mathcal{E}_+/m_e c^2 \geq 1$ is the energy of the positron and $\alpha = e^2/(\hbar c)$ is as usual the fine structure constant, and we recall the corresponding formula for the center of mass reference frame. In article Section 3.2 we recall the main steps in the classical Breit–Wheeler work Breit and Wheeler (1934) on the production of a real electron–positron pair in the collision of two photons, following the same method used by Dirac and leading to the evaluation of the total cross-section $\sigma_{\gamma\gamma}$ in the center of mass of the system

$$\sigma_{\gamma\gamma} = \frac{\pi}{2}\left(\frac{\alpha\hbar}{m_e\, c}\right)^2 (1-\hat\beta^2)\left[2\hat\beta(\hat\beta^2-2)+(3-\hat\beta^4)\ln\left(\frac{1+\hat\beta}{1-\hat\beta}\right)\right], \quad \text{with} \quad \hat\beta = \frac{c|\mathbf{p}|}{\mathcal{E}},$$

where $\hat\beta$ is the reduced velocity of the electron or the positron. In Section 3.3 of the article we recall the basic higher order processes, compared to the Dirac and Breit–Wheeler ones, leading to pair creation. In Section 3.4 in the following review we recall the famous Klein paradox Klein (1929); Sauter (1931) and the possible tunneling between the positive and negative energy states leading to the concept of level crossing and pair creation by analogy to the Gamow tunneling Gamow (1931) in the nuclear potential barrier. We then turn to the celebrated Sauter work Sauter (1931) showing the possibility of creating a pair in a uniform electric field E. We recover in Section 3.5.1 of the review a JWKB approximation in order to reproduce and improve on the Sauter result by obtaining the classical Sauter exponential term as well as the prefactor

$$\frac{\Gamma_{\rm JWKB}}{V} \simeq D_s\frac{\alpha E^2}{2\pi^2\hbar}e^{-\pi E_c/E},$$

where $D_s = 2$ for a spin-1/2 particle and $D_s = 1$ for spin-0, V is the volume. Finally, in review Section 3.5.2 the case of a simultaneous presence of an electric and a magnetic field B is presented leading to the estimate of pair production rate

$$\frac{\Gamma_{\text{JWKB}}}{V} \simeq \frac{\alpha\beta\varepsilon}{\pi\hbar} \coth\left(\frac{\pi\beta}{\varepsilon}\right) \exp\left(-\frac{\pi E_c}{\varepsilon}\right), \quad \text{spin} - 1/2 \text{ particle}$$

and

$$\frac{\Gamma_{\text{JWKB}}}{V} \simeq \frac{\alpha\beta\varepsilon}{2\pi\hbar} \sinh^{-1}\left(\frac{\pi\beta}{\varepsilon}\right) \exp\left(-\frac{\pi E_c}{\varepsilon}\right), \quad \text{spin} - 0 \text{ particle},$$

where

$$\begin{aligned} \varepsilon &\equiv \sqrt{(S^2 + P^2)^{1/2} + S}, \\ \beta &\equiv \sqrt{(S^2 + P^2)^{1/2} - S}, \end{aligned}$$

where the scalar S and the pseudoscalar P are

$$S \equiv \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2); \quad P \equiv \frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} = \mathbf{E} \cdot \mathbf{B},$$

where $\tilde{F}^{\mu\nu} \equiv \epsilon^{\mu\nu\lambda\kappa} F_{\lambda\kappa}$ is the dual field tensor.

3.3. Nonlinear electrodynamics and rate of pair creation

In article Section 4 we first recall the seminal work of Hans Euler Euler (1936) pointing out for the first time the necessity of nonlinear character of electromagnetism introducing the classical Euler Lagrangian

$$\mathcal{L} = \frac{\mathbf{E}^2 - \mathbf{B}^2}{8\pi} + \frac{1}{\alpha} \frac{1}{E_0^2} \left[a_E \left(\mathbf{E}^2 - \mathbf{B}^2\right)^2 + b_E \left(\mathbf{E} \cdot \mathbf{B}\right)^2 \right],$$

where

$$a_E = -1/(360\pi^2), \quad b_E = -7/(360\pi^2),$$

a first order perturbation to the Maxwell Lagrangian. In review article Section 4.2 we review the alternative theoretical approach of nonlinear electrodynamics by Max Born Born (1934) and his collaborators, to the more ambitious attempt to obtain the correct nonlinear Lagrangian of Electro-Dynamics. The motivation of Born was to attempt a theory free of divergences in the observable properties of an elementary particle, what has become known as 'unitarian' standpoint versus the 'dualistic' standpoint in description of elementary particles and fields. We recall how the Born Lagrangian was formulated

$$\mathcal{L} = \sqrt{1 + 2S - P^2} - 1,$$

and one of the first solutions derived by Born and Infeld Born and Infeld (1934). We also recall one of the interesting aspects of the courageous approach of Born had been to formulate this Lagrangian within a unified theory of gravitation and electromagnetism following Einstein program. Indeed, we also recall the very interesting solution within the Born theory obtained by Hoffmann Hoffmann (1935); Hoffmann and Infeld (1937). Still in the work of Born Born (1934) the seminal idea of describing the nonlinear vacuum properties of this novel electrodynamics by an effective dielectric constant and magnetic permeability functions of the field arisen. We then review in Section 4.3.1 of the article the work of Heisenberg and Euler Heisenberg and Euler (1936) adopting the general approach of Born and generalizing to the presence of a real and imaginary part of the electric permittivity and magnetic permeability. They obtain an integral expression of the effective Lagrangian given by

$$\begin{aligned} \Delta\mathcal{L}_{\rm eff} &= \frac{e^2}{16\pi^2\hbar c}\int_0^\infty e^{-s}\frac{ds}{s^3}\Big[is^2\,\bar{E}\bar{B}\frac{\cos(s[\bar{E}^2-\bar{B}^2+2i(\bar{E}\bar{B})]^{1/2})+\text{c.c.}}{\cos(s[\bar{E}^2-\bar{B}^2+2i(\bar{E}\bar{B})]^{1/2})-\text{c.c.}} \\ &\quad + \left(\frac{m_e^2c^3}{e\hbar}\right)^2 + \frac{s^2}{3}(|\bar{B}|^2-|\bar{E}|^2)\Big], \end{aligned}$$

where $\bar{E}, \bar{B}$ are the dimensionless reduced fields in the unit of the critical field E_c,

$$\bar{E} = \frac{|\mathbf{E}|}{E_c}, \quad \bar{B} = \frac{|\mathbf{B}|}{E_c}.$$

obtaining the real part and the crucial imaginary term which relates to the pair production in a given electric field. It is shown how these results give as a special case the previous result obtained by Euler (Eq. (4.1.3) in the review). In Section 4.3.2 of the following article the work by Weisskopf Weisskopf (1936) working on a spin-0 field fulfilling the Klein–Gordon equation, in contrast to the spin 1/2 field studied by Heisenberg and Euler, confirms the Euler-Heisenberg result. Weisskopf obtains explicit expression of pair creation in an arbitrary strong magnetic field and in an electric field described by $\bar{E}$ and $\bar{B}$ expansion.

For the first time Heisenberg and Euler provided a description of the vacuum properties by the characteristic scale of strong field E_c and the effective Lagrangian of nonlinear electromagnetic fields. In 1951, Schwinger Schwinger (1951, 1954a,b) made an elegant quantum field theoretic reformulation of this discovery in the QED framework. This played an important role in understanding the properties of the QED theory in strong electromagnetic fields.

The QED theory in strong coupling regime, i.e., in the regime of strong electromagnetic fields, is still a vast arena awaiting for experimental verification as well as of further theoretical understanding.

3.4. Pair production and annihilation in QED

In the review article in Section 5 after recalling some general properties of QED in Section 5.1 and some basic processes in Section 5.2 we proceed to the consideration of the Dirac and the Breit–Wheeler processes in QED in Secton 5.3. Then we discuss some higher order processes, namely double pair production in Section 5.4, electron-nucleus bremsstrahlung and pair production by a photon in the field of a nucleus in Section 5.5, and finally pair production by two ions in Section 5.6. In Section 5.7 the classical result for the vacuum to vacuum decay via pair creation in uniform electric field by Schwinger is recalled

$$\frac{\Gamma}{V} = \frac{\alpha E^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi E_c}{E}\right).$$

This formula generalizes and encompasses the previous results reviewed in our report: the JWKB results, discussed in Section 3.5, and the Sauter exponential factor (Eq. (3.5.11) in the review), and the Heisenberg-Euler imaginary part of the effective Lagrangian. We then recall the generalization of this formula to the case of a constant electromagnetic fields. Such results were further generalized to spatially nonuniform and time-dependent electromagnetic fields by Nikishov (1970), Vanyashin and Terent'ev (1965), Popov (1971, 1972b, 2001a), Narozhnyi and Nikishov (1970) and Batalin and Fradkin (1970a). We then conclude this argument by giving the real and imaginary parts for the effective Lagrangian for arbitrary constant electromagnetic field recently published by Ruffini and Xue (2006). This result generalizes the previous result obtained by Weisskopf in strong fields. In weak field it gives the Euler-Heisenberg effective Lagrangian. As we will see in the Section 7.2 of the review much attention has been given experimentally to the creation of pairs in the rapidly changing electric fields. A fundamental contribution in this field studying pair production rates in an oscillating electric field was given by Brezin and Itzykson (1970) and we recover in review Section 5.8 their main results which apply both to the case of bosons and fermions. We recall how similar results were independently obtained two years later by Popov Popov (1972a). In Section 5.10 of the article we recall an alternative physical process considering the quantum theory of the interaction of free electron with the field of a strong electromagnetic waves: an ultrarelativistic electron absorbs multiple photons and emits only a single photon in the reaction Bula et al. (1996):

$$e + n\omega \to e' + \gamma.$$

This process appears to be of the great relevance as we will see in the next Section for the nonlinear effects originating from laser beam experiments. Particularly important appears to be the possibility outlined by Burke et al. (1997) that the high-energy photon γ created in the first process propagates through the laser field, it interacts with laser photons $n\omega$ to produce an electron–positron pair

$$\gamma + n\omega \rightarrow e^+ + e^-.$$

We also refer to the papers by Nikishov and Ritus (1964a,b, 1965, 1967, 1979); Narozhnyǐ et al. (1965) studying the dependence of this process on the status of the polarization of the photons.

We point out the great relevance of departing from the case of the uniform electromagnetic field originally considered by Sauter, Heisenberg and Euler, and Schwinger. We also recall some of the classical works of Brezin and Itzykson and Popov on time varying fields. The space variation of the field was also considered in the classical papers of Nikishov and Narozhny as well as in the work of Wang and Wong. Finally, we recall the work of Khriplovich Khriplovich (2000) studying the vacuum polarization around a Reissner–Nordström black hole. A more recent approach using the worldline formalism, sometimes called the string-inspired formalism, was advanced by Dunne and Schubert Schubert (2001a); Dunne and Schubert (2005a).

3.5. Semi-classical description of pair production in a general electric field

In review Section 6, after recalling studies of pair production in inhomogeneous electromagnetic fields in the literature by Dunne and Schubert (2005a); Dunne et al. (2006); Dunne and Wang (2006); Kim and Page (2002, 2006, 2007), we present a brief review of our recent work Kleinert et al. (2008) where the general formulas for pair production rate as functions of either crossing energy level or classical turning point, and total production rate are obtained in external electromagnetic fields which vary either in one space direction $E(z)$ or in time $E(t)$. In Sections 6.1 and 6.2, these formulas are explicitly derived in the JWKB approximation and generalized to the case of three-dimensional electromagnetic configurations. We apply these formulas to several cases of such inhomogeneous electric field configurations, which are classified into two categories. In the first category, we study two cases: a semi-confined field $E(z) \neq 0$ for $z \lesssim \ell$ and the Sauter field

$$E(z) = E_0/\cosh^2(z/\ell), \quad V(z) = -\sigma_s\, m_e c^2 \tanh(z/\ell),$$

where ℓ is width in the z-direction, and

$$\sigma_s \equiv eE_0\ell/m_ec^2 = (\ell/\lambda_C)(E_0/E_c).$$

In these two cases the pairs produced are not confined by the electric potential and can reach an infinite distance. The resultant pair production rate varies as a function of space coordinate. The result we obtained is drastically different from the Schwinger rate in homogeneous electric fields without any boundary. We clearly show that the approximate application of the Schwinger rate to electric fields limited within finite size of space overestimates the total number of pairs produced, particularly when the finite size is comparable with the Compton wavelength λ_C, see article Figs. 6.2 and 6.3 where it is clearly shown how the rate of pair creation far from being constant goes to zero at both boundaries. The same situation is also found for the case of the semi-confined field $z(z) \neq 0$ for $|z| \lesssim \ell$, see Eq. (6.3.34). In the second category, we study a linearly rising electric field $E(z) \sim z$, corresponding to a harmonic potential $V(z) \sim z^2$, see Figs. 6.1. In this case the energy spectra of bound states are discrete and thus energy crossing levels for tunneling are discrete. To obtain the total number of pairs created, using the general formulas for pair production rate, we need to sum over all discrete energy crossing levels, see Eq. (6.4.11), provided these energy levels are not occupied. Otherwise, the pair production would stop due to the Pauli principle.

3.6. Phenomenology of electron-positron pair creation and annihilation

In Section 7 of the review we focus on the phenomenology of electron–positron pair creation and annihilation experiments. There are three different aspects which are examined: the verification of the process (3.0.1) initially studied by Dirac, the process (3.7.1) studied by Breit and Wheeler, and then the classical work of vacuum polarization process around a supercritical nucleus, following the Sauter, Euler, Heisenberg and Schwinger work. We first recall in Section 7.1 how the process (3.0.1) predicted by Dirac was almost immediately discovered by Klemperer Klemperer (1934). Following this discovery the electron–positron collisions have become possibly the most prolific field of research in the domain of particle physics. The crucial step experimentally was the creation of the first electron–positron collider the "Anello d'Accumulazione" (AdA) was built by the theoretical proposal of Bruno Touschek in Frascati (Rome) in 1960 Bernardini (2004). Following the success of AdA (luminosity $\sim 10^{25}/(\text{cm}^2\ \text{sec})$, beam energy $\sim$0.25GeV), it was decided to build in the Frascati National Laboratory a storage ring of the same kind, Adone. Electron-positron colliders have been built and proposed for this purpose all over the world (CERN, SLAC, INP, DESY, KEK and IHEP).

The aim here is just to recall the existence of this enormous field of research which appeared following the original Dirac idea. In the review the main cross-sections (7.1.1) and (7.1.2) are recalled and the diagram (Fig. 7.1) summarizing this very great success of particle physics is presented. While the Dirac process (3.0.1) has been by far one of the most prolific in physics, the Breit–Wheeler process (3.7.1) has been one of the most elusive for direct observations. In Earth-bound experiments the major effort today is directed to evidence this phenomenon in very strong and coherent electromagnetic field in lasers. In this process collision of many photons may lead in the future to pair creation. This topic is discussed in Section 7.2. Alternative evidence for the Breit–Wheeler process can come from optically thick electron–positron plasma which may be created either in the future in Earth-bound experiments, or currently observed in astrophysics, see Section 10. One additional way to probe the existence of the Breit–Wheeler process is by establishing in astrophysics an upper limits to observable high-energy photons, as a function of distance, propagating in the Universe as pioneered by Nikishov Nikishov (1961), see Section 7.4. We then recall in Section 7.3 how the crucial experimental breakthrough came from the idea of John Madey Deacon et al. (1977) of self-amplified spontaneous emission in an undulator, which results when charges interact with the synchrotron radiation they emit (Tremaine et al. (2002)). Such X-ray free electron lasers have been constructed among others at DESY and SLAC and focus energy onto a small spot hopefully with the size of the X-ray laser wavelength $\lambda \simeq O(0.1)$nm (Nuhn and Pellegrini (2000)), and obtain a very large electric field $E \sim 1/\lambda$, much larger than those obtainable with any optical laser of the same power. This technique can be used to achieve a very strong electric field near to its critical value for observable electron–positron pair production in vacuum. No pair can be created by a single laser beam. It is then assumed that each X-ray laser pulse is split into two equal parts and recombined to form a standing wave with a frequency ω. We then recall how for a laser pulse with wavelength λ about $1\mu m$ and the theoretical diffraction limit $\sigma_{\rm laser} \simeq \lambda$ being reached, the critical intensity laser beam would be

$$I^c_{\rm laser} = \frac{c}{4\pi}E_c^2 \simeq 4.6 \cdot 10^{29} W/\text{cm}^2.$$

In review Section 7.2.1 we recall the theoretical formula for the probability of pair production in time-alternating electric field in two limiting cases of large frequency and small frequency. It is interesting that in the limit of large field and small frequency the production rate approach the one of the Sauter, Heisenberg, Euler and Schwinger, discussed in Section 5. In the following Section 7.2.2 we recall the actually reached experimental limits quoted by Ringwald Ringwald (2001) for a X-ray laser and give a reference to the relevant literature. In Section 7.2.3 we summarize some of the most recent theoretical estimates for pair production by a circularly polarized laser beam by Narozhny, Popov and their collaborators. In this case the field invariants (3.5.23) are not vanishing and pair creation can be achieved by a single laser beam. They computed the total number of electron–positron pairs produced as a function of intensity and focusing parameter of the laser. Particularly interesting is their analysis of the case of two counter-propagating focused laser pulses with circular polarizations, pair production becomes experimentally observable when the laser intensity $I_{\rm laser} \sim 10^{26} W/{\rm cm}^2$ for each beam, which is about $1 \sim 2$ orders of magnitude lower than for a single focused laser pulse, and more than 3 orders of magnitude lower than the critical intensity (7.2.4). Equally interesting are the considerations which first appear in treating this problem that the back reaction of the pairs created on the field has to be taken into due account. We give the essential references and we will see in Section 9 how indeed this feature becomes of paramount importance in the field of astrophysics. We finally review in Section 7.2.4 the technological situation attempting to increase both the frequency and the intensity of laser beams.

The difficulty of evidencing the Breit–Wheeler process even when the high-energy photon beams have a center of mass energy larger than the energy-threshold $2m_ec^2 = 1.02$ MeV was clearly recognized since the early days. We discuss the crucial role of the effective nonlinear terms originating in strong electromagnetic laser fields: the interaction needs not to be limited to initial states of two photons Reiss (1962, 1971). A collective state of many interacting laser photons occurs. We turn then in Section 7.3 of the review to an even more complex and interesting procedure: the interaction of an ultrarelativistic electron beam with a terawatt laser pulse, performed at SLAC Kotseroglou et al. (1996), when strong electromagnetic fields are involved. A first nonlinear Compton scattering process occurs in which the ultrarelativistic electrons absorb multiple photons from the laser field and emit a single photon via the process (5.9.1). The theory of this process has been given in Section 5.10. The second is a drastically improved Breit–Wheeler process (5.9.2) by which the high-energy photon γ, created in the first process, propagates through the laser field and interacts with laser photons $n\omega$ to produce an electron–positron pair Burke et al. (1997). In Section 7.3.1 we describe the status of this very exciting experiments which give the first evidence for the observation in the laboratory of the Breit–Wheeler process although in a somewhat indirect form. Having determined the theoretical basis as well as attempts to verify experimentally the Breit–Wheeler formula we turn in Section 7.4 to a most important application of the Breit–Wheeler process in the framework of cosmology. As pointed out by Nikishov Nikishov (1961) the existence of background photons in cosmology puts a stringent cutoff on the maximum trajectory of the high-energy photons in cosmology.

Having reviewed both the theoretical and observational evidence of the Dirac and Breit–Wheeler processes of creation and annihilation of electron–positron pairs we turn then to one of the most conspicuous field of theoretical and experimental physics dealing with the process of electron–positron pair creation by vacuum polarization in the field of a heavy nuclei. This topic has originated one of the vastest experimental and theoretical physics activities in the last forty years, especially by the process of collisions of heavy ions. We first review in Section 7.5 of the article the $Z = 137$ catastrophe, a collapse to the center, in semi-classical approach, following the Pomeranchuk work Pomeranchuk and Smorodinskii (1945) based on the imposing the quantum conditions on the classical treatment of the motion of two relativistic particles in circular orbits. We then proceed showing in Section 7.5.3 how the introduction of the finite size of the nucleus, following the classical work of Popov and Zeldovich Zeldovich and Popov (1971), leads to the critical charge of a nucleus of $Z_{cr} = 173$ above which a bare nucleus would lead to the level crossing between the bound state and negative energy states of electrons in the field of a bare nucleus. We then review in Section 7.5.5 the recent theoretical progress in analyzing the pair creation process in a Coulomb field, taking into account radial dependence and time variability of electric field. We finally recall in Section 7.6 the attempt to use heavy-ion collisions to form transient superheavy "quasimolecules": a long-lived metastable nuclear complex with $Z > Z_{cr}$. It was expected that the two heavy ions of charges respectively Z_1 and Z_2 with $Z_1 + Z_2 > Z_{cr}$ would reach small inter-nuclear distances well within the electron's orbiting radii. The electrons would not distinguish between the two nuclear centers and they would evolve as if they were bounded by nuclear "quasimolecules" with nuclear charge $Z_1 + Z_2$. Therefore, it was expected that electrons would evolve quasi-statically through a series of well defined nuclear "quasimolecules" states in the two-center field of the nuclei as the inter-nuclear separation decreases and then increases again. When heavy-ion collision occurs the two nuclei come into contact and some deep inelastic reaction occurs determining the duration Δt_s of this contact. Such "sticking time" is expected to depend on the nuclei involved in the reaction and on the beam energy. Theoretical attempts have been proposed to study the nuclear aspects of heavy-ion collisions at energies very close to the Coulomb barrier and search for conditions, which would serve as a trigger for prolonged nuclear reaction times, to enhance the amplitude of pair production. The sticking time Δt_s should be larger than $1 \sim 2 \cdot 10^{-21}$ sec Greiner and Reinhardt (1999) in order to have significant pair production. Up to now no success has been achieved in justifying theoretically such a long sticking time. In reality the characteristic sticking time has been found of the order of $\Delta t \sim 10^{-23}$ sec, hundred times shorter than the needed to activate the pair creation process. We finally recall in Section 7.6.2 of the review the Darmstadt-Brookhaven dialogue between the Orange and the Epos groups and the Apex group at Argonne in which the claim for discovery of electron–positron pair creation by vacuum polarization in heavy-ion collisions was finally retracted. Out of the three fundamental processes addressed in this report, the Dirac electron–positron annihilation and the Breit–Wheeler electron–positron creation from two photons have found complete theoretical descriptions within Quantum Electro-Dynamics. The first one is very likely the best tested process in physical science, while the second has finally obtained the first indirect experimental evidence. The third process, the one of the vacuum polarization studied by Sauter, Euler, Heisenberg and Schwinger, presents in Earth-bound experiments presents a situation “terra incognita”.

3.7. The Breit-Wheeler cutoff in high-energy Gamma-rays

The Breit-Wheeler process for the photon-photon pair production is one of most relevant elementary processes in high energy astrophysics (see review Sec. 7.4). In addition to the importance of this process in dense radiation fields of compact objects (Bonometto and Rees, 1971), the essential role of this process in the context of intergalactic absorption of high-energy γ-rays was first pointed out by Nikishov (Nikishov, 1961; Gould and Schréder, 1967). The spectra of TeV radiation observed from distant ($d > 100$ Mpc) extragalactic objects suffer essential deformation during the passage through the intergalactic medium, caused by energy-dependent absorption of primary γ-rays at interactions with the diffuse extragalactic background radiation, for the optical depth $\tau_{\gamma\gamma}$ most likely significantly exceeding one (Gould and Schréder, 1967; Stecker et al., 1992; Vassiliev, 2000; Coppi and Aharonian, 1999). A relevant broad-band information about the cosmic background radiation (CBR) is important for the interpretation of the observed high-energy γ spectra (Aharonian et al., 2000; Kneiske et al., 2002; Dwek and Krennrich, 2005; Aharonian et al., 2006). For details see Hauser and Dwek (2001); Aharonian (2003). In this section, we are particularly interested in such absorption effect of high-energy γ-ray, originated from cosmological sources, interacting with the Cosmic Microwave Background (CMB) photons. Fazio and Stecker (Fazio and Stecker, 1970; Stecker et al., 1977) were the first who calculated the cutoff energy versus redshift for cosmological γ-rays. This calculation was applied to further study of the optical depth of the Universe to high-energy γ-rays (MacMinn and Primack, 1996; Kneiske et al., 2004; Stecker et al., 2006). With the Fermi telescope, such study turns out to be important to understand the spectrum of high-energy γ-ray originated from GRBs' sources at cosmological distance, we therefore offer the details of theoretical analysis as follow.

Breit-Wheeler cross-section in arbitrary frame

Breit and Wheeler (1934) studied the process

$$\gamma_1 + \gamma_2 \to e^+ + e^-, \tag{3.7.1}$$

in the center of mass of the system, the momenta of the electron and positron are equal and opposite $\mathbf{p}_1 = -\mathbf{p}_2$. The same thing holds for the momenta of the photons in the initial state: $\mathbf{k}_1 = -\mathbf{k}_2$. As a consequence, the energies of electron and positron are equal: $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}$, and so are the energies of the photons: $\hbar\omega_1 = \hbar\omega_2 = \mathcal{E}_\gamma = \mathcal{E}$. They found the total cross-section in the center of mass of the system:

$$\sigma_{\gamma\gamma} = \frac{\pi}{2}\left(\frac{\alpha\hbar}{m\,c}\right)^2 (1-\hat{\beta}^2)\left[2\hat{\beta}(\hat{\beta}^2-2)+(3-\hat{\beta}^4)\ln\left(\frac{1+\hat{\beta}}{1-\hat{\beta}}\right)\right], \quad \text{with} \quad \hat{\beta} = \frac{c|\mathbf{p}|}{\mathcal{E}}, \tag{3.7.2}$$

where $\mathbf{p}$ and $\hat{\beta}$ are respectively momentum and the reduced velocity of an electron or positron. The necessary kinematic condition in order for the process (3.7.1) taking place is that the energy of two colliding photons is larger than the energetic threshold $2m_ec^2$, i.e.,

$$\mathcal{E}_\gamma > m_ec^2. \tag{3.7.3}$$

The cross-section in line (3.7.2) can be easily generalized to an arbitrary reference frame $\mathcal{K}$, in which the two photons k_1 and k_2 are moving in opposite directions; for Lorentz invariance of $(k_1 \cdot k_2)$, one has $\omega_1\omega_2 = \mathcal{E}_\gamma^2$. Since

$$\mathcal{E}_\gamma = \mathcal{E} = m_ec^2/\sqrt{1-\hat{\beta}^2}, \tag{3.7.4}$$

to obtain the total cross-section in the arbitrary frame $\mathcal{K}$, we must therefore make the following substitution (Landau and Lifshitz, 1975),

$$\hat{\beta} \to \sqrt{1 - m_e^2c^4/(\omega_1\omega_2)}, \tag{3.7.5}$$

in Eq. (3.7.2). For $\mathcal{E} \gg m_ec^2$, the total effective cross-section is approximately proportional to

$$\sigma_{\gamma\gamma} \simeq \pi\left(\frac{\alpha\hbar}{m_ec}\right)^2\left(\frac{m_ec^2}{\mathcal{E}}\right)^2 = \pi r_e^2\left(\frac{m_ec^2}{\mathcal{E}}\right)^2, \tag{3.7.6}$$

where $r_e = \left(\frac{\alpha\hbar}{m_ec}\right)$ is the electron classical radius and $\pi r_e^2 \simeq 2.5 \cdot 10^{-25}\text{cm}^2$.

Opacity of high-energy GRB photons colliding with CMB photons

We study the Breit-Wheeler process (3.7.1) to the case that high-energy GRB photons ω_1, originated from GRBs sources at cosmological distance z, on their way traveling to us, collide with CMB photons ω_2 in the rest frame of CMB photons, leading to electron-positron pair production. We calculate the opacity and mean free-path of these high-energy GRB photons, find the energy-range of absorption as a function of the cosmological red-shift z.

In general, a high-energy GRB photon with a give energy ω_1, collides with background photons in all possible energies ω_2. We assume that i-type background photons have the spectrum distribution $f_i(\omega_2/T_i)$, where T_i is the characteristic energy scale of the distribution, the opacity is then given by

$$\tau^i_{\gamma\gamma}(\omega_1, z) = \int dr \int_{m_e^2c^4/\omega_1}^{\infty} \frac{\omega_2^2 d\omega_2}{\pi^2} f_i(\omega_2/T_i)\sigma_{\gamma\gamma}(\frac{\omega_1\omega_2}{m_e^2c^4}), \tag{3.7.7}$$

where $m_e^2c^4/\omega_1$ is the energy-threshold (3.7.3) above which the Breit-Wheeler process (3.7.1) can occurs and the cross-section $\sigma_{\gamma\gamma}(x)$ is given by Eqs. (3.7.2), depending only on $x = \frac{\omega_1\omega_2}{m_e^2c^4}$. The total opacity is then given by

$$\tau^{\rm total}_{\gamma\gamma}(\omega_1, z) = \sum_i \tau^i_{\gamma\gamma}(\omega_1, z), \tag{3.7.8}$$

which the sum is over all types of photon background in the Universe. The high-energy photons traveling path $\int dr$ is given by ,

$$\int_t^{t_0} \frac{dt'}{R(t')} = \int_0^{r(t)} \frac{dr}{(1-kr^2)^{1/2}} = \int_0^{r(t)} dr, \tag{3.7.9}$$

where $R(t)$ is the scalar factor, t_0 is the present time and t corresponds to epoch of the red-shift z for a flat ($k = 0$) Freemann Universe. Using the relationship $z + 1 = R_0/R(t)$, we change integrand variable from t' to the red-shift z,

$$dt' = -\frac{dz}{(z'+1)H(z')}, \tag{3.7.10}$$

so that we have

$$\int_0^{r(t)} dr = \int_t^{t_0} \frac{dt'}{R(t')} = \frac{1}{R_0}\int_0^z \frac{dz}{H(z)}, \tag{3.7.11}$$

where $H(z) = \dot{R}(t)/R(t_0)$ is the Hubble function, obeyed the Friedmann equation

$$H(z) = H_0[\Omega_M(z+1)^3 + \Omega_\Lambda]^{1/2}, \quad \Omega_M + \Omega_\Lambda = 1, \tag{3.7.12}$$

$\Omega_M \simeq 0.3$ and $\Omega_M \simeq 0.7$.

In the case of CMB photons in a black-body distribution $1/(e^{\omega_2/T} - 1)$ with the temperature T, the opacity is given by

$$\tau_{\gamma\gamma}(\omega_1, z) = \int dr \int_{m_e^2c^4/\omega_1}^{\infty} \frac{d\omega_2}{\pi^2} \frac{\omega_2^2}{e^{\omega_2/T} - 1} \sigma_{\gamma\gamma}(\frac{\omega_1\omega_2}{m_e^2c^4}), \tag{3.7.13}$$

where the Boltzmann constant $k_B = 1$. To simply Eq. (3.7.13), we set $x = \frac{\omega_1\omega_2}{m_e^2c^4}$,

$$\tau_{\gamma\gamma}(\omega_1, z) = \int dr \left(\frac{m_e^2c^4}{\omega_1}\right)^3 \int_1^\infty \frac{dx}{\pi^2} \frac{x^2}{\exp\frac{xm_e^2c^4}{\omega_1 T} - 1}\sigma_{\gamma\gamma}(x). \tag{3.7.14}$$

In terms of CMB temperature and GRB-photons energy at the present time,

$$T = (z+1)T^0; \quad \omega_{1,2} = (z+1)\omega_{1,2}^0, \tag{3.7.15}$$

we obtain,

$$\tau_{\gamma\gamma}(\omega_1^0, z) = \frac{1}{R_0}\int_0^z \frac{dz'}{H(z')}\frac{1}{(z+1)^3}\left(\frac{m_e^2c^4}{\omega_1^0}\right)^3 \int_1^\infty \frac{dx}{\pi^2}\frac{x^2}{\exp(x/\theta)-1}\sigma_{\gamma\gamma}(x), \tag{3.7.16}$$

where

$$\theta = x^0(z+1)^2; \quad x^0 = \frac{\omega_1^0 T^0}{m_e^2c^4}, \tag{3.7.17}$$

and x^0 is the energy ω_1^0 in unit of $m_ec^2(m_ec^2/T^0) = 1.15\cdot 10^{15}$eV. For the purpose of numerical calculations, we rewrite the expression,

$$\begin{aligned}\tau_{\gamma\gamma}(x^0, z) = {} & \frac{\pi r_e^2}{R_0H_0/c}\left(\frac{T^0}{x^0}\right)^3 \int_0^z \frac{dz'}{[\Omega_M(z'+1)^3+\Omega_\Lambda]^{1/2}}\frac{1}{(z'+1)^3}\times \\ & \times\int_1^\infty \frac{dx}{2\pi^2}\frac{x^2 f_{\gamma\gamma}(x)}{\exp(x/\theta)-1} = \\ & = \frac{23.8}{R_0h}\left(\frac{1}{x^0}\right)^3 \int_0^z \frac{dz'}{[\Omega_M(z'+1)^3+\Omega_\Lambda]^{1/2}}\frac{1}{(z'+1)^3}\times \\ & \times\int_1^\infty \frac{dx}{2\pi^2}\frac{x^2 f_{\gamma\gamma}(x)}{\exp(x/\theta)-1},\end{aligned} \tag{3.7.18}$$

where $R_0 = 1$, present Hubble constant $h = H_0/100\text{km}/sec/\text{Mpc}$ and

$$f_{\gamma\gamma}(x) \;=\; (1-\hat\beta^2)\Big[2\hat\beta(\hat\beta^2-2) + (3-\hat\beta^4)\ln\big(\frac{1+\hat\beta}{1-\hat\beta}\big)\Big], \quad \hat\beta = \sqrt{1-1/x}.$$

The $\tau_{\gamma\gamma}(\omega_1^0, z) = 1$ give the relationship $\omega_1^0 = \omega_1^0(z)$ that separates the absorbed regime $\tau_{\gamma\gamma}(\omega_1^0, z) > 1$ and transparent regime $\tau_{\gamma\gamma}(\omega_1^0, z) < 1$ in the $\omega_1^0 - z$ plane.

The numerical result is shown in Fig. 3.1. It clearly shows the following properties:

1. for the redshift z smaller than a critical value $z_c \simeq 0.1$ ($z < z_c$), the CMB photons are transparent $\tau_{\gamma\gamma}(\omega_1^0, z) < 1$ to GRB photons in any energy

bands, this indicates a minimal mean-free path of photons traveling in CMB photons background;

2. for the redshift z larger than the value ($z > z_c$), there are two branches of solutions for $\tau_{\gamma\gamma}(\omega_1^0, z) = 1$, respectively corresponding to the different energy-dependence of the cross-section (3.7.2): the cross-section increases with the center-mass-energy $x = \mathcal{E}_\gamma^2/(m_e c^2)^2$ from the energy-threshold $x = 1$ to $x \simeq 1.99$, and decreases (3.7.6) from $x \simeq 1.99$ to $x \to \infty$. The turn point ($z \simeq 0.1, \omega_1^0 \simeq 1.15 \cdot 10^{15}$eV) from one solution to another is determined by the maximal cross-section at $x \simeq 1.99$. Due to these two solutions, CMB photons are transparent to GRB photons of large and small energies, opaque to those GRB photons in an intermediate energy-range large for a given finite z-value;

3. CMB photons are transparent to very low-energy GRB photons $\omega_1^0 < 10^{12}$eV, i.e., $x^0 < 10^{-3}$, due to their energies are below the energetic threshold for the Breit-Wheeler process (3.7.1). In addition, CMB photons are transparent to very large-energy GRB photons $\omega_1^0 > 10^{18}$eV, i.e., $x^0 > 10^3$, due to the cross-section of Breit-Wheeler process (3.7.1) is very small for extremely high-energy photons. For very large $z \sim 10^3$, the Universe becomes completely opaque and photon distribution cannot be described by the black body spectrum, we disregard this regime.

Due to the fact that there are other radiation backgrounds (3.7.7), the background of CMB photons gives the lowest bound of opacity, absorption limit, to GRB photons with respect to the Breit-Wheeler process (3.7.1). Finally, we point out that Fazio and Stecker (Fazio and Stecker, 1970; Stecker et al., 1977) gave only asymptotic form of small-energy solution indicated in Fig. (3.1).

3.8. The extraction of blackholic energy from a black hole by vacuum polarization processes

We turn then to astrophysics, where, in the process of gravitational collapse to a black hole and in its outcomes these three processes will be for the first time verified on a much larger scale, involving particle numbers of the order of 10^{60}, seeing both the Dirac process and the Breit–Wheeler process at work in symbiotic form and electron–positron plasma created from the "blackholic energy" during the process of gravitational collapse. It is becoming more and more clear that the gravitational collapse process to a Kerr–Newman black hole is possibly the most complex problem ever addressed in physics and astrophysics. What is most important for this report is that it gives for the first time the opportunity to see the above three processes simultaneously at work under ultrarelativistic special and general relativistic regimes. The process of gravitational collapse is characterized by the

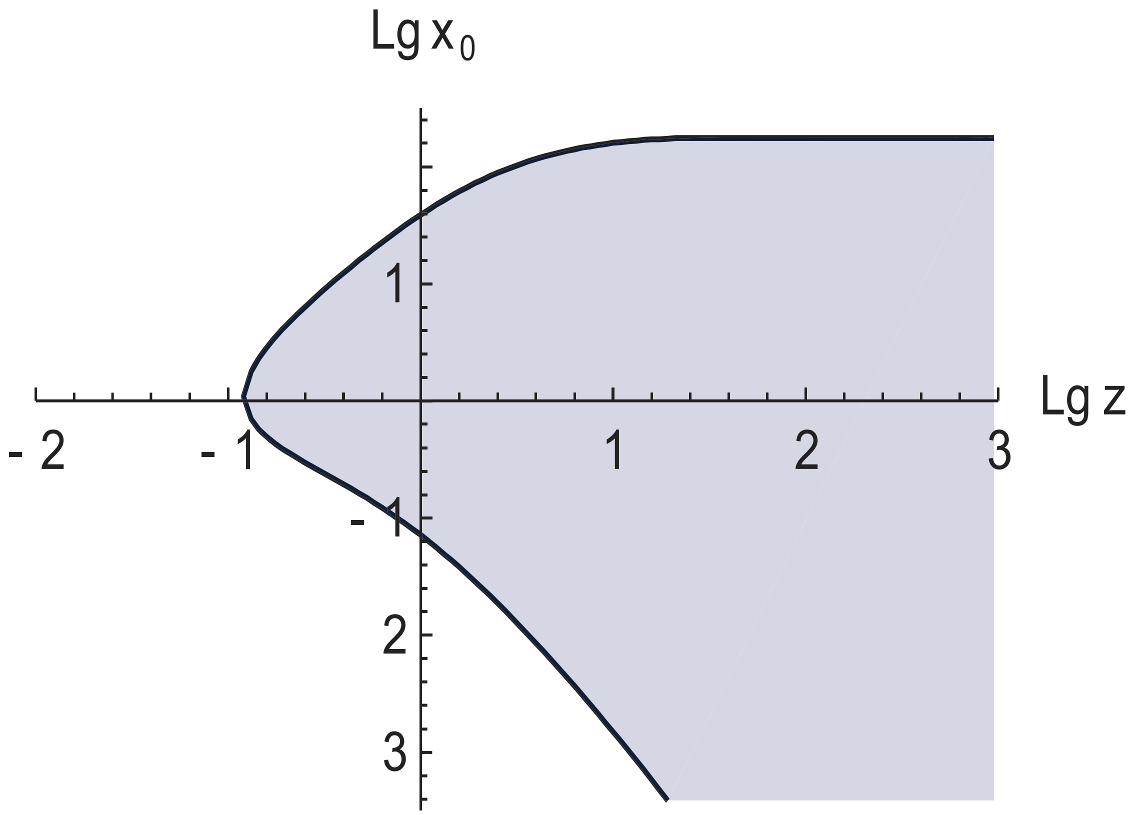

Figure 3.1.: This is a Log-Log plot for GRB photon energy x^0 (in unit of $1.11 \cdot 10^{15}$) vs redshift z. For $z > z_c \simeq 0.1$, the line that bounds shadow area indicates two solutions for the opacity $\tau_{\gamma\gamma} = 1$: (i) large-energy solution for $\omega_1^0 > 1.15 \cdot 10^{15}$eV; (ii) small-energy solution for $\omega_1^0 < 1.15 \cdot 10^{15}$eV, which separate the optically thick regime (shadow area) $\tau_{\gamma\gamma}(\omega_1^0, z) > 1$ and optically thin regime $\tau_{\gamma\gamma}(\omega_1^0, z) < 1$.

timescale $\Delta t_g = GM/c^3 \simeq 5 \cdot 10^{-6} M/M_\odot$ sec and the energy involved are of the order of $\Delta E = 10^{54} M/M_\odot$ ergs. It is clear that this is one of the most energetic and most transient phenomena in physics and astrophysics and needs for its correct description such a highly time varying treatment. Our approach in Section 8 is to gain understanding of this process by separating the different components and describing 1) the basic energetic process of an already formed black hole, 2) the vacuum polarization process of an already formed black hole, 3) the basic formula of the gravitational collapse recovering the Tolman-Oppenheimer-Snyder solutions and evolving to the gravitational collapse of charged and uncharged shells. This will allow among others to obtain a better understanding of the role of irreducible mass of the black hole and the maximum blackholic energy extractable from the gravitational collapse. We will as well address some conceptual issues between general relativity and thermodynamics which have been of interest to theoretical physicists in the last forty years. Of course in these brief chapter we will be only recalling some of these essential themes and refer to the literature where in-depth analysis can be found. In Section 8.1 we recall the Kerr–Newman metric and the associated electromagnetic field. We then recall the classical work of Carter Carter (1968) integrating the Hamilton-Jacobi equations for charged particle motions in the above given metric and electromagnetic field. We then recall in Section 8.2 the introduction of the effective potential techniques in order to obtain explicit expression for the trajectory of a particle in a Kerr–Newman geometry, and especially the introduction of the reversible–irreversible transformations which lead then to the Christodoulou-Ruffini mass formula of the black hole

$$M^2c^4 = \left(M_{\rm ir}c^2 + \frac{c^2Q^2}{4GM_{\rm ir}} \right)^2 + \frac{L^2c^8}{4G^2M_{\rm ir}^2},$$

where $M_{\rm ir}$ is the irreducible mass of a black hole, Q and L are its charge and angular momentum. We then recall in article Section 8.3 the positive and negative root states of the Hamilton–Jacobi equations as well as their quantum limit. We finally introduce in Section 8.4 the vacuum polarization process in the Kerr–Newman geometry as derived by Damour and Ruffini Damour and Ruffini (1975) by using a spatially orthonormal tetrad which made the application of the Schwinger formalism in this general relativistic treatment almost straightforward. We then recall in Section 8.5 the definition of a dyadosphere in a Reissner–Nordström geometry, a region extending from the horizon radius

$$r_+ \quad = \quad 1.47 \cdot 10^5 \mu (1 + \sqrt{1 - \xi^2}) \text{ cm}$$

out to an outer radius

$$r^\star = \left(\frac{\hbar}{m_e c}\right)^{1/2}\left(\frac{GM}{c^2}\right)^{1/2}\left(\frac{m_p}{m_e}\right)^{1/2}\left(\frac{e}{q_p}\right)^{1/2}\left(\frac{Q}{\sqrt{G}M}\right)^{1/2} = \\ = 1.12 \cdot 10^8 \sqrt{\mu\xi} \text{ cm},$$

where the dimensionless mass and charge parameters $\mu = \frac{M}{M_\odot}$, $\xi = \frac{Q}{(M\sqrt{G})} \leq 1$. In Section 8.6 of the review the definition of a dyadotorus in a Kerr–Newman metric is recalled. We have focused on the theoretically well defined problem of pair creation in the electric field of an already formed black hole. Having set the background for the blackholic energy we recall some fundamental features of the dynamical process of the gravitational collapse. In Section 8.7 we address some specific issues on the dynamical formation of the black hole, recalling first the Oppenheimer-Snyder solution Oppenheimer and Snyder (1939) and then considering its generalization to the charged nonrotating case using the classical work of W. Israel and V. de la Cruz Israel (1966); De la Cruz and Israel (1967). In Section 8.7.1 we recover the classical Tolman-Oppenheimer-Snyder solution in a more transparent way than it is usually done in the literature. In the Section 8.7.2 we are studying using the Israel-de la Cruz formalism the collapse of a charged shell to a black hole for selected cases of a charged shell collapsing on itself or collapsing in an already formed Reissner–Nordström black hole. Such elegant and powerful formalism has allowed to obtain for the first time all the analytic equations for such large variety of possibilities of the process of the gravitational collapse. The theoretical analysis of the collapsing shell considered in the previous section allows to reach a deeper understanding of the mass formula of black holes at least in the case of a Reissner–Nordström black hole. This allows as well to give in Section 8.8 of the review an expression of the irreducible mass of the black hole only in terms of its kinetic energy of the initial rest mass undergoing gravitational collapse and its gravitational energy and kinetic energy T_+ at the crossing of the black hole horizon r_+

$$M_{\text{ir}} = M_0 - \tfrac{M_0^2}{2r_+} + T_+.$$

Similarly strong, in view of their generality, are the considerations in Section 8.8.2 which indicate a sharp difference between the vacuum polarization process in an overcritical $E \gg E_c$ and undercritical $E \ll E_c$ black hole. For $E \gg E_c$ the electron–positron plasma created will be optically thick with average particle energy 10 MeV. For $E \ll E_c$ the process of the radiation will be optically thin and the characteristic energy will be of the order of 10^{21} eV. This argument will be further developed in a forthcoming report. In Section 8.9 we show how the expression of the irreducible mass obtained in the previous Section leads to a theorem establishing an upper limit to 50% of the total mass energy initially at rest at infinity which can be extracted from any process of gravitational collapse independent of the details. These results also lead to some general considerations which have been sometimes claimed in reconciling general relativity and thermodynamics.

