Electron-positron pairs in physics and astrophysics

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3. Brief description

3.1. Abstract

Due to the interaction of physics and astrophysics we are witnessing in these years a splendid synthesis of theoretical, experimental and observational results originating from three fundamental physical processes. They were originally proposed by Dirac, by Breit and Wheeler and by Sauter, Heisenberg, Euler and Schwinger. For almost seventy years they have all three been followed by a continued effort of experimental verification on Earth-based experiments. The Dirac process, $e^+e^- \to 2\gamma$, has been by far the most successful. It has obtained extremely accurate experimental verification and has led as well to an enormous number of new physics in possibly one of the most fruitful experimental avenues by introduction of storage rings in Frascati and followed by the largest accelerators worldwide: DESY, SLAC etc. The Breit–Wheeler process, $2\gamma \to e^+e^-$, although conceptually simple, being the inverse process of the Dirac one, has been by far one of the most difficult to be verified experimentally. Only recently, through the technology based on free electron X-ray laser and its numerous applications in Earth-based experiments, some first indications of its possible verification have been reached. The vacuum polarization process in strong electromagnetic field, pioneered by Sauter, Heisenberg, Euler and Schwinger, introduced the concept of critical electric field $E_c = m_e^2c^3/(e\hbar)$. It has been searched without success for more than forty years by heavy-ion collisions in many of the leading particle accelerators worldwide.

The novel situation today is that these same processes can be studied on a much more grandiose scale during the gravitational collapse leading to the formation of a black hole being observed in Gamma Ray Bursts (GRBs). This report is dedicated to the scientific race. The theoretical and experimental work developed in Earth-based laboratories is confronted with the theoretical interpretation of space-based observations of phenomena originating on cosmological scales. What has become clear in the last ten years is that all the three above mentioned processes, duly extended in the general relativistic framework, are necessary for the understanding of the physics of the gravitational collapse to a black hole. Vice versa, the natural arena where these processes can be observed in mutual interaction and on an unprecedented scale, is indeed the realm of relativistic astrophysics.

We systematically analyze the conceptual developments which have fol-

lowed the basic work of Dirac and Breit–Wheeler. We also recall how the seminal work of Born and Infeld inspired the work by Sauter, Heisenberg and Euler on effective Lagrangian leading to the estimate of the rate for the process of electron–positron production in a constant electric field. In addition of reviewing the intuitive semi-classical treatment of quantum mechanical tunneling for describing the process of electron–positron production, we recall the calculations in *Quantum Electro-Dynamics* of the Schwinger rate and effective Lagrangian for constant electromagnetic fields. We also review the electron–positron production in both time-alternating electromagnetic fields, studied by Brezin, Itzykson, Popov, Nikishov and Narozhny, and the corresponding processes relevant for pair production at the focus of coherent laser beams as well as electron beam–laser collision. We finally report some current developments based on the general JWKB approach which allows to compute the Schwinger rate in spatially varying and time varying electromagnetic fields.

We also recall the pioneering work of Landau and Lifshitz, and Racah on the collision of charged particles as well as experimental success of AdA and ADONE in the production of electron–positron pairs.

We then turn to the possible experimental verification of these phenomena. We review: (A) the experimental verification of the $e^+e^- \to 2\gamma$ process studied by Dirac. We also briefly recall the very successful experiments of e^+e^- annihilation to hadronic channels, in addition to the Dirac electromagnetic channel; (B) ongoing Earth based experiments to detect electron–positron production in strong fields by focusing coherent laser beams and by electron beam–laser collisions; and (C) the multiyear attempts to detect electron–positron production in Coulomb fields for a large atomic number $Z > 137$ in heavy ion collisions. These attempts follow the classical theoretical work of Popov and Zeldovich, and Greiner and their schools.