3.9. Thermalization of the mildly relativistic pair plasma

We then turn in Section 10 of the review to the last physical process needed in ascertaining the reaching of equilibrium of an optically thick electron–positron plasma. The average energy of electrons and positrons we illustrate is $0.1 < \epsilon < 10$ MeV. These bounds are necessary from the one hand to have significant amount of electron–positron pairs to make the plasma optically thick, and from the other hand to avoid production of other particles such as muons. As we will see in the next report these are indeed the relevant parameters for the creation of ultrarelativistic regimes to be encountered in pair creation process during the formation phase of a black hole. We then review the problem of evolution of optically thick, nonequilibrium electron–positron plasma, towards an equilibrium state, following Aksenov et al. (2007, 2008). These results have been mainly obtained by two of us (RR and GV) in recent publications and all relevant previous results are also reviewed in this Section 10. We have integrated directly relativistic Boltzmann equations with all binary and triple interactions between electrons, positrons and photons two kinds of equilibrium are found: kinetic and thermal ones. Kinetic equilibrium is obtained on a timescale of few $(\sigma_T n_\pm c)^{-1}$, where σ_T and $n_\pm$ are Thomson's cross-section and electron–positron concentrations respectively, when detailed balance is established between all binary interactions in plasma. Thermal equilibrium is reached on a timescale of few $(\alpha\sigma_T n_\pm c)^{-1}$, when all binary and triple, direct and inverse interactions are balanced. In Section 10.1 basic plasma parameters are illustrated. The computational scheme as well as the discretization procedure are discussed in Section 10.2. Relevant conservation laws are given in Section 10.3. Details on binary interactions, consisting of Compton, Møller and Bhabha scatterings, Dirac pair annihilation and Breit–Wheeler pair creation processes, and triple interactions, consisting of relativistic bremsstrahlung, double Compton process, radiative pair production and three photon annihilation process, are presented in Section 10.5 and 10.6, respectively. In Section 10.5 collisional integrals with binary interactions are computed from first principles, using QED matrix elements. In Section 10.7 Coulomb scattering and the corresponding cutoff in collisional integrals are discussed. Numerical results are presented in Section 10.8 where the time dependence of energy and number densities as well as chemical potential and temperature of electron–positron-photon plasma is shown, together with particle spectra. The most interesting result of this analysis is to have differentiate the role of binary and triple interactions. The detailed balance in binary interactions following the classical work of Ehlers Ehlers (1973) leads to a distribution function of the form of the Fermi-Dirac for electron–positron pairs or of the Bose-Einstein for the photons. This is the reason we refer in the text to such conditions as the Ehlers equilibrium conditions. The crucial role of the direct and inverse three-body interactions is well summarized in fig. 10.1, panel A from which it is clear that the inverse three-body interactions are essential in reaching thermal equilibrium. If the latter are neglected, the system deflates to the creation of electron–positron pairs all the way down to the threshold of 0.5MeV. This last result which is referred as the Cavallo–Rees scenario Cavallo and Rees (1978) is simply due to improper neglection of the inverse triple reaction terms (see Appendix 10).

3.10. Plasma oscillations in uniform electric fields

The conditions encountered in the vacuum polarization process around black holes lead to a number of electron–positron pairs created of the order of 10^{60} confined in the dyadosphere volume, of the order of a few hundred times to the horizon of the black hole. Under these conditions the plasma is expected to be optically thick and is very different from the nuclear collisions and laser case where pairs are very few and therefore optically thin. We turn then in Section 9, to discuss a new phenomenon: the plasma oscillations, following the dynamical evolution of pair production in an external electric field close to the critical value. In particular, we will examine: (i) the back reaction of pair production on the external electric field; (ii) the screening effect of pairs on the electric field; (iii) the motion of pairs and their interactions with the created photon fields. In review Secs. 9.1 and 9.2, we review semi-classical and kinetic theories describing the plasma oscillations using respectively the Dirac-Maxwell equations and the Boltzmann-Vlasov equations. The electron–positron pairs, after they are created, coherently oscillate back and forth giving origin to an oscillating electric field. The oscillations last for at least a few hundred Compton times. We review the damping due to the quantum decoherence. The energy from collective motion of the classical electric field and pairs flows to the quantum fluctuations of these fields. This process is quantitatively discussed by using the quantum Boltzmann-Vlasov equation in Sections 9.4 and 9.5. The damping due to collision decoherence is quantitatively discussed in Sections 9.6 and 9.7 by using Boltzmann-Vlasov equation with particle collisions terms. This damping determines the energy flows from collective motion of the classical electric field and pairs to the kinetic energy of non-collective motion of particles of these fields due to collisions. In Section 9.7, we particularly address the study of the influence of the collision processes $e^+e^- \rightleftarrows \gamma\gamma$ on the plasma oscillations in supercritical electric field Ruffini et al. (2003b). It is shown that the plasma oscillation is mildly affected by a small number of photons creation in the early evolution during a few hundred Compton times (see Fig. 9.4 of the review). In the later evolution of 10^{3-4} Compton times, the oscillating electric field is damped to its critical value with a large number of photons created. An equipartition of number and energy between electron–positron pairs and photons is reached (see Fig. 9.4). In Section 9.8, we introduce an approach based on the following three equations: the number density continuity equation, the energy-momentum conservation equation and the Maxwell equations. We describe the plasma oscillation for both overcritical electric field $E > E_c$ and under-critical electric field $E < E_c$ Ruffini et al. (2007b). In additional of reviewing the result well known in the literature for $E > E_c$ we review some novel result for the case $E < E_c$. It was traditionally assumed that electron–positron pairs, created by the vacuum polarization process, move as charged particles in external uniform electric field reaching arbitrary large Lorentz factors. It is reviewed how recent computations show the existence of plasma oscillations of the electron–positron pairs also for $E \lesssim E_c$. For both cases we quote the maximum Lorentz factors $\gamma_{\max}$ reached by the electrons and positrons as well as the length of oscillations. Two specific cases are given. For $E_0 = 10E_c$ the length of oscillations 10 $\hbar/(m_e c)$, and $E_0 = 0.15E_c$ the length of oscillations 10^7 $\hbar/(m_e c)$. We also review the asymptotic behavior in time, $t \to \infty$, of the plasma oscillations by the phase portrait technique. Finally we review some recent results which differentiate the case $E > E_c$ from the one $E < E_c$ with respect to the creation of the rest mass of the pair versus their kinetic energy. For $E > E_c$ the vacuum polarization process transforms the electromagnetic energy of the field mainly in the rest mass of pairs, with moderate contribution to their kinetic energy.

3.11. Dyadosphere formed in gravitational collapses

In Refs. Ruffini et al. (2003b,a), first initiating with supercritical electric fields on the core surface, we study electron-positron pair production and oscillation together with gravitational collapse. We use the exact solution of Einstein–Maxwell equations describing the gravitational collapse of a thin charged shell. Recall that the region of space–time external to the core is Reissner–Nordström with line element

$$ds^2 = -\alpha^2 dt^2 + \alpha^{-2} dr^2 + r^2 d\Omega^2 \tag{3.11.1}$$

in Schwarzschild like coordinate (t, r, θ, ϕ), where $\alpha^2 = 1 - 2M/r + Q^2/r^2$; M is the total energy of the core as measured at infinity and Q is its total

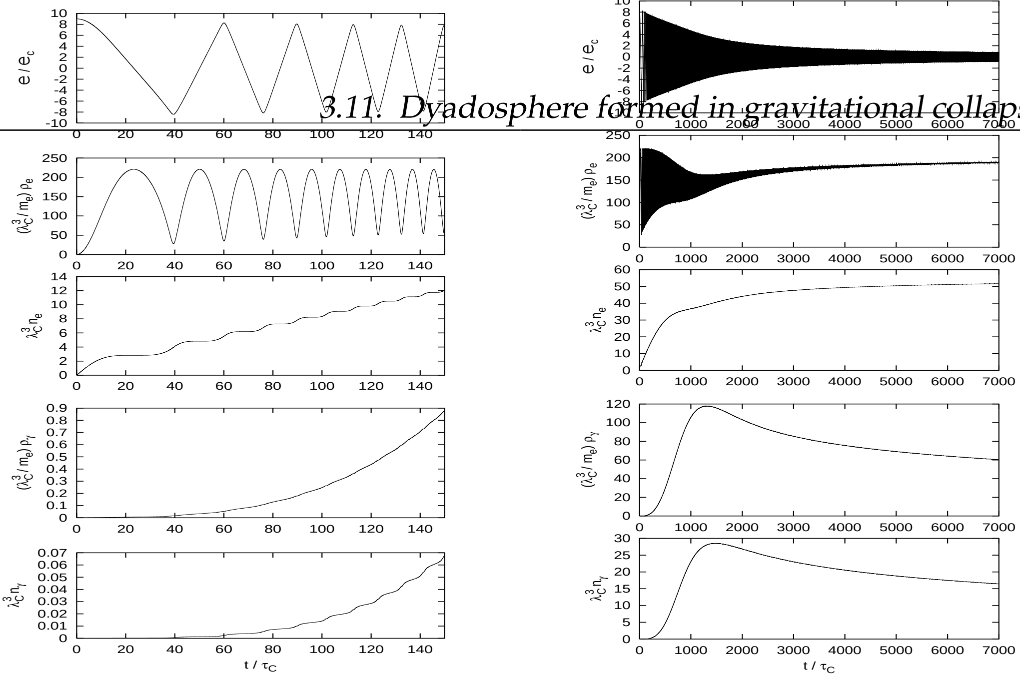

Figure 3.2.: In left figure: We plot for $t < 150\tau_C$, from the top to the bottom panel: a) electromagnetic field strength; b) electrons energy density; c) electrons number density; d) photons energy density; e) photons number density as functions of time. The right figure: We plot for $t < 7000\tau_C$ as the same quantities as in left.

charge. Let us label with r_0 and t_0 the radial and time–like coordinate of the core surface, and the equation of motion of the core is Israel (1966); De la Cruz and Israel (1967); Bekenstein (1971):

$$\frac{dr_0}{dt_0} = -\frac{\alpha^2(r_0)}{\Omega(r_0)}\sqrt{\Omega^2(r_0) - \alpha^2(r_0)}, \quad \Omega(r_0) = \frac{M}{M_0} - \frac{M_0^2+Q^2}{2M_0 r_0}; \tag{3.11.2}$$

M_0 being the rest mass of the shell. The analytical solutions of Eq. (3.11.2) were found $t_0 = t_0(r_0)$, and the core collapse speed $V^*(r_0)$ as a function of r_0 is plotted in Fig. 3.3, where we indicate $V^*_{\rm ds} \equiv V^*|_{r_0=r_{\rm ds}}$ as the velocity of the core at the Dyadosphere radius $r_{\rm ds}$.

We now turn to the pair creation and plasma oscillation taking place in the classical electric and gravitational fields during the gravitational collapse of a charged overcritical stellar core. As already show in Fig. 3.2, (i) the electric field oscillates with lower and lower amplitude around 0; (ii) electrons and positrons oscillates back and forth in the radial direction with ultra relativistic velocity, as result the oscillating charges are confined in a thin shell whose radial dimension is given by the elongation Δl of the oscillations. In Fig. 3.4, we plot the elongation Δl as a function of time and electron mean velocity v as a function of the elongation during the first half period Δt of oscillation. This shows precisely the characteristic time Δt and size Δl of charge confinement due to plasma oscillation.

In the time Δt the charge oscillations prevent a macroscopic current from flowing through the surface of the core. Namely in the time Δt the core moves

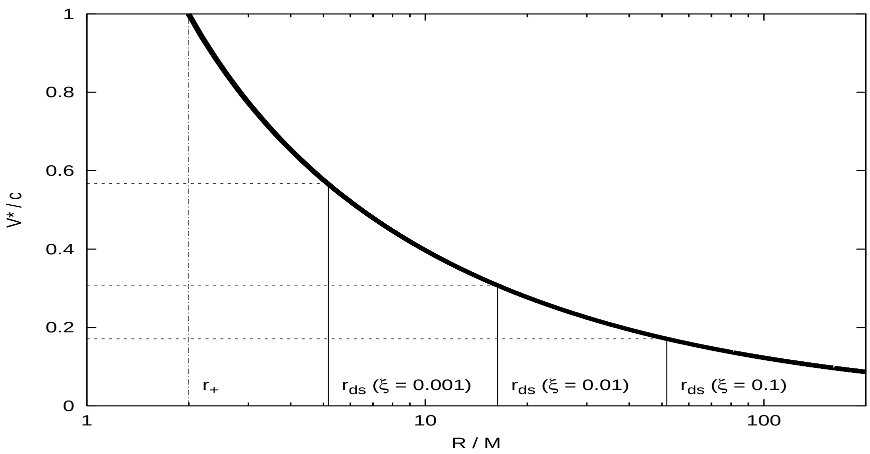

Figure 3.3.: Collapse velocity of a charged stellar core of mass $M_0 = 20M_\odot$ as measured by static observers as a function of the radial coordinate of the core surface. Dyadosphere radii for different charge to mass ratios ($\xi = 10^{-3}, 10^{-2}, 10^{-1}$) are indicated in the plot together with the corresponding velocity.

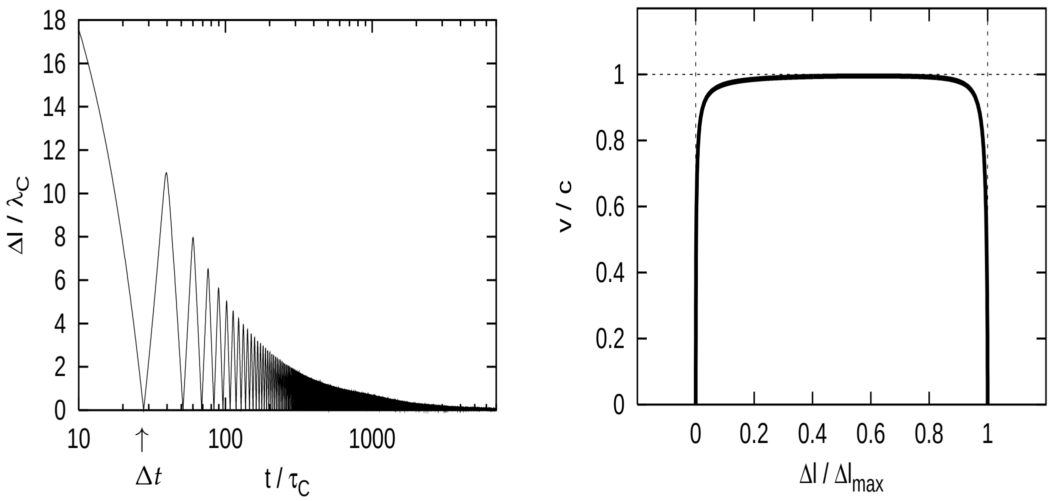

Figure 3.4.: In left figure: Electrons elongation as function of time in the case $r = r_{ds}/3$. The oscillations are damped in a time of the order of $10^3 - 10^4\tau_C$. The right figure: Electrons mean velocity as a function of the elongation during the first half oscillation. The plot summarize the oscillatory behaviour: as the electrons move, the mean velocity grows up from 0 to the speed of light and then falls down at 0 again.

inwards of

$$\Delta r^* = V^* \Delta t \gg \Delta l. \tag{3.11.3}$$

Since the plasma charges are confined within a region of thickness Δl, due to Eq. (3.11.3) no charge "reaches" the surface of the core which can neutralize it and the initial charge of the core remains untouched. For example in the case $M = 20 M_\odot, \xi = 0.1$, and $r = \frac{1}{3} r_{\rm ds}$, we have

$$\Delta l \lesssim 30 \lambda_{\rm C}, \quad \Delta t \sim 10^3 \tau_{\rm C}, \quad V^* \sim 0.3c, \tag{3.11.4}$$

and $\Delta r^* \gg \Delta l$. We conclude that the core is not discharged or, in other words, the electric charge of the core is stable against vacuum polarization and electric field $E = Q/r_0^2$ is amplified during the gravitational collapse. As a consequence, an enormous amount ($N \sim Q r_{\rm ds}/e\lambda_{\rm C}$ as claimed in Refs. Preparata et al. (1998, 2003); Ruffini and Xue (2008b,a)) of pairs is left behind the collapsing core and Dyadosphere Ruffini and Xue (2008a); Preparata et al. (1998, 2003) is formed.

Recently, we study this pair-production process in a neutral collapsing core, rather than a charged collapsing core, as described above. Neutral stellar cores at or over nuclear densities are described by positive charged baryon cores and negative charged electron gas since they possess different masses and interactions (equations of state). In static case, the equilibrium configuration of positive charged baryon cores and negative charged electron gas described by Thomas-Fermi equation shows an overcritical electric field on the surface of baryon core. Based on such an initial configuration and a simplified model of spherically collapsing cores, we approximately integrate the Einstein-Maxwell equations and the equations for the particle number and energy-momentum conservations. It is shown that in gravitational core-collapse, such an electric field dynamically evolves in the space-time and electron-positron pairs are produced and gravitational energy is converted to electron-positron energy. This important result has been published in Physics Review D. The details on this topic can be found in Appendix C.

The e^+e^- pairs generated by the vacuum polarization process around the core are entangled in the electromagnetic field Ruffini et al. (2003a), and thermalize in an electron–positron–photon plasma on a time scale $\sim 10^4 \tau_{\rm C}$ Ruffini et al. (2003b) (see Fig. 3.2). As soon as the thermalization has occurred, the hydrodynamic expansion of this electrically neutral plasma starts Ruffini et al. (1999, 2000). While the temporal evolution of the $e^+e^-\gamma$ plasma takes place, the gravitationally collapsing core moves inwards, giving rise to a further amplified supercritical field, which in turn generates a larger amount of e^+e^- pairs leading to a yet higher temperature in the newly formed $e^+e^-\gamma$ plasma. We report progress in this theoretically challenging process which is marked by distinctive and precise quantum and general relativistic effects. As presented in Ref. Ruffini et al. (2003a): we follow the dynamical phase of the formation of Dyadosphere and of the asymptotic approach to the horizon by examining the time varying process at the surface of the gravitationally collapsing core. The details on this topic can be found in Appendix A

3.12. Plasma oscillations and radiation in nonuniform electric fields

We also study electron-positron pair oscillation in spatially inhomogeneous and bound electric fields by integrating the equations of energy-momentum and particle-number conservations and Maxwell equations. The space and time evolutions of the pair-induced electric field, electric charge- and current-densities are calculated. The results show non-vanishing electric charge-density and the propagation of pair-induced electric fields, that are different from the case of homogeneous and unbound electric fields. The space and time variations of pair-induced electric charges and currents emit an electromagnetic radiation. We obtain the narrow spectrum and intensity of this radiation, whose peak $\omega_{\rm peak}$ locates in the region around 4 keV for electric field strength $\sim E_c$. We discuss their relevances to both the laboratory experiments for electron and positron pair-productions and the astrophysical observations of compact stars with an electromagnetic structure.

The origin of electron-positron pairs being created strong electric field and their oscillations has been considered in Ruffini et al. (2007b). There it was shown that plasma oscillations occur not only for overcritical electric field, but also for undercritical electric field, provided the electric field is maintained on spatial distances larger than the distance of oscillations determined explicitly in Ruffini et al. (2007b).

In the paper by Han et al. (2010) the spectrum of electromagnetic radiation seen by far observer for initial phase of oscillations has been computed. It was shown there that the spectrum contain a narrow feature which corresponds to the frequency of plasma oscillations. We revisited the approach of Ruffini et al. (2007b) and showed that for the case of uniform external electric field it is possible to reduce the system of four first order ordinary differential equations governing the dynamics of particle number density, energy density, momentum and electric field to just one second order equation.

Then in the paper by Han et al. (2010); Benedetti et al. (2011) we analyzed the frequency of oscillations, and found that the frequency of oscillations coincides up to a factor close to unity with the plasma frequency, which is strongly time dependent due to pair creation process. Analytical arguments suggest that the frequency of oscillations should asymptotically reach the plasma frequency, and this fact has been demonstrated. The results of this work allow simple estimation of the frequency of plasma oscillations, and then of the spectrum of electromagnetic radiation generated by these oscillations. For the details of this parts, see Appendex B.

3.13. Fractional Effective Action at strong electromagnetic fields

In 1931 Sauter Sauter (1931) and four years later Heisenberg and Euler Heisenberg and Euler (1936) provided a first description of the vacuum properties of QED. They identified a characteristic scale of strong field $E_c = m_e^2 c^3/e\hbar$, at which the field energy is sufficient to create electron positron pairs from the vacuum, and calculated an effective Lagrangian that will replace the Maxwell Lagrangian at strong fields. In 1951, Schwinger Schwinger (1951, 1954a,b) gave an elegant quantum-field theoretic reformulation of their result in the spinor and scalar QED framework (see also Nikishov (1970); Batalin and Fradkin (1970b)). The description was further extended to space-time dependent electromagnetic fields in Refs. Popov (1972d,c, 2001b); Narozhnyi and Nikishov (1970); Schubert (2001b); Dunne and Schubert (2005b); Kleinert and Xue (2013). The monographs Itzykson and Zuber (2006); Kleinert (2008); Greiner et al. (1985); Grib et al. (1980); Fradkin et al. (1991) and the recent review articles Dunne and Schubert (2000); Dunne (2005); Ruffini et al. (2010) can be consulted for more detailed calculations, discussions and bibliographies. Since then, the properties of QED in strong electromagnetic fields have become vast arena of theoretical research, awaiting experimental verification as well as further theoretical understanding.

An interesting aspect of effective field theories in the strong-field limit has recently been emphasized in a completely different class of quantum field theories. These have the property of developing in the strong-field limit an anomalous power behavior. It is experimentally observable at the critical point in second-order phase transitions, and for this reason such a power behavior is also called *critical behavior*. Such a behavior arises if the the so-called beta function (also called the Stueckelberg–Petermann function or the Gell-Mann–Low function) Stueckelberg and Petermann (1953); Gell-Mann and Low (1954), which governs the logarithmic growth of the coupling strength for varying energy scale, has a fixed point in the infrared. In such theories, it is possible to take the theory to the limit of infinite coupling strength. The effective action can usually be calculated in perturbation theory as a power series in the fields. The coefficients are the one-particle irreducible n-point vertex functions of the theory. In the limit of large field strength, this power series can be shown to develop an anomalous power behavior with irrational exponents Kleinert and Frohlinde (2001); Kleinert. Also the gradient terms in this effective action show anomalous powers Kleinert (2012).

In QED such a fixed point is presently believed to be absent Suslov (2001), even though many authors have in the past argued that it may exist Johnson et al. (1963, 1964); Maris et al. (1964, 1965); Frishman (1965) and could ultimately explain the numerical value of the fine structure constant. In this study we shall not assume the existence of such a fixed point, but point out that at strong fields, the effective action exhibits nevertheless a power behavior that is typical for critical phenomena.

We conclude that in the strong fields expansion, the leading order behavior of the Euler-Heisenberg effective Lagrangian is logarithmic, and can be formulated as a power law for three different cases:

1. $|S/P| \gg 1$,
2. ϵ, $\beta \gg E_c$ and $\epsilon/\beta \sim \mathcal{O}(1)$,
3. $|P/S| \gg 1$.

The the general form is the same for scalar and spinor QED. The only difference is a factor of four in the anomalous power δ.

We have not been able to conclude a result for $S, P \gg E_c^2$. This case is equivalent to $|\vec{E}| \gg |\vec{B}| \gg E_c$ or $|\vec{B}| \gg |\vec{E}| \gg E_c$ while the fields are almost parallel. If we combine the result

$$\mathcal{L}^{\Re} = \frac{1}{2} E_c^{-\delta} (\vec{E}^2 - \vec{B}^2) |\vec{E}^2 - \vec{B}^2|^{\delta/2} + \dots, \tag{3.13.1}$$

for the cases 1. and the result

$$\mathcal{L}^{\Re} = \frac{1}{2} E_c^{-\delta} (\vec{E}^2 - \vec{B}^2) |\vec{E} \cdot \vec{B}|^{\delta/2} + \dots, \tag{3.13.2}$$

for the cases 3. with the anomalous power $\delta := e^2/12\pi$, we can conjecture the more general result:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} E_c^{-2\delta} (\vec{E}^2 - \vec{B}^2) \left(|\vec{E}^2 - \vec{B}^2| |\vec{E} \cdot \vec{B}| \right)^{\delta/2} + \dots. \tag{3.13.3}$$

This correctly reduces to the cases 1. and 3. in the respective limits and thus is more general. As a result, Eq. (3.13.3) defines a fractional formulation for the QED in the regime of strong fields. Thus our finding exhibits an interesting similarity to the fractional quantum field theory discussed in Kleinert (2012).

The Euler-Heisenberg-Lagrangian is obtained in the configuration of constant electromagnetic fields. Nevertheless, for the case of smooth and slow variations of electromagnetic fields in space and time, it can be approximately used to study interesting effects like light-by-light scattering, photon splitting or electron-positron pair production (for reviews see Dunne (2005); Ruffini et al. (2010)). This implies that the fractional QED obtained in this article could find some applications in the regime of strong electromagnetic fields. This is particularly important for the recent rapid developments of experimental facilities using novel strong laser sources to reach the field strength and intensity of theoretical interest. Such facilities include the Extreme Light Infrastructure (ELI)[1], the Exawatt Center for Extreme Light studies (XCELS)[2], or the High Power laser Energy Research (HiPER)[3] facility, which are planned to exceed powers of 100 PW. Both theoretical and experimental studies of the QED of strong electromagnetical fields at the Sauter-Euler-Heisenberg scale E_c promise to become increasingly fascinating in the coming years.

3.14. Einstein-Euler-Heisenberg theory and charged black holes

Taking into account the Euler-Heisenberg effective Lagrangian of one-loop nonperturbative quantum electrodynamics (QED) contributions, we formulate the Einstein-Euler-Heisenberg theory and study the solutions of nonrotating black holes with electric and magnetic charges in spherical geometry. In the limit of strong and weak electromagnetic fields of black holes, we calculate the black hole horizon radius, area, and total energy up to the leading order of QED corrections and discuss the black hole irreducible mass, entropy, and maximally extractable energy as well as the Christodoulou-Ruffini mass formula. We find that these black hole quantities receive the QED corrections, in comparison with their counterparts in the Reissner-Nordström solution. The QED corrections show the screening effect on black hole electric charges and the paramagnetic effect on black hole magnetic charges. As a result, the black hole horizon area, irreducible mass, and entropy increase; however, the black hole total energy and maximally extractable energy decrease, compared with the Reissner-Nordström solution. In addition, we show that the condition for extremely charged black holes is modified due to the QED correction. The reason is that the QED vacuum polarization gives rise to the screening effect on the black hole electric charge and the paramagnetic effect on the black hole magnetic charge. It is mentioned that in the Einstein-Euler-Heisenberg theory, it is worthwhile to study Kerr-Newman black holes, whose electric field **E** and magnetic field **B** are determined by the black hole mass M, charge Q, and angular momentum a Newman et al. (1965). In addition, it will be interesting to study the QED corrections in black hole physics by taking into account the one-loop photon-graviton amplitudes of the effective Lagrangian (E.3.11) Drummond and Hathrell (1980) and its generalizations Gilkey (1975); Bastianelli et al. (2000); Barvinsky and Vilkovisky (1985); Gusev (2009); Bastianelli et al. (2009). We leave these studies for future work. For the details of this part, see Appendix E.

[1] http://www.extreme-light-infrastructure.eu/
[2] http://www.xcels.iapras.ru/
[3] http://www.hiper-laser.org/

3.15. Electron and positron pair-production in gravitational collapses

We attempt to study possible electric processes in the dynamical perturbations of neutral stellar cores. These dynamical perturbations can be caused by either the gravitational collapse or pulsation of neutral stellar cores. The basic equations are the Einstein-Maxwell equations and those governing the particle number and energy-momentum conservation

$$\begin{aligned}(\bar{n}_{e,B}U^{\nu}_{e,B})_{;\nu} &= 0,\\ G_{\mu\nu} &= -8\pi G(T_{\mu\nu}+T^{\rm em}_{\mu\nu}),\\ (T^{\nu}{}_{\mu})_{;\nu} &= -F_{\mu\nu}J^{\nu},\\ F^{\mu\nu}{}_{;\nu} &= 4\pi J^{\mu},\end{aligned} \tag{3.15.1}$$

in which the Einstein tensor $G_{\mu\nu}$, the electromagnetic field $F^{\mu\nu}$ (satisfying $F_{[\alpha\beta,\gamma]}=0$) and its energy-momentum tensor $T^{\rm em}_{\mu\nu}$ appear; $U^{\nu}_{e,B}$ and $\bar{n}_{e,B}$ are, respectively, the four velocities and proper number-densities of the electrons and baryons. The electric current density is

$$J^{\mu}=e\bar{n}_pU^{\mu}_B-e\bar{n}_eU^{\mu}_e, \tag{3.15.2}$$

where $\bar{n}_p$ is the proper number-density of the positively charged baryons. The energy-momentum tensor $T^{\mu\nu}=T^{\mu\nu}_e+T^{\mu\nu}_B$ is taken to be that of two simple perfect fluids representing the electrons and the baryons, each of the form

$$T^{\mu\nu}_{e,B} = \bar{p}_{e,B}g^{\mu\nu}+(\bar{p}_{e,B}+\bar{\rho}_{e,B})U^{\mu}_{e,B}U^{\nu}_{e,B}, \tag{3.15.3}$$

where $\bar{\rho}_{e,B}(r,t)$ and $\bar{p}_{e,B}(r,t)$ are the respective proper energy densities and pressures. Baryon fluid and electron fluid are separately described for the reason that in addition to baryons being much more massive than electrons, the EOS of baryons $\bar{p}_B=\bar{p}_B(\bar{\rho}_B)$ is very different from the electron one $\bar{p}_e=\bar{p}_e(\bar{\rho}_e)$ due to the strong interaction. Therefore, in the dynamical perturbations of neutral stellar cores, one should not expect that the space-time evolution of number density, energy density, four velocity, and pressure of baryon fluid be identical to the space-time evolution of counterparts of electron fluid. The difference of space-time evolutions of two fluids results in the electric current J^{μ} and field $F^{\mu\nu}$, possibly leading to some electric processes. In a simplified model for the dynamical perturbations of neutral stellar cores, we approximately study possible electric processes by assuming that the equilibrium configurations of neutral stellar cores are initial configurations.

As a result, we find that total electric field, electron number density, energy density, and pressure oscillate around their equilibrium configurations. These oscillations with frequency $\omega = \tau_{\rm osci}^{-1} \sim 1.5 m_e$ around the equilibrium configuration take place in a thin layer of a few Compton lengths around the boundary of baryon core, which undergo the dynamical perturbations caused by the gravitational collapse or pulsation. Suppose that the dynamical perturbation of the baryon core is caused by either the gravitational collapse or pulsation of the baryon core, that gains the gravitational energy. Then, in this oscillating process, energy transforms from the dynamical perturbation of the baryon core to the electron fluid via an oscillating electric field. This can been seen from the energy conservation along a flow line of the electron fluid for $v_e \neq v_p$

$$U_e^\mu (T^\nu{}_\mu)_{;\nu} = e\bar{n}_p F_{\mu\nu} U_e^\mu U_B^\nu = e\bar{n}_p \gamma_e \gamma_B (v_p - v_e) g_{rr} E. \tag{3.15.4}$$

The energy densities of the oscillating electric field and electron fluid are converted from one to another in the oscillating process with frequencies $\omega \sim \tau_{\rm osci}^{-1} \sim 1.5 m_e$ around the equilibrium configuration. Oscillating electric fields $E(r,t) > E_{\rm eq}(r)$, this leads to electron-positron pair production in strong electric fields and converts electric energy into the energy of electron-positron pairs, provided the pair-production rate $\tau_{\rm pair}^{-1} \approx 6.6 m_e$ is faster than the oscillating frequency $\omega = \tau_{\rm osci}^{-1}$.

It is an assumption that the gravitationally collapsing process is represented by the sequence of events: the baryon core starts to collapse from rest by gaining gravitational energy, the increasing Coulomb energy results in decreasing kinetic energy and slowing down the collapse process, the electric processes mentioned above convert the Coulomb energy into the radiative energy of electron-positron pairs, and as a result the baryon core restarts to accelerate the collapse process by further gaining gravitational energy. This indicates that in the gravitationally collapsing process, the gravitational energy must be partly converted into the radiative energy of electron-positron pairs. By summing over all events in the sequence of the gravitationally collapsing process, we approximately estimate the total number and energy of electron-positron pairs produced in the range $R_c \sim 5 \times 10^5 - 10^7$cm: from 10^{56}–10^{57} and 10^{52}–10^{53} erg to 10^{55}–10^{56} and 10^{51}–10^{52} erg for different ratios of charged and neutral baryon numbers. These electron-positron pairs undergo the plasma oscillation in strong electric fields and annihilate to photons to form a neutral plasma of photons and electron-positron pairs Ruffini et al. (2003b,a). This is reminiscent of the vacuum polarization of a charged black hole Damour and Ruffini (1975); Cherubini et al. (2009) and the Dyadosphere supposed to be dynamically created during gravitational collapse in Refs. Ruffini and Xue (2008a); Preparata et al. (1998, 2003). For the details of this part, see Appendix C.

3.16. Gravitational and electric energies in gravitational collapses

In our previous work "Electron and positron pair-production in gravitational collapses" (PHYSICAL REVIEW D 86, 084004 (2012)), we present a study of strong oscillating electric fields and electron-positron pair-production in gravitational collapse of a neutral stellar core at or over nuclear densities. In order to understand the back-reaction of such electric energy building and radiating on collapse, we adopt a simplified model describing the collapse of a spherically thin capacitor to give an analytical description how gravitational energy is converted to both kinetic and electric energies in collapse. It is shown that (i) averaged kinetic and electric energies are the same order, about an half of gravitational energy of spherically thin capacitor in collapse; (ii) caused by radiating and rebuilding electric energy, gravitational collapse undergoes a sequence of "on and off" hopping steps in the microscopic Compton scale. Although such a collapse process is still continuous in terms of macroscopic scales, it is slowed down as kinetic energy is reduced and collapsing time is about an order of magnitude larger than that of collapse process eliminating electric processes. These results indicate that it is essential to take into account, rather than ignore, electric processes in more realistic models for studying gravitational collapse of neutral stellar core at or over the nuclear density. For the details of this part, see Appendix D.

4. Publications (before 2005)

1. R. Ruffini and J. A. Wheeler, “Introducing the black hole ”, Phys. Today, January (1971) 178.

   This article proved to be popular and was written with the intention of communicating some of the major processes made in understanding the final configurations of collapsed stars to the largest possible audience. In this article, the authors summarized the results of their students’ work with particular emphasis on the work of D. Christodoulou (graduate student of R. Ruffini’s at that time) together with some of their most significant new results. Moreover, it was emphasized that of all the procedures for identifying a collapsed object in space at a great distance, the most promising consisted of analyzing a close binary system in which one member is a normal star and the other a black hole. The X–ray emission associated with the transfer of material from the normal star to the collapsed object would then be of greatest importance in determining the properties of the collapsed object. This article has been reprinted many times and has been translated into many languages (Japanese, Russian, and Greek, among others). It has created much interest in the final configuration of stars after the endpoint of their thermonuclear evolution. The analysis of the possible processes leading to the formation of a black hole, via either a one–step process of a multistep process, was also presented for the first time in this article.

2. D. Christodoulou and R. Ruffini, “Reversible Transformations of a Charged Black Hole”, Phys. Rev. D4 (1971) 3552.

   A formula is derived for the mass of a black hole as a function of its ”irreducible mass,” its angular momentum, and its charge. It is shown that 50% of the mass of an extreme charged black hole can be converted into energy as contrasted with 29% for an extreme rotating black hole.

3. T. Damour and R. Ruffini, “Quantum electrodynamical effects in Kerr-Newman geometries”, Phys. Rev. Lett. 35 (1975) 463.

   Following the classical approach of Sauter, of Heisenberg and Euler and of Schwinger the process of vacuum polarization in the field of a ”bare” Kerr-Newman geometry is studied. The value of the critical strength of the electromagnetic fields is given together with an analysis of the feedback of the discharge on the geometry. The relevance of this analysis for current astrophysical observations is mentioned.

4. J. Ferreirinho, R. Ruffini and L. Stella, "On the relativistic Thomas-Fermi model", Phys. Lett. B 91, (1980) 314. The relativistic generalization of the Thomas-Fermi model of the atom is derived. It approaches the usual nonrelativistic equation in the limit $Z \ll Z_{\rm crit}$, where Z is the total number of electrons of the atom and $Z_{\rm crit} = (3\pi/4)^{1/2}\alpha^{-3/2}$ and α is the fine structure constant. The new equation leads to the breakdown of scaling laws and to the appearance of a critical charge, purely as a consequence of relativistic effects. These results are compared and contrasted with those corresponding to N self-gravitating degenerate relativistic fermions, which for $N \approx N_{\rm crit} = (3\pi/4)^{1/2}(m/m_p)^3$ give rise to the concept of a critical mass against gravitational collapse. Here m is the mass of the fermion and $m_p = (\hbar c/G)^{1/2}$ is the Planck mass.

5. R. Ruffini and L. Stella,"Some comments on the relativistic Thomas-Fermi model and the Vallarta-Rosen equation", Phys. Lett. B 102 (1981) 442. Some basic differences between the screening of the nuclear charge due to a relativistic cloud of electrons in a neutral atom and the screening due to vacuum polarization effects induced by a superheavy ion are discussed.

6. G. Preparata, R. Ruffini and S.-S. Xue, "The dyadosphere of black holes and gamma-ray bursts", Astron. Astroph. Lett. 337 (1998) L3.

   The "dyadosphere" has been defined (Ruffini, Preparata et al.) as the region outside the horizon of a black hole endowed with an electromagnetic field (abbreviated to EMBH for "electromagnetic black hole") where the electromagnetic field exceeds the critical value, predicted by Heisenberg & Euler for e^+e^- pair production. In a very short time ($\sim O(\hbar/(mc^2))$), a very large number of pairs is created there. We here give limits on the EMBH parameters leading to a Dyadosphere for $10M_\odot$ and $10^5M_\odot$ EMBH's, and give as well the pair densities as functions of the radial coordinate. We here assume that the pairs reach thermodynamic equilibrium with a photon gas and estimate the average energy per pair as a function of the EMBH mass. These data give the initial conditions for the analysis of an enormous pair-electromagnetic-pulse or "P.E.M. pulse" which naturally leads to relativistic expansion. Basic energy requirements for gamma ray bursts (GRB), including GRB971214 recently observed at $z = 3.4$, can be accounted for by processes occurring in the dyadosphere. In this letter we do not address the problem of forming either the EMBH or the dyadosphere: we establish some inequalities which must be satisfied during their formation process.

7. R. Ruffini, "On the dyadosphere of black holes", at the XLIXth Yamada Conference on "Black Holes and High-Energy Astrophysics", H. Sato Ed., Univ. Acad. Press, Tokyo, 1998.

   The "dyadosphere" (from the Greek word "duas-duados" for pairs) is here defined as the region outside the horizon of a black hole endowed with an electromagnetic field (abbreviated to EMBH for "electromagnetic black hole") where the electromagnetic field exceeds the critical value, predicted by Heisenberg and Euler for electron-positron pair production. In a very short time, a very large number of pairs is created there. I give limits on the EMBH parameters leading to a Dyadosphere for 10 solar mass and 100000 solar mass EMBH's, and give as well the pair densities as functions of the radial coordinate. These data give the initial conditions for the analysis of an enormous pair-electromagnetic-pulse or "PEM-pulse" which naturally leads to relativistic expansion. Basic energy requirements for gamma ray bursts (GRB), including GRB971214 recently observed at z=3.4, can be accounted for by processes occurring in the dyadosphere.

8. R. Ruffini, J. D. Salmonson, J. R. Wilson and S.-S. Xue, "On the Pair Electromagnetic Pulse of a Black Hole with Electromagnetic Structure", Astron. Astroph. 350 (1999) 334.

   Starting from a nonequilibrium configuration we analyse the essential role of the direct and the inverse binary and triple interactions in reaching an asymptotic thermal equilibrium in a homogeneous isotropic electron-positron-photon plasma. We focus on energies in the range 0.1–10 MeV. We numerically integrate the integro-partial differential relativistic Boltzmann equation with the exact QED collisional integrals taking into account all binary and triple interactions in the plasma. We show that first, when detailed balance is reached for all binary interactions on a timescale $t_k \lesssim 10^{-14}$sec, photons and electron-positron pairs establish kinetic equilibrium. Successively, when triple interactions fulfill the detailed balance on a timescale $t_{eq} \lesssim 10^{-12}$sec, the plasma reaches thermal equilibrium. It is shown that neglecting the inverse triple interactions prevents reaching thermal equilibrium. Our results obtained in the theoretical physics domain also find application in astrophysics and cosmology.

9. R. Ruffini, J. D. Salmonson, J. R. Wilson and S.-S. Xue, "On Evolution of the Pair-Electromagnetic Pulse of a Charge Black Hole ", Astron. Astrophys. Suppl. Ser. 138 (1999) 511.

   Using hydrodynamic computer codes, we study the possible patterns of relativistic expansion of an enormous pair-electromagnetic-pulse (P.E.M. pulse); a hot, high density plasma composed of photons, electron-positron pairs and baryons deposited near a charged black hole (EMBH). On the bases of baryon-loading and energy conservation, we study the bulk Lorentz factor of expansion of the P.E.M. pulse by both numerical and analytical methods.

10. R. Ruffini, J. D. Salmonson, J. R. Wilson and S.-S. Xue, "On the pair-electromagnetic pulse from an electromagnetic Black Hole surrounded by a Baryonic Remnant ", Astron. Astrophys 359, 855-864 (2000).

    The interaction of an expanding Pair-Electromagnetic pulse (PEM pulse) with a shell of baryonic matter surrounding a Black Hole with electromagnetic structure (EMBH) is analyzed for selected values of the baryonic mass at selected distances well outside the dyadosphere of an EMBH. The dyadosphere, the region in which a super critical field exists for the creation of electron-positron pairs, is here considered in the special case of a Reissner-Nordstrom geometry. The interaction of the PEM pulse with the baryonic matter is described using a simplified model of a slab of constant thickness in the laboratory frame (constant-thickness approximation) as well as performing the integration of the general relativistic hydrodynamical equations. The validation of the constant-thickness approximation, already presented in a previous paper Ruffini, et al.(1999) for a PEM pulse in vacuum, is here generalized to the presence of baryonic matter. It is found that for a baryonic shell of mass-energy less than 1% of the total energy of the dyadosphere, the constant-thickness approximation is in excellent agreement with full general relativistic computations. The approximation breaks down for larger values of the baryonic shell mass, however such cases are of less interest for observed Gamma Ray Bursts (GRBs). On the basis of numerical computations of the slab model for PEM pulses, we describe (i) the properties of relativistic evolution of a PEM pulse colliding with a baryonic shell; (ii) the details of the expected emission energy and observed temperature of the associated GRBs for a given value of the EMBH mass; 10^3 solar masses, and for baryonic mass-energies in the range 10^{-8} to 10^{-2} the total energy of the dyadosphere.

11. G. Preparata, R. Ruffini and S.-S. Xue,"The role of the screen factor in GRBs ", Il Nuovo Cimento B115 (2000) 915.

    We derive the screen factor for the radiation flux from an optically thick plasma of electron-positron pairs and photons, created by vacuum polarization process around a black hole endowed with electromagnetic structure.

12. C. L. Bianco, R. Ruffini and S.-S. Xue, "The elementary spike produced by a pure e^+e^- pair-electromagnetic pulse from a Black Hole: The PEM Pulse ", Astron. Astrophys. 368 (2001) 377.

    In the framework of the model that uses black holes endowed with electromagnetic structure (EMBH) as the energy source, we study how an elementary spike appears to the detectors. We consider the simplest possible case of a pulse produced by a pure e^+e^- pair-electro-magnetic plasma, the PEM pulse, in the absence of any baryonic matter. The resulting time profiles show a *Fast-Rise-Exponential-Decay* shape, followed by a power-law tail. This is obtained without any special fitting procedure, but only by fixing the energetics of the process taking place in a given EMBH of selected mass, varying in the range from 10 to 10^3 $M_\odot$ and considering the relativistic effects to be expected in an electron-positron plasma gradually reaching transparency. Special attention is given to the contributions from all regimes with Lorentz γ factor varying from $\gamma = 1$ to $\gamma = 10^4$ in a few hundreds of the PEM pulse travel time. Although the main goal of this paper is to obtain the elementary spike intensity as a function of the arrival time, and its observed duration, some qualitative considerations are also presented regarding the expected spectrum and on its departure from the thermal one. The results of this paper will be comparable, when data will become available, with a subfamily of particularly short GRBs not followed by any afterglow. They can also be propedeutical to the study of longer bursts in presence of baryonic matter currently observed in GRBs.

13. R. Ruffini and L. Vitagliano, "Irreducible mass and energetics of an electromagnetic black hole ", Phys. Lett. B545 (2002) 233.

    The mass-energy formula for a black hole endowed with electromagnetic structure (EMBH) is clarified for the nonrotating case. The irreducible mass $M_{\rm irr}$ is found to be independent of the electromagnetic field and explicitly expressable as a function of the rest mass, the gravitational energy and the kinetic energy of the collapsing matter at the horizon. The electromagnetic energy is distributed throughout the entire region extending from the horizon of the EMBH to infinity. We discuss two conceptually different mechanisms of energy extraction occurring respectively in an EMBH with electromagnetic fields smaller and larger than the critical field for vacuum polarization. For a subcritical EMBH the energy extraction mechanism involves a sequence of discrete elementary processes implying the decay of a particle into two oppositely charged particles. For a supercritical EMBH an alternative mechanism is at work involving an electron-positron plasma created by vacuum polarization. The energetics of these mechanisms as well as the definition of the spatial regions in which thay can occur are given. The physical implementations of these ideas are outlined for ultrahigh energy cosmic rays UHECR) and gamma ray bursts (GRBs).

14. C. Cherubini, R. Ruffini and L. Vitagliano, "On the electromagnetic field of a charged collapsing spherical shell in general relativity ", Phys. Lett. B545 (2002) 226.

    A new exact solution of the Einstein-Maxwell equations for the gravitational collapse of a shell of matter in an already formed black hole is given. Both the shell and the black hole are endowed with electromagnetic structure and are assumed spherically symmetric. Implications for current research are outlined.

15. R. Ruffini, L. Vitagliano and S.-S. Xue, "On Plasma Oscillations in Strong Electric Fields ", Phys. Lett. B559 (2003) 12.

    We describe the creation and evolution of electron-positron pairs in a strong electric field as well as the pairs annihilation into photons. The formalism is based on generalized Vlasov equations, which are numerically integrated. We recover previous results about the oscillations of the charges, discuss the electric field screening and the relaxation of the system to a thermal equilibrium configuration. The timescale of the thermalization is estimated to be $\sim 10^3 - 10^4 \hbar/m_e c^2$.

16. R. Ruffini, L. Vitagliano and S.-S. Xue, "Electron-positron-photon plasma around a collapsing star ", (invited talk) in Proc. of the 28^{th} Joint ICFA Conference on Quantum Aspects of Beam Physics and Other Critical Issues of Beams in Physics and Astrophysics, January 7–11, 2003, Hiroshima University, Higashi–Hiroshima, Japan, Pisin Chen Ed., World Scientific, Singapore.

    We describe electron-positron pairs creation around an electrically charged star core collapsing to an electromagnetic black hole (EMBH), as well as pairs annihilation into photons. We use the kinetic Vlasov equation formalism for the pairs and photons and show that a regime of plasma oscillations is established around the core. As a byproduct of our analysis we can provide an estimate for the thermalization time scale.

17. G. Preparata, R. Ruffini and S.-S. Xue, "On the Dyadosphere of Black Hole", J. Korean Phys.Soc. 42 (2003) S99-S104 (astro-ph/0204080).

    Basic energy requirements of Gamma Ray Burst(GRB) sources can be easily accounted for by a pair creation process occurring in the "Dyadosphere" of a Black Hole endowed with an electromagnetic field (abbreviated to EMBH for "electromagnetic Black Hole"). This includes the recent observations of GRB971214 by Kulkarni et al. The "Dyadosphere" is defined as the region outside the horizon of an EMBH where the electromagnetic field exceeds the critical value for e^+e^- pair production. In a very short time $\sim O(\hbar mc^2)$, very large numbers of pairs are created there. Further evolution then leads naturally to a relativistically expanding pair-electromagnetic-pulse (PEM-pulse). Specific examples of Dyadosphere parameters are given for 10 and 10^5 solar mass EMBH's. This process does occur for EMBH with charge-to-mass ratio larger than 2.210^{-5} and strictly smaller than one. From a fundamental point of view, this process represents the first mechanism proved capable of extracting large amounts of energy from a Black Hole with an extremely high efficiency (close to 100%).

18. R. Ruffini and L. Vitagliano, "Energy Extraction From Gravitational Collapse to Static Black Holes ", Int. J. Mod. Phys. D12 (2003) 121.

    The mass–energy formula of black holes implies that up to 50% of the energy can be extracted from a static black hole. Such a result is reexamined using the recently established analytic formulas for the collapse of a shell and expression for the irreducible mass of a static black hole. It is shown that the efficiency of energy extraction process during the formation of the black hole is linked in an essential way to the gravitational binding energy, the formation of the horizon and the reduction of the kinetic energy of implosion. Here a maximum efficiency of 50% in the extraction of the mass energy is shown to be generally attainable in the collapse of a spherically symmetric shell: surprisingly this result holds as well in the two limiting cases of the Schwarzschild and extreme

Reissner-Nordström space-times. Moreover, the analytic expression recently found for the implosion of a spherical shell onto an already formed black hole leads to a new exact analytic expression for the energy extraction which results in an efficiency strictly less than 100% for any physical implementable process. There appears to be no incompatibility between General Relativity and Thermodynamics at this classical level.

5. Publications (2005-2013)

1. R. Ruffini, F. Fraschetti, L. Vitagliano and S.-S. Xue,"Observational signatures of an electromagnetic overcritical gravitational collapse ", Int. Journ. Mod. Phys. D14 (2005) 131.

   We present theoretical predictions for the spectral, temporal and intensity signatures of the electromagnetic radiation emitted during the process of the gravitational collapse of a stellar core to a black hole, during which electromagnetic field strengths rise over the critical value for e^+e^- pair creation. The last phases of this gravitational collapse are studied, leading to the formation of a black hole with a subcritical electromagnetic field, likely with zero charge, and an outgoing pulse of initially optically thick e^+e^--photon plasma. Such a pulse reaches transparency at Lorentz gamma factors of 10^2–10^4. We find a clear signature in the outgoing electromagnetic signal, drifting from a soft to a hard spectrum, on very precise time-scales and with a very specific intensity modulation. The relevance of these theoretical results for the understanding of short gamma-ray bursts is outlined.

2. Federico Fraschetti, Remo Ruffini, Luca Vitagliano, and She-Sheng Xue, "Theoretical predictions of spectral evolution of short GRBs ", in Venice (Italy), June 5-9, 2006, IL NUOVO CIMENTO Vol. 121 (2006) 1477.

   We present the properties of spectrum of radiation emitted during gravitational collapse in which electromagnetic field strengths rise over the critical value for e^+e^- pair creation. A drift from soft to a hard energy and a high energy cut off have been found; a comparison with a pure black body spectrum is outlined.

3. R. Ruffini and S.-S. Xue, "Effective Lagrangian of QED", Journal of the Korean physical society, Vol. 49, No. 2, august 2006, pp. 715.

   From the Euler-Heisenberg formula we calculate the exact real part of the one-loop effective Lagrangian of Quantum Electrodynamics in a constant electromagnetic field, and determine its strong-field limit.

4. C. Cherubini, A. Geralico, J. Rueda and R. Ruffini, "On the "Dyadotorus" of Kerr-Newman space time ", Phys. Rev. D 79 124002 (2009).

   We present the geometrical properties of the region where vacuum polarization precess occur int he Kerr-Newman space time. We find that the shape of the region can be ellipsoid-like or torus-like depending on the charge of the black hole.

5. H. Kleinert, R. Ruffini and S.-S. Xue, "Electron-positron pair-production in nonuniform electric fields", Phys. Rev. D 78 (2008) 025011.

   Treating the production of electron and positron pairs in vacuum as quantum tunneling, at the semiclassical level $O(\hbar)$, we derive a general expression, both exponential and pre-exponential factors, of the pair-production rate in nonuniform electric fields varying only in one direction. In particularly we discuss the expression for the case when produced electrons (or positrons) fill into bound states of electric potentials with discrete spectra of energy-level crossings. This expression is applied to the examples of the confined field $E(z) \neq 0, |z| \lesssim \ell$, half-confined field $E(z) \neq 0, z \gtrsim 0$, and linear increasing field $E(z) \sim z$, as well as the Coulomb field $E(r) = eZ/r^2$ for a nucleus with finite size $r_{\rm n}$ and large $Z \gg 1$.

6. R. Ruffini, G. V. Vereshchagin and S.-S. Xue, "Vacuum polarization and plasma oscillations", Phys. Lett. A 371(2007) 399 (arXiv:0706.4363).

   We evidence the existence of plasma oscillations of electrons-positron pairs created by the vacuum polarization in an uniform electric field with $E < E_c$. Our general treatment, encompassing also the traditional, well studied case of $E > E_c$, shows the existence in both cases of a maximum Lorentz factor acquired by electrons and positrons and allows determination of the a maximal length of oscillation. We quantitatively estimate how plasma oscillations reduce the rate of pair creation and increase the time scale of the pair production. These results are particularly relevant in view of the experimental progress in approaching the field strengths $E < E_c$.

7. A. G. Aksenov, R. Ruffini, and G. V. Vereshchagin, "Thermalization of a nonequilibrium electron-positron-photon plasma ", Phys.Rev.Lett. 99 (2007) 125003 .

   Starting from a nonequilibrium configuration we analyse the essential role of the direct and the inverse binary and triple interactions in reaching an asymptotic thermal equilibrium in a homogeneous isotropic electron-positron-photon plasma. We focus on energies in the range 0.1–10 MeV. We numerically integrate the integro-partial differential relativistic Boltzmann equation with the exact QED collisional integrals taking into account all binary and triple interactions in the plasma. We show that first, when detailed balance is reached for all binary interactions on a timescale $t_k \lesssim 10^{-14}$sec, photons and electron-positron pairs establish kinetic equilibrium. Successively, when triple interactions fulfill the detailed balance on a timescale $t_{eq} \lesssim 10^{-12}$sec, the plasma reaches thermal equilibrium. It is shown that neglecting the inverse triple interactions prevents reaching thermal equilibrium. Our results obtained in the theoretical physics domain also find application in astrophysics and cosmology.

8. R. Ruffini, M. Rotondo and S.-S. Xue, "Electrodynamics for Nuclear Matter in Bulk ", Int. Journ. Mod. Phys. D Vol. 16, No. 1 (2007) 1-9.

   A general approach to analyze the electrodynamics of nuclear matter in bulk is presented using the relativistic Thomas-Fermi equation generalizing to the case of $N \simeq (m_{\rm Planck}/m_n)^3$ nucleons of mass m_n the approach well tested in very heavy nuclei ($Z \simeq 10^6$). Particular attention is given to implement the condition of charge neutrality globally on the entire configuration, versus the one usually adopted on a microscopic scale. As the limit $N \simeq (m_{\rm Planck}/m_n)^3$ is approached the penetration of electrons inside the core increases and a relatively small tail of electrons persists leading to a significant electron density outside the core. Within a region of 10^2 electron Compton wavelength near the core surface electric fields close to the critical value for pair creation by vacuum polarization effect develop. These results can have important consequences on the understanding of physical process in neutron stars structures as well as on the initial conditions leading to the process of gravitational collapse to a black hole.

9. V. Popov, M. Rotondo, R. Ruffini and S.-S. Xue, "Analytic treatment of the electrodynamics for nuclear matter in bulk", to be submitted to Phy. Rev. C.

   Using the relativistic Thomas-Fermi equation, we present an analytic treatment of the electrodynamic properties of nuclear matter in bulk. Following the works of Migdal and Popov we generalize to the case of a massive core with the mass number $A \sim 10^{57}$ the analytic approach well tested in very heavy nuclei with $A \sim 10^6$. Attention is given to implement the condition of charge neutrality globally on the entire configuration, versus the one usually adopted on a microscopic scale. It is confirmed that also in this limit A, an electric field develops near the core surface of magnitude close to the critical value of vacuum polarization. It is shown that such a configuration is energetically favorable with respect to the one which obeys local charge neutrality. These results can have important consequences on the understanding of the physical process in neutron stars as well as on the initial conditions leading to the process of gravitational collapse to a black hole.

10. R. Ruffini, M. Rotondo and S.-S. Xue, "Neutral nuclear core vs super charged one ", in Proceedings of the Eleventh Marcel Grossmann Meeting, R. Jantzen, H. Kleinert, R. Ruffini (eds.), (World Scientific, Singapore, 2008).

    Based on the Thomas-Fermi approach, we describe and distinguish the electron distributions around extended nuclear cores: (i) in the case that cores are neutral for electrons bound by protons inside cores and proton and electron numbers are the same; (ii) in the case that super charged cores are bare, electrons (positrons) produced by vacuum polarization are bound by (fly into) cores (infinity).

11. R. Ruffini and S.-S. Xue, "Dyadosphere formed in gravitational collapse ", AIP Conf. Proc. 1059 (2008) 72.

    We first recall the concept of Dyadosphere (electron-positron-photon plasma around a formed black holes) and its motivation, and recall on (i) the Dirac process: annihilation of electron-positron pairs to photons; (ii) the Breit-Wheeler process: production of electron-positron pairs by photons with the energy larger than electron-positron mass threshold; the Sauter-Euler-Heisenberg effective Lagrangian and rate for the process of electron-positron production in a constant electric field. We present a general formula for the pair-production rate in the semi-classical treatment of quantum mechanical tunneling. We also present in the *Quantum Electro-Dynamics* framework, the calculations of the Schwinger rate and effective Lagrangian for constant electromagnetic fields. We give a review on the electron-positron plasma oscillation in constant electric fields, and its interaction with photons leading to energy and number equipartition of photons, electrons and positrons. The possibility of creating an overcritical field in astrophysical condition is pointed out. We present the discussions and calculations on (i) energy extraction from gravitational collapse; (ii) the formation of Dyadosphere in gravitational collapsing process, and (iii) its hydrodynamical expansion in Reissner Nordström geometry. We calculate the spectrum and flux of photon radiation at the point of transparency, and make predictions for short Gamma-Ray Bursts.

12. Jorge A. Rueda, Remo Ruffini, and S.-S. Xue, "On the electrostatic structure of neutron stars", AIP Conference Proceedings Volume 1205, page 143 (2009), International Conference in Honor of Ya.B. Zeldovich's 95th Anniversary, Minsk, (Belarus), 20-23 April 2009.

    We consider neutron stars composed by, (1) a core of degenerate neutrons, protons, and electrons above nuclear density; (2) an inner crust of nuclei in a gas of neutrons and electrons; and (3) an outer crust of nuclei in a gas of electrons. We use for the strong interaction model for the baryonic matter in the core an equation of state based on the phenomenological Weizsacker mass formula, and to determine the properties of the inner and the outer crust below nuclear saturation density we adopt the well–known equation of state of Baym–Bethe–Pethick. The integration of the Einstein–Maxwell equations is carried out under the constraints of β–equilibrium and global charge neutrality. We obtain baryon densities that sharply go to zero at nuclear density and electron densities matching smoothly the electron component of the crust. We show that a family of equilibrium configurations exists fulfilling overall neutrality and characterized by a non–trivial electrodynamical structure at the interface between the core and the crust. We find that the electric field is overcritical and that the thickness of the transition surface–shell separating core and crust is of the order of the electron Compton wavelength.

13. Jorge A. Rueda H., B. Patricelli, M. Rotondo, R. Ruffini, and S. S. Xue,

"The Extended Nuclear Matter Model with Smooth Transition Surface", to be published in the Proceedings of The 3rd Stueckelberg Workshop on Relativistic Field Theories, Pescara-Italy (2008).

The existence of electric fields close to their critical value $E_c = m_e^2 c^3/(e\hbar)$ has been proved for massive cores of 10^7 up to 10^{57} nucleons using a proton distribution of constant density and a sharp step function at its boundary. We explore the modifications of this effect by considering a smoother density profile with a proton distribution fulfilling a Woods-Saxon dependence. The occurrence of a critical field has been confirmed. We discuss how the location of the maximum of the electric field as well as its magnitude is modified by the smoother distribution.

14. B. Patricelli, M. Rotondo and R. Ruffini, "On the Charge to Mass Ratio of Neutron Cores and Heavy Nuclei", AIP Conference Proceedings, Vol. 966 (2008), pp. 143-146.

    We determine theoretically the relation between the total number of protons N_p and the mass number A (the charge to mass ratio) of nuclei and neutron cores with the model recently proposed by Ruffini et al. (2007) and we compare it with other N_p versus A relations: the empirical one, related to the Periodic Table, and the semi-empirical relation, obtained by minimizing the Weizsäcker mass formula. We find that there is a very good agreement between all the relations for values of A typical of nuclei, with differences of the order of per cent. Our relation and the semi-empirical one are in agreement up to $A \approx 10^4$ for higher values, we find that the two relations differ. We interpret the different behavior of our theoretical relation as a result of the penetration of electrons (initially confined in an external shell) inside the core, that becomes more and more important by increasing A; these effects are not taken into account in the semi-empirical mass-formula.

15. M. Rotondo, R. Ruffini and S.-S Xue, "On the Electrodynamical properties of Nuclear matter in bulk", AIP Conference Proceedings, Vol. 966 (2008), pp. 147-152.

    We analyze the properties of solutions of the relativistic Thomas-Fermi equation for globally neutral cores with radius of the order of $R \approx 10$ Km, at constant densities around the nuclear density. By using numerical tecniques as well as well tested analytic procedures developed in the study of heavy ions, we confirm the existence of an electric field close to the critical value $E_c = m_e^2 c^3/e\hbar$ in a shell $\Delta R \approx 10^4 \hbar/m_\pi c$ near the core surface. For a core of ≈ 10 Km the difference in binding energy reaches 10^{49} ergs. These results can be of interest for the understanding of very heavy nuclei as well as physics of neutron stars, their formation processes and further gravitational collapse to a black hole.

16. B. Patricelli, M. Rotondo, J. A. Rueda H. and R. Ruffini, “The Electrodynamics of the Core and the Crust components in Neutron Stars”, AIP Conference Proceedings, Vol. 1059 (2008), pp. 68-71.

    We study the possibility of having a strong electric field (E) in Neutron Stars. We consider a system composed by a core of degenerate relativistic electrons, protons and neutrons, surrounded by an oppositely charged leptonic component and show that at the core surface it is possible to have values of E of the order of the critical value for electron-positron pair creation, depending on the mass density of the system. We also describe Neutron Stars in general relativity, considering a system composed by the core and an additional component: a crust of white dwarf - like material. We study the characteristics of the crust, in particular we calculate its mass M_{crust}. We propose that, when the mass density of the star increases, the core undergoes the process of gravitational collapse to a black hole, leaving the crust as a remnant; we compare M_{crust} with the mass of the baryonic remnant considered in the fireshell model of GRBs and find that their values are compatible.

17. R. Ruffini, “The Role of Thomas-Fermi approach in Neutron Star Matter”, to be published in the Proceedings of the 9^{th} International Conference “Path Integrals - New trends and perspectives”, Max Planck Institute for the Physics of Complex Systems, Dresden, Germany, Semptember 23 - 28 2007, World Scientific 207 - 218 (2008), eds. W. Janke and A. Pelster

    The role of the Thomas-Fermi approach in Neutron Star matter cores is presented and discussed with special attention to solutions globally neutral and not fulfilling the traditional condition of local charge neutrality. A new stable and energetically favorable configuration is found. This new solution can be of relevance in understanding unsolved issues of the gravitational collapse processes and their energetics.

18. R. Ruffini and S.-S. Xue, “Electron-positron pairs production in a macroscopic charged core”, Phys. Lett. B 696 (2011) 416.

    Classical and semi-classical energy states of relativistic electrons bounded by a massive and charged core with the charge-mass-radio Q/M and macroscopic radius R_c are discussed. We show that the energies of semi-classical (bound) states can be much smaller than the negative electron mass-energy ($-mc^2$), and energy-level crossing to negative energy continuum occurs. Electron-positron pair production takes place by quantum tunneling, if these bound states are not occupied. Electrons fill into these bound states and positrons go to infinity. We explicitly calculate the rate of pair-production, and compare it with the rates of electron-positron production by the Sauter-Euler-Heisenberg-Schwinger in a constant electric field. In addition, the pair-production rate for the electro-gravitational balance ratio $Q/M = 10^{-19}$ is much larger than the pair-production rate due to the Hawking processes.

19. W.-B. Han, R. Ruffini and S.-S. Xue, "Electron-positron pair oscillation in spatially inhomogeneous electric fields and radiation ", Physics Letters B, Vol. 691 (2010), pp. 99-104.

    It is known that strong electric fields produce electron and positron pairs from the vacuum, and due to the back-reaction these pairs oscillate back and forth coherently with the alternating electric fields in time. We study this phenomenon in spatially inhomogeneous and bound electric fields by integrating the equations of energy-momentum and particle-number conservations and Maxwell equations. The space and time evolutions of the pair-induced electric field, electric charge- and current-densities are calculated. The results show non-vanishing electric charge-density and the propagation of pair-induced electric fields, that are different from the case of homogeneous and unbound electric fields. The space and time variations of pair-induced electric charges and currents emit an electromagnetic radiation. We obtain the narrow spectrum and intensity of this radiation, whose peak $\omega_{\rm peak}$ locates in the region around 4 keV for electric field strength $\sim E_c$. We discuss their relevances to both the laboratory experiments for electron and positron pair-productions and the astrophysical observations of compact stars with an electromagnetic structure.

20. A. Benedetti, W.-B. Han, R. Ruffini, G. V. Vereshchagin, "On the frequency of oscillations in the pair plasma generated by a strong electric field ", Phys. Lett. B698:75-79,2011.

    We study the frequency of the plasma oscillations of electron-positron pairs created by the vacuum polarization in an uniform electric field with strength E in the range $0.2E_c < E < 10E_c$. Following the approach adopted in [1] we work out one second order ordinary differential equation for a variable related to the velocity from which we can recover the classical plasma oscillation equation when $E \to 0$. Thereby, we focus our attention on its evolution in time studying how this oscillation frequency approaches the plasma frequency. The time-scale needed to approach to the plasma frequency and the power spectrum of these oscillations are computed. The characteristic frequency of the power spectrum is determined uniquely from the initial value of the electric field strength. The effects of plasma degeneracy and pair annihilation are discussed.

21. M. Rotondo, Jorge A. Rueda, R. Ruffini, S.-S. Xue, "The self-consistent general relativistic solution for a system of degenerate neutrons, protons and electrons in beta-equilibrium ", Physics Letters B, Volume 701, Issue 5, p. 667-671 (2011).

    We present the self-consistent treatment of the simplest, nontrivial, self-gravitating system of degenerate neutrons, protons and electrons in β-equilibrium within relativistic quantum statistics and the Einstein-Maxwell equations. The impossibility of imposing the condition of local charge neutrality on such systems is proved, consequently overcoming the traditional Tolman-Oppenheimer-Volkoff treatment. We emphasize the crucial role of imposing the constancy of the generalized Fermi energies. A new approach based on the coupled system of the general relativistic Thomas-Fermi-Einstein-Maxwell equations is presented and solved. We obtain an explicit solution fulfilling global and not local charge neutrality by solving a sophisticated eigenvalue problem of the general relativistic Thomas-Fermi equation. The value of the Coulomb potential at the center of the configuration is $eV(0) \simeq m_\pi c^2$ and the system is intrinsically stable against Coulomb repulsion in the proton component. This approach is necessary, but not sufficient, when strong interactions are introduced.

22. M. Rotondo, Jorge A. Rueda, R. Ruffini, S.-S. Xue, "On Compressed Nuclear Matter: from Nuclei to Neutron Stars ", International Journal of Modern Physics D, Volume 20, Issue 10, pp. 1789-1796 (2011).

    We address the description of neutron-proton-electron degenerate matter in beta equilibrium subjected to compression both in the case of confined nucleons into a nucleus as well as in the case of deconfined nucleons. We follow a step-by-step generalization of the classical Thomas-Fermi model to special and general relativistic regimes, which leads to a unified treatment of beta equilibrated neutron-proton-electron degenerate matter applicable from the case of nuclei all the way up to the case of white-dwarfs and neutron stars. New gravito-electrodynamical effects, missed in the traditional approach for the description of neutron star configurations, are found as a consequence of the new set of general relativistic equilibrium equations.

23. M. Rotondo, Jorge A. Rueda, R. Ruffini, S.-S. Xue, "The relativistic Feynman-Metropolis-Teller theory for white-dwarfs in general relativity ", Physical Review D, vol. 84, Issue 8, 084007 (2011).

    The recent formulation of the relativistic Thomas-Fermi model within the Feynman-Metropolis-Teller theory for compressed atoms is applied to the study of general relativistic white dwarf equilibrium configurations. The equation of state, which takes into account the β-equilibrium, the nuclear and the Coulomb interactions between the nuclei and the surrounding electrons, is obtained as a function of the compression by considering each atom constrained in a Wigner-Seitz cell. The contribution of quantum statistics, weak, nuclear, and electromagnetic interactions is obtained by the determination of the chemical potential of the Wigner-Seitz cell. The further contribution of the general relativistic equilibrium of white dwarf matter is expressed by the simple formula $\sqrt{g_{00}}\mu_{\rm ws}$= constant, which links the chemical potential of the Wigner-Seitz cell $\mu_{\rm ws}$ with the general relativistic gravitational potential g_{00} at each point of the configuration. The configuration outside each Wigner-Seitz cell is strictly neutral and therefore no global electric field is necessary to warranty the equilibrium of the white dwarf. These equations modify the ones used by Chandrasekhar by taking into due account the Coulomb interaction between the nuclei and the electrons as well as inverse β-decay. They also generalize the work of Salpeter by considering a unified self-consistent approach to the Coulomb interaction in each Wigner-Seitz cell. The consequences on the numerical value of the Chandrasekhar-Landau mass limit as well as on the mass-radius relation of ^{4}He, ^{12}C, ^{16}O and ^{56}Fe white dwarfs are presented. All these effects should be taken into account in processes requiring a precision knowledge of the white dwarf parameters.

24. M. Rotondo, Jorge A. Rueda, R. Ruffini, S.-S. Xue, "On the relativistic Thomas-Fermi treatment of compressed atoms and compressed nuclear matter cores of stellar dimensions ", Physics Review C83, 045805 (2011).

    The Feynman, Metropolis and Teller treatment of compressed atoms is extended to the relativistic regimes. Each atomic configuration is confined by a Wigner-Seitz cell and is characterized by a positive electron Fermi energy. The non-relativistic treatment assumes a point-like nucleus and infinite values of the electron Fermi energy can be attained. In the relativistic treatment there exists a limiting configuration, reached when the Wigner-Seitz cell radius equals the radius of the nucleus, with a maximum value of the electron Fermi energy $(E_e^F)_{max}$, here expressed analytically in the ultra-relativistic approximation. The corrections given by the relativistic Thomas-Fermi-Dirac exchange term are also evaluated and shown to be generally small and negligible in the relativistic high density regime. The dependence of the relativistic electron Fermi energies by compression for selected nuclei are compared and contrasted to the non-relativistic ones and to the ones obtained in the uniform approximation. The relativistic Feynman, Metropolis, Teller approach here presented overcomes some difficulties in the Salpeter approximation generally adopted for compressed matter in physics and astrophysics. The treatment is then extrapolated to compressed nuclear matter cores of stellar dimensions with $A \simeq (m_{\rm Planck}/m_n)^3 \sim 10^{57}$ or $M_{core} \sim M_\odot$. A new family of equilibrium configurations exists for selected values of the electron Fermi energy varying in the range $0 < E_e^F \leq (E_e^F)_{max}$. Such configurations fulfill global but not local charge neutrality. They have electric fields on the core surface, increasing for decreasing values of the electron Fermi energy reaching values much larger than the critical value $E_c = m_e^2 c^3/(e\hbar)$, for $E_e^F = 0$. We compare and contrast our results with the ones of Thomas-Fermi model in strange stars.

25. Belvedere, Riccardo; Pugliese, Daniela; Rueda, Jorge A.; Ruffini, Remo; Xue, She-Sheng, "Neutron star equilibrium configurations within a fully relativistic theory with strong, weak, electromagnetic, and gravitational interactions ", Nuclear Physics A, Volume 883, p. 1-24, 2012.

    We formulate the equations of equilibrium of neutron stars taking into account strong, weak, electromagnetic, and gravitational interactions within the framework of general relativity. The nuclear interactions are described by the exchange of the sigma, omega, and rho virtual mesons. The equilibrium conditions are given by our recently developed theoretical framework based on the Einstein-Maxwell-Thomas-Fermi equations along with the constancy of the general relativistic Fermi energies of particles, the "Klein potentials", throughout the configuration. The equations are solved numerically in the case of zero temperatures and for selected parametrization of the nuclear models. The solutions lead to a new structure of the star: a positively charged core at supranuclear densities surrounded by an electronic distribution of thickness $\sim \hbar/(m_e c)$ of opposite charge, as well as a neutral crust at lower densities. Inside the core there is a Coulomb potential well of depth $\sim m_\pi c^2/e$. The constancy of the Klein potentials in the transition from the core to the crust, impose the presence of an overcritical electric field $\sim (m_\pi/m_e)^2 E_c$, the critical field being $E_c = m_e^2 c^3/(e\hbar)$. The electron chemical potential and the density decrease, in the boundary interface, until values $\mu_e^{\rm crust} < \mu_e^{\rm core}$ and $\rho_{\rm crust} < \rho_{\rm core}$. For each central density, an entire family of core-crust interface boundaries and, correspondingly, an entire family of crusts with different mass and thickness, exist. The configuration with $\rho_{\rm crust} = \rho_{\rm drip} \sim 4.3 \times 10^{11}$ g/cm^3 separates neutron stars with and without inner crust. We present here the novel neutron star mass-radius for the case $\rho_{\rm crust} = \rho_{\rm drip}$ and compare and contrast it with the one obtained from the Tolman-Oppenheimer-Volkoff treatment.

26. H. Kleinert and S.-S. Xue, "Vacuum pair-production in a classical electric field and an electromagnetic wave ", Annals of Physics 333 (2013) 104.

    Using semiclassical WKB-methods, we calculate the rate of electronC positron pair-production from the vacuum in the presence of two external fields, a strong (space- or time-dependent) classical field and a monochromatic electromagnetic wave. We discuss the possible medium effects on the rate in the presence of thermal electrons, bosons, and neutral plasma of electrons and protons at a given temperature and chemical potential. Using our rate formula, we calculate the rate enhancement due to a laser beam, and discuss the possibility that a significant enhancement may appear in a plasma of electrons and protons with self-focusing properties.

27. Jorge A. Rueda, R. Ruffini, S.-S. Xue, "The Klein first integrals in an equilibrium system with electromagnetic, weak, strong and gravitational interactions ", Nuclear Physics A, Volume 872, Issue 1, 286-295 (2011).

    The isothermal Tolman condition and the constancy of the Klein potentials originally expressed for the sole gravitational interaction in a single fluid are here generalized to the case of a three quantum fermion fluid duly taking into account the strong, electromagnetic, weak and gravitational interactions. The set of constitutive equations including the Einstein-Maxwell-Thomas-Fermi equations as well as the ones corresponding to the strong interaction description are here presented in the most general relativistic isothermal case. This treatment represents an essential step to correctly formulate a self-consistent relativistic field theoretical approach of neutron stars.

28. H. Kleinert, E. Strobel and S.-S. Xue, "Fractional Effective Action at strong electromagnetic fields ", Physics Review D88, 025049 (2013).

    In 1936, Weisskopf showed that for vanishing electric or magnetic fields the strong-field behavior of the one loop Euler-Heisenberg effective Lagrangian of quantum electro dynamics (QED) is logarithmic. Here we generalize this result for different limits of the Lorentz invariants $(\vec{E}^2 - \vec{B}^2)$ and $(\vec{B} \cdot \vec{E})$. The logarithmic dependence can be interpreted as a lowest-order manifestation of an anomalous power behavior of the effective Lagrangian of QED, with critical exponents $(\delta = e^2/(12\pi))$ for spinor QED, and $(\delta_S = \delta/4)$ for scalar QED.

29. A. Benedetti, R. Ruffini and G. Vereshchagin, "Phase space evolution of pairs created in strong electric fields ", Phys. Lett. A377, 206-215 (2013).

    We study the process of energy conversion from overcritical electric field into electron-positron-photon plasma. We solve numerically Vlasov-Boltzmann equations for pairs and photons assuming the system to be homogeneous and anisotropic. All the 2-particle QED interactions between pairs and photons are described by collision terms. We evidence several epochs of this energy conversion, each of them associated to a specific physical process. Firstly pair creation occurs, secondly back reaction results in plasma oscillations. Thirdly photons are produced by electron-positron annihilation. Finally particle interactions lead to completely equilibrated thermal electron-positron-photon plasma.

30. W.-B. Han, R. Ruffini and S.-S. Xue, "Electron and positron pair production in gravitational collapse ", Physics Review D86, 084004 (2012).

    Neutral stellar core at or over nuclear densities is described by a positive charged baryon core and negative charged electron fluid since they possess different masses and interactions. Based on a simplified model of a gravitationally collapsing or pulsating baryon core, we approximately integrate the Einstein-Maxwell equations and the equations for the number and energy-momentum conservation of complete degenerate electron fluid. We show possible electric processes that lead to the production of electron-positron pairs in the boundary of a baryon core and calculate the number and energy of electron-positron pairs. This can be relevant for understanding the energetic sources of supernovae and gamma-ray bursts.

31. S.-P. Kim, H. W. Lee and R. Ruffini, "Schwinger Pair Production in Pulsed Electric Fields ", arXiv:1207.5213 (2012).

    We numerically investigate the temporal behavior and the structure of longitudinal momentum spectrum and the field polarity effect on pair production in pulsed electric fields in scalar quantum electrodynamics (QED). Using the evolution operator expressed in terms of the particle and antiparticle operators, we find the exact quantum states under the influence of electric pulses and measure the number of pairs of the Minkowski particle and antiparticle. The number of pairs, depending on the configuration of electric pulses, exhibits rich structures in the longitudinal momentum spectrum and undergoes diverse dynamical behaviors at the onset of the interaction but always either converges to a momentum-dependent constant or oscillates around a momentum-dependent time average after the completion of fields.

32. R. Ruffini, Y.-B. Wu and S.-S. Xue, "Einstein-Euler-Heisenberg theory and charged black holes ", Physics Review D88, 085004 (2013).

    Taking into account the Euler-Heisenberg effective Lagrangian of one-loop nonperturbative quantum electrodynamics (QED) contributions, we formulate the Einstein-Euler-Heisenberg theory and study the solutions of nonrotating black holes with electric and magnetic charges in spherical geometry. In the limit of strong and weak electromagnetic fields of black holes, we calculate the black hole horizon radius, area, and total energy up to the leading order of QED corrections and discuss the black hole irreducible mass, entropy, and maximally extractable energy as well as the Christodoulou-Ruffini mass formula. We find that these black hole quantities receive the QED corrections, in comparison with their counterparts in the Reissner-Nordström solution. The QED corrections show the screening effect on black hole electric charges and the paramagnetic effect on black hole magnetic charges. As a result, the black hole horizon area, irreducible mass, and entropy increase; however, the black hole total energy and maximally extractable energy decrease, compared with the Reissner-Nordström solution. In addition, we show that the condition for extremely charged black holes is modified due to the QED correction.

33. R. Ruffini and S.-S. Xue, "Gravitational and electric energies in collapse of spherically thin capacitor ", Physics Letters A377 (2013) 2450.

    We adopt a simplified model describing the collapse of a spherically thin capacitor to give an analytical description how gravitational energy is converted to both kinetic and electric energies in collapse. It is shown that (i) averaged kinetic and electric energies are the same order, about an half of gravitational energy of spherically thin capacitor in collapse; (ii) caused by radiating and rebuilding electric energy, gravitational collapse undergoes a sequence of "on and off" hopping steps in the microscopic Compton scale. Although such a collapse process is still continuous in terms of macroscopic scales, it is slowed down as kinetic energy is reduced and collapsing time is about an order of magnitude larger than that of collapse process eliminating electric processes.

6. Invited talks in international conferences

1. Yamada conference "On the dyadosphere of black holes" in Kyoto Japan, April 1998 .
2. International workshop on Gamma Ray Bursts, Rome (1998) .
3. 19th Texas Symposium, Dec. 1998
4. "Exploring the Universe", a Festschrift in honour of Riccardo Giacconi, (2000).
5. Fluctuating Paths and Fields - Dedicated to Hagen Kleinert on the Occasion of His 60th Birthday, Berlin 2001.
6. The ESO workshop on "Black Holes in Binaries and Galactic Nuclei", in honour of Prof. R. Giacconi, (2000) .
7. Marcel Grossmann Meetings IX (Rome) (2000), X (Brazil) (2003) and XI Berlin (2006).
8. International conference in the quantum aspect of beam physics in Hiroshima Japan (2003)
9. "Frontiers in Astroparticle Physics and Cosmology", 6th RESCEU International Symposium, Tokyo 2003.
10. International Conference "Analysis, manifolds and geometric structures in physics", in Honour of Y. Choquet-Bruhat, Isola d'Elba June 24th-26th, 2004 .
11. Brazilian School of Cosmology and Gravitation X (2002), XI (2004) and XII (2006) (Portobello, Brazile).
12. Relativistic Astrophysics and Cosmology - Einstein's Legacy meeting, November 7-11, 2005,
13. 35th COSPAR scientific assembly (Paris, 2004) and 36th COSPAR scientific assembly (Beijing , 2006).

14. 9th International Conference Path Integrals - New Trends and Perspectives, DRESDEN, Germany 23 - 28 September 2007
15. APS April meeting, April 12-15 2008, Saint Louis (USA).
16. V Italian-Sino Workshop, May 28- June 1 2008, Taipei (Taiwan).
17. III Stueckelberg Workshop, July 8-18 2008, Pescara (Italy).
18. XIII Brazilian School of Cosmology and Gravitation, July 20-August 2 2008, Rio de Janeiro (Brazil).
19. Path Integrals - New Trends and Perspectives, September 23 - 28 2007, Dresden (Germany)
20. APS April meeting, April 14-17 2007, Jacksonville (USA).
21. The first Sobral Meeting, May 26-29, 2009 Fortaleza (Cear) Brazi
22. Zeldovich Meeting, April 20-23, 2009 Minsk - (BELARUS).
23. XI Marcel Grossmann Meeting on General Relativity, July 23-29 2006, Berlin (Germany).
24. The first Galileo - Xu Guangqi Meeting October 26-30, 2009 - Shanghai - (CHINA).
25. 11th Italian-Korean Meeting November 2-4, 2009 - Seoul - (KOREA).
26. Christchurch Meeting December 16-18, 2009 - Christchurch - (New Zealand).
27. Annual Meeting of the Korean Physical Society October 19-22, 2010 - Seoul - (KOREA).
28. The second Galileo - Xu Guangqi Meeting July 12-18, 2010 - Ventimiglia and Nice - (Italy and France).
29. 12th Italian-Korean Meeting July 4-8, 2011, Pescara, Italy .
30. The third Galileo - Xu Guangqi Meeting October 12-16, 2011 Beijing (China).
31. The first LeCosPA Symposium: Towards Ultimate Understanding of the Universe, Feb 6-9, 2012, Taipei Taiwan.
32. The meeting for Italian-Korean cooperation, Nov 5-6, 2012, Seoul, South Korea.
33. The 13th MG meeting, July 1-7, 2012, Stockholm, Sweden.

34. The Scientific meeting of ICRANet, June, 2013, Pescara, Italy.
35. The meeting for 9th Italian-Korean meeting, July 12-18, 2013, Seoul, South Korea.

7. APPENDICES

A. Dyadosphere (electron-positron-photon plasma) formation in gravitational collapse.

The e^+e^- pairs generated by the vacuum polarization process around the core are entangled in the electromagnetic field Ruffini et al. (2003a), and thermalize in an electron–positron–photon plasma on a time scale $\sim 10^4 \tau_C$ Ruffini et al. (2003b) (see Fig. 3.2). As soon as the thermalization has occurred, the hydrodynamic expansion of this electrically neutral plasma starts Ruffini et al. (1999, 2000). While the temporal evolution of the $e^+e^-\gamma$ plasma takes place, the gravitationally collapsing core moves inwards, giving rise to a further amplified supercritical field, which in turn generates a larger amount of e^+e^- pairs leading to a yet higher temperature in the newly formed $e^+e^-\gamma$ plasma. We report progress in this theoretically challenging process which is marked by distinctive and precise quantum and general relativistic effects. As presented in Ref. Ruffini et al. (2003a): we follow the dynamical phase of the formation of Dyadosphere and of the asymptotic approach to the horizon by examining the time varying process at the surface of the gravitationally collapsing core.

It is worthy to remark that the time–scale of hydrodynamic evolution ($t \sim 0.1s$) is, in any case, much larger than both the time scale needed for "all pairs to be created" ($\sim 10^3 \tau_C$), and the thermalization time–scale ($\sim 10^4 \tau_C$, see Fig. 3.2) and therefore it is consistent to consider pair production, plus thermalization, and hydrodynamic expansion as separate regimes of the system. We assume the initial condition that the Dyadosphere starts to be formed at the instant of gravitational collapse $t_{\rm ds} = t_0 \left(r_{\rm ds}\right) = 0$, and $r_{\rm ds} = R_c$ the radius of massive nuclear core. Having formulated the core collapse in General Relativity in Eq. (3.11.2), we discretize the gravitational collapse of a spherically symmetric core by considering a set of events ($N-$events) along the world line of a point of fixed angular position on the collapsing core surface. Between each of these events we consider a spherical shell of plasma of constant coordinate thickness Δr so that:

1. Δr is assumed to be a constant which is small with respect to the core radius;

2. Δr is assumed to be large with respect to the mean free path of the particles so that the statistical description of the $e^+e^-\gamma$ plasma can be used;

3. There is no overlap among the slabs and their union describes the entirety of the process.

We check that the final results are independent of the special value of the chosen Δr and N.

In each slab the processes of e^+e^--pair production, oscillation with electric field and thermalization with photons are considered. While the average of the electric field $\mathcal{E}$ over one oscillation is 0, the average of $\mathcal{E}^2$ is of the order of $\mathcal{E}_c^2$, therefore the energy density in the pairs and photons, as a function of r_0, is given by

$$\epsilon_0\left(r_0\right) = \tfrac{1}{8\pi}\left[\mathcal{E}^2\left(r_0\right) - \mathcal{E}_{\rm c}^2\right] = \tfrac{\mathcal{E}_{\rm c}^2}{8\pi}\left[\left(\tfrac{r_{\rm ds}}{r_0}\right)^4 - 1\right]. \tag{A.0.1}$$

For the number densities of e^+e^- pairs and photons at thermal equilibrium we have $n_{e^+e^-} \simeq n_\gamma$; correspondingly the equilibrium temperature T_0, which is clearly a function of r_0 and is different for each slab, is such that Ruffini et al. (1999, 2000)

$$\epsilon\left(T_0\right) \equiv \epsilon_\gamma\left(T_0\right) + \epsilon_{e^+}\left(T_0\right) + \epsilon_{e^-}\left(T_0\right) = \epsilon_0, \tag{A.0.2}$$

with ϵ and n given by Fermi (Bose) integrals (with zero chemical potential):

$$\epsilon_{e^+e^-}\left(T_0\right) = \tfrac{2}{\pi^2\hbar^3}\int_{m_e}^{\infty}\tfrac{\left(E^2-m_e^2\right)^{1/2}}{\exp(E/kT_0)+1}E^2 dE, \quad \epsilon_\gamma\left(T_0\right) = \tfrac{\pi^2}{15\hbar^3}\left(T_0\right)^4, \tag{A.0.3}$$

$$n_{e^+e^-}\left(T_0\right) = \tfrac{1}{\pi^2\hbar^3}\int_{m_e}^{\infty}\tfrac{\left(E^2-m_e^2\right)^{1/2}}{\exp(E/kT_0)+1}E dE, \quad n_\gamma\left(T_0\right) = \tfrac{2\zeta(3)}{\hbar^3}\left(T_0\right)^3. \tag{A.0.4}$$

From the conditions set by Eqs. (A.0.2), (A.0.3), (A.0.4), we can now turn to the dynamical evolution of the $e^+e^-\gamma$ plasma in each slab. We use the covariant conservation of energy momentum and the rate equation for the number of pairs in the Reissner–Nordström geometry external to the core:

$$\nabla_a T^{ab} = 0, \tag{A.0.5}$$

$$\nabla_a\left(n_{e^+e^-}u^a\right) = \overline{\sigma v}\left[n_{e^+e^-}^2\left(T\right) - n_{e^+e^-}^2\right], \tag{A.0.6}$$

where $T^{ab} = (\epsilon + p)\, u^a u^b + p g^{ab}$ is the energy–momentum tensor of the plasma with proper energy density ϵ and proper pressure p, u^a is the fluid 4−velocity, $n_{e^+e^-}$ is the number of pairs, $n_{e^+e^-}\left(T\right)$ is the equilibrium number of pairs and $\overline{\sigma v}$ is the mean of the product of the e^+e^- annihilation cross-section and the thermal velocity of pairs. In each slab the plasma remains at thermal equilibrium in the initial phase of the expansion and the right hand side of the rate Eq. (A.0.6) is effectively 0.

If we denote by ξ^a the static Killing vector field normalized at unity at spacial infinity and by $\{\Sigma_t\}_t$ the family of space-like hypersurfaces orthogonal to ξ^a (t being the Killing time) in the Reissner–Nordström geometry, from Eqs. (A.0.6), the following integral conservation laws can be derived

$$\int_{\Sigma_t} \xi_a T^{ab} d\Sigma_b = E, \quad \int_{\Sigma_t} n_{e^+e^-} u^b d\Sigma_b = N_{e^+e^-}, \tag{A.0.7}$$

where $d\Sigma_b = \alpha^{-2}\xi_b r^2 \sin\theta dr d\theta d\phi$ is the vector surface element, E the total energy and $N_{e^+e^-}$ the total number of pairs which remain constant in each slab. We then have

$$\left[(\epsilon + p)\,\gamma^2 - p\right] r^2 = \mathfrak{E}, \quad n_{e^+e^-}\gamma\alpha^{-1}r^2 = \mathfrak{N}_{e^+e^-}, \tag{A.0.8}$$

where $\mathfrak{E}$ and $\mathfrak{N}_{e^+e^-}$ are constants and

$$\gamma \equiv \alpha^{-1}u^a\xi_a = \left[1 - \alpha^{-4}\left(\frac{dr}{dt}\right)^2\right]^{-1/2} \tag{A.0.9}$$

is the Lorentz γ factor of the slab as measured by static observers. We can rewrite Eqs. (A.0.7) for each slab as

$$\left(\frac{dr}{dt}\right)^2 = \alpha^4 f_{r_0}, \tag{A.0.10}$$

$$\left(\frac{r}{r_0}\right)^2 = \left(\frac{\epsilon+p}{\epsilon_0}\right)\left(\frac{n_{e^+e^-0}}{n_{e^+e^-}}\right)^2\left(\frac{\alpha}{\alpha_0}\right)^2 - \frac{p}{\epsilon_0}\left(\frac{r}{r_0}\right)^4, \tag{A.0.11}$$

$$f_{r_0} = 1 - \left(\frac{n_{e^+e^-}}{n_{e^+e^-0}}\right)^2\left(\frac{\alpha_0}{\alpha}\right)^2\left(\frac{r}{r_0}\right)^4 \tag{A.0.12}$$

where pedex $_0$ refers to quantities evaluated at selected initial times $t_0 > 0$, having assumed $r\,(t_0) = r_0$, $dr/dt|_{t=t_0} = 0$, $T\,(t_0) = T_0$.

Eq. (A.0.10) is only meaningful when $f_{r_0}\,(r) \geq 0$. From the structural analysis of such equation it is clearly identifiable a critical radius r_0 such that:

- for any slab initially located at $r_0 > \bar{R}$ we have $f_{r_0}\,(r) \geq 0$ for any value of $r \geq r_0$ and $f_{r_0}\,(r) < 0$ for $r \lesssim r_0$; therefore a slab initially located at a radial coordinate $r_0 > \bar{R}$ moves outwards,
- for any slab initially located at $r_0 < \bar{R}$ we have $f_{r_0}\,(r) \geq 0$ for any value of $r_+ < r \leq r_0$ and $f_{r_0}\,(r) < 0$ for $r \gtrsim r_0$; therefore a slab initially located at a radial coordinate $r_0 < \bar{R}$ moves inwards and is trapped by the gravitational field of the collapsing core.

We define the surface $r = \bar{R}$, the *Dyadosphere trapping surface* (DTS). The radius $\bar{R}$ of DTS is generally evaluated by the condition $\left.\frac{df_{\bar{R}}}{dr}\right|_{r=\bar{R}} = 0$. $\bar{R}$ is so close to the horizon value r_+ that the initial temperature T_0 satisfies $kT_0 \gg m_e c^2$ and we can obtain for $\bar{R}$ an analytical expression. Namely the ultra relativistic approximation of all Fermi integrals, Eqs. (A.0.3) and (A.0.4), is justified and we have $n_{e^+e^-}(T) \propto T^3$ and therefore $f_{r_0} \simeq 1 - (T/T_0)^6 (\alpha_0/\alpha)^2 (r/r_0)^4$ $(r \leq \bar{R})$. The defining equation of $\bar{R}$, together with (A.0.12), then gives

$$\bar{R} = 2M\left[1 + \left(1 - 3Q^2/4M^2\right)^{1/2}\right] > r_+. \tag{A.0.13}$$

In the case of an EMBH with $M = 20M_\odot$, $Q = 0.1M$, we compute:

- the fraction of energy trapped in DTS:

$$\bar{E} = \int_{r_+<r<\bar{R}} \alpha\epsilon_0 d\Sigma \simeq 0.53 \int_{r_+<r<r_{\rm ds}} \alpha\epsilon_0 d\Sigma; \tag{A.0.14}$$

- the world–lines of slabs of plasma for selected r_0 in the interval $(\bar{R}, r_{\rm ds})$ (see left figure in Fig. A.1);
- the world–lines of slabs of plasma for selected r_0 in the interval $(r_+, \bar{R})$ (see Fig. A.2).

At time $\bar{t} \equiv t_0(\bar{R})$ when the DTS is formed, the plasma extends over a region of space which is almost one order of magnitude larger than the Dyadosphere and which we define as the *effective Dyadosphere*. The values of the Lorentz γ factor, the temperature and e^+e^- number density in the effective Dyadosphere are given in the right figure in Fig. A.1.

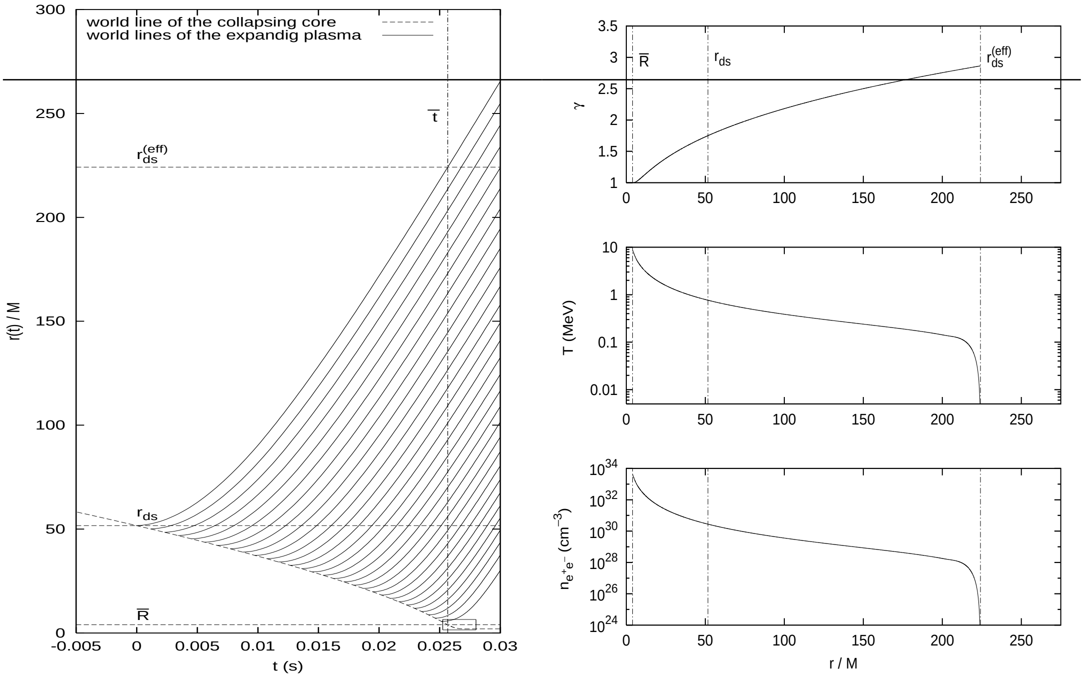

Figure A.1.: In left figure: World line of the collapsing charged core (dashed line) as derived from Eq. (3.11.2); world lines of slabs of plasma for selected radii r_0 in the interval $(\bar{R}, r_{\rm ds})$. At time $\bar{t}$ the expanding plasma extends over a region which is almost one order of magnitude larger than the Dyadosphere. The small rectangle in the right bottom is enlarged in Fig. A.2. The right figure: Physical parameters in the effective Dyadosphere: Lorentz γ factor, proper temperature and proper e^+e^- number density as functions at time $\bar{t}$.

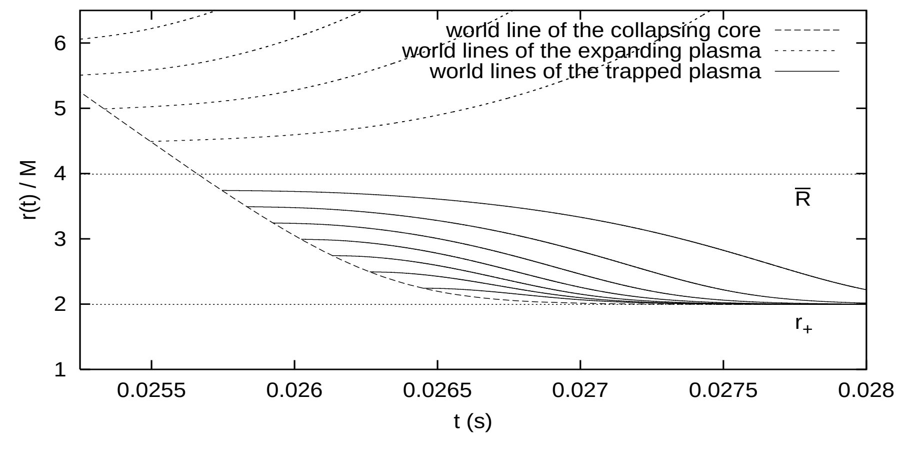

Figure A.2.: Enlargement of the small rectangle in the right bottom of left figure in Fig. A.1. World–lines of slabs of plasma for selected radii r_0 in the interval $(r_+, \bar{R})$.

B. Electron-positron pair oscillation in spatially inhomogeneous electric fields and radiation

Introduction. As reviewed in the recent report Ruffini et al. (2010), since the pioneer works by Sauter Sauter (1931), Heisenberg and Euler Heisenberg and Euler (1936) in 1930's, then by Schwinger Schwinger (1951) in 1950's, it has been well known that positron-electron pairs are produced from the vacuum in external electric fields. In a constant electric field E_0 in dependent of space and time, the pair-creation rate per unit volume is given by Heisenberg and Euler (1936),

$$S \equiv \frac{dN}{dVdt} = \frac{m_e^4}{4\pi^3}\left(\frac{E_0}{E_c}\right)^2 \exp\left(-\pi\frac{E_c}{E_0}\right), \tag{B.0.1}$$

where the critical field $E_c \equiv m_e^2c^3/(e\hbar)$, the Plank's constant $\hbar$, the speed of light c, the electron mass m_e, the absolute value of electron charge e and the fine structure constant $\alpha = e^2/\hbar c$ (in this article we use the natural units $\hbar = c = 1$, unless otherwise specified). The pair-production rate (B.0.1) is significantly large for strong electric fields $E \gtrsim E_c \simeq 1.3 \cdot 10^{16}\mathrm{V/cm}$. The critical field will probably be reached by recent advanced laser technologies in laboratory experiments Ringwald (2001); Tajima and Mourou (2002); Gordienko et al. (2005), X-ray free electron laser (XFEL) facilities[1], optical high-intensity laser facilities such as Vulcan or ELI[2], and SLAC E144 using nonlinear Compton scattering Burke et al. (1997). On the other hand, strong overcritical electric fields ($E \geq 10E_c$) can be created in astrophysical environments, for instance, quark stars Usov (1998); Usov et al. (2005) and neutron stars Ruffini et al. (2007a)-Popov et al. (2009).

The back-reaction and screening effects of electron and positron pairs on external electric fields lead to the phenomenon of plasma oscillations: electrons and positrons moving back and forth coherently with alternating electric fields. This means that external electric fields are not eliminated within the Compton time $\hbar/m_ec^2$ of pair-production process, rather oscillate collec-

[1] http://www.xfel.eu
[2] http://www.extreme-light-infrastructure.eu

tively with the motion of pairs in a much longer timescale.

In a constant electric field E_0 (B.0.1), the phenomenon of plasma oscillations is studied in the two frameworks Ruffini et al. (2010): (1) the semi-classical QED with quantized Dirac field and classical electric field Kluger et al. (1991, 1992); Cooper and Mottola (1989); (2) the kinetic description using the Boltzmann-Vlasov and Maxwell equations Biro et al. (1984); Gatoff et al. (1987); Cooper et al. (1993); Ruffini et al. (2003b, 2007b). In the second framework, the Boltzmann-Vlasov equation is used to obtain the equations for the continuity and energy-momentum conservations Gatoff et al. (1987).

Ref. Ruffini et al. (2007b) shows the evidence of plasma oscillation in undercritical field ($E < E_c$) and the relation between the kinetic energy and numbers of oscillating pairs in a given electric field strength E_0. Taking into account the creation and annihilation process $e^+ + e^- \Leftrightarrow \gamma + \gamma$, it is shown Ruffini et al. (2003b) that the plasma oscillation in an overcritical field is led to a plasma of photons, electrons and positions with the equipartition of their number- and energy-densities. The phenomenon of plasma oscillations is studied in connection with pair creation in heavy ions collisions Biro et al. (1984)-Cooper et al. (1993), the laser field Ringwald (2001)-Hebenstreit et al. (2008), and gravitational collapse Ruffini et al. (2003a). It is worthwhile to emphasize that the plasma oscillation occurs not only at overcritical field-strengths $E_0 \gtrsim E_c$ (see for instance Refs. Kluger et al. (1991, 1992); Ruffini et al. (2003b)), but also undercritical field-strengths $E_0 \lesssim E_c$ (see Ref. Ruffini et al. (2007b)), and plasma oscillation frequency is related to field-strength E_0, while the number of oscillating pairs depends on the pair-production rate (B.0.1). More details can be found in the recent review article Ruffini et al. (2010).

The realistic ultra-strong electric fields are not only vary with space and time, but also confined in a finite region. In this letter, studying the plasma oscillations in spatially inhomogeneous electric field, we present the evidence of electric fields propagation, leading to electromagnetic radiation with a peculiar narrow spectrum in the keV-region, which should be distinctive and experimentally observable.

In the kinetic description for the plasma fluids of positrons (+) or electrons (−), whose single-particle spectrum $p^0_\pm = (\mathbf{p}^2_\pm + m_e^2)^{1/2}$, we define the number-densities $n_\pm(t, \mathbf{x})$ and "averaged" velocities $\mathbf{v}_\pm(t, \mathbf{x})$ of the fluids:

$$n_\pm(t,\mathbf{x}) \equiv \int \frac{d^3\mathbf{p}_\pm}{(2\pi)^3} f_\pm(t,\mathbf{p}_\pm,\mathbf{x}), \tag{B.0.2}$$

$$\mathbf{v}_\pm(t,\mathbf{x}) \equiv \frac{1}{n_\pm} \int \frac{d^3\mathbf{p}_\pm}{(2\pi)^3} \left(\frac{\mathbf{p}_\pm}{p^0_\pm}\right) f_\pm(t,\mathbf{p}_\pm,\mathbf{x}), \tag{B.0.3}$$

where $f_\pm(t, \mathbf{p}_\pm, \mathbf{x})$ is the distribution function in the phase space. The four-velocities of the electron and positron fluids $U^\mu_\pm = \gamma_\pm(1, \mathbf{v}_\pm)$, the Lorentz factor $\gamma_\pm = (1-|\mathbf{v}_\pm|^2)^{-1/2}$, and the comoving number-densities $\bar{n}_\pm = n_\pm(\gamma_\pm)^{-1}$, where we choose the laboratory frame where pairs are created at rest. The collision-less plasma fluid of electrons and positrons coupling to electromagnetic fields is governed by the continuity, energy-momentum conservation and Maxwell equations:

$$\frac{\partial\left(\bar{n}_\pm U^\mu_\pm\right)}{\partial x^\mu} = S, \tag{B.0.4}$$

$$\frac{\partial T^{\mu\nu}_\pm}{\partial x^\nu} = -F^\mu_\sigma(J^\sigma_\pm + J^\sigma_{\pm\mathrm{pola}}), \tag{B.0.5}$$

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = -4\pi(J^\mu_{\mathrm{cond}} + J^\mu_{\mathrm{pola}} + J^\mu_{\mathrm{ext}}), \tag{B.0.6}$$

where is the pair-production rate, $J^\mu_\pm = \pm e\bar{n}_\pm U^\mu_\pm$ electric currents and the energy-momentum tensors Weinberg (1972)

$$T^{\mu\nu}_\pm = \bar{p}_\pm g^{\mu\nu} + (\bar{p}_\pm + \bar{\epsilon}_\pm)U^\mu_\pm U^\nu_\pm, \tag{B.0.7}$$

and the pressure $\bar{p}_\pm$ and comoving energy-density $\bar{\epsilon}_\pm$ is related by the equation of state, in general $0 \le \bar{p}_\pm \le \bar{\epsilon}_\pm/3$. In the laboratory frame, the fluid energy-density $\epsilon_\pm \equiv T^{00}$ and momentum-density $p^i_\pm \equiv T^{i0}$ are given by

$$\epsilon_\pm = (\bar{\epsilon}_\pm + \bar{p}_\pm \mathbf{v}^2_\pm)\gamma^2_\pm, \quad \mathbf{p}_\pm = (\bar{\epsilon}_\pm + \bar{p}_\pm)\gamma^2_\pm \mathbf{v}_\pm. \tag{B.0.8}$$

In Eqs. (B.0.5,B.0.6) F^μ_σ is the tensor of electromagnetic fields $(\mathbf{E}, \mathbf{B})$, the conducting four-current density

$$J^\mu_{\mathrm{cond}} \equiv e(\bar{n}_+ U^\mu_+ - \bar{n}_- U^\mu_-), \quad \partial_\mu J^\mu_{\mathrm{cond}} = 0, \tag{B.0.9}$$

and polarized four-current density $J^\mu_{\mathrm{pola}} = \sum_\pm J^\mu_{\pm\mathrm{pola}}$ and $J^\mu_{\pm\mathrm{pola}} = \left(\rho^\pm_{\mathrm{pola}}, \mathbf{J}^\pm_{\mathrm{pola}}\right)$ Gatoff et al. (1987); Kajantie and Matsui (1985)

$$F^\nu_\mu J^\mu_{\pm\mathrm{pola}} = \Sigma^\nu_\pm, \quad \Sigma^\nu_\pm \equiv \int \frac{d^3\mathbf{p}_\pm}{(2\pi)^3 p^0_\pm} p^\nu_\pm \mathcal{S}, \tag{B.0.10}$$

and $S = \int d^3\mathbf{p}_\pm/[(2\pi)^3 p^0_\pm]\mathcal{S}$. Using "averaged" velocities (B.0.3) of the fluids, we approximately have

$$\mathbf{J}^\pm_{\mathrm{pola}} \simeq \frac{m_e\gamma_\pm S}{|\mathbf{E}|}\hat{\mathbf{E}}, \quad \rho^\pm_{\mathrm{pola}} \simeq \pm\frac{m_e\gamma_\pm|\mathbf{v}_\pm|S}{|\mathbf{E}|}, \tag{B.0.11}$$

where the magnetic field $\mathbf{B} = 0$. In Eq. (B.0.6), $J^\mu_{\mathrm{ext}} = (\rho_{\mathrm{ext}}, \mathbf{J}_{\mathrm{ext}})$ is an external electric current.

Basic equations of motion. For simplicity to start with, we consider the electric field $\mathbf{E}_{\rm ext}$ created by a capacitor made of two parallel plates, one carries an external charge $+Q$ and another $-Q$. The sizes of two parallel plats are L_x and L_y, which are much larger than their separation ℓ in the $\hat{\mathbf{z}}$-direction, i.e., $L_x \gg \ell$ and $L_y \gg \ell$. For $|z| \sim \mathcal{O}(\ell)$, the system has an approximate translation symmetry in the (x, y) plane. As results the electric field $\mathbf{E}_{\rm ext}(x, y, z) \approx E_{\rm ext}(z)\hat{\mathbf{z}}$ and $\mathbf{B}_{\rm ext}(x, y, z) \approx 0$, is approximately homogeneous in the (x, y) plane and confined within the capacitor. In addition, $\partial\mathbf{E}_{\rm ext}/\partial t \approx 0$, namely, this electric field is assumed to be continuously supplied by an external source $(+Q, -Q)$ or slowly varying. In order to do calculations we model this electric field as the one-dimensional Sauter electric field in the $\hat{\mathbf{z}}$-direction

$$E_{\rm ext}(z) = E_0/\cosh^2(z/\ell), \quad \sigma \equiv eE_0\ell/m_e c^2 = (\ell/\lambda_C)(E_0/E_c), \qquad \text{(B.0.12)}$$

where the λ_C is Compton wavelength, the external electric charge is given by $\partial E_{\rm ext}(z)/\partial z = 4\pi\rho_{\rm ext}$ and the external electric current vanishes $J_{\rm ext} = 0$ for the field being static $\partial E_{\rm ext}/\partial t = 0$. In the electric field configuration (B.0.12) and $\mathbf{B} \approx 0$, the "averaged" velocities $v_\pm$ of electrons and positrons fluids are in the $\hat{\mathbf{z}}$-direction,

$$U^\mu_\pm = \gamma_\pm\,(1, 0, 0, \pm v_\pm)\,, \qquad \text{(B.0.13)}$$

and the total fluid current- and charge-densities (B.0.6) $J^\mu = (\rho, \mathbf{J})$ are

$$J_z = en_+v_+ + en_-v_- + \frac{m_e(\gamma_+ + \gamma_-)S}{E}, \qquad \text{(B.0.14)}$$

$$\rho = e\,(n_+ - n_-) + \frac{m_e(\gamma_+v_+ - \gamma_-v_-)S}{E}. \qquad \text{(B.0.15)}$$

The system can be approximately treated as a $1+1$ dimensional system in terms of space-time variables (z, t), and Eqs. (B.0.4-B.0.6) become for zero pressure [3],

$$\frac{\partial n_\pm}{\partial t} \pm \frac{\partial n_\pm v_\pm}{\partial z} = S, \tag{B.0.16}$$

$$\frac{\partial \epsilon_\pm}{\partial t} \pm \frac{\partial p_\pm}{\partial z} = e n_\pm v_\pm E + m_e \gamma_\pm S, \tag{B.0.17}$$

$$\frac{\partial p_\pm}{\partial t} \pm \frac{\partial p_\pm v_\pm}{\partial z} = e n_\pm E + m_e \gamma_\pm v_\pm S, \tag{B.0.18}$$

$$\frac{\partial E}{\partial t} = -4\pi J_z, \tag{B.0.19}$$

$$\frac{\partial E}{\partial z} = 4\pi(\rho + \rho_{\rm ext}). \tag{B.0.20}$$

The total electric field $E(z,t)$ in Eqs. (B.0.14-B.0.20) is the superposition of two components:

$$E(z,t) = E_{\rm ext}(z) + E_{\rm ind}(z,t), \tag{B.0.21}$$

where the space- and time-dependent $E_{\rm ind}(z,t)$ is the electric field created by electron and positron pairs. We call $J_z(z,t)$ (B.0.14), $\rho(z,t)$ (B.0.15) and $E_{\rm ind}(z,t)$ pair-induced electric current, charge and field.

As for the pair-production rate S in Eqs. (B.0.16-B.0.19), instead of the pair-production rate (B.0.1) for a constant field E_0, we adopt the following z-dependent formula for the pair-production rate in the Sauter field (B.0.12), obtained by using the WKB-method to calculate the probability of quantum-mechanical tunneling Kleinert et al. (2008),

$$S(z) = \frac{m_e^4}{4\pi^3}\frac{E_0 E(z)}{E_c^2 \tilde{G}[0,\mathcal{E}]} e^{-\pi G[0,\mathcal{E}]E_c/E_0}, \tag{B.0.22}$$

where $G(0,\mathcal{E})$ and $\tilde{G}(0,\mathcal{E})$ are functions of the energy-level crossings $\mathcal{E}(z)$ and we approximately adopt $E(z) \approx E_0/G(0,\mathcal{E}) \approx E_0/\tilde{G}(0,\mathcal{E})$ in Eq. (B.0.22) in order to do feasible numerical calculations. As shown by the Fig. 2 in Ref. Kleinert et al. (2008), the deviation of the pair-production rate (B.0.22) due to this approximation is small. The formula (B.0.22) is derived for the static Sauter field (B.0.12). However, analogously to the discussions for

[3] For an electric field $E \sim E_c$, the number-density of electron-positron pairs is small and the pressure of pairs can be neglected. While for an over electric field $E \gg E_c$, the number-density of pairs is large and the collisions and annihilation of pairs into photons are important, leading to the energy equipartition of electron, positrons and photons. In this case, the pressure, effective temperature and equation of state have to be considered.For an electric field $E \sim E_c$, the number-density of electron-positron pairs is small and the pressure of pairs can be neglected. While for an over electric field $E \gg E_c$, the number-density of pairs is large and the collisions and annihilation of pairs into photons are important, leading to the energy equipartition of electron, positrons and photons. In this case, the pressure, effective temperature and equation of state have to be considered.

the plasma oscillations in spatially homogeneous fields Cooper et al. (1993)-Ruffini et al. (2007b), it can be approximately used for a time-varying electric field $E(z,t)$ (B.0.21), provided the time-dependent component $E_{\rm ind}(z,t)$, created by electron-positron pair-oscillations, varies much slowly compared with the rate of electron-positron pair-productions $\mathcal{O}(m_e c^2/\hbar)$. This can be justified by the inverse adiabaticity parameter Greiner et al. (1985)-Popov (1973),

$$\eta = \frac{m_e}{\omega}\frac{E_0}{E_c} \gg 1, \tag{B.0.23}$$

where ω is the frequency of pair-oscillations.

Eqs. (B.0.16,B.0.17,B.0.18) describe the motion of electron-positron plasma coupling to the electric field E and source S of pair-productions. The Maxwell equations (B.0.19,B.0.20) describe the motion of the electric field (B.0.21) coupled to the current- and charge-densities (B.0.15), leading to the wave equation of the propagating electric field $E_{\rm ind}(z,t)$ Jackson (1998),

$$\frac{\partial^2 E_{\rm ind}}{\partial t^2} - \frac{1}{c^2}\frac{\partial^2 E_{\rm ind}}{\partial z^2} = 4\pi\left(\frac{\partial \rho}{\partial z} + \frac{1}{c^2}\frac{\partial J_z}{\partial t}\right), \tag{B.0.24}$$

where we use $\partial E_{\rm ext}/\partial z = 4\pi\rho_{\rm ext}$ and $\partial E_{\rm ext}/\partial t = 0$. This wave equation shows the propagating electric field $E_{\rm ind}(z,t)$ in the region $\mathcal{R}$ where the non-vanishing current J_z and charge ρ are, and both the propagation and polarization of the electric field are in the $\hat{\mathbf{z}}$-direction. This implies a wave transportation of electromagnetic energies inside the region $\mathcal{R}$. Since the current- and charge-densities (ρ, J_z) are functions of the field $E(t,z)$ (B.0.21), the wave equation is highly nonlinear, the dispersion relation of the field is very complex and the velocity of field-propagation is not the speed of light.

Numerical integrations. Given the parameters $E_0 = E_c$ and $\ell = 10^5\lambda_C$ of the Sauter field (B.0.12) as an initial electric field $E_{\rm ext}$, we numerically integrate Eqs.(B.0.16-B.0.19) in the spatial region $\mathcal{R}$: $-\ell/2 \leq z \leq \ell/2$ and time interval $\mathcal{T}$: $0 \leq t \leq 3500\tau_C$, where τ_C is the Compton time. The value $\mathcal{T} \leq 3500\tau_C$ is chosen so that the adiabatic condition (B.0.23) is satisfied, and the spatial range $\mathcal{R}$ is determined by the capacity of computer for numerical calculations. The electric field strength E_0 is chosen around the critical value E_c, so that the semiclassical pair-production rate (B.0.22) can be approximately used. Actually, E_0, ℓ and $\mathcal{T}$ are attributed to the characteristics of external ultra-strong electric fields $E_{\rm ext}$ established by either experimental setups or astrophysical conditions.

In Figs. B.1 and B.2, we respectively plot the time- and space-evolution of the total electric fields $E(z,t)$ (B.0.21) as functions of t and z at three different spatial points and times. As discussed in Figure captions, numerical results show the properties of the electric field wave $E_{\rm ind}(z,t)$ propagating in the plasma of oscillating electron-positron pairs, as described by the wave equation (B.0.24). This electric field wave propagates along the directions in which external electric field-strength decreases. The wave propagation is rather complex, depending on the space and time variations of the net charge density $\rho(z,t)$ and current density $j_z(z,t)$, as shown in Figs. B.4-B.5. The net charge density ρ oscillates (see Figs. B.3 and B.4) proportionally to the field-gradient (B.0.20) and at the center $z=0$ the charge density and field-gradient are zero independent of time evolution (see Fig. B.4). However, the total charge of pairs $Q=\int_{\mathcal{R}} d^3x\rho$ must be zero at any time, as required by the neutrality. The electric current $j_z(z,t)$ alternating in space and time follows the space and time evolution of the electric field $E(z,t)$ see Eq. (B.0.19), as shown in Figs. B.5 and B.6.

We recall the discussions of the plasma oscillations in the case of spatially homogeneous electric field E_0 without boundary Ruffini et al. (2003b, 2007b). Due to the spatial homogeneity of electric fields and pair-production rate S (B.0.1), the number-densities $n_\pm(t,\mathbf{x})=n(t)$ (B.0.2), "averaged" velocities $|\mathbf{v}_\pm(t,\mathbf{x})|=v(t)$ (B.0.3) and energy-momenta $\epsilon_\pm(t,\mathbf{x})=\epsilon(t), |\mathbf{p}_\pm(t,\mathbf{x})|=p(t)$ (B.0.8) are spatially homogeneous so that the charge density (B.0.15) $\rho\equiv 0$ identically vanishes and current (B.0.14) $J_z=J_z(t)$. All spatial derivative terms in Eqs. (B.0.16-B.0.18) and Eq. (B.0.24) vanish and Eq. (B.0.20) becomes irrelevant. As results, the plasma oscillations described is the oscillations of electric fields and currents with respect time at each spatial point, and the electric field has no any spatial correlation and does not propagate.

In contrary to the plasma oscillation in homogeneous fields, the presence of such field-propagation in inhomogeneous fields is due to: (i) non-vanishing field-gradient $\partial_z E$ (B.0.20) and net charge-density ρ (B.0.15), as shown in Figs. B.3 and B.4, give the spatial correlations of the fields at neighboring points; (ii) the stronger field-strength, the larger field-oscillation frequency is, as shown in Fig. B.1; (iii) at the center $z=0$ the field-strength is largest and the field-oscillation is most rapid, and the field-oscillations at points $|z|>0$ are slower and in retard phases, as shown in Fig. B.2. The point (i) is essential, the charge density ρ oscillates (see Figs. B.3 and B.4) proportionally to the field-gradient Eq. (B.0.20) and at the center $z=0$ the charge density and field-gradient are zero independent of time evolution (see Fig. B.4). Such field-propagation is reminiscent of the drift motion of particles driven by a field-gradient ("ponderomotive") force, which is a cycle-averaged force on a charged particle in a spatially inhomogeneous oscillating electromagnetic field Boot and R.-S.-Harvie (1957); Kibble (1966); Hopf et al. (1976).

Radiation fields. As numerically shown in Fig. B.1-B.6, the propagation of the electric field wave $E_{\rm ind}(z,t)$ inside the region $\mathcal{R}$ is rather complex, due to th high non-linearity of wave equation (B.0.24). Nevertheless, the electromagnetic radiation fields $\mathbf{E}_{\rm rad}$ and $\mathbf{B}_{\rm rad}$ far away from the region $\mathcal{R}$ are completely determined and could be experimentally observable. At the space-time point $(t,\mathbf{x})$ of an observer, the electromagnetic radiation fields $\mathbf{E}_{\rm rad}(z,t)$ and $\mathbf{B}_{\rm rad}(z,t)$, emitted by the variations of electric charge density $\rho(\mathbf{x}',t')$ and current-density $\mathbf{J}(\mathbf{x}',t')$ in the region $\mathcal{R}$ ($\mathbf{x}'\in\mathcal{R}$) and time t' ($t'\in\mathcal{T}$), are given

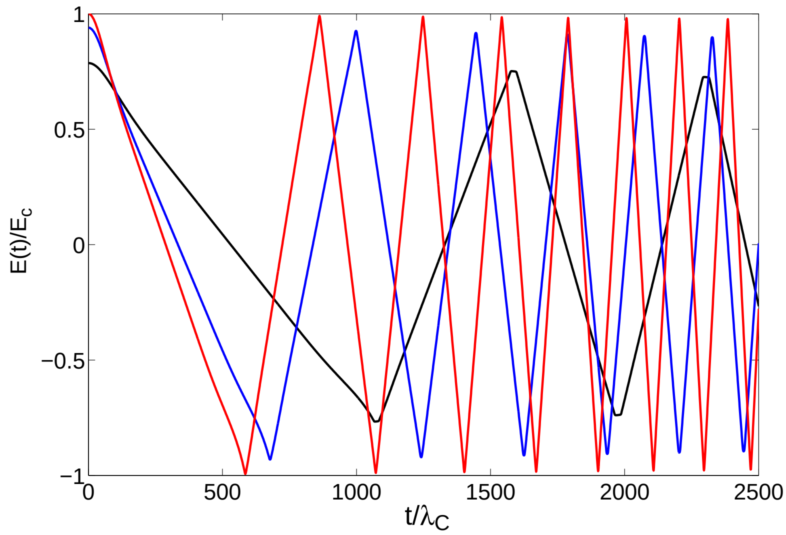

Figure B.1.: Electric fields $E(z,t)$ are plotted as functions of t at three different points: $z=0$ (red), $z=\ell/4$ (blue) and $z=\ell/2$ (black). Analogously to the plasma oscillation in homogeneous fields, the stronger initial field-strength, the larger field-oscillation frequency is, i.e., $\omega(z=0)>\omega(z=\ell/4)>\omega(z=\ell/2)$, where $\omega(z)$ is the field oscillating frequency at the spatial point z.

by Jackson (1998)

$$\mathbf{E}_{\rm rad}(t,\mathbf{x}) = -\int_{\mathcal{R}} d^3\mathbf{x}'\Big\{\frac{\hat{\mathbf{R}}}{R^2}\left[\rho(t',\mathbf{x}')\right]_{\rm ret} + \frac{\hat{\mathbf{R}}}{cR}\left[\frac{\partial\rho(t',\mathbf{x}')}{\partial t'}\right]_{\rm ret} + \frac{1}{c^2R}\left[\frac{\partial\mathbf{J}(t',\mathbf{x}')}{\partial t'}\right]_{\rm ret}\Big\}, \tag{B.0.25}$$

$$\mathbf{B}_{\rm rad}(t,\mathbf{x}) = \int_{\mathcal{R}} d^3\mathbf{x}'\left\{\left[\mathbf{J}(t',\mathbf{x}')\right]_{\rm ret}\times\frac{\hat{\mathbf{R}}}{cR^2} + \left[\frac{\partial\mathbf{J}(t',\mathbf{x}')}{\partial t'}\right]_{\rm ret}\times\frac{\hat{\mathbf{R}}}{c^2R}\right\}. \tag{B.0.26}$$

where the subscript "ret" indicates $t'=t-R/c$, $R=|\mathbf{x}-\mathbf{x}'|$. In the radiation zone $|\mathbf{x}|\gg|\mathbf{x}'|$ and $R\approx|\mathbf{x}|$, where is far away from the plasma oscillation region $\mathcal{R}$, the radiation fields (B.0.25,B.0.26) approximately are

$$\mathbf{E}_{\rm rad}(t,\mathbf{x}) \approx -\frac{1}{c^2|\mathbf{x}|}\int d^3\mathbf{x}'\left[\frac{\partial\mathbf{J}(t',\mathbf{x}')}{\partial t'}\right]_{\rm ret}, \tag{B.0.27}$$

$$\mathbf{B}_{\rm rad}(t,\mathbf{x}) \approx \hat{\mathbf{R}}\times\mathbf{E}_{\rm rad}(t,\mathbf{x}), \tag{B.0.28}$$

where we use the charge conservation (B.0.9) and total neutrality condition of pairs $\int_{\mathcal{R}} d^3\mathbf{x}'\rho(t',\mathbf{x}')=0$. The first terms in Eqs. (B.0.25,B.0.26) are the Coulomb-type fields decaying away as $\mathcal{O}(1/|\mathbf{x}|^2)$. The Fourier transforms of

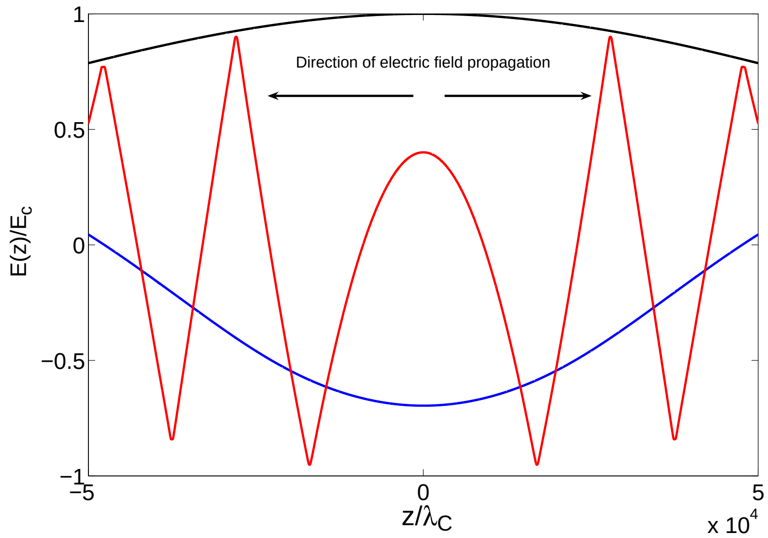

Figure B.2.: Electric fields $E(z,t)$ are plotted as functions of z at three different times in the Compton unit: $t = 1$(black), $t = 500$ (blue) and $t = 1500$ (red). As shown in Fig. B.1, the electric field $E(z,t)$ oscillation at the center ($z = 0$) is most rapid, and gets slower and slower at spatial points ($|z| > 0$) further away from the center. This implies the electric field wave propagating in the space, and the directions of propagations are indicated.

Eqs. (B.0.27) and (B.0.28) are

$$\tilde{\mathbf{E}}_{\rm rad}(\omega,\mathbf{x}) \approx -\frac{e^{-ik|\mathbf{x}|}}{c^2|\mathbf{x}|}\tilde{\mathbf{D}}(\omega), \quad \tilde{\mathbf{B}}_{\rm rad}(\omega,\mathbf{x}) \approx \hat{\mathbf{R}} \times \tilde{\mathbf{E}}_{\rm rad}(\omega,\mathbf{x}) \tag{B.0.29}$$

$$\tilde{\mathbf{D}}(\omega) \equiv \int_{\mathcal{R}} d^3\mathbf{x}' \int_{\mathcal{T}} dt' e^{i\omega t'} \left[\frac{\partial \mathbf{J}(t',\mathbf{x}')}{\partial t'}\right], \tag{B.0.30}$$

where the wave number $k = \omega/c$ and the numerical integration (B.0.30) is carried out overall the space-time evolution of the electric current $\mathbf{J}(\mathbf{x}',t')$ (see Figs. B.6 and B.5). For definiteness we thinks of the oscillation currents occurring for some finite interval of time $\mathcal{T}$ or at least falling off for remote past and future times, so that the total energy radiated is finite, thus the energy radiated per unit solid angle per frequency interval is given by Jackson (1998)

$$\frac{d^2 I}{d\omega d\Omega} = 2|\tilde{\mathbf{D}}(\omega)|^2. \tag{B.0.31}$$

The squared amplitude $|\tilde{\mathbf{D}}(\omega)|^2$ as a function of ω gives the spectrum of the radiation (see Fig. B.7), which is very narrow as expected with a peak locating at $\omega_{\rm peak} \approx 0.08 m_e = 4$keV for $E_0 = E_c$, consistently with the plasma

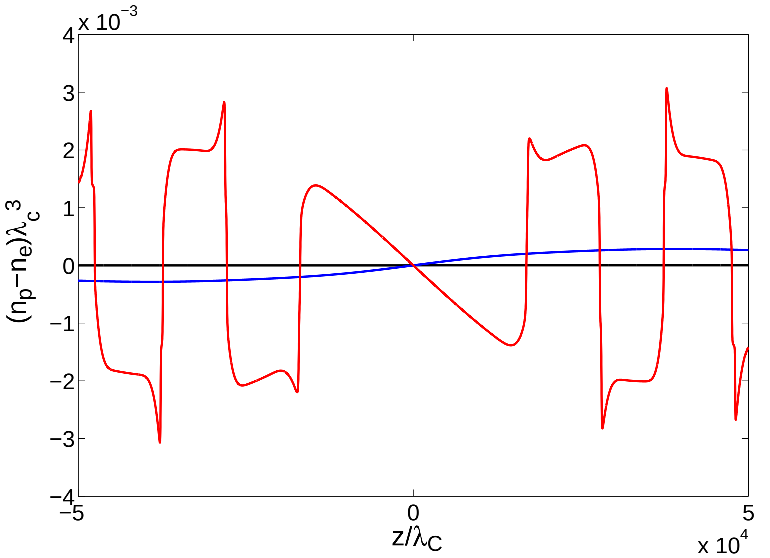

Figure B.3.: The net charge density $\rho(z, t)$ [see Eq. (B.0.15)] as a function of z at three different times: $t = 1$ (black, nearly zero), $t = 500$ (blue) and $t = 1500$ (red). It is shown that the net charged density value $|\rho(z, t)|$ is zero at the center where the initial electric field gradient vanishes [see Eq. (B.0.20)], whereas it increases as the initial electric field gradient increases for $|z| > 0$.

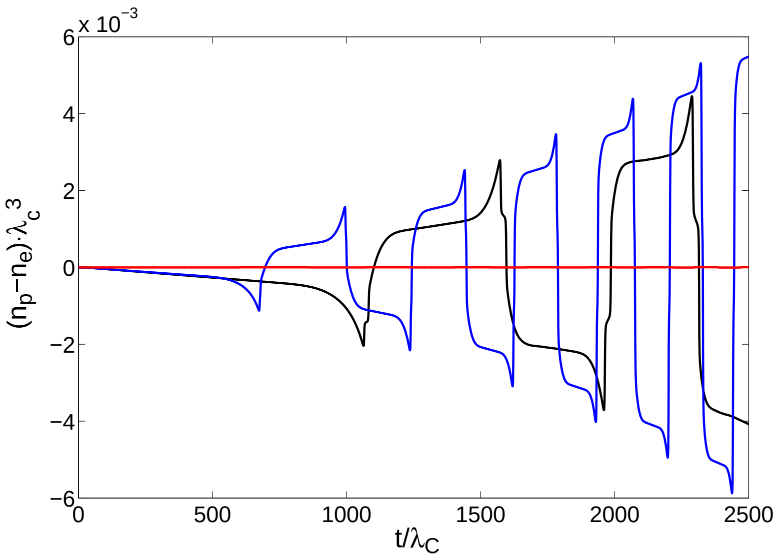

Figure B.4.: The net electric charge density $\rho(z, t)$ [see Eq. (B.0.15)] as a function of t at three different points: $z = 0$ (red, nearly zero) , $z = \ell/4$ (blue) and $z = \ell/2$ (black). It is shown that the net electric charge density $\rho(z, t)$ (except the center $z = 0$) increases as time.

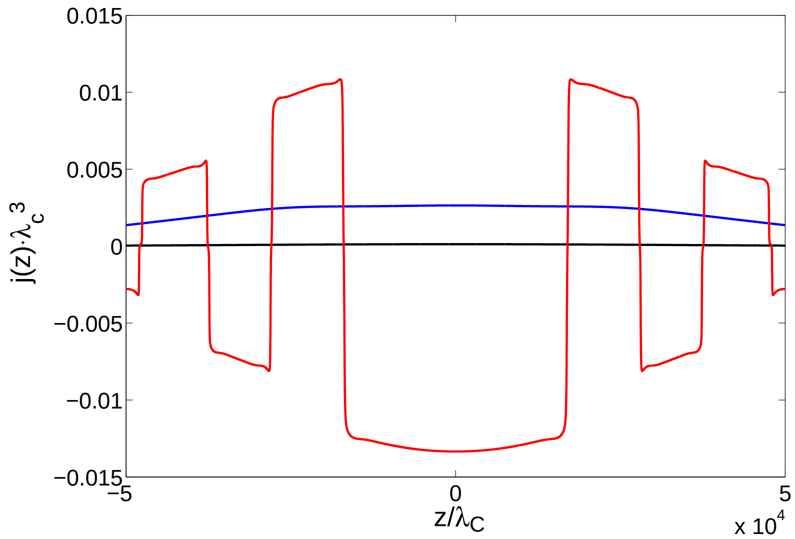

Figure B.5.: Electric current densities $j_z(z,t)$ [see Eq. (B.0.14)] as functions of z at three different times: $t = 1$ (black), $t = 500$ (blue) and $t = 1500$ (red). Following Eq. (B.0.19), the electric current alternates following the alternating electric field (see Fig. B.1), the plateaus indicate the current saturation for $v \sim c$ and its spatial distribution is determined by the initial electric field $E_{\rm ext}(z)$.

oscillation frequency (see Fig. B.1). The energy-spectrum and its peak are shifted to high-energies as the initial electric field-strength increases, and the relation between the spectrum peak location and the electric field-strength is shown in Fig. B.8. In addition, the energy-spectrum and its peak are also shifted to high-energies as the temporary duration $\mathcal{T}$ of plasma oscillations increases (see Fig. B.1). In calculations, the temporary duration $\mathcal{T} = 3500\tau_C$ is chosen, not only to satisfy the adiabaticity condition Eq. (B.0.23) [4], but also to be in the time duration when the oscillatory behavior is distinctive (see Figs. B.1,B.4,B.6), since the oscillations of pair-induced currents damp and pairs annihilate into photons Ruffini et al. (2003b). The radiation intensity (B.0.31) depends on the strength, spatial dimension and temporal duration of strong external electric fields, created by either experimental setups or astrophysical conditions.

Conclusions and remarks. We show the space and time evolutions of pair-induced electric charges, currents and fields in strong external electric fields bounded within a spatial region. These results imply the wave propagation

[4] We check the two cases $E_0 = E_c$ and $E_0 = 10E_c$, and find for the first oscillation $\eta = 865$ and $\eta = 487$ respectively. As can be seen for the Fig. B.1 the frequencies ω of pair-oscillations increase with time which means the parameter η becoming smaller. Eventually it may reach unity so the formula (B.0.22) becomes inapplicable.

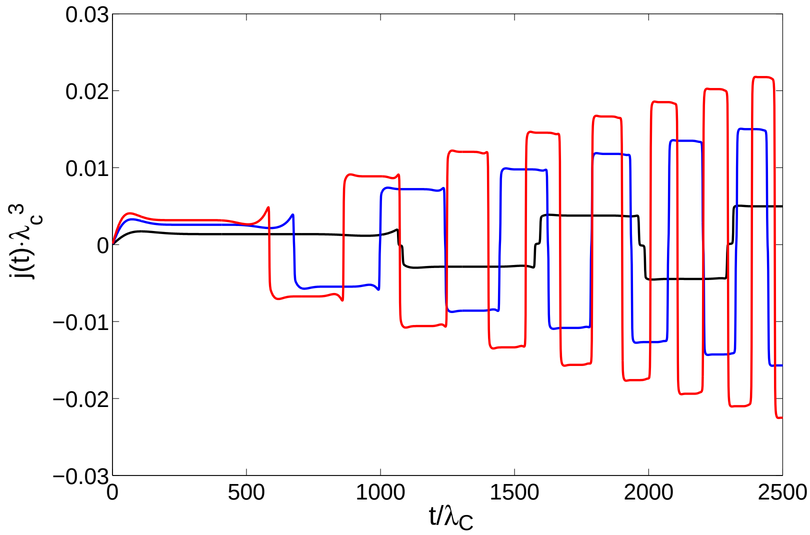

Figure B.6.: Electric current densities $j_z(z,t)$ [see Eq. (B.0.14)] as functions of t at three different points: $z = \ell/2$ (black), $z = \ell/4$ (blue) and $z = 0$ (red). The plateaus (see also Fig. B.6) for the current saturation values increases as time, mainly due to the number-densities $n_\pm$ of electron-positron pairs increase with time. In addition, they are maximal at the center $z = 0$ where the initial electric field is maximal, and decrease as the initial electric field $E_{\rm ext}(z)$ decreasing for $|z| > 0$.

of the pair-induced electric field and wave-transportation of the electromagnetic energy in the strong field region. Analogously to the electromagnetic radiation emitted from an alternating electric current, the space and time variations of pair-induced electric currents and charges emit an electromagnetic radiation. We show that this radiation has a the peculiar energy-spectrum (see Fig. B.7) that is clearly distinguishable from the energy-spectra of the bremsstrahlung radiation, electron-positron annihilation and other possible background events. This possibly provides a distinctive way to detect the radiative signatures for the production and oscillation of electron-positron pairs in ultra-strong electric fields that can be realized in either ground laboratories or astrophysical environments.

As mentioned in introduction, the critical electric field E_c will be reached soon in ground laboratories and sensible methods to detect signatures of pair-productions become important. Recently, the momentum signatures of pair-production is found Hebenstreit et al. (2009) in a time-varying electric field $E(t)$ with sub-cycle structure. On the other hand, space-based telescopes the Swift-BAT NASA (2004), NuSTAR caltech (2010) and Astro-H japan (2010) focusing high-energy X-ray missions, will also give possibilities of detecting X-ray radiation signature, discussed in this paper, from compact stars with

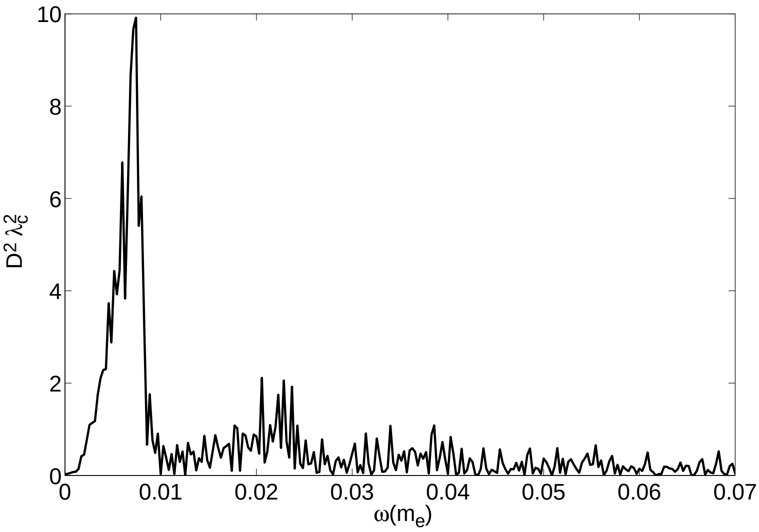

Figure B.7.: In the Compton unit, normalizing $\tilde{D}(\omega)$ [see Eq. (B.0.30)] by the volume $\mathcal{V} \equiv \int d^3\mathbf{x}'$ of the radiation source $\mathbf{J}(t', \mathbf{x}')$, we plot $|\tilde{D}(\omega)|^2$ [see Eq. (B.0.31)] representing the narrow energy-spectrum of the radiation field $\mathbf{E}_{\rm rad}$ and peak locates at the frequency $\omega_{\rm peak} \approx 0.08 m_e$.

electromagnetic structure.

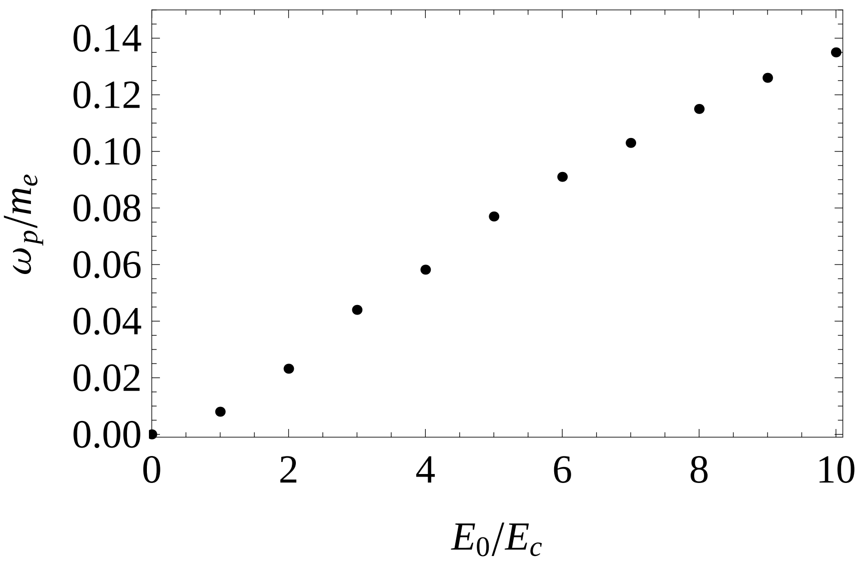

Figure B.8.: The peak frequency ω_{peak} of the radiation approximately varies from 4keV to 70 keV as the initial electric field strength E_0 varies from E_c to $10E_c$. The values for very large field-strengths $E_0/E_c > 1$ possibly receive corrections, since the semiclassical pair-production rate (B.0.22) is approximately adopted and the pressure term (see footnote on page 1259) is not properly taken into account.

C. Electron and positron pair production in gravitational collapse

C.1. Introduction.

In the gravitational collapse or pulsation of neutral stellar cores at densities comparable to the nuclear density, complex dynamical processes are expected to take place. These involve both macroscopic processes such as gravitational and hydrodynamical processes, as well as microscopic processes due to the strong and electroweak interactions. The time and length scales of macroscopic processes are much larger than those of the microscopic processes. Despite the existence of only a few exact solutions of Einstein's equations for simplified cases, macroscopic processes can be studied rather well by numerical algorithms. In both analytical solutions and numerical simulations it is rather difficult to simultaneously analyze both macroscopic and microscopic processes characterized by such different time and length scales. In these approaches, microscopic processes are approximately treated as local and instantaneous processes that are effectively represented by a model-dependent parametrized equation of state (EOS). We call this *approximate locality*.

Applying *approximate locality* to electric processes, as required by the charge conservation, one is led to *local neutrality*: positive and negative charge densities are exactly equal over all space and time. As a consequence, all electric fields and processes are eliminated. An internal electric field (charge separation) *must* be developed Olson and Bailyn (1975, 1976); Rotondo et al. (2011a,b) in a totally neutral system of proton and electron fluids in a gravitational field. If the electric field (process) is weak (slow) enough, *approximate locality* is applicable. However, this should be seriously questioned when the electric field (process) is strong (rapid). For example, neutral stellar cores reach the nuclear density where positive charged baryons interact via the strong interaction while electrons do not, in addition to their widely different masses. As a result, their pressure, number, and energy density are described by different EOS, and a strong electric field (charge separation) on the baryon core surface is realized Usov (1998); Popov et al. (2009) in an electrostatic equilibrium state.

Furthermore, either gravitationally collapsing or pulsating of the baryon core leads to the dynamical evolution of electrons. As a consequence, the strong electric field dynamically evolves in space and time, and some electromagnetic processes can result if their reaction rates are rapid enough, for example, the electron-positron pair-production process of Sauter-Heisenberg-Euler-Schwinger (see the review Ruffini et al. (2010)) for electric fields $E \gtrsim E_c \equiv m_e^2 c^3/(e\hbar)$. If this indeed occurs, gravitational and pulsating energies of neutral stellar cores are converted into the observable energy of electron-positron pairs via the space and time evolution of electric fields. In this chapter, we present our studies of this possibility (the natural units $\hbar = c = 1$ are adopted, unless otherwise specified).

C.2. Basic equations for dynamical evolution.

We attempt to study possible electric processes in the dynamical perturbations of neutral stellar cores. These dynamical perturbations can be caused by either the gravitational collapse or pulsation of neutral stellar cores. The basic equations are the Einstein-Maxwell equations and those governing the particle number and energy-momentum conservation

$$\begin{aligned}(\bar{n}_{e,B} U^\nu_{e,B})_{;\nu} &= 0, \\ G_{\mu\nu} &= -8\pi G (T_{\mu\nu} + T^{\rm em}_{\mu\nu}), \\ (T^\nu{}_\mu)_{;\nu} &= -F_{\mu\nu} J^\nu, \\ F^{\mu\nu}{}_{;\nu} &= 4\pi J^\mu,\end{aligned} \tag{C.2.1}$$

in which the Einstein tensor $G_{\mu\nu}$, the electromagnetic field $F^{\mu\nu}$ (satisfying $F_{[\alpha\beta,\gamma]} = 0$) and its energy-momentum tensor $T^{\rm em}_{\mu\nu}$ appear; $U^\nu_{e,B}$ and $\bar{n}_{e,B}$ are, respectively, the four velocities and proper number-densities of the electrons and baryons. The electric current density is

$$J^\mu = e\bar{n}_p U^\mu_B - e\bar{n}_e U^\mu_e, \tag{C.2.2}$$

where $\bar{n}_p$ is the proper number-density of the positively charged baryons. The energy-momentum tensor $T^{\mu\nu} = T^{\mu\nu}_e + T^{\mu\nu}_B$ is taken to be that of two simple perfect fluids representing the electrons and the baryons, each of the form

$$T^{\mu\nu}_{e,B} = \bar{p}_{e,B} g^{\mu\nu} + (\bar{p}_{e,B} + \bar{\rho}_{e,B}) U^\mu_{e,B} U^\nu_{e,B}, \tag{C.2.3}$$

where $\bar{\rho}_{e,B}(r,t)$ and $\bar{p}_{e,B}(r,t)$ are the respective proper energy densities and pressures.

In this chapter, baryons indicate hadrons, or their constituents (quarks) that carry baryon numbers. Electrons indicate all negatively charged leptons. Baryon fluid and electron fluid are separately described for the reason that in addition to baryons being much more massive than electrons, the EOS of baryons $\bar{p}_B = \bar{p}_B(\bar{\rho}_B)$ is very different from the electron one $\bar{p}_e = \bar{p}_e(\bar{\rho}_e)$ due to the strong interaction. Therefore, in the dynamical perturbations of neutral stellar cores, one should not expect that the space-time evolution of number density, energy density, four velocity, and pressure of baryon fluid be identical to the space-time evolution of counterparts of electron fluid. The difference of space-time evolutions of two fluids results in the electric current (C.2.2) and field $F^{\mu\nu}$, possibly leading to some electric processes. In a simplified model for the dynamical perturbations of neutral stellar cores, we approximately study possible electric processes by assuming that the equilibrium configurations of neutral stellar cores are initial configurations.

C.3. Equilibrium configurations.

In Refs. Olson and Bailyn (1975, 1976); Rotondo et al. (2011a,b), the equilibrium configurations of neutral stellar cores, whose densities are smaller than nuclear density $n_{\rm nucl}$, are studied on the basis of hydrostatic dynamics of baryon and electron fluids in the presence of long-ranged gravitational and Coulomb forces. In these equilibrium configurations, very weak electric fields $E \ll E_c$ are present, resulted from the balance between attractive gravitational force and repulsive Coulomb force. This electric field is too weak to make important electric processes, for example, electron-positron pair productions. We are interested in the case where strong electric fields are present. This leads us to consider strong electric fields in the surface layer of baryon cores of compact stars (quark or neutron stars) at or over the nuclear density. In this case, we assume that baryons form a rigid core of radius R_c and density

$$\frac{\bar{n}_{B,p}(r)}{\bar{n}_{B,p}} = \left[\exp\frac{r-R_c}{\zeta}+1\right]^{-1}, \; \bar{n}_{B,p} \approx \frac{N_{B,p}}{(4\pi R_c^3/3)}, \tag{C.3.1}$$

where $\bar{n}_p/\bar{n}_B \approx N_p/N_B < 1$, $N_B(N_p)$ is the number of total (charged) baryons and $\bar{n}_{B,p} \gtrsim n_{\rm nucl} \approx 1.4\times 10^{38}{\rm cm}^{-3}$. The baryon core has a sharp boundary ($r \sim R_c$) of the width $\zeta \sim m_\pi^{-1}$ due to the strong interaction. The line element is Bekenstein (1971); Mashhoon and Partovi (1979)

$$\begin{aligned} ds^2 &= -g_{tt}dt^2 + g_{rr}dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\,, \\ g_{rr}^{-1}(r) &= 1 - 2GM(r)/r + GQ^2(r)/r^2\,, \end{aligned} \tag{C.3.2}$$

where mass $M(r)$, charge $Q(r)$ and radial electric field $E(r) = Q(r)/r^2$.

Electrons form a complete degenerate fluid and their density $n_e^{\rm eq}(r)$ obeys the following Poisson equation and equilibrium condition Popov et al. (2009);

Rueda et al. (2011):

$$\begin{aligned} &\frac{d^2V_{\rm eq}}{dr^2}+\left[\frac{2}{r}-\frac{1}{2}\frac{d}{dr}\ln(g_{tt}g_{rr})\right]\frac{dV_{\rm eq}}{dr}\\ &\qquad=-4\pi e g_{rr}(\bar{n}_pU_p^t-n_e^{\rm eq}U_e^t),\\ &E_e^F=g_{tt}^{1/2}\sqrt{|P_e^F|^2+m_e^2}-m_e-eV_{\rm eq}={\rm const.}, \end{aligned} \tag{C.3.3}$$

where $U_p^t=U_e^t=(1,0,0,0)$, E_e^F, and $P_e^F=(3\pi^2 n_e^{\rm eq})^{1/3}$ are the Fermi energy and momentum, $V_{\rm eq}(r)$ and $E_{\rm eq}=-(g_{rr})^{-1/2}\partial V_{\rm eq}(r)/\partial r$ are the static electric potential and field. In the ultrarelativistic case $|P_e^F|\gg m_e$, we numerically integrate Eq. (C.3.3) with boundary conditions:

$$\begin{aligned} &n_e^{\rm eq}(r)|_{r\ll R_c}=n_B\\ &n_e^{\rm eq}(r)|_{r\gg R_c}=\frac{dn_e^{\rm eq}(r)}{dr}\Big|_{r\gg R_c}=\frac{dn_e^{\rm eq}(r)}{dr}\Big|_{r\ll R_c}=0. \end{aligned} \tag{C.3.4}$$

As a result, we obtain on the baryon core boundary $r\approx R_c$, the nontrivial charge-separation $(n_p-n_e^{\rm eq})/n_B$ and overcritical electric field $E_{\rm eq}/E_c>0$ in a thin layer of a few electron Compton length λ_e [the curves ($t=0$) in Fig. C.1]. This is due to the sharpness boundary ($\zeta\sim m_\pi^{-1}$) of the baryon core (C.3.1) at the nuclear density, as discussed for compact stars Usov (1998); Popov et al. (2009). Note that all electronic energy-levels Kleinert et al. (2008)

$$\mathcal{E}_{\rm occupied}=e\int g_{rr}^{1/2}drE_{\rm eq}(r) \tag{C.3.5}$$

are fully occupied and pair-production is not permitted due to Pauli blocking, although electric fields in the surface layer are over critical. We want to understand the space and time evolution of the electric field in this thin layer and its consequence in the dynamical perturbations of baryon cores, which can be caused by either the gravitational collapse or pulsation of baryon cores.

C.4. Modeling dynamical perturbations of baryon cores.

It is rather difficult to solve the dynamical system (C.2.1-C.2.3) with the EOS $\bar{p}_B=\bar{p}_B(\bar{\rho}_B)$ and $\bar{p}_e=\bar{p}_e(\bar{\rho}_e)$ for the gravitational collapse or pulsation of baryon core and electron fluid, and to examine possible electromagnetic processes. The main difficulty comes from the fact that the time and length scales of gravitational and electromagnetic processes differ by many orders of magnitude. In order to gain some physical insight into the problem, we are bound to split the problem into three parts: (i) first, we adopt a simplified model to describe the dynamical perturbations of baryon cores; (ii) second, we examine how electron fluid responds to this dynamical perturbation of baryon cores; (iii) third, we check whether the resulted strong electric fields can lead to very rapid electromagnetic processes, for example, electron-positron pair production.

As for the first part, we adopt the following simplified model. Suppose that at the time $t = 0$ the baryon core is in the equilibrium configuration (C.3.1) with the radius R_c and starts dynamical perturbations with an inward velocity $\dot{R}_c(t)$ or pulsation frequency $\omega_{\rm pulsa} \simeq \dot{R}_c/R_c$. The rate of dynamical perturbations of baryon cores is defined as $\tau^{-1}_{\rm coll} = \dot{R}_c/R_c \lesssim c/R_c$. We further assume that in these dynamical perturbations, baryon cores are rigid, based on the argument that as the baryon core density $\bar{n}_{B,p}$ (C.3.1) increases, the EOS of baryons $\bar{p}_B = \bar{p}_B(\bar{\rho}_B)$ due to the strong interaction is such that the baryon core profile (C.3.1) and boundary width $\zeta \sim m_\pi^{-1}$ are maintained in the nuclear relaxation rate $\tau^{-1}_{\rm stro} \sim m_\pi$, which is much larger than $\tau^{-1}_{\rm coll}$. Thus, due to these properties of strong interaction, the dynamical perturbation of the baryon core induces an inward charged baryon current-density

$$J^r_B(R_c) = e\bar{n}_p(R_c)U^r_B(R_c), \tag{C.4.1}$$

on the sharp boundary of baryon core density (C.3.1) at R_c, where the baryon density $\bar{n}_{B,p}(R_c) = 0.5\bar{n}_{B,p}$ and the four-velocity $U^r_B(R_c) \neq 0$. We have not yet been able, from the first principle of strong interaction theory, to derive this boundary property (C.4.1) of baryon cores undergoing dynamical perturbations, which essentially are assumptions in the present chapter, and the boundary density $\bar{n}_{B,p}(R_c)$ and the boundary four-velocity $U^r_B(R_c)$ are two parameters depending on dynamical perturbations. This is in the same situation that so far one has not yet been able, from the first principle of strong interaction theory, to derive the sharp boundary profile (C.3.1) of baryon core densities of static compact stars Usov (1998); Popov et al. (2009). However, we have to point out that the boundary properties (C.3.1) and (C.4.1) of the baryon core undergoing dynamical perturbations are rather technical assumptions for the following numerical calculations of dynamical evolution of electron fluid and electric processes in the Compton time and length scales. These assumptions could be abandoned if we were able to simultaneously make numerical integration of differential equations for both dynamical perturbations of baryon cores at macroscopic length scale and strong and electric processes at microscopic length scale.

C.5. Dynamical evolution of electron fluid

In this section, we attempt to examine how the electron fluid around the boundary layer of the baryon core responds to the dynamical perturbations of the baryon core described by the boundary properties (C.3.1) and (C.4.1). Given these boundary properties at different values of baryon core radii R_c, we describe electrons and electric fields around the boundary layer of baryon core by Maxwell's equations, the electron number and energy-momentum conservation laws (C.2.1) in the external metric field (C.3.2). In addition, we assume that the electron fluid is completely degenerate, and its EOS is given by

$$\begin{aligned} \bar{\rho}_e(t,r) &= 2\int_0^{P_e^F} p^0 d^3\mathbf{p}/(2\pi)^3, \\ \bar{p}_e(t,r) &= \frac{1}{3}\frac{2}{(2\pi)^3}\int_0^{P_e^F} \frac{\mathbf{p}^2}{p^0} d^3\mathbf{p}\,, \end{aligned} \tag{C.5.1}$$

where the single-particle spectrum is $p^0 = (\mathbf{p}^2 + m_e^2)^{1/2}$ and the Fermi momentum is $P_e^F = (3\pi^2 \bar{n}_e)^{1/3}$. In the present chapter, for the sake of simplicity, we set the temperature of electron fluid to be zero and neglect all temperature effects, which may be important and will be studied in future.

The electron fluid has four velocity $U_e^\mu = (U^t, U^r)_e$, radial velocity $v_e \equiv (U^r/U^t)_e$, $U_e^t = g_{tt}^{-1/2}\gamma_e$ and Lorentz factor $\gamma_e \equiv (1 + U_r U^r)_e^{1/2} = [1 + (g_{rr}/g_{tt})v_e^2]^{-1/2}$. In the rest frame at a given radius r it has the number density $n_e = \bar{n}_e \gamma_e$, energy density $\epsilon_e = (\bar{\rho}_e + \bar{p}_e v_e^2)\gamma_e^2$, momentum density $P_e = (\bar{\rho}_e + \bar{p}_e)\gamma_e^2 v_e$, and $v_e = P_e/(\epsilon_e + \bar{p}_e)$. In the rest frame, the number and energy-momentum conservation laws for the electron fluid, and Maxwell's equations are given by

$$\left(n_e g_{tt}^{-1/2}\right)_{,t} + \left(n_e v_e g_{tt}^{-1/2}\right)_{,r} = 0, \tag{C.5.2}$$

$$\begin{aligned} (\epsilon_e)_{,t} + (P_e)_{,r} + \frac{1}{2g_{tt}}\left[\frac{\partial g_{rr}}{\partial t}P_e v_e - \frac{\partial g_{tt}}{\partial t}(\epsilon_e + \bar{p}_e)\right] \\ = -e n_e v_e E g_{tt}^{-1/2}, \end{aligned} \tag{C.5.3}$$

$$\begin{aligned} \left(P_e \frac{g_{rr}}{g_{tt}}\right)_{,t} + \left(\bar{p}_e + P_e v_e \frac{g_{rr}}{g_{tt}}\right)_{,r} \\ + \frac{\epsilon_e + \bar{p}_e}{2g_{tt}}\left(\frac{\partial g_{tt}}{\partial r} - \frac{\partial g_{rr}}{\partial r}v_e^2\right) = -e n_e E g_{tt}^{-1/2}, \end{aligned} \tag{C.5.4}$$

$$(E)_{,t} = -4\pi e(n_p v_p - n_e v_e) g_{tt}^{-1/2}, \tag{C.5.5}$$

where $(\cdots)_{,x} \equiv (-g)^{-1/2}\partial(-g)^{1/2}(\cdots)/\partial x$, and in the line (C.5.5), the boundary velocity v_p of the baryon core comes from the baryon current-density

(C.4.1). We have the boundary four-velocity U_B^r of the baryon core,

$$v_p = v_B \equiv (U^r/U^t)_B, \quad U_B^t = g_{tt}^{-1/2}\gamma_B, \tag{C.5.6}$$

and the Lorentz factor

$$\gamma_B \equiv (1 + U_r U^r)_B^{1/2} = [1 + (g_{rr}/g_{tt})v_p^2]^{-1/2}, \tag{C.5.7}$$

at the baryon core boundary R_c.

In the static case for $v_p = v_e = 0$, Eqs. (C.5.1-C.5.5) are equivalent to Eq. (C.3.3). Provided an initial equilibrium configuration (C.3.3) and proper boundary conditions, we numerically integrate these five equations (C.5.1-C.5.5) to obtain five variables $n_e(t,r), \epsilon_e(t,r), P_e(t,r), \bar{p}(t,r)$ and $E(t,r)$ describing the electric processes around the baryon core boundary.

C.6. Oscillations of electron fluid and electric field.

We consider the baryon core of mass $M = 10M_\odot$ and radius $R_c \sim 10^7$cm at the nuclear density $n_{\rm nucl}$, and select its boundary velocity $v_p = 0.2c$ to represent possible dynamical perturbations of baryon cores. In the proper frame of a rest observer at the core radius R_c, where $g_{tt}(R_c) \approx g_{rr}^{-1}(R_c)$, we chose the surface layer boundaries $\xi_- \approx -\lambda_e, \xi_+ \approx 3.5\lambda_e$, at which $E_{\rm eq}(\xi_\pm) \approx 0$ and proper thickness $\ell = \xi_+ - \xi_-$, and numerically integrate Eqs. (C.5.1-C.5.5) for the electron fluid. Numerical results are presented in Figs. C.1 and C.2, showing that total electric field

$$E(t,r) = E_{\rm eq}(r) + \tilde{E}(t,r), \tag{C.6.1}$$

where electron number density, energy density, and pressure oscillate around their equilibrium configurations Han et al. (2010). This is due to the fact that electrons do not possess the strong interaction and their mass is much smaller than the baryon one, as a result, the current density of electron fluid in the boundary layer does not exactly follow the baryon core current density (C.4.1). Instead, triggered by the baryon core current (C.4.1), total electric fields $E(t,r)$ deviate from $E_{\rm eq}(r)$ and increase, which breaks the equilibrium condition (C.3.3), namely, the balance between pressure and electric force acting on electrons, $dP_e^F/dr + eE_{\rm eq} = 0$. Accelerated by increasing electric fields, electrons outside the core start to move inwards following the collapsing baryon core. This leads to the increase of the electron pressure (C.5.1) and the decrease of the electric fields. On the contrary, increasing electron pressure pushes electrons backwards, and bounces them back. Overcritical electric fields work against the pressure of ultrarelativistic electrons. As a consequence, oscillations with frequency $\omega = \tau_{\rm osci}^{-1} \sim 1.5m_e$ around the equilibrium configuration take place in a thin layer of a few Compton lengths

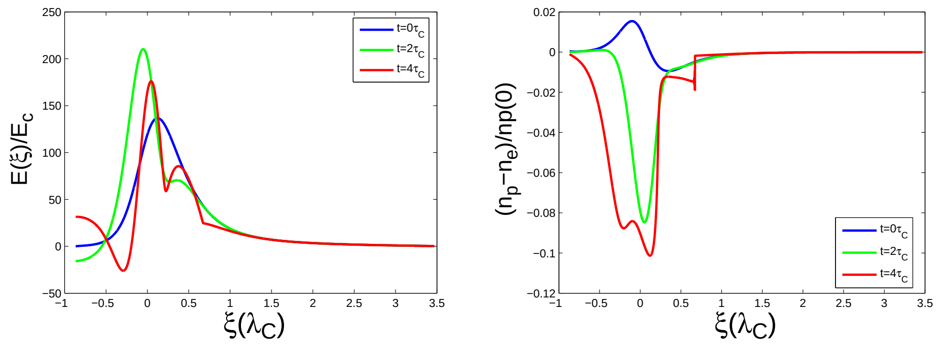

Figure C.1.: The space and time evolution of the electric field (left) and charge-separation (right) around the boundary layer of the baryon core, $M = 10M_{\odot}$, $R_c \approx 10^7$cm, and $v_p = 0.2c$. The coordinate is $\xi \equiv r - R_c$.

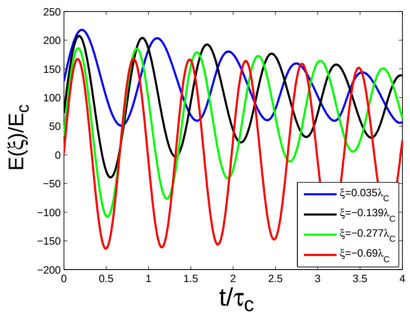

Figure C.2.: Time evolution of electric fields at different radial positions around the boundary layer of the baryon core, $M = 10M_{\odot}$, $R_c \approx 10^7$cm and $v_p = 0.2c$. The coordinate is $\xi \equiv r - R_c$.

around the boundary of baryon core. These are the main results presented in this chapter. We would like to point out that these results should not depend on the boundary properties (C.3.1) and (C.4.1) that we assume for the dynamical perturbations of baryon cores. The reason is that both electron and proton fluids in baryon cores are at or over nuclear density, and their Fermi momenta are the order of the pion mass m_π; therefore, electric fields must be at or over critical value $E_c = m_e^2/e$ to do work against motion of charge separation between positively charged baryon and electron fluids, and the frequency of oscillation because of the backreaction should also be the order of m_e. It is worthwhile that these results are further checked by full numerical calculations without assuming the boundary properties (C.3.1) and (C.4.1) of baryon cores, which undergo the dynamical perturbations caused by the gravitational collapse or pulsation.

Suppose that the dynamical perturbation of the baryon core is caused by either the gravitational collapse or pulsation of the baryon core, that gains the gravitational energy. Then, in this oscillating process, energy transforms from the dynamical perturbation of the baryon core to the electron fluid via an oscillating electric field. This can been seen from the energy conservation (C.2.1) along a flow line of the electron fluid for $v_e \neq v_p$

$$U_e^\mu (T^\nu{}_\mu)_{;\nu} = e\bar{n}_p F_{\mu\nu} U_e^\mu U_B^\nu = e\bar{n}_p \gamma_e \gamma_B (v_p - v_e) g_{rr} E, \tag{C.6.2}$$

although we have not yet explicitly proved it. The energy density of the oscillating electric field is

$$\epsilon_{\rm osci} \equiv [E^2(t,r) - E^2_{\rm eq}(r)]/(8\pi). \tag{C.6.3}$$

The energy densities of the oscillating electric field and electron fluid are converted from one to another in the oscillating process with frequencies $\omega \sim \tau^{-1}_{\rm osci} \sim 1.5m_e$ around the equilibrium configuration. However, the oscillating electron fluid has to relax to the new equilibrium configuration determined by Eqs. (C.3.3) and (C.3.4) with a smaller baryon core radius $R'_c < R_c$. As a result, the oscillating electric field must damp out and its lifetime $\tau_{\rm relax}$ is actually a relaxation time to the new equilibrium configuration. As shown in Fig. C.2 the relaxation rate $\tau^{-1}_{\rm relax} \sim 0.05m_e$. We notice very different time scales of strong interacting processes, electric interacting processes and dynamical perturbations of baryon cores: $\tau^{-1}_{\rm stro} \gg \tau^{-1}_{\rm osci} \gg \tau^{-1}_{\rm relax} \gg \tau^{-1}_{\rm coll}$.

Moreover, when $E(r,t) > E_{\rm eq}(r)$ (see Fig. C.1), the unoccupied electronic energy-level can be obtained by Kleinert et al. (2008)

$$\begin{aligned}\mathcal{E}_{\rm unocuppied} &= e\int g_{rr}^{1/2} dr E(t,r) - \mathcal{E}_{\rm ocuppied} \\ &= e\int g_{rr}^{1/2} dr \tilde{E}(t,r),\end{aligned} \tag{C.6.4}$$

see Eq. (C.3.5). This leads to pair production in strong electric fields and converts electric energy into the energy of electron-positron pairs, provided the pair-production rate $\tau^{-1}_{\rm pair}$ is faster than the oscillating frequency $\omega = \tau^{-1}_{\rm osci}$. Otherwise, the energy of oscillating electric fields would completely be converted into the electrostatic Coulomb energy of the new equilibrium configuration of electron fluid, which cannot be not radiative.

C.7. Electron-positron pair production

We turn to the pair-production rate in spatially inhomogeneous and temporally oscillating electric fields $E(t,r)$. Although the oscillating frequency ω is rather large, the pair-production rate $\tau^{-1}_{\rm pair}$ can be even larger due to the very strong electric fields $E(t,r)$. The pair-production rate can be approximately calculated by the formula for static fields. The validity of this approximation is justified (see Ruffini et al. (2010); Brezin and Itzykson (1970)) by the adiabaticity parameter $\eta^{-1} = (\omega/m_e)(E_c/E_{\max}) \ll 1$, where $E_{\max}$ is the maximal value of the electric field on the baryon core surface $r \simeq R_c$. Therefore we adopt Eqs. (38) and (39) and (64)-(66) in Ref. Kleinert et al. (2008) for the Sauter electric field to estimate the density of the pair-production rate in the proper frame at the core radius R_c

$$\mathcal{R}_{\rm pair} \approx \frac{e^2 E\tilde{E}}{4\pi^3\,\bar{G}_0(\sigma)} e^{-\pi(E_c/E)G_0(\sigma)} \sim \frac{e^2 E\tilde{E}}{4\pi^3}, \tag{C.7.1}$$

where $\tilde{E}$ (instead of E) in the prefactor accounts for the unoccupied electric energy levels, $G_0(\sigma) \to 0$ and $\bar{G}_0(\sigma) \to 1$ for $\sigma = (\ell/\lambda_e)(E/E_c) \gg 1$. The electron-positron pairs screen the oscillating field $\tilde{E}$ so that the number of pairs can be estimated by $\mathcal{N}_{\rm pair} \approx 4\pi R_c^2(\tilde{E}/e)$. The pair-production rate is $\tau^{-1}_{\rm pair} \approx \mathcal{R}_{\rm pair}(4\pi R_c^2\ell)/\mathcal{N}_{\rm pair} \sim \alpha m_e(\ell/\lambda_e)(E/E_c) \simeq 6.6 m_e > \tau^{-1}_{\rm osci}$. The number density of pairs is estimated by $n_{\rm pair} \approx \mathcal{N}_{\rm pair}/(4\pi R_c^2\ell)$. Assuming the energy density $\epsilon_{\rm osci}$ of oscillating fields is totally converted into the pair energy density, we have the pair mean energy $\bar{\epsilon}_{\rm pair} \equiv \epsilon_{\rm osci}/n_{\rm pair}$. Using the parameters $v_p \approx 0.2c$, $R_c \approx 10^7$cm, and $M = 10M_\odot$, we obtain $\epsilon_{\rm osci} \approx 4.3\times 10^{28}\,{\rm ergs/cm^3}, n_{\rm pair} \approx 1.1\times 10^{33}/{\rm cm}^3$, and $\bar{\epsilon}_{\rm pair} \approx 24.5$MeV. These estimates are preliminary without considering the efficiency of pair-productions, possible suppression due to strong magnetic fields, and possible enhancement due to finite temperature effect.

C.8. Gravitational collapse and Dyadosphere

Up to now, we have not discussed how the dynamical perturbations of baryon cores can be caused by either the gravitational collapse or pulsation of baryon cores. Actually, we have not been able to completely integrate the dynamical equations discussed in Sec. C.2 for the reasons discussed in Secs. C.1 and C.4. Nevertheless, we attempt to use the results of electric field oscillation and pair production obtained in Secs. C.5, C.6 and C.7 to gain some physical insight into what and how electric processes could possibly occur in the gravitational collapse of baryon cores. For this purpose and in order to do some quantitative calculations, we first model the gravitational collapse of baryon cores by the following assumptions:

1. the gravitationally collapsing process is made of the sequence of events (in time) occurring at different radii R_c of the baryon core;

2. at each event the baryon core maintains its density profile and sharp boundary as described by Eqs. (C.3.1) and (C.4.1).

The first assumption is based on the arguments that (i) in the electric processes discussed in Sec. C.6, the charge-mass ratio Q/M of the baryon core can possibly be approaching to 1, then the collapse process of the baryon core is slowing down and its kinetic energy is vanishing because the attractive gravitational energy gained is mostly converted into the repulsive Coulomb energy of the baryon core; (ii) then this Coulomb energy can be possibly converted into the radiative energy of electron-positron pairs as discussed in Sec. C.7, and the baryon core restarts acceleration by gaining gravitational energy. We have already discussed the second assumption in Secs. C.4 and C.6. Here we want to emphasize that (i) the sharp boundary properties (C.3.1) and (C.4.1) in the second assumption are technically used in order to numerically calculate the dynamics of electron fluid in the thin shell around the baryon boundary (Secs. C.5, C.6 and C.7); (ii) in the gravitational collapse or pulsation of neutral stellar cores at or over nuclear density, these sharp boundary properties (C.3.1) and (C.4.1) should be abandoned in a more realistic model of simultaneously integrating dynamical equations of electron and baryon fluids over the entire stellar core at macroscopic scales. This turns out to be much more complicated and we will focus on this study in the future.

On the basis of these assumptions, the boundary velocity $v_p(R_c)$ (C.5.6) and boundary radius R_c [or boundary density $\bar{n}_{B,p}(R_c)$ (C.3.1)] of the baryon core at or over the nuclear density are no longer independent parameters, instead they should be related by the gravitational collapse equation of the baryon core. We adopt a simplified model for the gravitational collapse of the baryon core by approximately using the collapsing equation for a thin shell Israel (1966); De la Cruz and Israel (1967); Bekenstein (1971); Cherubini et al. (2002); Ruffini and Vitagliano (2002)

$$\left(\frac{\Omega}{F}\right)^2 \left(\frac{dR_c}{dt}\right)^2 = \left[1 + \frac{GM}{2R_c}(1 - \xi_Q^2)\right]^2 - 1, \tag{C.8.1}$$

where at different radii R_c of the baryon core, we define the charge-mass ratio

$$\xi_Q \equiv Q^{\rm eq}/(G^{1/2}M) < 1; \quad Q^{\rm eq} = R_c^2 E_{\rm eq}, \tag{C.8.2}$$

and

$$\begin{aligned} \Omega &\equiv 1 - (M/2R_c)(1 + \xi_Q^2) \\ F &\equiv 1 - (2M/R_c) + (Q^{\rm eq}/R_c)^2. \end{aligned} \tag{C.8.3}$$

The collapsing Eq. (C.8.1) for the collapsing velocity $\dot{R}_c$ is based on the condition that at each collapsing radius R_c, the shell starts to collapse from rest. As a result, using these Eqs. (C.8.1-C.8.3) we describe the sequence of events in the gravitationally collapsing process in terms of the collapsing velocities $v_p = \dot{R}_c = dR_c/dt$ defined by (C.5.6) and (C.5.7) at different collapsing radii

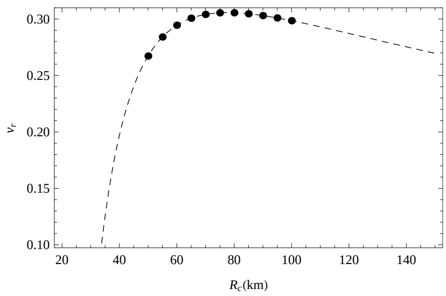

Figure C.3.: The estimate of the core collapsing velocity $v_p \equiv \dot{R}_c = dR_c/dt$ at different collapsing radii R_c for the baryon core of mass $M = 10\,M_\odot$.

R_c of the baryon core, as shown in Fig. C.3. Thus, at each event the induced inward charged baryon current-density (C.4.1) is given by

$$J^r_B = e\bar{n}_p U^r_B \approx e\bar{n}_p(\dot{R}_c\Omega/F), \tag{C.8.4}$$

as a function of the collapsing radius R_c. The strength of this charged baryon current density (C.8.4) depends also on the ratio of the charged baryon number and total baryon number (N_p/N_B), which varies in the gravitational collapsing process because of the β processes Mohammadi et al. (2012). In this chapter, the β processes are not considered and the charged baryon (proton) number N_p is constant; we select two values $N_p/N_B \approx 1/38$ or $N_p/N_B \approx 1/380$ for the charged baryon current density Eq. (C.8.4). The collapsing process rate is $\tau^{-1}_{\rm coll} = \dot{R}_c/R_c \lesssim c/R_c$. If the dynamical perturbation of the baryon core is caused by the gravitational core pulsation, the pulsation frequency can be expressed as $\omega_{\rm pulsa} \simeq \dot{R}_c/R_c = \tau_{\rm coll}$.

In the sequence of the gravitationally collapsing process, at each event characterized by $[R_c, v_p(R_c)]$, we first solve Eqs. (C.3.3) and (C.3.4) of the equilibrium configuration to obtain the number density ($n_e^{\rm eq}$) and electric field ($E_{\rm eq}$) as the initial configuration of the electron fluid and electric field. Then, with this initial configuration we numerically solve the dynamical equations (C.5.1-C.5.5) to obtain the dynamical evolution of electron fluid and electric field within the thin shell (a few Compton lengths) around the baryon core boundary, described by Eqs. (C.3.1) and (C.8.4). As a result, based on the analysis presented in Sec. C.7 we calculate the energy and number densities of the electron-positron pairs produced at each event in the sequence of the gravitationally collapsing process. These results are plotted in Figs. C.4. Limited by numerical methods, we cannot do calculations for smaller radii.

In addition, at each event in the sequence of the gravitationally collapsing process, using the Gauss law, $Q = R_c^2 E$, we calculate the charge-mass ratio Q/M averaged over oscillations of electric fields, $Q/M < 1$ as shown in Fig. C.5. The averaged charge-mass ratio Q/M is not very small, rather

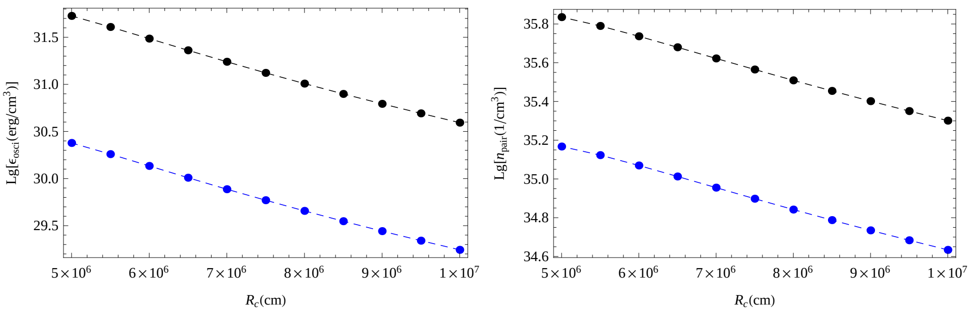

Figure C.4.: The energy (left) and number (right) densities of electron-positron pairs at selected values of collapsing radii R_c for $M = 10M_\odot$ and $N_p/N_B \approx 1/38$ (upper); $1/380$ (lower). We select $R_c^{\max} \sim 10^7$cm so that $\bar{n}_B \sim n_{\rm nucl}$.

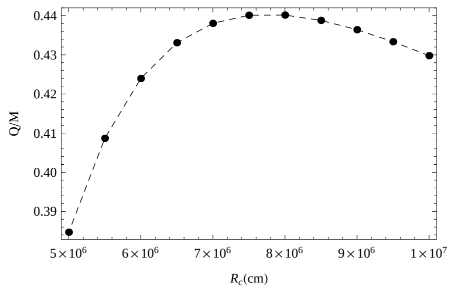

Figure C.5.: The charge-mass ratio Q/M averaged over oscillations of electric fields is plotted at different collapsing radii R_c for the baryon core of mass $M = 10\, M_\odot$.

about 0.4 (see Fig. C.5), implying the possible validity of the first assumption we made that the gravitational collapsing process is approximately made of a sequence of events. In principle, at $Q/M = 1$ the gravitational collapsing process should stop, whereas the gravitational collapsing process is continuous for $Q/M = 0$ without considering electric interactions.

It is clear that the ratio N_p/N_B becomes larger, the charged baryon current density (C.4.1) or (C.8.4) becomes larger, and all effects of electrical processes we discussed in Secs. C.5, C.6 and C.7 become larger. As shown in Figs. C.4, for the ratio $N_p/N_B \approx 1/38$, the energy density of electron-positron pairs is about 10^{31} ergs/cm^3, and the number density of electron-positron pairs is about $10^{35.6}$ /cm^3. The mean energy of electron-positron pairs is $\bar{\epsilon}_{\rm pair} \equiv \epsilon_{\rm osci}/n_{\rm pair} \sim$ 10–50 MeV. While, for the ratio $N_p/N_B \approx 1/380$, the energy density of electron-positron pairs is about 10^{30} ergs/cm^3, the number density of electron-positron pairs is about $10^{34.6}$ /cm^3, and the mean energy of electron-positron pairs $\bar{\epsilon}_{\rm pair} \equiv \epsilon_{\rm osci}/n_{\rm pair} \sim$ 10–50 MeV does not change very much.

It this chapter, it is an assumption that the gravitationally collapsing process is represented by the sequence of events: the baryon core starts to collapse from rest by gaining gravitational energy, the increasing Coulomb energy results in decreasing kinetic energy and slowing down the collapse process, the electric processes discussed in Secs. C.5, C.6 and C.7 convert the Coulomb energy into the radiative energy of electron-positron pairs, and as a result the baryon core restarts to accelerate the collapse process by further gaining gravitational energy. This indicates that in the gravitationally collapsing process, the gravitational energy must be partly converted into the radiative energy of electron-positron pairs. However, we have not been able so far to calculate all processes with very different time and length scales from one event to another in the sequence, so that it is impossible to quantitatively obtain the rate of the conversion of the gravitational energy to the energy of electron-positron pairs. Nevertheless, by summing over all events in the sequence of the gravitationally collapsing process, we approximately estimate the total number and energy of electron-positron pairs produced in the range $R_c \sim 5 \times 10^5 - 10^7$cm: 10^{56}–10^{57} and 10^{52}–10^{53} erg for the ratio $N_p/N_B \approx 1/38$; 10^{55}–10^{56} and 10^{51}–10^{52} erg for the ratio $N_p/N_B \approx 1/380$. These electron-positron pairs undergo the plasma oscillation in strong electric fields and annihilate to photons to form a neutral plasma of photons and electron-positron pairs Ruffini et al. (2003b,a). This is reminiscent of the vacuum polarization of a charged black hole Damour and Ruffini (1975); Cherubini et al. (2009) and a sphere of electron-positron pairs and photons, called a Dyadosphere that is supposed to be dynamically created during gravitational collapse in Refs. Ruffini and Xue (2008a); Preparata et al. (1998, 2003).

C.9. Summary and remarks.

In the simplified model for the baryon cores of neutral compact stars, we show possible electric processes for the production of electron-positron pairs within the thin shell (a few Compton lengths) around the boundary of baryon cores that undergo gravitationally collapsing or pulsating processes, depending on the balance between attractive gravitational energy and repulsive electric and internal energies (see the numerical results in Ref. Ghezzi (2005); Ghezzi and Letelier (2007)). This indicates a possible mechanism that the gravitational energy is converted into the energy of electron-positron pairs in either baryon core collapse or pulsation.

In theory, this is a well-defined problem based on the Einstein-Maxwell equations, particle-number and energy-momentum conservation (C.2.1)-(C.2.3), and equations of states, as well as the Sauter-Heisenberg-Euler-Schwinger mechanism. However, in practice, it is a rather complicated problem that one has to deal with various interacting processes with very different time and length scales. The approach we adopt in this chapter is the adiabatic approximation: the interacting processes with very small rates are considered to be adiabatic processes in comparison with the interacting processes with very large rates. Therefore, we try to split the problem of rapid microscopic processes from the problem of slow macroscopic processes, and focus on studying rapid microscopic processes in the background of adiabatic (slowly varying) macroscopic processes. The adiabatic approximation we adopted here is self-consistently and quantitatively justified by process rates

$$\tau_{\rm strong}^{-1} \gg \tau_{\rm pair}^{-1} > \tau_{\rm osci}^{-1} \gg \tau_{\rm relax}^{-1} \gg \tau_{\rm coll}^{-1}, \tag{C.9.1}$$

studied in this chapter. In addition to the adiabatic approximation, we have not considered in this over simplified model the hydrodynamical evolution of baryon cores, the back-reaction of oscillations and pair-production on the collapsing or pulsating processes, and the dynamical evolution of the electron-positron pairs and photons. Needless to say, these results should be further checked by numerical algorithms integrating the full Einstein-Maxwell equations and proper EOS of particles in gravitational collapse. Nevertheless, the possible consequences of these electromagnetic processes discussed in this chapter are definitely interesting and could be possibly relevant and important for understanding energetic sources of supernovae and gamma-ray bursts.

D. Gravitational and electric energies in gravitational collapse

D.1. Introduction

In the gravitational collapse of neutral stellar cores at densities comparable to the nuclear density, both macroscopic processes of gravitational and hydrodynamical interactions and microscopic processes of the strong and electroweak interactions occur. In theoretical principle, these can be well described by the Einstein-Maxwell equations and the equations for the number and energy-momentum conservations of particles, duly taking into account their interactions. In practical calculations of analytical or numerical approach, however, it is rather difficult to simultaneously analyze both macroscopic and microscopic processes for the reason that the time and length scales of macroscopic processes are much larger than those of the microscopic processes. The approximation normally adopted is that microscopic processes are treated as local and instantaneous processes which are effectively represented by a model-dependent parameterized equation of state (EOS). We call this *approximate locality*.

Applying the *approximate locality* to electric processes, as required by the charge conservation, one is led to *local neutrality*: positive and negative charge densities are exactly equal overall space and time. As a consequence, all electric processes are completely eliminated in the assumption of the *approximate locality*. On the other hand, it is well known that an internal electric field (charge-separation) *must* be developed Olson and Bailyn (1975, 1976); Rotondo et al. (2011a,b) in a totally neutral system of proton and electron fluids in the presence of gravitational fields. If the electric field (process) is weak (slow) enough, the *approximate locality* is applicable. However, this should be seriously questioned when the electric field (process) is strong (rapid) in the case that neutral stellar cores reach the nuclear density where positive charged baryons interact via strong interactions that do not associate to negative charged electrons, in addition to widely different gravitational masses of baryons and electrons. In fact, strong electric fields are created on the baryon core surface in an electrostatic equilibrium state Usov (1998); Popov et al. (2009). Furthermore, it is shown in Ref. Han et al. (2012), either pulsating or gravitationally collapsing of the baryon core results in the dynamical evolution of electrons, as a consequence, the strong electric field dynamically evolves in space and time, and leads to the electron-positron pair-production process of Sauter-Heisenberg-Euler-Schwinger (see the review Ruffini et al. (2010)) for overcritical electric fields $E \gtrsim E_c \equiv m_e^2 c^3/(e\hbar)$. When this occurs in gravitational collapses of neutral stellar cores, some part of the gravitational energy of neutral stellar cores converts to the observable energy of electron-positron pairs, as a result the kinetic and internal energies of neutral stellar cores are reduced.

As mentioned above, the difficulties of dealing with such a problem come from very different space-time scales of macroscopic and microscopic processes. We are forced to properly split the problem into three parts: (i) microscopic processes of electrodynamics; (ii) macroscopic processes of gravitational collapses; (iii) the back-reaction of microscopic processes on macroscopic processes. In Ref. Han et al. (2012), we study the first part of the problem: microscopic processes of electrodynamics for strong electric field oscillations and pair-productions, which form a radiative electric energy, in a postulated space-time world line of gravitational collapse. However, the back-reaction of such radiative electric energy on collapse is not considered. In this chapter, we start to quantitatively understand the second and third parts of the problem in a simplified model how gravitational, electric and kinetic energies of neutral stellar cores transfer from one to another in gravitational collapses, to see the possibility of converting the gravitational energy to the electromagnetic energy by the "breaking process" of reducing kinetic energy Ruffini and Vitagliano (2003). The Planck units $G = \hbar = c = 1$ are adopted, unless otherwise specified.

D.2. Einstein-Maxwell Equations and conservation laws of two fluids

The gravitational collapse of neutral stellar cores is generally described by the Einstein-Maxwell equations and those governing the particle number and energy-momentum conservations

$$\begin{aligned} G_{\mu\nu} &= -8\pi G(T_{\mu\nu} + T^{\rm em}_{\mu\nu}), \quad & F^{\mu\nu}{}_{;\nu} &= 4\pi J^\mu, \\ (T^\nu{}_\mu)_{;\nu} &= -F_{\mu\nu}J^\nu, \quad & (\bar{n}_{e,B}U^\nu_{e,B})_{;\nu} &= 0, \end{aligned} \tag{D.2.1}$$

in which appear the Einstein tensor $G_{\mu\nu}$, the electromagnetic field $F^{\mu\nu}$ (satisfying $F_{[\alpha\beta,\gamma]} = 0$) and its energy-momentum tensor

$$T^{\rm em}_{\mu\nu} = \frac{1}{4\pi}\left(F_\mu^{\ \rho}F_{\rho\nu} - \frac{1}{4}g_{\mu\nu}F^{\rho\sigma}F_{\rho\sigma}\right); \tag{D.2.2}$$

$U^\nu_{e,B}$ and $\bar{n}_{e,B}$ are respectively the four-velocities and proper number-densities of electrons and baryons,

$$J^\mu = e\bar{n}_p U^\mu_B - e\bar{n}_e U^\mu_e \tag{D.2.3}$$

is the electric current density, and $\bar{n}_p < \bar{n}_B$ the proper number-density of the positively charged baryons. The energy-momentum tensor $T^{\mu\nu} = T^{\mu\nu}_e + T^{\mu\nu}_B$ is taken to be that of two simple perfect fluids representing electrons and the baryons, each of the form

$$T^{\mu\nu}_B = \bar{p}_B g^{\mu\nu} + (\bar{p}_B + \bar{\rho}_B) U^\mu_B U^\nu_B, \tag{D.2.4}$$

$$T^{\mu\nu}_e = \bar{p}_e g^{\mu\nu} + (\bar{p}_e + \bar{\rho}_e) U^\mu_e U^\nu_e, \tag{D.2.5}$$

where $\bar{\rho}_{e,B}$ and $\bar{p}_{e,B}$ are the respective proper energy densities and pressures. In this scenario, electrons and baryons are respectively described by two perfect fluids at or over the nuclear density, and they couple each other via the electromagnetic interaction.

Baryon fluid and electron fluid must be separately described for the reasons that in addition to the different kinematics of baryons and electrons, the most important differences between their dynamics are: (i) baryons are much more massive than electrons in terms of the long-range gravitational force and baryon cores undergo relativistically collapsing processes; (ii) at or over the nuclear density $\bar{n}_{\rm nucl}$, the electron pressure is much larger that baryon one, and baryons interact each other via the short-range strong force that does not act on electrons. Electron and baryon fluids interact via the long-range electromagnetic force, when two fluids are at or over the nuclear density, this interaction between two fluids becomes rather strong, as will be specified below. Note that we ignore the short-range weak interactions for the β-process in this chapter. The long-range gravitational and electromagnetic forces are explicitly present in Eqs. (D.2.1-D.2.3). Instead, the short-range strong interaction is taken into account by pressure and energy density in the proper frame (see Ref. Weinberg (1972)),

$$\bar{p}_B = \frac{1}{3}\sum_{i=1}^{3} T^{ii} = \frac{1}{3}\sum_B \delta^3(\mathbf{x} - \mathbf{x}_B)\frac{\mathbf{p}_B^2}{E_B}, \tag{D.2.6}$$

$$\bar{\rho}_B = T^{tt} = \sum_B \delta^3(\mathbf{x} - \mathbf{x}_B) E_B \tag{D.2.7}$$

where $E_B = E_B(\mathbf{p}_B)$ is the energy spectrum of baryons, duly taking into account their short-range strong interactions (nuclear potential) at a given density $\bar{n}_B \gtrsim \bar{n}_{\rm nucl}$. Electrons' pressure and energy density are analogously given by Eqs. (D.2.6) and (D.2.7) by replacing the subscript $B \to e$, however, the spectrum $E_e = E_e(\mathbf{p}_e)$ is different from baryon one, due to the fact that electrons are blind with the short-range strong interactions. As a result, the baryon and electron EOS $\bar{p}_B = \bar{p}_B(\bar{\rho}_B)$ and $\bar{p}_e = \bar{p}_e(\bar{\rho}_e)$ are different, moreover, the space-time gradients $\nabla\bar{p}_{e,B}$ and $\partial\bar{p}_{e,B}/\partial t$ are different.

We turn now to discuss how the short-range strong interaction effect on the baryon fluid velocity $v^i_B = (U^i/U^t)_B$. In the Newtonian limit, Eqs. (D.2.1-D.2.4) lead to the Euler equation (see Ref. Weinberg (1972))

$$\frac{\partial \mathbf{v}_B}{\partial t} + (\mathbf{v}_B \cdot \nabla)\mathbf{v}_B = -\frac{1-\mathbf{v}^2_B}{\bar{\rho}_B + \bar{p}_B}\left[\nabla\bar{p}_B + \mathbf{v}_B\frac{\partial \bar{p}_B}{\partial t}\right] + \text{terms of long–range forces.} \tag{D.2.8}$$

The first term in the right-handed side of Eq. (D.2.8) indicates the force due to the space-time gradients of baryon fluid pressure. This implies that the space-time gradients of baryon fluid velocity $\mathbf{v}_B(\mathbf{x}, t)$ should have the rates of short-range strong interactions, which are proportional to the inverses of π, σ, ρ and ω meson masses ($\sim m^{-1}_{\pi,\sigma,\rho,\omega,\cdots}$), depending on values of the baryon density $\bar{n}_B(\mathbf{x}, t)$. These nuclear reaction rates must be larger than the rate ($\gtrsim m_e^{-1}$) of electromagnetic interactions. In other words, the baryon fluid and electron fluid have the different values of the incompressibility so that they have different rates (frequencies) of reactions in space and time. However, this still remains as an argument, because we has not so far been able to quantitatively calculate the space-time gradients of baryon fluid pressure by Eqs. (D.2.6) and (D.2.7), then to obtain the space-time gradients of baryon fluid velocity by Euler equation (D.2.8) together with the Einstein-Maxwell field equations.

In the following, we attempt to address our attention to the issue how the gravitational energy gained by the baryon fluid in collapses is transfered to the electromagnetic energy and how kinetic and internal energies are reduced as a consequence of total energy conservation. The energy conservation (D.2.1) along a flow line of the electron fluid yields

$$U^\mu_e (T^\nu{}_\mu)_{;\nu} = e\bar{n}_p F_{\mu\nu} U^\mu_e U^\nu_B = e\bar{n}_p \gamma_e \gamma_B (v_B - v_e) g_{rr} E, \tag{D.2.9}$$

where e and E are electric charge and field, the fluid velocity $v_{(e,B)} \equiv v^r_{(e,B)} = (U^r/U^t)_{(e,B)}$ and Lorentz factor $\gamma_{(e,B)} \equiv (1 + U_r U^r)^{1/2}_{(e,B)}$ in the spherical geometry

$$ds^2 = -g_{tt}dt^2 + g_{rr}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \,. \tag{D.2.10}$$

Eq. (D.2.9) indicates that the dynamical evolutions of the baryon fluid caused by the gravitational or strong interactions can transfer the energy that the baryon fluid gains to the electron fluid via an electric field, provided $v_e \not\equiv v_B$. As explained in the introductory section, for the reason that the differential equations governing macroscopic processes (e.g. gravitational collapse) and the differential equations governing microscopic processes (e.g. electrodynamic pair-production, nuclear reaction) have very different space-time scales at least of the order of 10^{17}, it is very difficult to simultaneously integrate these differential equations and quantitatively show the energy transformation as indicated by Eq. (D.2.9) in the realistic case of gravitational collapses. In order to overcome these difficulties and make steps toward the understanding of the issue, on the basis of some assumptions and approximations, we decouple the differential equations governing macroscopic processes from the differential equations governing microscopic processes as follows.

1. The first, we study the static case of compact stars at/over the nuclear density, e.g., baryons and electrons of neutral compact stars are in their equilibrium states. The local equilibrium profile of baryons must be determined by the strong interaction, whereas the local equilibrium profile of electrons must be determined instead by the electromagnetic interaction. In the Thomas-Fermi model, an overcritical "equilibrium" electric fields are found Usov (1998); Popov et al. (2009) on the surface of baryon cores. These results provide the initial configurations for the dynamical space-time evolution of baryon core and electron fluid in the gravitational collapse or pulsation.

2. Because of the dynamics of gravitational collapse or pulsation, the baryon core deviates from its equilibrium state. We postulate that due to the nuclear rigidity of baryon cores, an inward velocity v_B and charged current J_B (Eqs. (9) in Han et al. (2012)) of baryon cores are introduced at the rate of the nuclear reaction scales, rather than the rate of the gravitational collapse, as already indicated in Eqs. (D.2.6,D.2.7,D.2.8).We asked the question how the electron fluid responses to this external baryon current J_B. In Ref. Han et al. (2012), by solving the microscopic kinetic transport equations (particle number and energy-momentum conservations) of the electron fluid as well as the Maxwell equation, we obtained the space-time evolution (non-equilibrium) of the electron fluid and overcritical electric fields in the Compton scale, and estimated the rate of pair-productions. These results are essentially due to the postulation that the inward baryon current J_B is introduced at the rate of the strong interaction scale, rather than the gravitational one. The rate of gravitational collapses is too slow to trigger these electrodynamic processes at the Compton scale. In addition, it should be pointed out that in these calculations we did not solve the differential equations for the electron fluid and the Maxwell equation together with the differential equation for the gravitational collapse. The baryon velocity v_B is treated as a parameter and its values are given by a simple collapsing equation of thin shell at different radii of gravitational collapse (Figure 3 in Ref. Han et al. (2012)). In summary, two important assumptions were

made: (i) the baryon core is treated as a giant nucleus and the deviation from its equilibrium state, represented by the baryon electric current $J_B \sim v_B$, is introduced at the rate of the strong introduction; (ii) the values of v_B are given by a simple collapsing model without considering dynamics of the gravitational collapses.

3. On the contrary, instead of solving the differential equations for microscopic electrical processes in a given dynamics of gravitational collapse, in this chapter we focus on solving differential equations for macroscopic gravitational collapse processes in a given dynamics of electric processes studied in Ref. Han et al. (2012), represented by an ansatz function. Our purpose is to see the back-reaction of microscopic electrical processes on macroscopic gravitational collapse processes. In order to gain some insight into this issue, we study the gravitational collapse of a spherically thin capacitor, which might present a thin layer of collapsing stellar cores. Although this spherically thin capacitor is totally neutral, it carries electric and gravitational energies. Using such a simplified model, we try to find an analytical description and make a step in understanding the issue how the gravitational energy is converted to electric, kinetic and internal energies in a neutral stellar core collapse.

This has been so far our approach to the electromagnetic processes in the gravitational collapse of neutral compact stars at/over nuclear density. This approach is clearly far from being complete. In order to quantitatively show that the production, oscillation and annihilation of electron-positron pairs with overcritical electric fields indeed dynamically take place, one must solve altogether the Maxwell equation and the quantum Boltzmann-Vlasov transport equations not only for the electrons fluid Ruffini et al. (2003b), but also for the baryon fluid with the strong interaction. We have not yet been able to model the strong interaction for doing these quantitative calculations. On the basis of the rates of various microscopic processes and interactions, we argue the possibility of the production, oscillation and annihilation of electron-positron pairs and dynamical evolution of overcritical electric fields (Eq. (26) in Ref. Han et al. (2012)). We clarify that in our model these electric processes are triggered by the rapid action rate of the baryon core due to the strong interaction, rather than the gravitational interaction. However, the question is to understand how to quantitatively describe and calculate the dynamics of strongly interacting baryon core in gravitational collapse, and how the baryon charged current J_B is introduced at the rate of strong interactions. This will be the subject for our future work.

D.3. A thin shell of spherical capacitor

The thin shell of spherical capacitor is composed by a layer of positively charged baryons and a layer of negatively charged electrons. The baryon layer is defined as a mathematically thin layer, while the electron layer is understood as a physically thin layer with a thickness "d" specified below. The total numbers of charged baryons and electrons are exactly equal so that the thin shell of spherical capacitor is totally neutral but carries non-vanishing the electric energy stored inside two spherical layers. The number-densities of two spherical layers are at least order of the nuclear density, as a consequence the radial separation "d" between two spherical layers must be a few orders of the Compton length λ_C. The reasons are the following: electric fields between two layers $E \approx e\bar{n}_{\rm nucl}d$ are overcritical and electric force acting on ultra-relativistic electrons balances their Fermi momenta $eEd \approx P^F_e \approx \bar{n}^{1/3}_{\rm nucl}$. Let the baryon layer locate at the Schwarzschild-like radial coordinate r_0 and electron layer distributes from r_0 to $r_0 + d$. The spherical capacitor can be physically considered as an infinitely thin shell for $d/r_0 \to 0$. The spherical capacitor is henceforth denoted by "the thin shell" in short.

As the baryon layer is mathematically thin, in Eq. (D.2.4) the baryon pressure $\bar{p}_B = 0$ and mass density $\bar{\rho}_B(x) = \bar{\rho}_B\delta^{(4)}(x, x_0)$, where $\bar{\rho}_B$ is the constant surface density in the proper frame of the baryon layer and the 4-dimensional Dirac distribution is defined as

$$\int \delta^{(4)}(x, x_0)\sqrt{-g}d^4x = 1,$$

where $g = \det\|g_{\mu\nu}\|$. Then we have ($d\Omega = \sin\theta d\theta d\phi$)

$$\int \bar{\rho}_B\delta^{(4)}(x, x_0)\, r^2 dr d\Omega d\tau = M_0, \tag{D.3.1}$$

where M_0 is the rest mass of the baryon layer, and τ is the proper time along the world surface $S : x_0 = x_0(\tau, \theta, \phi)$ of the baryon layer. S divides the space-time into two complementary static space-times: an internal one $\mathcal{M}_-$ and an external one $\mathcal{M}_+$. Their time-like Killing vectors are denoted by ξ^μ_- and ξ^μ_+. $\mathcal{M}_+$ is foliated by the family $\{\Sigma^+_t : t_+ = t\}$ of space-like hypersurfaces of constant t_+.

On the other hand, introducing the orthonormal tetrad

$$\boldsymbol{\omega}^{(0)}_\pm = (g^\pm_{tt})^{1/2}dt, \quad \boldsymbol{\omega}^{(1)}_\pm = (g^\pm_{rr})^{-1/2}dr, \quad \boldsymbol{\omega}^{(2)} = rd\theta, \quad \boldsymbol{\omega}^{(3)} = r\sin\theta d\phi, \tag{D.3.2}$$

we describe the electric field $\boldsymbol{E} = E\boldsymbol{\omega}^{(1)}$ and electromagnetic tensor $(T^{\rm em})_t{}^t = E^2/(8\pi)$ and $(T^{\rm em})_i{}^i = -E^2/(8\pi)$ inside the thin shell ($r_0 \le r \le r_0 + d$). The electric energy of the thin shell, measured by an observer at rest at infinity, is obtained by evaluating the Killing integral

$$\int_{\Sigma_t^+} \xi_+^\mu T_{\mu\nu}^{\rm em} d\Sigma_+^\nu = 4\pi \int_r^\infty r^2 dr\, (T^{\rm em})_t{}^t \equiv \frac{Q_{\rm eff}^2(r)}{2r}, \tag{D.3.3}$$

where $d\Sigma_+^\nu$ is the surface element vector of the space-like hypersurfaces Σ_t^+ in $\mathcal{M}_+$. In Eq. (D.3.3), we introduce the quantity $Q_{\rm eff}^2(r) \neq 0$ for $r_0 \leq r \leq r_0 + d$ to characterize the electric energy stored inside the thin shell. $Q_{\rm eff}^2(r) = 0$ for $r > r_0 + d$ and $r < r_0$. The total electric energy inside the thin shell is given by

$$\mathcal{E}_{\rm em}(r_0) = \frac{Q_{\rm eff}^2(r_0)}{2r_0}, \tag{D.3.4}$$

where the quantity $Q_{\rm eff}^2(r_0)$ parametrizes the total electric energy stored inside the thin shell that locates at radius $r_0(t_0)$ and time t_0. $Q_{\rm eff}(r)$ does not represent an electric charge carried by the thin shell. We express the repulsive electric energy (D.3.3) or (D.3.4) in the same form of the Coulomb energy of a spherical charged layer for the reason that it is useful to study the collapse equation of the thin shell in next section.

The energy-momentum tensor (D.2.5) of the electron layer has a physical distribution over the size "d" of the thin shell. Analogously to Eq. (D.3.3), we define the total energy of the electron layer as

$$\mathcal{E}_{\rm electron}(r_0) \equiv \int_{\Sigma_t^+} \xi_+^\mu (T_e)_{\mu\nu} d\Sigma_+^\nu = 4\pi \int_{r_0}^\infty r^2 dr\, (T_e)_t{}^t, \tag{D.3.5}$$

where $(T_e)^{tt} = (\bar\rho_e + \bar p_e \langle \mathbf{v}_e^2 \rangle)/(1 - \langle \mathbf{v}_e^2 \rangle)$ and $\mathbf{v}_e$ is the electron fluid velocity. In Ref. Han et al. (2012), it is shown that the electron fluid velocity $\mathbf{v}_e$ is ultra-relativistically oscillating back and forth collectively with oscillating electric fields inside the thin shell, $\langle \mathbf{v}_e^2 \rangle$ indicates the averaged value over rapid oscillations in the Compton scale. In Eq. (D.3.5), the rest mass of the electron layer is negligible, compared with its internal energy for ultra-relativistically oscillating electrons. Moreover, at or over the nuclear density, electron Fermi momenta $P_e^F \sim m_\pi$ in the proper frame of the electron fluid is rather smaller than the baryon mass $m_{_B}$. Therefore, compared with the rest mass of baryon layer M_0, we neglect the internal energy of electron layer $\mathcal{E}_{\rm electron}(r_0)$ of Eq. (D.3.5) in this chapter.

Here, we disregard the detailed space-time oscillations of electric field and electron fluid in the Compton length scale, leading to the energy radiation in the form of electron-positron pairs. Instead, we attempt to properly model the quantity $Q_{\rm eff}^2(r_0)$ to represent these microscopic processes of building the electric energy (D.3.4) and radiating it away from the thin shell, so as to study the back-reaction of these microscopic processes on the macroscopic process of gravitational collapse of the thin shell.

D.4. Collapse of spherically thin capacitor

A lot of attention has been focused on the exact solution of thin charged shell in gravitational collapse Israel (1966); De la Cruz and Israel (1967); Bekenstein (1971); Cherubini et al. (2002); Ruffini and Vitagliano (2002). Following the line presented in Refs. Cherubini et al. (2002) and Ruffini and Vitagliano (2002) for finding an exact solution of thin charged shell in gravitational collapse, we try to approximately solve the Einstein equations (D.2.1,D.2.2) for the gravitational collapse of the spherically thin capacitor (the thin shell). We have $g^-_{tt} = (g^-_{rr})^{-1} \equiv f_-$ and $g^+_{tt} \approx (g^+_{rr})^{-1} \equiv f_+$, where the sign "$\approx$" indicates for the range $r_0 \geq r \geq r_0 + d$, where we neglect the charge and mass-energy distributions of the electron layer. From the G_{tt} Einstein equation, we get

$$ds^2 = \begin{cases} -f_+ dt_+^2 + f_+^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) & \text{in } \mathcal{M}_+ \\ -f_- dt_-^2 + f_-^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) & \text{in } \mathcal{M}_- \end{cases}, \tag{D.4.1}$$

where

$$f_+ = 1 - \tfrac{2M}{r} + \tfrac{Q^2_{\rm eff}(r)}{r^2}, \quad \text{and} \quad f_- = 1; \tag{D.4.2}$$

t_- and t_+ are the Schwarzschild-like time coordinates in $\mathcal{M}_-$ and $\mathcal{M}_+$ respectively. M is the total mass-energy of the thin shell, measured by an observer at rest at infinity. Indicating by $t_{0\pm}$ the Schwarzschild-like time coordinate of the thin shell, from the G_{tr} Einstein equation we have

$$\tfrac{M_0}{2}\left[f_+(r_0)\tfrac{dt_{0+}}{d\tau} + f_-(r_0)\tfrac{dt_{0-}}{d\tau}\right] = M - \tfrac{Q^2_{\rm eff}}{2r_0}, \tag{D.4.3}$$

where we introduce the notation $Q^2_{\rm eff} \equiv Q^2_{\rm eff}(r_0)$. The remaining Einstein equations are identically satisfied. From (D.4.3) we have that the inequality

$$M - \tfrac{Q^2_{\rm eff}}{2r_0} > 0, \tag{D.4.4}$$

holds since the left-handed side of Eq. (D.4.3) is clearly positive. We define the four-velocity U^μ of the thin shell as the four-velocity U^μ_B of the baryon layer, for the reasons discussed in the paragraphs where Eqs. (D.2.6-D.2.8) are. From (D.4.3) and the normalization condition of the four-velocity of the thin shell $U_\mu U^\mu = -1$,

$$\left[-f_\pm(r_0)\tfrac{dt_{0\pm}}{d\tau} + f_\pm(r_0)\tfrac{dt_{0\pm}}{d\tau}\right] = -1, \tag{D.4.5}$$

we find

$$\left(\frac{dr_0}{d\tau}\right)^2 = \frac{1}{M_0^2}\left(M \pm \frac{M_0^2}{2r_0} - \frac{Q_{\rm eff}^2}{2r_0}\right)^2 - f_{\mp}\left(r_0\right), \tag{D.4.6}$$

$$\frac{dt_{0\pm}}{d\tau} = \frac{1}{M_0 f_{\pm}(r_0)}\left(M \mp \frac{M_0^2}{2r_0} - \frac{Q_{\rm eff}^2}{2r_0}\right), \tag{D.4.7}$$

in the space-times $\mathcal{M}_{\pm}$. Eqs. (D.4.1-D.4.7) completely describe a 3-parameter (M, $Q_{\rm eff}^2$, M_0) family of solutions of the Einstein equations. As we will see, for the description of the collapse we can choose either $\mathcal{M}_{-}$ or $\mathcal{M}_{+}$. The two descriptions are equivalent and relevant for the physical interpretation of the solutions.

For astrophysical applications, see for example Ref. Ruffini et al. (2003a), we attempt to approximately solve the equation of motion of the thin shell and obtain the trajectory $r_0 = r_0\left(t_{0+}\right)$ as a function of the time coordinate t_{0+} relative to the space-time region $\mathcal{M}_{+}$. In the following we drop the $+$ index from t_{0+}. From (D.4.6) and (D.4.7) we have the equation of motion of the thin shell

$$\begin{aligned} \frac{dr_0}{dt_0} &= \frac{dr_0}{d\tau}\frac{d\tau}{dt_0} = \pm\frac{F}{\Omega}\sqrt{\Omega^2 - F}, \\ \frac{dr_0}{d\tau} &= \pm\sqrt{\Omega^2 - F} \end{aligned} \tag{D.4.8}$$

where $F \equiv f_{+}\left(r_0\right)$ of Eq. (D.4.2),

$$\Omega \equiv \Gamma - \frac{M_0^2 + Q_{\rm eff}^2}{2M_0 r_0}, \quad \Gamma \equiv \frac{M}{M_0}. \tag{D.4.9}$$

Since we are interested in an imploding thin shell, only the minus sign case in (D.4.8) will be studied. We can give the following physical interpretation of Γ. For $M \geq M_0$, Γ coincides with the Lorentz factor of the imploding thin shell at infinity; from (D.4.8) it satisfies

$$\Gamma = \frac{1}{\sqrt{1 - \left(\frac{dr_0}{dt_0}\right)^2_{r_0=\infty}}} \geq 1. \tag{D.4.10}$$

We rewrite equation of motion (D.4.8) as

$$\left(\frac{dr_0}{d\tau}\right)^2 = \left[\Gamma + \frac{M_0}{2r_0}(1 - \xi^2)\right]^2 - 1,$$

or

$$\left(\frac{\Omega}{F}\right)^2\left(\frac{dr_0}{dt_0}\right)^2 = \left[\Gamma + \frac{M_0}{2r_0}(1 - \xi^2)\right]^2 - 1, \tag{D.4.11}$$

where $\Omega \equiv \Gamma - (M_0/2r_0)(1+\xi^2)$ and we define an effective "charge-mass-ratio"

$$\xi \equiv \frac{Q_{\rm eff}}{M_0}. \tag{D.4.12}$$

Actually ξ^2 represents the ratio of electric energy and gravitational energy of the thin shell. For the case $\Gamma = 1$ ($M = M_0$), i.e., the thin shell collapses at rest from infinity. Eq. (D.4.4) requires $M_0 \geq Q^2_{\rm eff}/2r_0$ to start gravitational collapse and Eq. (D.4.11) requires $\xi < 1$ to continue gravitational collapse. When $\xi = 1$, gravitational collapse stops and kinetic energy of the thin shell vanishes as will be seen below. The trajectory of the thin shell is given by the solution:

$$\int dt_0 = -\int \frac{\Omega}{F\sqrt{\Omega^2 - F}} dr_0. \tag{D.4.13}$$

to the equation of motion (D.4.8).

To understand the total energy conservation of the thin shell in gravitational collapse, we use the solution (D.4.6) in the flat space-time $\mathcal{M}_-$,

$$\left(M_0 \frac{dr_0}{d\tau}\right)^2 = \left(M + \frac{M_0^2}{2r_0} - \frac{Q^2_{\rm eff}}{2r_0}\right)^2 - M_0^2, \tag{D.4.14}$$

we can interpret $-\frac{M_0^2}{2r_0}$ as the gravitational attractive energy of the thin shell and $\frac{Q^2_{\rm eff}}{2r_0}$ is its repulsive electric energy. Introducing the total four-momentum of the shell $P^\mu = M_0 U^\mu$ and its radial component $P \equiv M_0 U^r = M_0 \frac{dr_0}{d\tau}$, the kinetic energy of the thin shell as measured by static observers in $\mathcal{M}_-$ is expressed as Ruffini and Vitagliano (2002)

$$T(r_0) \equiv -P_\mu \xi^\mu_- - M_0 = \sqrt{P^2 + M_0^2} - M_0. \tag{D.4.15}$$

Then from Eqs. (D.4.14,D.4.15) we have

$$\begin{aligned} M(r_0) &= -\frac{M_0^2}{2r_0} + \frac{Q^2_{\rm eff}}{2r_0} + \sqrt{P^2 + M_0^2} \\ &= M_0 + T(r_0) - \frac{M_0^2}{2r_0} + \frac{Q^2_{\rm eff}}{2r_0}, \end{aligned} \tag{D.4.16}$$

where we choose the positive root solution due to the constraint (D.4.4). Eq. (D.4.16) is the total energy-conservation of the thin shell, whose rest mass M_0, kinetic energy $T(r_0)$, gravitational energy $-\frac{M_0^2}{2r_0}$, and electric energy $\frac{Q^2_{\rm eff}}{2r_0}$ depends on the radial coordinate $r_0(t_0)$ in gravitational collapse.

In the following discussion, we consider the shell is at rest at infinity and starts to gravitational collapse, $T(r_0) = 0$, $-\frac{M_0^2}{2r_0} = 0$ and $\frac{Q^2_{\rm eff}}{2r_0} = 0$ at $r_0 \to \infty$.

The initial energy of the thin shell $M(r_0 \to \infty) = M_0$, i.e., $\Gamma = 1$. The total shell energy $M(r_0) = M_0$ is conserved in the entire collapsing process.

D.5. Collapse of the thin shell with varying electric energy

In Ref. Han et al. (2012), assuming that in gravitational collapses, the baryon layer induces an inward current-density

$$J^r_{_B}(r_0) = e\bar{n}_p U^r_{_B} \approx e\bar{n}_p(\dot{r}_0\Omega/F), \quad \dot{r}_0 = dr_0/dt_0, \tag{D.5.1}$$

at the rate of strong interaction scales, we show that triggered by this baryon current (D.5.1), the current-density $J^r_e = e\bar{n}_e U^r_e$ of the electron layer oscillates collectively with overcritical electric fields E at frequency $\omega_{\rm osci} = \tau^{-1}_{\rm osci} \simeq 1.5\, m_e$, leading to the production of electron-positron pairs at rate $\tau^{-1}_{\rm pair} \simeq 6.6\, m_e$. Selecting values $J^r_{_B}(r_0)$ and $\dot{r}_0$ of Eq. (D.5.1) at different collapsing radii, we calculated Han et al. (2012) the averaged energy and number densities of electron-positron pairs produced, as well as the averaged electric energy (Coulomb energy) of oscillating overcritical electric fields. In addition, our results presented in Refs. Ruffini et al. (2003b,a) show that these electron-positron pairs annihilate to photons and the ultra-dense plasma of electron-positron pairs and photons is formed with the equipartition of energy and number of electron-positron pairs and photons, beside this plasma undergoes the hydrodynamical expansion and the photon radiation occurs. This indicates that the electric energy is established by the electron-positron oscillations collectively with overcritical electric fields, then dissipated by electron-positron annihilations to photons radiating away. Clearly, these results and discussions are based on the postulation that the baryon current of Eq. (D.5.1) introduced by the strong interaction in a gravitational collapse process triggers all electric processes, provided that the reaction rates of processes satisfy the inequality of Eq. (26) in Ref. Han et al. (2012). In the light of the total energy conservation in gravitational collapses and Eq. (D.2.9), we further postulate that the electric energy of these electric processes is converted from the gravitational energy, as a consequence, the gravitational energy gained by the collapsing baryon core is transfered to the photon radiation energy. In future work, we are bound to show this energy conversion by solving the equations of gravitational collapses altogether with the equations of electric processes and nuclear processes. In the present chapter, we attempt to study the back-reaction effect of this energy conversion on the gravitational collapse.

In the simplified model of collapsing thin shell, we represent $\frac{Q^2_{\rm eff}}{2r_0}$ the electric energy established by electron-positron pair production and oscillation with overcritical electric fields, then dissipated by electron-positron annihilations to photons radiating away at the collapsing radius r_0. The time variation rate of this electric energy $\frac{Q^2_{\rm eff}}{2r_0}$ is characterized by the frequency $\omega_{\rm osci} \simeq 1.5m_e$ Han et al. (2012). On the other hand, from collapse equation (D.4.11) for $\Gamma = 1$, it is shown that the collapsing velocity (dr_0/dt_0) varies between zero and its maximal value as the "charge-mass-ratio" ξ varies from 1 and 0, corresponding to the microscopic processes of the electric energy $\frac{Q^2_{\rm eff}}{2r_0}$ built up and completely radiating away. In order to see the back-reaction of this radiative electric energy on the gravitational collapse of the thin shell, we model the electric energy $\frac{Q^2_{\rm eff}}{2r_0}$ by an ansatz function for varying "charge-mass-ratio" ξ in the collapse equation (D.4.11)

$$\xi = \xi^{\rm max}|\sin(\omega_{\rm osci}r_0)| + \xi^{\rm min}, \quad r_0 = r_0(t_0). \tag{D.5.2}$$

As indicated by the results of Ref. Han et al. (2012) for $M_0 = 20M_\odot$, we adopt values $\omega_{\rm osci} \simeq 1.5m_e$, $\xi^{\rm max} = 0.6$ and $\xi^{\rm min} = 0.1$ for illustrating the back-reaction effect. This postulates that at the collapsing radius $r_0(t_0)$ of the baryon layer, the microscopic processes of the electric energy $\frac{Q^2_{\rm eff}}{2r_0}$ built up and radiating away are in the rate of the Compton scale $\omega_{\rm osci} \simeq 1.5m_e$ and effectively described by a simple function of Eq. (D.5.2), and $\xi^{\rm min} \neq 0$ representing the part of the electric energy that does not radiate away from the shin shell. Whereas the case ($\xi \equiv 0$) represents the collapse of a neutral thin shell without carrying any electric energy.

We express r_0 and t_0 in units of GM_0 and GM_0/c, then $\omega r_0 = 1.5(m_eGM_0)r_0$, $\lambda_C/GM_0 = 1.05 \times 10^{-16}$, $20GM_\odot/c^2 \simeq 10^{-4}$ second and $M_0 = 20GM_\odot/c \simeq 3 \times 10^6$ cm. Plotting the velocity $\dot{r}_0 = dr_0/dt_0$ of Eq. (D.4.11) in Fig. D.1, we find that in collapse process, the thin shell velocity is oscillating between zero and the envelop curve, which represents the collapsing velocity of the thin shell carrying the electric energy described by $\xi^{\rm min} \neq 0.1$. This result shows a sequence of "on and off" collapsing steps: the thin shell at rest starts to move inwards due to the gravitational attraction of the baryon layer, and stops due to the repulsion of the electric energy $\frac{Q^2_{\rm eff}}{2r_0}$ built up to $\xi = 1$, then restarts to move inwards again due to the electric energy $\frac{Q^2_{\rm eff}}{2r_0}$ partially radiating away in the form of electron-positron pairs and photons. The frequency of this "on and off" hopping sequence is about $\omega_{\rm osci} \sim m_e$, the Compton scale. The collapse process is still continuous in terms of macroscopic scale. However, as will be seen soon, the time scale and kinetic energy of collapses are changed.

The averaged collapsing velocity of the thin shell of Eq. (D.5.2) is smaller than the collapsing velocity (envelop curve) for the case $\xi = 0$. As a result, the time duration of collapse process becomes longer. Assuming that the thin shell is at rest at the radius $R_0 = 30M_0$ and starts to collapse, we plot in Fig. D.2 the time coordinate t_0 of Eq. (D.4.13) as a function of the radial coordinate r_0 of the collapsing thin shell, in comparison with that of the case $\xi = 0$. The blue line for the case $\xi = 0$ shows that the collapsing shell takes time $\sim 10^2\, GM_0/c^2$ to approach the horizon, whereas the red line for the case ξ of Eq. (D.5.2) shows that the collapsing thin shell takes time $\sim 10^3\, GM_0/c^2$ to approach the horizon. The collapsing time for the case ξ of Eq. (D.5.2) is about 10 times longer than the collapsing time for the case $\xi = 0$. This result is not sensitive to the value of the frequency $\omega_{\rm osci}$ in the Compton scale and the detailed form of an oscillating function (D.5.2) of the frequency $\omega_{\rm osci}$.

It should be pointed out that in this simplified toy model of thin shell collapsing, to evidently illustrate the back-reaction effect that slows down the collapsing process in comparison with the free fall collapsing process in the same plot (see Fig. D.2), we select the initial radius $R_0 = 30M_0$ at which the thin shell starts to collapse. As discussed, the baryon core must be at (over) the nuclear density and the mean distance between baryons is about one Fermi (smaller than one Fermi), where the strong interaction plays an important role. This is the one of necessary conditions for the electric processes of production and oscillation of electron-positron pairs together with "non-equilibrium" overcritical electric fields to occur. Under this consideration, the initial radius R_0 of the baryon core starting to collapse should be smaller than $30M_0$. However, in this simplified toy model of thin shell collapsing, the surface density of the baryon thin shell is over the nuclear density at the initial radius $R_0 = 30M_0$. Nevertheless, the necessary condition of baryon cores being at/over the nuclear density should be duly taken into account, when we study the back-reaction in a more realistic model describing the gravitational collapse of neutral stellar cores.

Using the velocity $\dot{r}_0 = dr_0/dt_0$ of Eqs. (D.4.8) and (D.4.11), we plot in Fig. D.3 the kinetic energy $T(r_0)$ of Eq. (D.4.15) and the gravitational energy $M_0^2/2r_0$ of the collapsing thin shell as a function of collapsing radius r_0. Following the total energy conservation of Eq. (D.4.16) and $M(r_0) = M_0$,

$$T(r_0) - \frac{M_0^2}{2r_0} + \frac{Q_{\rm eff}^2}{2r_0} = 0, \tag{D.5.3}$$

the electric energy $\frac{Q_{\rm eff}^2}{2r_0}$ is given by the difference between gravitational energy and kinetic energy, as shown in Fig. D.3. In the collapse process, the kinetic energy $T(r_0)$ and electric energy $\frac{Q_{\rm eff}^2}{2r_0}$ are rapidly oscillating, following the ansatz function (D.5.2) with the frequency $\omega_{\rm osci}$ of microscopic processes. Averaging over these rapid oscillations, we obtain the averaged values of the kinetic energy and electric energy, which are approximately equal to an half of gravitational energy:

$$\langle T(r_0)\rangle \approx \langle \frac{Q_{\rm eff}^2}{2r_0}\rangle \approx \frac{1}{2}\frac{M_0^2}{2r_0}. \tag{D.5.4}$$

This implies that the averaged electric energy radiating away from the thin shell is about an half of the gravitational energy gained by the collapsing thin shell in the collapsing process. When the black hole horizon is reached, using Eq. (D.4.16), the irreducible mass of black hole is introduced Ruffini and Vitagliano (2002)

$$M = M_{\rm ir} + \tfrac{Q^2_{\rm eff}}{2r_+}, \quad \text{and} \quad M_{\rm ir} = M_0 - \tfrac{M_0^2}{2r_+} + T(r_+), \tag{D.5.5}$$

where $\frac{Q^2_{\rm eff}}{2r_+}$ is the total electric energy of the thin shell approaching the horizon r_+. Suppose that the electric energy $\frac{Q^2_{\rm eff}}{2r_+}$ completely radiates away, a black hole is formed with the horizon $r_0 \to r_+ = 2M_0$ for $F \equiv f_+(r_0) \to 0$. In this case, the total electric energy radiating away from the thin shell is about an half of gravitational energy of the thin shell

$$\langle \frac{Q^2_{\rm eff}}{2r_+} \rangle \approx \frac{1}{2}\left(\frac{M_0^2}{2r_+}\right) = \frac{1}{8}M_0, \tag{D.5.6}$$

and the irreducible mass of the formed black hole is about

$$M_{\rm ir} = M_0 - \tfrac{M_0^2}{2r_+} + \langle T(r_+) \rangle \approx \frac{7}{8}M_0, \tag{D.5.7}$$

$$M_0 = M_{\rm ir} + \langle \frac{Q^2_{\rm eff}}{2r_+} \rangle, \tag{D.5.8}$$

which implies about 1/8 of the gravitational energy extracted in gravitational collapses.

D.6. Summary and remarks

In this chapter, on the basis of a simple model for describing the gravitational collapse of a spherically thin capacitor, we analytically study how the gravitational energy gained in collapse converts to the kinetic energy and electric energy, the latter can be radiated away. Using an ansatz function for the effective "charge-mass-ratio" (D.4.12) to model the microscopic processes that create this electric energy and radiate it away in the Compton scale, we study how the back-reaction of such radiative electric energy on the macroscopic process of gravitational collapse. We find that the rebuilding and radiating of repulsive electric energy cause the collapse process undergoing a sequence of "on and off" hopping steps in the microscopic Compton scale. Although such a collapse process is still continuous in the macroscopic scales, it is slowed down as the kinetic energy is reduced and collapsing time is about an order of magnitude larger than that of collapse process eliminating electric pro-

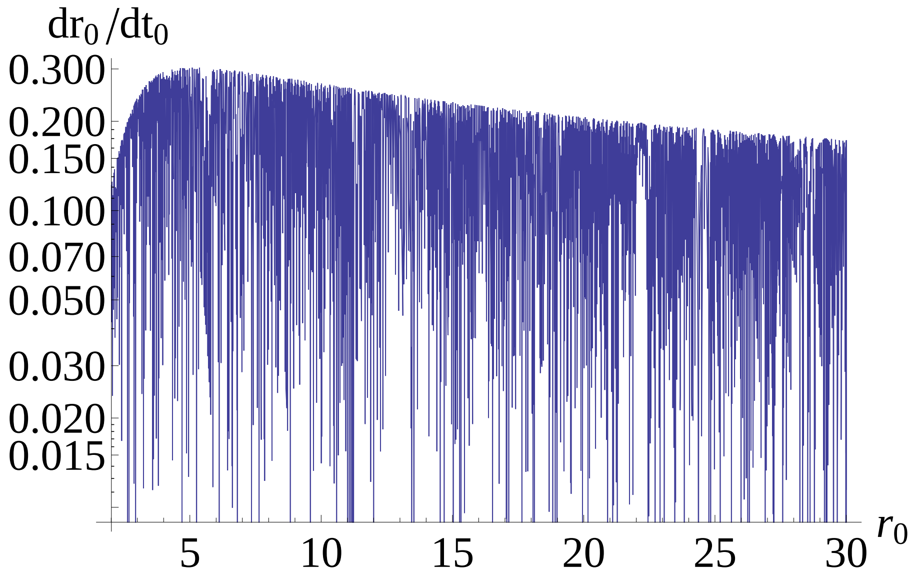

Figure D.1.: In unit of the speed of light c, the collapse velocity (dr_0/dt_0) is plotted (fast oscillating lines in blue) as a function of radius r_0 of the collapsing thin shell. The thin shell is at rest at the radius $R_0 = 30\,GM_0$ and starts to collapse. The thin shell mass $M_0 = 20M_\odot$.

cesses. The averaged kinetic and electric energies are the same order, about an half of gravitational energy in collapse.

These results are obtained from an over simplified model for both macroscopic and microscopic processes. Nevertheless they indicate that apart from an electromagnetic energy radiation, the microscopic processes of electrodynamics have significant back-reaction and effects on gravitational collapsing processes in macroscopic scales. It is thus essential to take into account, rather than ignore, electric processes in more realistic models for studying gravitational collapse of neutral stellar core at/over the nuclear density, even though calculations are very complicate.

To end this chapter, we would like to mention the relevance of these results to our previous studies of energetic budget and time duration of Gamma-Ray Bursts (GRBs) as a signal of the final stage of gravitational collapse of massive stellar cores. The total electromagnetic energy extractable from a charged black hole Damour and Ruffini (1975); Ruffini and Xue (2008a); Preparata et al. (1998, 2003) (from the collapse of a neutral stellar core Han et al. (2012)) is a fraction of its mass, which reasonably accounts for the energetic budget of GRBs. In addition, the time duration T_{90} of electromagnetic radiation is about 10^{-2} second obtained Ruffini et al. (1999, 2000) by solving hydrodynamical equations with an initial configuration of electro-positron pairs and photons sphere (dyadosphere) around a charged black hole. This time duration scale is elongated to be an order of magnitude larger $\sim 10^{-1}$ second

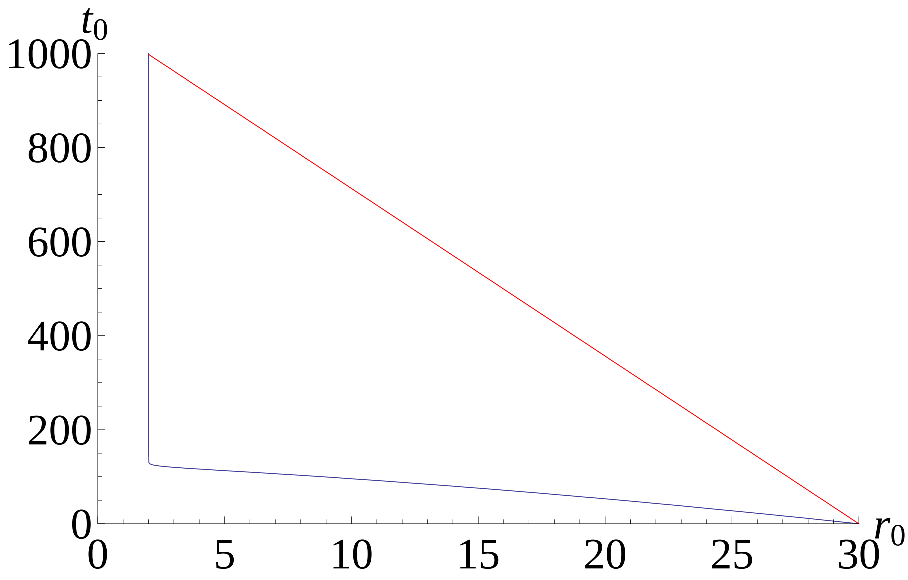

Figure D.2.: In thin shell collapsing process, the time coordinate t_0 is plotted as a function of radial coordinate r_0 of the thin shell. t_0 and r_0 are in unit of GM_0. The red line is for ξ of Eq. (D.5.2) and the blue for $\xi = 0$. The shell is at rest at the radius $R_0 = 30\,GM_0$ and starts to collapse. The thin shell mass $M_0 = 20M_\odot$.

Ruffini et al. (2003a); Fraschetti et al. (2006); Ruffini et al. (2005) by considering both the dynamical formation and hydrodynamical evolution of dyadosphere in a collapsing charged core. The results of this chapter imply that due to the back-reaction of the dynamical formation and hydrodynamical evolution of dyadosphere on collapsing neutral stellar cores at or over the nuclear density, the slowing down of gravitational collapsing processes should elongate this time duration scale by another factor of 10, i.e., $T_{90} \sim 1$ second that reasonably accounts for the time duration of short GRBs.

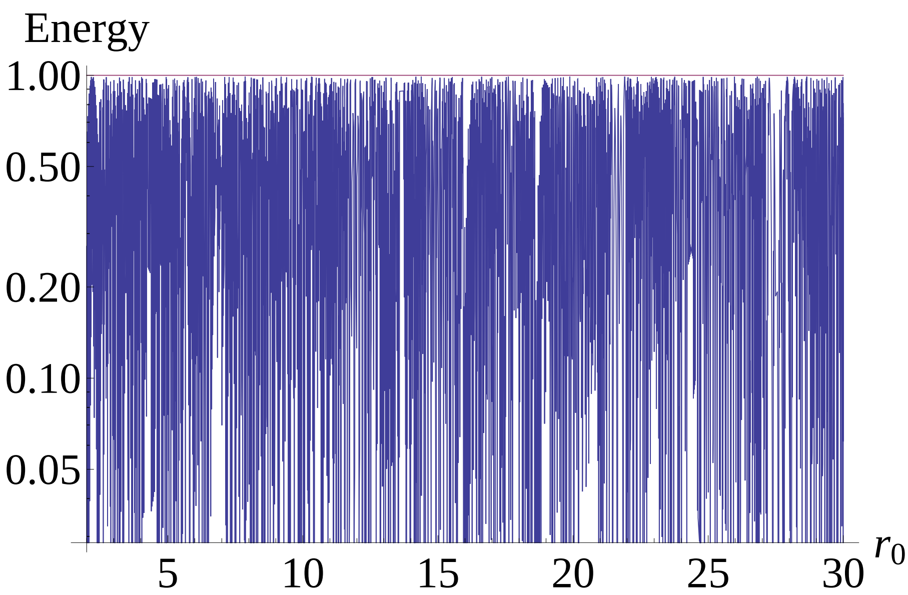

Figure D.3.: In unit of the gravitational energy $M_0^2/(2r_0)$, the gravitational energy (constant red line at 1) and kinetic energy (fast oscillating lines in blue) and electric energy (fast oscillating lines in white) of the thin shell are plotted as a function of collapsing radius r_0.

E. Einstein-Euler-Heisenberg theory and charged black holes

E.1. Introduction

For several decades the nonlinear electromagnetic generalization of the Reissner-Nordström solution of the Einstein-Maxwell equations has attracted a great deal of attention. The most popular example is the gravitating Born-Infeld (BI) theory Born and Infeld (1934). The static charged black holes in gravitating nonlinear electrodynamics were studied in the 1930s Hoffmann (1935); Hoffmann and Infeld (1937). The discovery that the string theory, as well as the D-brane physics, leads to Abelian and non-Abelian BI-like Lagrangians in its low-energy limit (see, e.g., Refs. Fradkin and Tseytlin (1985); Abouelsaood et al. (1987); Tseytlin (1997)), has renewed the interest in these kinds of nonlinear actions. Asymptotically flat, static, spherically symmetric black hole solutions for the Einstein-Born-Infeld theory were obtained in the literature Garcia et al. (1984); Demianski (1986).

Generalization of the exact solutions of spherically symmetric Born-Infeld black holes with a cosmological constant in arbitrary dimensions has been considered Fernando and Krug (2003); Dey (2004); Cai et al. (2004), as well as in other gravitational backgrounds Wiltshire (1998); Aiello et al. (2004). Many other models of nonlinear electrodynamics leading to static and spherically symmetric structures have been considered in the last decades, such as the theory with a nonlinear Lagrangian of a general function of the gauge invariants ($F^{\mu\nu}F_{\mu\nu}$ and $F_{\mu\nu}\tilde{F}^{\mu\nu}$) Diaz-Alonso and Rubiera-Garcia (2010b,a, 2011a,b) or a logarithmic function of the Maxwell invariant ($F^{\mu\nu}F_{\mu\nu}$) Soleng (1995), and the theory with a generalized nonlinear Lagrangian De Oliveira (1994) which can lead to the BI Lagrangian and the weak-field limit of the Euler-Heisenberg effective Lagrangian Heisenberg and Euler (1936). The static and spherically symmetric black hole, whose gravity coupled to the nonlinear electrodynamics of the weak-field limit of the Euler-Heisenberg effective Lagrangian as a low-energy limit of the Born-Infeld theory, was studied in Ref. Yajima and Tamaki (2001). Some attempts in the obtention of regular (singularity-free) static and spherically symmetric black hole solutions in gravitating nonlinear electrodynamics have been made Ayón-Beato and García (1998, 1999); Cirilo Lombardo (2009); Burinskii and Hildebrandt (2002); Dymnikova (2004), and the unusual properties of these solutions have been discussed in Refs. Novello et al. (2000); Bronnikov (2001). Generalization of spherically symmetric black holes in higher dimension in the theory with a nonlinear Lagrangian of a function of power of the Maxwell invariant has been considered in the literature Hassaïne and C. Martínez (2007, 2008); González et al. (2009); Mazharimousavi et al. (2010). Finally, we mention that rotating black branes Dehghani and Rastegar Sedehi (2006); Dehghani et al. (2007) and rotating black strings Hendi (2010) in the Einstein-Born-Infeld theory have been also considered.

The effective Lagrangian of nonlinear electromagnetic fields has been formulated for the first time by Heisenberg and Euler using the Dirac electron-positron theory Heisenberg and Euler (1936). Schwinger reformulated this nonperturbative one-loop effective Lagrangian within the quantum electrodynamics (QED) framework Schwinger (1951). This effective Lagrangian characterizes the phenomenon of vacuum polarization. Its imaginary part describes the probability of the vacuum decay via the electron-positron pair production. If electric fields are stronger than the critical value $E_c = m^2c^3/e\hbar$, the energy of the vacuum can be lowered by spontaneously creating electron-positron pairs Heisenberg and Euler (1936); Schwinger (1951); Sauter (1931). For many decades, both theorists and experimentalists have been interested in the aspects of the electron-positron pair production from the QED vacuum and the vacuum polarization by an external electromagnetic field (see, e.g., Refs. Ruffini et al. (2010); ELI).

As a fundamental theory, QED gives an elegant description of the electromagnetic interaction; moreover, it has been experimentally verified. Therefore, it is important to study the QED effects in black hole physics. As a result of one-loop nonperturbative QED, the Euler-Heisenberg effective Lagrangian deserves to attract more attention in the topic of generalized black hole solutions mentioned above. In this chapter, we adopt the contribution from the Euler-Heisenberg effective Lagrangian to formulate the Einstein-Euler-Heisenberg theory, and study the solutions of electrically and magnetically charged black holes in spherical geometry. We calculate and discuss the QED corrections to the black hole horizon area, entropy, total energy, and the maximally extractable energy.

The chapter is organized as follows. In Sec. E.2, we first recall the Euler-Heisenberg effective Lagrangian. We formulate the Einstein-Euler-Heisenberg theory in Sec. E.3. The study of electrically charged black holes in the weak electric field case is presented in Sec. E.4. The study of magnetically charged black holes in both weak and strong magnetic field cases is presented in Sec. E.5. Then we present the study of black holes with both electric and magnetic charges in the Einstein-Euler-Heisenberg theory in Sec. E.6. A summary is given in Sec. E.7. The use of units with $\hbar = c = 1$ is throughout the chapter.

E.2. The Euler-Heisenberg effective Lagrangian

The QED one-loop effective Lagrangian was obtained by Heisenberg and Euler Heisenberg and Euler (1936) for constant electromagnetic fields,

$$\Delta\mathcal{L}_{\rm eff} = \frac{1}{2(2\pi)^2}\int_0^\infty \frac{ds}{s^3}\Big[e^2\varepsilon\beta s^2\coth(e\varepsilon s)\cot(e\beta s) \\ -1-\frac{e^2}{3}(\varepsilon^2-\beta^2)s^2\Big]e^{-is(m_e^2-i\eta)}, \tag{E.2.1}$$

as a function of two invariants: the scalar S and the pseudoscalar P,

$$S \equiv -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(\mathbf{E}^2-\mathbf{B}^2) \equiv \varepsilon^2-\beta^2, \\ P \equiv -\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \mathbf{E}\cdot\mathbf{B} \equiv \varepsilon\beta, \tag{E.2.2}$$

where the field strength is $F^{\mu\nu}$, $\tilde{F}^{\mu\nu} \equiv \epsilon^{\mu\nu\lambda\kappa}F_{\lambda\kappa}/2$, and

$$\varepsilon = \sqrt{(S^2+P^2)^{1/2}+S}, \tag{E.2.3}$$

$$\beta = \sqrt{(S^2+P^2)^{1/2}-S}. \tag{E.2.4}$$

The effective Lagrangian reads

$$\mathcal{L}_{\rm eff} = \mathcal{L}_M + \Delta\mathcal{L}_{\rm eff}, \tag{E.2.5}$$

where $\mathcal{L}_M = S$ is the Maxwell Lagrangian. Its imaginary part is related to the decay rate of the vacuum per unit volume Heisenberg and Euler (1936); Schwinger (1951),

$$\frac{\Gamma}{V} = \frac{\alpha\varepsilon^2}{\pi^2}\sum_{n=1}\frac{1}{n^2}\frac{n\pi\beta/\varepsilon}{\tanh n\pi\beta/\varepsilon}\exp\left(-\frac{n\pi E_c}{\varepsilon}\right) \tag{E.2.6}$$

for fermionic fields, and

$$\frac{\Gamma}{V} = \frac{\alpha\varepsilon^2}{2\pi^2}\sum_{n=1}\frac{(-1)^n}{n^2}\frac{n\pi\beta/\varepsilon}{\sinh n\pi\beta/\varepsilon}\exp\left(-\frac{n\pi E_c}{\varepsilon}\right) \tag{E.2.7}$$

for bosonic fields; here, $E_c = \frac{m_e^2 c^3}{e\hbar}$ is the critical field. Using the expressions Gradshteyn and Ryzhik (1994)

$$e\varepsilon s \coth(e\varepsilon s) = \sum_{n=-\infty}^{\infty} \frac{s^2}{(s^2 + \tau_n^2)}, \quad \tau_n \equiv n\pi/e\varepsilon, \tag{E.2.8}$$

$$e\beta s \cot(e\beta s) = \sum_{m=-\infty}^{\infty} \frac{s^2}{(s^2 - \tau_m^2)}, \quad \tau_m \equiv m\pi/e\beta, \tag{E.2.9}$$

one obtains the real part of the Euler-Heisenberg effective Lagrangian (E.2.1) (see Refs. Ruffini et al. (2010); Ruffini and Xue (2006); Mielniczuk (1982); Valluri et al. (1993); Cho and Pak (2001); Kleinert et al. (2013)),

$$(\Delta\mathcal{L}_{\rm eff}^{\cos})_{\mathcal{P}} = \frac{1}{2(2\pi)^2} \sum_{n,m=-\infty}^{\infty} \frac{1}{\tau_m^2 + \tau_n^2} \left[\bar{\delta}_{m0} J(i\tau_m m_e^2) - \bar{\delta}_{n0} J(\tau_n m_e^2)\right] \tag{E.2.10}$$

$$= -\frac{1}{(2\pi)^2} \left[\sum_{n=1}^{\infty} \frac{e\beta}{\tau_n} \coth(e\beta\tau_n) J(\tau_n m_e^2) - \sum_{m=1}^{\infty} \frac{e\epsilon}{\tau_m} \coth(e\epsilon\tau_m) J(i\tau_m m_e^2)\right] \tag{E.2.11}$$

The symbol $\bar{\delta}_{ij} \equiv 1 - \delta_{ij}$ denotes the complimentary Kronecker δ, which vanishes for $i = j$, and

$$J(z) \equiv \mathcal{P} \int_0^{\infty} ds \frac{s e^{-s}}{s^2 - z^2} = -\frac{1}{2}\left[e^{-z}{\rm Ei}(z) + e^{z}{\rm Ei}(-z)\right]. \tag{E.2.12}$$

Here, $\mathcal{P}$ indicates the principle value integral, and ${\rm Ei}(z)$ is the exponential-integral function,

$${\rm Ei}(z) \equiv \mathcal{P} \int_{-\infty}^{z} dt \frac{e^t}{t} = \log(-z) + \sum_{k=1}^{\infty} \frac{z^k}{k k!}. \tag{E.2.13}$$

Using the series and asymptotic representation of the exponential-integral function ${\rm Ei}(z)$ for large z corresponding to weak electromagnetic fields ($\varepsilon/E_c \ll 1$, $\beta/E_c \ll 1$),

$$J(z) = -\frac{1}{z^2} - \frac{6}{z^4} - \frac{120}{z^6} - \frac{5040}{z^8} - \frac{362880}{z^{10}} + \cdots, \tag{E.2.14}$$

the weak-field expansion of Eq. (E.2.10) is

$$(\Delta\mathcal{L}_{\rm eff})_{\mathcal{P}} = \frac{2\alpha^2}{45 m_e^4}\left\{4S^2 + 7P^2\right\} + \frac{64\pi\alpha^3}{315 m_e^8}\left\{16S^3 + 26SP^2\right\} + \cdots, \tag{E.2.15}$$

which is expressed in terms of a powers series of weak electromagnetic fields up to $O(\alpha^3)$, the first term was obtained by Heisenberg and Euler in their original article Heisenberg and Euler (1936).

On the other hand, using the series and asymptotic representation of the exponential-integral function $\mathrm{Ei}(z)$ for small $z \ll 1$ Gradshteyn and Ryzhik (1994) corresponding to strong electromagnetic fields ($\varepsilon/E_c \gg 1$, $\beta/E_c \gg 1$),

$$J(z) = -\frac{1}{2}\left[e^z \ln(z) + e^{-z}\ln(-z)\right] - \frac{1}{2}\gamma\left[e^z + e^{-z}\right] + \mathcal{O}(z), \tag{E.2.16}$$

the leading terms in the strong-field expansion of Eqs. (E.2.10) and (E.2.11) are given by (see Refs. Ruffini et al. (2010); Ruffini and Xue (2006); Kleinert et al. (2013); Kleinert (2011))

$$(\Delta\mathcal{L}_{\text{eff}}^{\cos})_{\mathcal{P}} = \frac{1}{2(2\pi)^2}\sum_{n,m=-\infty}^{\infty}\frac{1}{\tau_m^2+\tau_n^2}\left[\bar{\delta}_{n0}\ln\left(\tau_n m_e^2\right) - \bar{\delta}_{m0}\ln\left(\tau_m m_e^2\right)\right] + \cdots \tag{E.2.17}$$

$$= \frac{1}{2(2\pi)^2}\left[\sum_{n=1}^{\infty}\frac{e\beta}{\tau_n}\coth\left(e\beta\tau_n\right)\ln\left(\tau_n m_e^2\right) - \sum_{m=1}^{\infty}\frac{e\epsilon}{\tau_m}\coth\left(e\epsilon\tau_m\right)\ln\left(\tau_m m_e^2\right)\right] \tag{E.2.18}$$

In the case of vanishing magnetic field $\mathbf{B} = 0$ and a strong electric field $E \gg E_c$ using $\lim_{z\to\infty} J(iz) = 0$ and $\lim_{z\to 0} z\coth(az) = 1/a$, Eq. (E.2.18) becomes (see Refs. Ruffini et al. (2010); Ruffini and Xue (2006); Kleinert et al. (2013))

$$(\Delta\mathcal{L}_{\text{eff}}^{\cos})_{\mathcal{P}} = \frac{e^2E^2}{4\pi^4}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[\ln\left(\frac{n\pi E_c}{E}\right)+\gamma\right] + \cdots \tag{E.2.19}$$

$$= \frac{e^2E^2}{24\pi^2}\left[\ln\left(\frac{\pi E_c}{E}\right)+\gamma\right] - \frac{e^2E^2}{4\pi^4}\zeta'(2) + \cdots, \tag{E.2.20}$$

with the Euler-Mascheroni constant $\gamma = 0.577216$, the Riemann zeta function $\zeta(k) = \sum_n 1/n^k$, and

$$\zeta'(2) = \frac{\pi^2}{6}\left[\gamma + \ln(2\pi) - 12\ln A\right] \simeq -0.937548, \tag{E.2.21}$$

with $A = 1.28243$ being the Glaisher constant. Similarly, in the case of vanishing electric field $\mathbf{E} = 0$ and a strong magnetic field $B \gg E_c$, Eq. (E.2.18) becomes (see Refs. Ruffini et al. (2010); Ruffini and Xue (2006); Kleinert et al. (2013))

$$(\Delta\mathcal{L}_{\text{eff}}^{\cos})_{\mathcal{P}} = -\frac{e^2B^2}{4\pi^4}\sum_{m=1}^{\infty}\frac{1}{n^2}\left[\ln\left(\frac{n\pi E_c}{B}\right)+\gamma\right] + \cdots \tag{E.2.22}$$

$$= -\frac{e^2B^2}{24\pi^2}\left[\ln\left(\frac{\pi E_c}{B}\right)+\gamma\right] + \frac{e^2B^2}{4\pi^4}\zeta'(2) + \cdots. \tag{E.2.23}$$

The ($n = 1$) term in Eq. (E.2.22) is the one obtained by Weisskopf Weisskopf (1936).

E.3. The Einstein-Euler-Heisenberg theory

Since the real part of the Euler-Heisenberg effective Lagrangian $(\Delta\mathcal{L}_{\rm eff}^{\cos})_{\mathcal{P}}$ of Eq. (E.2.10) is expressed in terms of Lorentz invariants (ε, β) or (S, P), the Euler-Heisenberg effective action in the curve space-time described by metric $g_{\mu\nu}$ can be written as

$$\mathcal{S}_{\rm EH} = \int d^4x\sqrt{-g}\mathcal{L}_{\rm EH}, \quad \mathcal{L}_{\rm EH} = [S + (\Delta\mathcal{L}_{\rm eff}^{\cos})_{\mathcal{P}}]\,. \tag{E.3.1}$$

The Einstein and Euler-Heisenberg action is then given by

$$\mathcal{S}_{\rm EEH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \mathcal{S}_{\rm EH}, \tag{E.3.2}$$

where R is the Ricci scalar.

The Einstein field equations are

$$G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = 8\pi G T^{\mu\nu}, \tag{E.3.3}$$

where the energy-momentum tensor is

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta\mathcal{S}_{\rm EH}}{\delta g_{\mu\nu}}. \tag{E.3.4}$$

The electromagnetic field equations and Bianchi identities are given by

$$\mathcal{D}_\mu P^{\nu\mu} = j^\nu, \quad \mathcal{D}_\mu \tilde{F}^{\mu\nu} = 0, \tag{E.3.5}$$

and the displacement fields $P^{\nu\mu}, D^i = P^{0i}$, and $H^i = -\epsilon^{ijk}P_{jk}$ are defined as

$$P^{\mu\nu} = \frac{\delta\mathcal{L}_{\rm EH}}{\delta F_{\mu\nu}}, \quad D^i = \frac{\delta\mathcal{L}_{\rm EH}}{\delta E_i}, \quad H^i = -\frac{\delta\mathcal{L}_{\rm EH}}{\delta B_i}. \tag{E.3.6}$$

Here, electromagnetic fields are treated as smooth varying fields over all space generated by external charge currents j^μ at infinity.

Using functional derivatives, we obtain

$$\begin{aligned}T^{\mu\nu} &= -g^{\mu\nu}\left[S + (\Delta\mathcal{L}_{\rm eff}^{\cos})_{\mathcal{P}}\right] + 2\left[\frac{\delta S}{\delta g_{\mu\nu}}\frac{\delta\mathcal{L}_{\rm EH}}{\delta S} + \frac{\delta P}{\delta g_{\mu\nu}}\frac{\delta\mathcal{L}_{\rm EH}}{\delta P}\right],\\ &= -g^{\mu\nu}\left[S + (\Delta\mathcal{L}_{\rm eff}^{\cos})_{\mathcal{P}}\right] + 2\left[(1+\mathcal{A}_S)\frac{\delta S}{\delta g_{\mu\nu}} + \mathcal{A}_P\frac{\delta P}{\delta g_{\mu\nu}}\right],\end{aligned} \tag{E.3.7}$$

where two invariants are defined as

$$\mathcal{A}_S \equiv \frac{\delta(\Delta\mathcal{L}_{\rm eff}^{\rm cos})_{\mathcal{P}}}{\delta S}; \quad \mathcal{A}_P \equiv \frac{\delta(\Delta\mathcal{L}_{\rm eff}^{\rm cos})_{\mathcal{P}}}{\delta P}. \tag{E.3.8}$$

It is straightforward to obtain

$$\frac{\delta S}{\delta g_{\mu\nu}} = \frac{1}{2}F^{\mu}{}_{\lambda}F^{\lambda\nu}, \quad \frac{\delta P}{\delta g_{\mu\nu}} = F^{\mu}{}_{\lambda}\tilde{F}^{\lambda\nu} = g^{\mu\nu}P, \tag{E.3.9}$$

and as a result, we rewrite Eq. (E.3.7) as

$$\begin{aligned} T^{\mu\nu} =& T^{\mu\nu}_M + g^{\mu\nu}\left[\mathcal{A}_P P - (\Delta\mathcal{L}_{\rm eff}^{\rm cos})_{\mathcal{P}}\right] + \mathcal{A}_S F^{\mu}{}_{\lambda}F^{\lambda\nu}, \\ =& T^{\mu\nu}_M(1+\mathcal{A}_S) + g^{\mu\nu}\left[\mathcal{A}_S S + \mathcal{A}_P P - (\Delta\mathcal{L}_{\rm eff}^{\rm cos})_{\mathcal{P}}\right], \end{aligned} \tag{E.3.10}$$

where $T^{\mu\nu}_M = -g^{\mu\nu}S + F^{\mu}{}_{\lambda}F^{\lambda\nu}$ is the energy-momentum tensor of the electromagnetic fields of the linear Maxwell theory. Equation (E.3.10) is in fact a general result, independent of the explicit form of nonlinear Lagrangian $(\Delta\mathcal{L}_{\rm eff}^{\rm cos})_{\mathcal{P}}$. Equations (E.3.1)-(E.3.10) in principle give a complete set of equations for Einstein and Euler-Heisenberg effective theory, together with total charge (Q), angular-momentum (L), and energy (M) conservations. In this chapter, adopting the Euler-Heisenberg effective Lagrangian (E.2.10), we explicitly calculate invariants $\mathcal{A}_S$ and $\mathcal{A}_P$ of Eq. (E.3.8) as well as the energy-momentum $T^{\mu\nu}$ of Eq. (E.3.10) in the following cases.

It is necessary to point out that in present chapter, we do not consider the couplings between photons and gravitons that are also induced by QED vacuum polarization effects at the level of one-fermion loop. Drummond and Hathrell obtained the photon effective action from the lowest term of one-loop vacuum polarization on a general curved background manifold; i.e., a graviton couples to two on-mass-shell photons through a fermionic loop Drummond and Hathrell (1980),

$$\begin{aligned} \mathcal{S}_{\rm DH} =& -\frac{\alpha}{720\pi m_e^2}\int d^4x\sqrt{-g}\,\big(5RF_{\mu\nu}F^{\mu\nu} - 26R_{\mu\nu}F^{\mu\sigma}F^{\nu}_{\sigma} \\ & + 2R_{\mu\nu\sigma\tau}F^{\mu\nu}F^{\sigma\tau} + 24\mathcal{D}_{\mu}F^{\mu\nu}\mathcal{D}_{\sigma}F^{\sigma}_{\nu}\big). \end{aligned} \tag{E.3.11}$$

Further studies of one-loop effective action (E.3.11) were made based on the approach of the heat-kernel or "inverse mass" expansion Gilkey (1975); Bastianelli et al. (2000), the approach of the so-called "derivative expansion" Barvinsky and Vilkovisky (1985, 1990a,b); Gusev (2009), and the consideration of the one-loop one particle irreducible of one graviton interacting with any number of photons Bastianelli et al. (2009). This effective action (E.3.11) was used to study the modified photon dispersion relation by a generic gravitational background Drummond and Hathrell (1980) and the possible consequences Latorre et al. (1995); Dittrich and Gies (1998); Shore (1996); Hollowood and Shore (2007, 2008b,a).

At the level of one-loop quantum corrections of the QED theory in the presence of gravitational field, the effective Lagrangian (E.3.11) should be considered as an addition to the Euler and Heisenberg effective Lagrangian (E.2.15) in the weak-field limit. In this chapter, we try to quantitatively study the QED corrections in spherically symmetric black holes with mass M and charge Q. In this case, the corrections from the Euler and Heisenberg effective Lagrangian (E.2.15) must be much larger than the one from the effective Lagrangian (E.3.11). Studying the discussion and result of Ref. Drummond and Hathrell (1980) for spherical symmetric black holes, we approximately estimate the ratio of Eqs. (E.2.15) and (E.3.11) around the horizon of black holes with mass M and charge Q. As a result, this ratio is $\sim 10^{-2}\left(\frac{Q}{M\sqrt{G}}\right)^2\frac{\alpha}{Gm_e^2} \gg 1$. It is not surprising that the electromagnetic coupling $e \sim 1/\sqrt{137}$ is much larger than the effective gravitational counterpart $Gm_e^2 \sim 10^{-45}$. Besides, it is expected that calculations involving both the Euler-Heisenberg effective Lagrangian (E.2.15) and Eq. (E.3.11) are much more complex and tedious. Nevertheless, it is interesting to investigate the effect of the photon-graviton amplitudes on black hole physics. In this chapter, for the sake of simplicity, we first consider only the Einstein-Euler-Heisenberg action (E.3.2) as a leading contribution in order to gain some physical insight into the QED corrections in black hole physics.

E.3.1. $\mathbf{B} = 0, \mathbf{E} \neq 0$ or $\mathbf{E} = 0, \mathbf{B} \neq 0$

We consider the case of $\mathbf{B} = 0$ and $\mathbf{E} \neq 0$, namely, $\beta = P = 0$, $\varepsilon = E = |\mathbf{E}|$, and $S = E^2/2$. $\mathcal{A}_P = 0$ and the effective Lagrangian Eq. (E.2.10) becomes

$$(\Delta\mathcal{L}_{\rm eff}^{\rm cos})_{\mathcal{P}} = -\frac{e^2E^2}{4\pi^4}\sum_{n=1}^{\infty}\frac{1}{n^2}J(n\pi E_c/E). \tag{E.3.12}$$

Using

$$\mathcal{P}\int_0^\infty ds\frac{e^{-s}}{(s^2-z^2)} = -\frac{1}{2z}\Big[e^{-z}\mathrm{Ei}(z) - e^{z}\mathrm{Ei}(-z)\Big], \tag{E.3.13}$$

we calculate

$$\frac{dJ(z)}{dz^2} = \mathcal{P}\int_0^\infty ds\frac{se^{-s}}{(s^2-z^2)^2} = \frac{1}{2z^2} - \mathcal{P}\int_0^\infty ds\frac{e^{-s}}{(s^2-z^2)} \tag{E.3.14}$$

and obtain

$$\mathcal{A}_S = -\frac{e^2}{2\pi^4}\sum_{n=1}^{\infty}\frac{1}{n^2}J(n\pi E_c/E)$$
$$-\frac{e^2}{4\pi^2}\zeta(2) + \frac{e^2}{4\pi}\frac{E_c}{E}\sum_{n=1}^{\infty}\frac{1}{n}\tilde{J}(n\pi E_c/E), \tag{E.3.15}$$

where

$$\tilde{J}(z) = e^{-z}\mathrm{Ei}(z) - e^{z}\mathrm{Ei}(-z). \tag{E.3.16}$$

Substituting these quantities into Eq. (E.3.10), we obtain the expression of the energy-momentum tensor $T^{\mu\nu}(\varepsilon)$. In the case of $\mathbf{E} = 0$ and $\mathbf{B} \neq 0$, the energy-momentum tensor $T^{\mu\nu}(\beta)$ can be straightforwardly obtained from $T^{\mu\nu}(\varepsilon)$ by the discrete duality transformation $\varepsilon \to i\beta$, i.e., $|\mathbf{E}| \to i|\mathbf{B}|$. In principle, using the complete Euler-Heisenberg effective Lagrangian $(\Delta\mathcal{L}^{\cos}_{\rm eff})_{\mathcal{P}}$ (E.2.10) for arbitrary electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$, one can obtain the energy-momentum tensor $T^{\mu\nu}(\varepsilon, \beta)$ of Eq. (E.3.10). For the reason of practical calculations, we consider the cases of weak and strong fields.

E.3.2. Weak- and strong-field cases

In the weak-field case, using Eq. (E.2.15) and calculating Eqs. (E.3.7)-(E.3.10), we obtain

$$\mathcal{A}_S = \frac{2\alpha^2}{45m_e^4}(8S) + \frac{64\pi\alpha^3}{315m_e^8}(48S^2 + 26P^2) + \cdots,$$
$$\mathcal{A}_P = \frac{2\alpha^2}{45m_e^4}(14P) + \frac{64\pi\alpha^3}{315m_e^8}(52SP) + \cdots, \tag{E.3.17}$$

and

$$T^{\mu\nu} = T^{\mu\nu}_M\left[1 + 8\left(\frac{2\alpha^2}{45m_e^4}\right)S\right] + g^{\mu\nu}\left(\frac{2\alpha^2}{45m_e^4}\right)\left[4S^2 + 7P^2\right] + \cdots, \tag{E.3.18}$$

up to the leading order.

In strong-field case $\varepsilon/E_c \gg 1$ and $\beta/E_c \gg 1$ using Eq. (E.2.17) and calculating Eqs. (E.3.7)-(E.3.10), we obtain

$$\mathcal{A}_S = \frac{1}{2(2\pi)^2}\frac{2}{\varepsilon^2+\beta^2}\sum_{n,m=-\infty}^{\infty}\frac{1}{(\tau_m^2+\tau_n^2)^2}\Big\{\bar{\delta}_{n0}\Big[(\tau_n^2-\tau_m^2)\ln(\tau_n m_e^2) - \frac{1}{2}(\tau_m^2+\tau_n^2)\Big]$$
$$-\bar{\delta}_{m0}\Big[(\tau_n^2-\tau_m^2)\ln(\tau_m m_e^2) + \frac{1}{2}(\tau_m^2+\tau_n^2)\Big]\Big\} + \cdots \tag{E.3.19}$$

and

$$\mathcal{A}_P = \frac{1}{2(2\pi)^2}\frac{2\varepsilon\beta}{\varepsilon^2+\beta^2}\sum_{n,m=-\infty}^{\infty}\frac{1}{(\tau_m^2+\tau_n^2)^2}\Big\{\bar{\delta}_{n0}\Big[\left(\frac{\tau_n^2}{\varepsilon^2}+\frac{\tau_m^2}{\beta^2}\right)\ln(\tau_n m_e^2)-\frac{1}{2}\frac{(\tau_m^2+\tau_n^2)}{\varepsilon^2}\Big]$$
$$-\bar{\delta}_{m0}\Big[\left(\frac{\tau_n^2}{\varepsilon^2}+\frac{\tau_m^2}{\beta^2}\right)\ln(\tau_m m_e^2)-\frac{1}{2}\frac{(\tau_m^2+\tau_n^2)}{\beta^2}\Big]\Big\}+\cdots. \qquad \text{(E.3.20)}$$

From Eq. (E.2.20) for $\mathbf{B}=0$ and a strong electric field, we obtain

$$\mathcal{A}_S = \frac{e^2}{24\pi^2}\left[2\ln\left(\frac{\pi E_c}{E}\right)+2\gamma-1\right]-\frac{e^2}{2\pi^4}\zeta'(2)+\cdots, \qquad \text{(E.3.21)}$$

and the energy-momentum tensor $T^{\mu\nu}$ of Eq. (E.3.10),

$$T^{\mu\nu} = T_M^{\mu\nu}\left\{1+\frac{e^2}{24\pi^2}\left[2\ln\left(\frac{\pi E_c}{E}\right)+2\gamma-1\right]-\frac{e^2}{2\pi^4}\zeta'(2)\right\}-g^{\mu\nu}\frac{e^2E^2}{48\pi^2}+\cdots. \qquad \text{(E.3.22)}$$

Analogously, from Eq. (E.2.23) for $\mathbf{E}=0$ and a strong magnetic field, we obtain

$$\mathcal{A}_S = \frac{e^2}{24\pi^2}\left[2\ln\left(\frac{\pi E_c}{B}\right)+2\gamma-1\right]-\frac{e^2}{2\pi^4}\zeta'(2)+\cdots, \qquad \text{(E.3.23)}$$

and the energy-momentum tensor

$$T^{\mu\nu} = T_M^{\mu\nu}\left\{1+\frac{e^2}{24\pi^2}\left[2\ln\left(\frac{\pi E_c}{B}\right)+2\gamma-1\right]-\frac{e^2}{2\pi^4}\zeta'(2)\right\}+g^{\mu\nu}\frac{e^2B^2}{48\pi^2}+\cdots. \qquad \text{(E.3.24)}$$

In the following sections, using the energy-momentum tensors $T^{\mu\nu}$ of Eqs. (E.3.18), (E.3.22), and (E.3.24), we try to study the solutions of the Einstein-Euler-Heisenberg theory for nonrotating (spherically symmetric), electrically or magnetically charged black holes.

E.4. Electrically charged black holes

In this section, we study a nonrotating (spherically symmetric) electrically charged black hole. In this spherical symmetry case, the gauge potential is

$$A_\mu(x) = [A_0(r),0,0,0], \qquad \text{(E.4.1)}$$

corresponding to the electric field $E(r)=-A_0'(r)=-\partial A_0(r)/\partial r$ in the radial direction, and the metric field is assumed to be

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 d\Omega; \quad f(r)\equiv 1-2Gm(r)/r. \qquad \text{(E.4.2)}$$

The metric function $f(r)$ and the electric field $E(r)$ fulfill the Einstein equations (E.3.3) and electromagnetic field equations (E.3.5) and their asymptotically flat solutions at $r \gg 1$,

$$A_0(r) \to -\frac{Q}{4\pi r}, \quad E(r) \to \frac{Q}{4\pi r^2}, \quad \frac{Gm(r)}{r} \to \frac{GM}{r} \tag{E.4.3}$$

satisfy the Gauss law, where Q and M are the black hole electric charge and mass seen at infinity.

In order to find the solution near to the horizon of the black hole by taking into account the QED effects, we approximately adopt the Euler-Heisenberg effective Lagrangian for constant fields that leads to the energy-momentum tensor (E.3.18) or (E.3.22) for $\mathbf{B} = 0$. This approximation is based on the assumption that the macroscopic electric field $E(r)$ is approximated as a constant field E over the microscopic scale of the electron Compton lengths. When the electric field of charged black holes are overcritical, electron-positron pair productions take place and the electric field is screened down to its critical value E_c (see Refs. Damour and Ruffini (1975); Preparata et al. (1998, 2003); Ruffini et al. (2008)). In this chapter, we study the QED effects on electrically charged black holes with spherical symmetry, whose electric field is much smaller than the critical field E_c. In this weak electric field case using Eq. (E.3.18) we obtain the energy-momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_M \left(1 + \frac{2\alpha E^2}{45\pi E_c^2}\right) + g^{\mu\nu}\frac{\alpha E^4}{90\pi E_c^2} + \cdots . \tag{E.4.4}$$

As a result, the (0-0) component of Einstein equations is

$$\frac{2m'(r)}{r^2} = 4\pi \left[E^2(r) + \frac{\alpha}{15\pi}E^4(r)/E_c^2\right], \tag{E.4.5}$$

which relates to the energy conservation. Analogously, using Eqs. (E.3.5) and (E.3.6) and the metric of Eq. (E.4.2), we obtain the field equation up to the leading order,

$$\frac{2\alpha}{45\pi}E^3(r)/E_c^2 + E(r) = \frac{Q}{4\pi r^2}, \tag{E.4.6}$$

which is the zero component of $\mathcal{D}_\mu P^{\nu\mu} = j^\nu$ of Eq. (E.3.5) in the spherical symmetry case. This equation relates to the total charge conservation.

A similar case was studied in Ref. Yajima and Tamaki (2001), in which, however, the effective Lagrangian [the first term in Eq. (E.2.15)] was considered as a low-energy limit of the Born-Infeld theory; the coefficients of the S^2 and P^2 terms in Eq. (E.2.15) are treated as free parameters, so as to either numerically or analytically study the properties of spherically symmetric black hole solutions in the Einstein-Euler-Heisenberg system. In the following, in order to analytically study the QED effects on the black hole solution, we use the Euler-Heisenberg effective Lagrangian (E.2.15) and find the black hole solution by a series expansion in powers of α. Introducing $\overline{E}(r) \equiv E(r)/E_c$, up to the first order of α, the solution to Eq. (E.4.6) is approximately given by

$$\overline{E}(r) = E_Q \left(1 - \frac{2\alpha}{45\pi} E_Q^2 + \cdots\right), \tag{E.4.7}$$

where $E_Q \equiv E_Q(r) \equiv Q/(4\pi r^2 E_c)$. We find that the electric field $E(r)$ is smaller than $Q/4\pi r^2$, due to the charge screening effect of the vacuum polarization. Substituting this solution (E.4.7) into the Einstein equation (E.4.5), we obtain the integration

$$m(r) = M - \int_r^\infty 4\pi r^2 dr \frac{1}{2}\left[E^2(r) + \frac{\alpha}{15\pi} E^4(r)/E_c^2\right]. \tag{E.4.8}$$

This equation clearly shows that the energy-mass function $m(r)$ of Eq. (E.4.2) is the total gravitational mass M (attractive) "screened down" by the electromagnetic energy (repulsive). In the Maxwell theory $(\Delta\mathcal{L}_{\rm eff}^{\cos})_{\mathcal{P}} = 0$ and $E(r) = Q/(4\pi r^2)$, we obtain the Reissner-Nordström solution $m(r) = M - Q^2/8\pi r$. In the Euler-Heisenberg system, it is not proper to make the integration in Eq. (E.4.8), since the integrand comes from the Euler-Heisenberg effective Lagrangian, which is valid only for constant fields. In order to gain some physical insight into the energy-mass function (E.4.8), we integrate Eq. (E.4.8) to the leading order of α,

$$m(r) \approx M - \frac{Q^2}{8\pi r}\left[1 - \frac{\alpha}{225\pi}\frac{1}{(4\pi)^2}\frac{Q^2}{r^4}\frac{1}{E_c^2}\right] = M - \frac{Q^2}{8\pi r}\left[1 - \frac{\alpha}{225\pi}E_Q^2\right], \tag{E.4.9}$$

which shows the QED correction to the Reissner-Nordström solution. Due to the QED vacuum polarization effect, the black hole charge Q is screened

$$Q \to Q\left[1 - \frac{\alpha}{225\pi}E_Q^2\right]^{1/2}. \tag{E.4.10}$$

As a consequence, the electrostatic energy of Eq. (E.4.9) is smaller than $Q^2/(8\pi r)$ in the Reissner-Nordström solution.

Moreover, we study the QED correction to the black hole horizon. For this purpose, we define the horizon radius r_H at which the function $f(r)$ of Eq. (E.4.2) vanishes, i.e., $f(r_H) = 0$, leading to

$$\frac{Gm(r_H)}{r_H} = \frac{1}{2}. \tag{E.4.11}$$

Using the energy-mass function $m(r)$ of Eq. (E.4.9), we obtain

$$\frac{GM}{r_H} - \frac{GQ^2}{8\pi r_H^2}\left[1 - \frac{\alpha}{225\pi}E_{Qh}^2\right] = \frac{1}{2}, \tag{E.4.12}$$

where $E_{Qh} \equiv E_Q(r_H)$. Up to the leading order of α, we obtain

$$r_{H+} = GM + \sqrt{G^2M^2 - \frac{GQ^2}{4\pi}\left[1 - \frac{\alpha}{225\pi}E_{Q+}^2\right]}, \tag{E.4.13}$$

$$r_{H-} = GM - \sqrt{G^2M^2 - \frac{GQ^2}{4\pi}\left[1 - \frac{\alpha}{225\pi}E_{Q-}^2\right]}, \tag{E.4.14}$$

where $E_{Q+} \equiv E_Q(r_{H+})$ and $E_{Q-} \equiv E_Q(r_{H-})$. Equation (E.4.13) shows that the black hole horizon radius r_{H+} becomes larger than the Reissner-Nordström one r_+ given by Eq. (E.4.13) for setting $\alpha = 0$. The black hole horizon area $4\pi r_{H+}^2$ becomes larger than the Reissner-Nordström one $4\pi r_+^2$ given by Eq. (E.4.13) for setting $\alpha = 0$. This is again due to the black hole charge Q screened by the QED vacuum polarization (E.4.10).

In the Reissner-Nordström solution, the extreme black hole solution is given by $r_+ = r_-$ or $4\pi GM^2 = Q^2$. In our case, this is given by $r_{H+} = r_{H-} = r_H$ yielding

$$G^2M^2 - \frac{GQ^2}{4\pi}\left[1 - \frac{\alpha}{225\pi}E_{Qh}^2\right] = 0. \tag{E.4.15}$$

From Eqs. (E.4.13) and (E.4.14), we obtain

$$\begin{aligned} 4\pi r_H^2 &= 4\pi G^2M^2 = GQ^2\left[1 - \frac{\alpha}{225\pi}E_{Qh}^2\right] \\ &= GQ^2\left[1 - \frac{\alpha}{225\pi}\frac{1}{G^2Q^2E_c^2}\right], \end{aligned} \tag{E.4.16}$$

$$r_{H_\pm} \approx Q\left[1 - \frac{\alpha}{225\pi}E_{Qh}^2\right]^{1/2} = Q\left[1 - \frac{\alpha}{225\pi}\frac{1}{(E_cQ)^2}\right]^{1/2}. \tag{E.4.17}$$

In Eq. (E.4.17) we adopt $G/4\pi = 1$. Due to the QED correction, the condition of extremely electrically charged black holes with spherical symmetry changes from $M = Q/4\pi$ to

$$M = \frac{Q}{4\pi}\left[1 - \frac{\alpha}{225\pi}\frac{1}{(E_cQ)^2}\right]^{1/2}. \tag{E.4.18}$$

This implies that for a given M, the black holes are allowed to carry more charge Q than the Reissner-Nordström case. These results show that when the black hole mass M is fixed, the horizon area and radius of the extremely electrically charged black hole are the same as the extreme Reissner-Nordström one. However, when the black hole charge Q is fixed, the black hole horizon area and radius are smaller than those of the extreme Reissner-Nordström black hole. The reason is that the charge screening effect decreases the electrostatic energy; hence, this leads to a smaller mass M for the extreme black hole.

Now we turn to the maximal energy extractable from a black hole. As pointed out in Ref. Christodoulou and Ruffini (1971), the surface area S_a of the black hole horizon is related to the irreducible mass M_{ir} of the black hole

$$S_a = 16\pi G^2 M_{ir}^2 = 4\pi r_{H+}^2, \tag{E.4.19}$$

where r_{H+} is given by Eq. (E.4.13). The surface area of the black hole horizon cannot be decreased by classical processes Christodoulou and Ruffini (1971); Christodoulou (1970); Hawking (1971). Any transformation of the black hole which leaves fixed the irreducible mass is called reversible Christodoulou and Ruffini (1971); Christodoulou (1970). Any transformation of the black hole which increases its irreducible mass, for instance, the capture of a particle with nonzero radial momentum at the horizon, is called irreversible. In irreversible transformations there is always some kinetic energy that is irretrievably lost behind the horizon. Note that transformations which arbitrarily close to reversible ones are the most efficient transformations for extracting energy from a black hole Christodoulou and Ruffini (1971); Christodoulou (1970). Following the same argument presented in Ref. Christodoulou and Ruffini (1971), and including the leading-order QED correction (E.4.9), we obtain the Christodoulou-Ruffini mass formula

$$M = M_{ir} + \frac{Q^2}{16\pi G M_{ir}}\left[1 - \frac{\alpha}{225\pi}E_{Q+}^2\right], \tag{E.4.20}$$

where the electrostatic energy of the black hole is reduced for the reason that the black hole charge is screened down by the QED vacuum polarization effect (E.4.10).

The properties of the surface area S_a of the black hole horizon and irreducible mass M_{ir} can also been understood from the concepts of information theory Bekenstein (1973). The black hole entropy S_{en} is introduced as the measure of information about a black hole interior which is inaccessible to an exterior observer and is proportional to the surface area S_a of the black hole horizon Bekenstein (1973)

$$S_{en} = S_a/4 = \pi r_{H+}^2. \tag{E.4.21}$$

The physical content of the concept of the black hole entropy derives from the generalized second law of thermodynamics: when common entropy in the black hole exterior plus the black hole entropy never decreases Bekenstein (1973). In the Einstein-Euler-Heisenberg theory, the black hole irreducible mass of Eq. (E.4.19) and entropy of Eq. (E.4.21) with the QED correction are determined by the horizon radius r_{H+} of Eq. (E.4.13) for charged black holes and Eq. (E.4.16) for extreme black holes.

Now we consider the physical interpretation of the electromagnetic term in Eq. (E.4.20). This term represents the maximal energy extractable from a black hole, which can be obtained by evaluating the conserved Killing integral Ruffini et al. (2010); Ruffini and Vitagliano (2002)

$$\int_{\Sigma_t^+} \xi_+^\mu T_{\mu\nu} d\Sigma^\nu = 4\pi \int_{r_{H+}}^{\infty} r^2 T_0^0 dr, \tag{E.4.22}$$

where Σ_t^+ is the spacelike hypersurface in the space-time region that is outside the horizon $r > r_{H+}$ described by the equation $t =$ constant, with $d\Sigma^\nu$ as its surface element vector. ξ_+^μ is the static Killing vector field. This electromagnetic term in Eq. (E.4.20) is the total energy of the electromagnetic field and includes its own gravitational binding energy. Using the energy-momentum tensor of Eq. (E.4.4) and weak-field solution (E.4.7), we obtain the maximal energy extractable from an electrically charged black hole

$$\varepsilon_{ex} = \frac{Q^2}{8\pi r_{H+}} \left[1 - \frac{\alpha}{225\pi} E_{Q+}^2\right]. \tag{E.4.23}$$

This shows that the black hole maximal extractable energy decreases in comparison with the Reissner-Nordström case ($Q^2/8\pi r_+$). This can be explained by the following: (i) the charge screening effect decreases the electrostatic energy; (ii) the black hole horizon radius r_{H+} of Eq. (E.4.13) increases, leading to the decrease of the maximally extractable energy, because the most efficient transformations that extract energy from a black hole occur near the horizon. For the extremely electrically charged black hole, the maximally extractable energy is the same as that in the Reissner-Nordström case, when the black hole mass M is fixed; however, it becomes smaller than the Reissner-Nordström one when the black hole electric charge Q is fixed.

E.5. Magnetically charged black holes

Now we turn to study the Einstein-Euler-Heisenberg theory (E.3.18) and (E.3.24) in the presence of the magnetic field **B**. As shown by Eq. (E.2.6), the magnetic field **B** does not contribute to the pair-production rate so that the process of the electron-positron pair production does not occur for a strong magnetic field **B**. For this reason, we consider black holes with strong magnetic fields. The conventional black hole with electric and magnetic fields is the rotating charged black hole of the Kerr-Newman black hole Newman et al. (1965). However, the solution to a rotating charged black hole in the Einstein-Euler-Heisenberg theory is rather complicated, and we do not consider it in this work. For the sake of simplicity, we study the nonrotating magnetically charged black hole with spherical symmetry in order to investigate the QED corrections in the presence of the magnetic field **B** in the Einstein-Euler-Heisenberg theory.

For a nonrotating magnetically charged black hole with magnetic charge Q_m, the tensor $F_{\mu\nu}$ compatible with spherical symmetry can involve only a radial magnetic field $F_{23} = -F_{32}$. In the Einstein-Maxwell theory, the field equations (E.3.5) give (see, e.g., Refs. Hawking and Ross (1995); Gibbons and Rasheed (1995))

$$F_{23} = \frac{Q_m \sin\theta}{4\pi}, \tag{E.5.1}$$

and the gauge potential will be (see, e.g., Refs. Hawking and Ross (1995))

$$A_\mu(x) = [0, 0, 0, Q_m(1-\cos\theta)/4\pi]. \tag{E.5.2}$$

The metric is similar to the one of nonrotating electrically charged black holes,

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 d\Omega, \quad f(r) \equiv 1 - 2Gm(r)/r, \tag{E.5.3}$$

where $m(r)$ is the mass-energy function. In the Einstein-Maxwell theory, the metric function $f(r)$ of magnetically charged black holes with spherical symmetry is given by (see, e.g., Refs. Hawking and Ross (1995))

$$f(r) = 1 - \frac{2GM}{r} + \frac{GQ_m^2}{4\pi r^2}, \tag{E.5.4}$$

where M is the black hole mass seen at infinity.

E.5.1. Weak magnetic field case

Using Eq. (E.3.18), we obtain the energy-momentum tensor for the weak magnetic field B case,

$$T^{\mu\nu} = T_M^{\mu\nu}\left(1 - \frac{2\alpha B^2}{45\pi E_c^2}\right) + g^{\mu\nu}\frac{\alpha B^4}{90\pi E_c^2} + \cdots. \tag{E.5.5}$$

Similar to the analysis of electrically charged black holes with spherical symmetry, we obtain the (0-0) component of Einstein equations,

$$\frac{2m'(r)}{r^2} = 4\pi\left[B^2(r) - \frac{\alpha}{45\pi}B^4(r)/E_c^2\right]. \tag{E.5.6}$$

For the magnetically charged black hole with spherical symmetry, only a radial magnetic field is present. The field equations (E.3.5) give $B(r) = Q_m/(4\pi r^2)$ (see, e.g., Refs. Yajima and Tamaki (2001); Bronnikov (2001)). Substituting

$B(r)$ into the Einstein equation (E.5.6), we obtain the mass-energy function

$$m(r) = M - \int_r^\infty 4\pi r^2 dr \frac{1}{2}\left[B^2(r) - \frac{\alpha}{45\pi} B^4(r)/E_c^2\right]. \tag{E.5.7}$$

Neglecting the QED correction of the Euler-Heisenberg effective Lagrangian, Eq. (E.5.7) gives $m(r) = M - Q_m^2/8\pi r$, which is the solution of the magnetically charged Reissner-Nordström black hole in the Einstein-Maxwell theory. Making the integration in Eq. (E.5.7), one obtains Yajima and Tamaki (2001)

$$m(r) = M - \frac{Q_m^2}{8\pi r}\left[1 - \frac{\alpha}{225\pi}\frac{1}{(4\pi)^2}\frac{Q_m^2}{r^4}\frac{1}{E_c^2}\right] = M - \frac{Q_m^2}{8\pi r}\left[1 - \frac{\alpha}{225\pi} B_Q^2\right], \tag{E.5.8}$$

where $B_Q \equiv B_Q(r) \equiv Q_m/(4\pi r^2 E_c)$. As shown in Eq. (E.5.8), taking into account the QED vacuum polarization effect, the total magnetostatic energy is smaller than $Q_m^2/8\pi r$ in the magnetically charged Reissner-Nordström case. This can be understood as follows. In the magnetic field $\mathbf{B}$ of the black holes, the vacuum polarization effect results in a positive magnetic polarization $\mathbf{M}$. Then the magnetic $\mathbf{H}$ field defined $\mathbf{B} = \mathbf{H} + \mathbf{M}$ is smaller than the magnetic field $\mathbf{B}$. The magnetostatic energy density $\varepsilon_{EM} \propto \mathbf{B}\cdot\mathbf{H}$ decreases. This shows that in weak magnetic fields, the vacuum polarization effect exhibits the paramagnetic property.

Compared to the result of the electrically charged black hole in the first order of α, Eqs. (E.4.9) and (E.5.8) have the same expression. One can obtain Eq. (E.5.8) by simply replacing E_Q in Eq. (E.4.9) by B_Q, namely, replacing Q by Q_m because of the duality symmetry (see, e.g., Ref. Hawking and Ross (1995)). Similar to the analysis of electric charged black holes, we obtain the horizon radii r_{H_+} and r_{H_-} of the magnetically charged black hole, up to the leading order of α,

$$r_{H+} = GM + \sqrt{G^2M^2 - \frac{GQ_m^2}{4\pi}\left[1 - \frac{\alpha}{225\pi} B_{Q+}^2\right]}, \tag{E.5.9}$$

$$r_{H-} = GM - \sqrt{G^2M^2 - \frac{GQ_m^2}{4\pi}\left[1 - \frac{\alpha}{225\pi} B_{Q-}^2\right]}, \tag{E.5.10}$$

where $B_{Q+} \equiv B_Q(r_{H+})$ and $B_{Q-} \equiv B_Q(r_{H-})$. The result (E.5.9) shows that the black hole horizon radius r_{H+} increases in comparison with the magnetically charged Reissner-Nordström one r_+. This is again due to the paramagnetic effect of the vacuum polarization that decreases the magnetostatic energy of the black hole.

Now we turn to the extreme black hole ($r_{H+} = r_{H-} = r_H$). Similarly, we have

$$G^2M^2 - \frac{GQ_m^2}{4\pi}\left[1 - \frac{\alpha}{225\pi} B_{Qh}^2\right] = 0, \tag{E.5.11}$$

where $B_{Qh} \equiv B_Q(r_H)$, and we obtain the black hole horizon area and radius

$$4\pi r_H^2 = 4\pi G^2 M^2 = G Q_m^2 \left[1 - \frac{\alpha}{225\pi} B_{Qh}^2\right] = G Q_m^2 \left[1 - \frac{\alpha}{225\pi} \frac{1}{G^2 Q_m^2 E_c^2}\right], \tag{E.5.12}$$

$$r_H \approx Q_m \left[1 - \frac{\alpha}{225\pi} B_{Qh}^2\right]^{1/2} = Q_m \left[1 - \frac{\alpha}{225\pi} \frac{1}{(E_c Q_m)^2}\right]^{1/2}. \tag{E.5.13}$$

In the second line, we adopt $G/4\pi = 1$. The QED correction changes the condition of extremely magnetically charged black holes with spherical symmetry from $M = Q_m/4\pi$ to

$$M = \frac{Q_m}{4\pi} \left[1 - \frac{\alpha}{225\pi} \frac{1}{(E_c Q_m)^2}\right]^{1/2}. \tag{E.5.14}$$

The properties of the horizon area and radius of the extremely magnetically charged black hole are the same as their counterparts in the extremely electrically charged black hole, given by the duality transformation $Q \leftrightarrow Q_m$.

Following the same argument presented in Ref. Christodoulou and Ruffini (1971), we obtain the Christodoulou-Ruffini mass formula

$$M = M_{ir} + \frac{Q_m^2}{16\pi G M_{ir}} \left[1 - \frac{\alpha}{225\pi} B_{Q+}^2\right] \tag{E.5.15}$$

for magnetically charged black holes with spherical symmetry in the Einstein-Euler-Heisenberg theory. One is able to obtain the irreducible mass M_{ir} by substituting Eq. (E.5.9) into Eq. (E.4.19), and the black hole entropy S_{en} by substituting Eq. (E.5.9) into Eq. (E.4.21). The irreducible mass M_{ir} and the black hole entropy S_{en} in terms of black hole horizon radius r_{H+} Eq. (E.5.9) have the same paramagnetic property in the presence of the QED vacuum polarization effect, as already discussed.

As shown in Eq. (E.5.15), the maximal energy extractable from a magnetically charged black hole is

$$\varepsilon_{ex} = \frac{Q_m^2}{8\pi r_{H+}} \left[1 - \frac{\alpha}{225\pi} B_{Q+}^2\right], \tag{E.5.16}$$

where r_{H+} is given by Eq. (E.5.9). The result shows that the maximal energy extractable from a magnetically charged black hole is smaller than $\frac{Q_m^2}{8\pi r_+}$ of the magnetically charged Reissner-Nordström black hole. The reasons are the following: (i) the vacuum polarization effect decreases the magnetostatic energy; (ii) the black hole horizon radius r_{H+} of Eq. (E.5.9) increases, therefore the maximally extractable energy decreases. The maximal energy extractable from an extremely magnetically charged black hole is the same as that from an extremely magnetically charged Reissner-Nordström black hole when the black hole mass M is fixed, while it decreases when the black hole magnetic charge Q_m is fixed, as we have already discussed at the end of Sec. E.4 for the case of the extremely electrically charged black hole.

E.5.2. Strong magnetic field case

In this section, we study the magnetically charged black holes with a strong magnetic field $B(r)$. From Eq. (E.3.24), we obtain the energy-momentum tensor of the magnetically charged black hole with spherical symmetry in the strong magnetic field case. Analogous to the weak magnetic field case of magnetically charged black holes with spherical symmetry, we obtain the (0-0) component of Einstein equations

$$\frac{2m'(r)}{4\pi r^2} = 4\pi \left\{ B^2(r) + \frac{e^2B^2}{12\pi^2}\left[\ln\left(\frac{\pi E_c}{B}\right) + \gamma - \frac{6}{\pi^2}\zeta'(2)\right]\right\}, \tag{E.5.17}$$

and the field equations (E.3.5) give $B(r) = Q_m/(4\pi r^2)$. Substituting this magnetic field $B(r)$ into the Einstein equation (E.5.17), we obtain

$$m(r) \approx M - \int_r^\infty 4\pi r^2 dr \frac{1}{2}\left\{B^2 + \frac{e^2B^2}{12\pi^2}\left[\ln\left(\frac{\pi E_c}{B}\right) + \gamma - \frac{6}{\pi^2}\zeta'(2)\right]\right\} \tag{E.5.18}$$

$$\approx M - \frac{Q_m^2}{8\pi r}\left\{1 + \frac{\alpha}{3\pi}\left[\ln\left(\frac{\pi}{B_Q}\right) + \gamma + 2 - \frac{6}{\pi^2}\zeta'(2)\right]\right\}. \tag{E.5.19}$$

This result is valid for $B \gg E_c$, for which the value of $\ln\left(\pi/B_Q\right) + \gamma + 2 - \frac{6}{\pi^2}\zeta'(2)$ is negative. As a result, Eq. (E.5.19) shows that the total magnetostatic energy in the presence of the vacuum polarization is smaller than $Q_m^2/8\pi r$ of the magnetically charged Reissner-Nordström black hole. Similar to the weak-field case, this is again due to the paramagnetic effect of the vacuum polarization that decreases the magnetostatic energy of black holes. In the strong magnetic field case, the QED vacuum polarization effect is much larger than the result (E.5.8) in the weak-field case, where the QED correction term in Eq. (E.5.8) is small for the smallness of $\alpha/(225\pi)$ and B_Q^2. This result (E.5.19) shows a significant QED effect of the vacuum polarization on the energy of magnetically charged black holes in the strong magnetic field case.

Now we turn to the study of the black hole horizon radius and area in the strong magnetic field case. Using the condition $f(r_H) = 0$, we obtain the horizon radii r_{H+} and r_{H-} up to the leading order of α,

$$r_{H+} = GM + \sqrt{G^2M^2 - \frac{GQ_m^2}{4\pi}\left[1 + \frac{\alpha}{3\pi}\mathcal{K}_{NR+}\right]}, \tag{E.5.20}$$

$$r_{H-} = GM - \sqrt{G^2M^2 - \frac{GQ_m^2}{4\pi}\left[1 + \frac{\alpha}{3\pi}\mathcal{K}_{NR-}\right]}, \tag{E.5.21}$$

where

$$\mathcal{K}_{NR+} = \ln\left(\frac{\pi}{B_{Q+}}\right) + \gamma + 2 - \frac{6}{\pi^2}\zeta'(2), \tag{E.5.22}$$

$$\mathcal{K}_{NR-} = \ln\left(\frac{\pi}{B_{Q-}}\right) + \gamma + 2 - \frac{6}{\pi^2}\zeta'(2). \tag{E.5.23}$$

Equation (E.5.20) shows that the horizon radius $r_{_{H+}}$ increases in comparison with the magnetically charged Reissner-Nordström one r_+. This is again due to the paramagnetic effect of the vacuum polarization that decreases the magnetostatic contribution to the total energy of black holes.

For the case of the extreme black hole ($r_{_{H+}} = r_{_{H-}} = r_{_H}$), we have

$$G^2M^2 - \frac{GQ_m^2}{4\pi}\left[1 + \frac{\alpha}{3\pi}\mathcal{K}_{NR}\right] = 0, \tag{E.5.24}$$

where

$$\mathcal{K}_{NR} = \ln\left(\frac{\pi}{B_{Qh}}\right) + \gamma + 2 - \frac{6}{\pi^2}\zeta'(2). \tag{E.5.25}$$

As a result, we obtain

$$4\pi r_{_H}^2 = 4\pi G^2M^2 = GQ_m^2\left[1 + \frac{\alpha}{3\pi}\mathcal{K}_{NR}\right], \tag{E.5.26}$$

$$r_{_H} \approx Q_m\left[1 + \frac{\alpha}{3\pi}\mathcal{K}_{NR}\right]^{1/2}. \tag{E.5.27}$$

Similar to the weak magnetic field case, the QED correction changes the condition of extremely magnetically charged black holes with spherical symmetry from $M = Q_m/4\pi$ to

$$M = \frac{Q_m}{4\pi}\left[1 + \frac{\alpha}{3\pi}\mathcal{K}_{NR}\right]^{1/2}. \tag{E.5.28}$$

These results show that the horizon area and radius of the extreme black hole are the same as their counterparts of the extremely magnetically charged Reissner-Nordström black hole, when the black hole mass M is fixed. Whereas, the black hole magnetic charge Q_m is fixed, Eqs. (E.5.26) and (E.5.27) show that the black hole horizon area and radius become smaller than their counterparts of extremely magnetically charged Reissner-Nordström black holes. We have discussed this behavior in Eqs. (E.4.15)-(E.4.18) for the case of extremely electrically charged black holes.

Analogously, we obtain the Christodoulou-Ruffini mass formula in the strong-field case of magnetically charged black holes,

$$M = M_{ir} + \frac{Q^2}{16\pi G M_{ir}}\left[1 + \frac{\alpha}{3\pi}\mathcal{K}_{NR+}\right]. \tag{E.5.29}$$

It is straightforward to obtain irreducible mass M_{ir} by substituting Eq. (E.5.20) into Eq. (E.4.19), and the black hole entropy S_{en} by substituting Eq. (E.5.20) into Eq. (E.4.21). Analogous to the case of the electrically charged black hole, the black hole irreducible mass M_{ir} and entropy S_{en} in the strong magnetic field case depend on the black hole horizon radius $r_{_{H+}}$ of Eqs. (E.5.20) and (E.5.26). Equation (E.5.29) indicates that the maximal energy extractable from a magnetically charged black hole is

$$\varepsilon_{ex} = \frac{Q_m^2}{8\pi r_{_{H+}}} \left[1 + \frac{\alpha}{3\pi}\mathcal{K}_{NR+}\right]. \tag{E.5.30}$$

The properties of the maximally extractable energy in the strong magnetic field case are similar to those of the magnetically charged black hole in the weak magnetic field case. However, the QED correction of the vacuum polarization effect to the energy of the magnetically charged black hole in the strong magnetic field case is much more significant in comparison with that in the weak magnetic field case.

E.6. Black holes with electric and magnetic charges

If the spherically symmetric (nonrotating) black hole is both electrically and magnetically charged, electric and magnetic fields do not vanish. As shown in Eq. (E.2.11), both invariants S and P contribute to the Euler-Heisenberg effective Lagrangian. The metric takes the same form as the metric of Eq. (E.4.2) for electrically charged black holes with spherical symmetry. In this case, the tensor $F_{\mu\nu}$ compatible with spherical symmetry can involve only a radial electric field $F_{01} = -F_{10}$ and a radial magnetic field $F_{23} = -F_{32}$, and the gauge potential is (see, e.g., Ref. Hawking and Ross (1995))

$$A_\mu(x) = [A(r), 0, 0, Q_m(1-\cos\theta)/4\pi]. \tag{E.6.1}$$

In the Einstein-Maxwell theory, $A(r) = -Q/(4\pi r)$, and the metric function $f(r)$ of Eq. (E.4.2) is given by (see, e.g., Ref. Hawking and Ross (1995))

$$f(r) = 1 - \frac{2GM}{r} + \frac{GQ^2}{4\pi r^2} + \frac{GQ_m^2}{4\pi r^2}. \tag{E.6.2}$$

In the Einstein-Euler-Heisenberg theory, we study the spherically symmetric black hole with electric and magnetic charges in the weak-field case. Using Eq. (E.3.18), we derive the energy-momentum tensor with a radial electric field E and a radial magnetic field B,

$$T^{\mu\nu} = T^{\mu\nu}_{M}\left[1+\frac{2\alpha}{45\pi E_c^2}(E^2-B^2)\right]+g^{\mu\nu}\frac{\alpha}{90\pi E_c^2}\left[(E^2-B^2)^2+7(E\cdot B)^2\right]+\cdots . \tag{E.6.3}$$

Analogous to the analysis of electrically/magnetically charged black holes with spherical symmetry, we obtain the (0-0) component of Einstein equations,

$$\frac{2m'(r)}{r^2} = 4\pi\left[E^2(r)+B^2(r)+\frac{\alpha}{15\pi}E^4(r)/E_c^2-\frac{\alpha}{45\pi}B^4(r)/E_c^2+\frac{\alpha}{9\pi E_c^2}E^2(r)B^2(r)\right]. \tag{E.6.4}$$

In addition, we obtain the field equations from Eq. (E.3.5) (see also Ref. Yajima and Tamaki (2001)),

$$E(r)+\frac{2\alpha}{45\pi}E^3(r)/E_c^2+\frac{\alpha B^2}{9\pi E_c^2}E(r) = \frac{Q}{4\pi r^2}, \tag{E.6.5}$$

$$B(r) = \frac{Q_m}{4\pi r^2}. \tag{E.6.6}$$

Note that the mixing terms of the electric and magnetic fields in Eqs. (E.6.4) and (E.6.5) come from the contribution of the invariant P. Introducing $\overline{E}(r) \equiv E(r)/E_c$, we have

$$\overline{E}(r) = E_Q - \frac{2\alpha}{45\pi}E_Q^3 - \frac{\alpha}{9\pi}B_Q^2E_Q + \cdots , \tag{E.6.7}$$

up to the first order of α. We substitute the solutions of (E.6.6) and (E.6.7) into the Einstein equation (E.6.4) and obtain the mass-energy function

$$m(r) = M - \int_r^\infty 4\pi r^2 dr\frac{1}{2}E_c^2\left[E_Q^2+B_Q^2-\frac{\alpha}{45\pi}E_Q^4-\frac{\alpha}{45\pi}B_Q^4-\frac{\alpha}{9\pi}B_Q^2E_Q^2\right]. \tag{E.6.8}$$

Disregarding the QED correction of the Euler-Heisenberg effective Lagrangian, Eq. (E.6.8) gives the solution $m(r) = M - Q^2/8\pi r - Q_m^2/8\pi r$ for the Reissner-Nordström black hole with electric and magnetic charges. Performing the integration in Eq. (E.6.8), we approximately obtain

$$m(r) = M - \frac{Q^2}{8\pi r}\left[1-\frac{\alpha}{225\pi}E_Q^2\right] - \frac{Q_m^2}{8\pi r}\left[1-\frac{\alpha}{225\pi}B_Q^2\right] + \frac{\alpha}{45\pi}\frac{Q_m^2}{8\pi r}E_Q^2. \tag{E.6.9}$$

In the limit $Q \gg Q_m$, Eq. (E.6.9) becomes Eq. (E.4.9) of the electrically charged black hole. On the contrary, in the limit $Q_m \gg Q$, Eq. (E.6.9) becomes Eq. (E.5.8) of the magnetically charged black hole. In order to study the effect of the P term in the Euler-Heisenberg effective Lagrangian, we consider the case with large P and small S, i.e., $Q_m \approx Q$. In this situation, Eq. (E.6.9) becomes

$$m(r) = M - \frac{Q^2}{8\pi r}\left[2 - \frac{7\alpha}{225\pi}E_Q^2\right], \tag{E.6.10}$$

for $Q_m = Q$, i.e., $S = 0$ and large P. Comparing to the cases of electrically/magnetically charged black holes, the QED correction to the black hole energy becomes larger, which results from the combination effects of the vacuum polarization on electric and magnetic charges of black holes in the Einstein-Euler-Heisenberg theory.

In the same way that has been discussed in previous sections, up to the leading order of α, we obtain the horizon radii r_{H+} and r_{H-} from Eq. (E.6.10),

$$r_{H+} = GM + \sqrt{G^2M^2 - \frac{GQ^2}{4\pi}\left[2 - \frac{7\alpha}{225\pi}E_{Q+}^2\right]}, \tag{E.6.11}$$

$$r_{H-} = GM - \sqrt{G^2M^2 - \frac{GQ^2}{4\pi}\left[2 - \frac{7\alpha}{225\pi}E_{Q-}^2\right]}, \tag{E.6.12}$$

and the Christodoulou-Ruffini mass formula

$$M = M_{ir} + \frac{Q^2}{16\pi G M_{ir}}\left[2 - \frac{7\alpha}{225\pi}E_{Q+}^2\right], \tag{E.6.13}$$

as well as the maximal energy extractable from a black hole

$$\varepsilon_{ex} = \frac{Q^2}{8\pi r_{H+}}\left[2 - \frac{7\alpha}{225\pi}E_{Q+}^2\right]. \tag{E.6.14}$$

Analogously, we obtain the irreducible mass M_{ir} by substituting Eq. (E.6.11) into Eq. (E.4.19), and the black hole entropy S_{en} by substituting Eq. (E.6.11) into Eq. (E.4.21). The irreducible mass M_{ir}, the black hole entropy S_{en}, and the maximal energy extractable from a black hole receive the same QED correction, but a factor of 7/2 larger, as compared with their counterparts in the case of either electrically or magnetically charged black holes in the weak-field case.

E.7. Summary

In this chapter, in addition to the Maxwell Lagrangian, we consider the contribution from the QED Euler-Heisenberg effective Lagrangian to formulate the Einstein-Euler-Heisenberg theory. On the basis of this theory, we study the horizon radius, area, total energy, entropy, and irreducible mass as well as the maximally extractable energy of spherically symmetric (nonrotating) black holes with electric and magnetic charges. Our calculations are made up to the leading order of the QED corrections in the limits of strong and weak fields. Our results show that the QED correction of the vacuum polarization results in the increase of the black hole horizon area, entropy and irreducible mass, as well as the decrease of the black hole total energy and maximally extractable energy. The reason is that the QED vacuum polarization gives rise to the screening effect on the black hole electric charge and the paramagnetic effect on the black hole magnetic charge. The condition of the extremely charged black hole $M = Q/4\pi$ or $M = Q_m/4\pi$ is modified [see Eqs. (E.4.18), (E.5.14), and (E.5.28)], which results from the screening and paramagnetic effects.

To end this chapter, we would like to mention that in the Einstein-Euler-Heisenberg theory, it is worthwhile to study Kerr-Newman black holes, whose electric field **E** and magnetic field **B** are determined by the black hole mass M, charge Q, and angular momentum a Newman et al. (1965). In addition, it will be interesting to study the QED corrections in black hole physics by taking into account the one-loop photon-graviton amplitudes of the effective Lagrangian (E.3.11) Drummond and Hathrell (1980) and its generalizations Gilkey (1975); Bastianelli et al. (2000); Barvinsky and Vilkovisky (1985); Gusev (2009); Bastianelli et al. (2009). We leave these studies for future work.

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