We then turn to astrophysics. We first review the basic work on the energetics and electrodynamical properties of an electromagnetic black hole and the application of the Schwinger formula around Kerr–Newman black holes as pioneered by Damour and Ruffini. We only focus on black hole masses larger than the critical mass of neutron stars, for convenience assumed to coincide with the Rhoades and Ruffini upper limit of 3.2 $M_\odot$. In this case the electron Compton wavelength is much smaller than the spacetime curvature and all previous results invariantly expressed can be applied following well established rules of the equivalence principle. We derive the corresponding rate of electron–positron pair production and introduce the concept of dyadosphere. We review recent progress in describing the evolution of optically thick electron–positron plasma in presence of supercritical electric field, which is relevant both in astrophysics as well as ongoing laser beam experiments. In particular we review recent progress based on the Vlasov-Boltzmann-Maxwell equations to study the feedback of the created electron–positron pairs on the original constant electric field. We evidence the existence of plasma oscillations and its interaction with photons leading to energy and number equipartition of photons, electrons and positrons. We finally review the recent progress obtained by using the Boltzmann equations to study the evolution of an electron–positron-photon plasma towards thermal equilibrium and determination of its characteristic timescales. The crucial difference introduced by the correct evaluation of the role of two and three body collisions, direct and inverse, is especially evidenced. We then present some general conclusions.

The results reviewed in this report are going to be submitted to decisive tests in the forthcoming years both in physics and astrophysics. To mention only a few of the fundamental steps in testing in physics we recall the starting of experimental facilities at the National Ignition Facility at the Lawrence Livermore National Laboratory as well as corresponding French Laser the Mega Joule project. In astrophysics these results will be tested in galactic and extragalactic black holes observed in binary X-ray sources, active galactic nuclei, microquasars and in the process of gravitational collapse to a neutron star and also of two neutron stars to a black hole giving origin to GRBs. The astrophysical description of the stellar precursors and the initial physical conditions leading to a gravitational collapse process will be the subject of a forthcoming report. As of today no theoretical description has yet been found to explain either the emission of the remnant for supernova or the formation of a charged black hole for GRBs. Important current progress toward the understanding of such phenomena as well as of the electrodynamical structure of neutron stars, the supernova explosion and the theories of GRBs will be discussed in the above mentioned forthcoming report. What is important to recall at this stage is only that both the supernovae and GRBs processes are among the most energetic and transient phenomena ever observed in the Universe: a supernova can reach energy of $\sim 10^{54}$ ergs on a time scale of a few months and GRBs can have emission of up to $\sim 10^{54}$ ergs in a time scale as short as of a few seconds. The central role of neutron stars in the description of supernovae, as well as of black holes and the electron–positron plasma, in the description of GRBs, pioneered by one of us (RR) in 1975, are widely recognized. Only the theoretical basis to address these topics are discussed in the present report.

3.2. The three fundamental contributions to the electron-positron pair creation and annihilation and the concept of critical electric field

The annihilation of electron–positron pair into two photons, and its inverse process – the production of electron–positron pair by the collision of two photons were first studied in the framework of quantum mechanics by P.A.M. Dirac and by G. Breit and J.A. Wheeler in the 1930s (Dirac (1930); Breit and Wheeler (1934)).

A third fundamental process was pioneered by the work of Fritz Sauter and Oscar Klein, pointing to the possibility of creating an electron–positron pair from the vacuum in a constant electromagnetic field. This became known as the 'Klein paradox' and such a process named as *vacuum polarization*. It would occur for an electric field stronger than the critical value

$$E_c \equiv \frac{m_e^2 c^3}{e\hbar} \simeq 1.3 \cdot 10^{16}\,\mathrm{V/cm}. \tag{3.2.1}$$

where m_e, e, c and $\hbar$ are respectively the electron mass and charge, the speed of light and the Planck's constant.

The experimental difficulties to verify the existence of such three processes became immediately clear. While the process studied by Dirac was almost immediately observed Klemperer (1934) and the electron–positron collisions became possibly the best tested and prolific phenomenon ever observed in physics. The Breit–Wheeler process, on the contrary, is still today waiting a direct observational verification. Similarly the vacuum polarization process defied dedicated attempts for almost fifty years in experiments in nuclear physics laboratories and accelerators all over the world, see Section 7 in the following article.

From the theoretical point of view the conceptual changes implied by these processes became immediately clear. They were by vastness and depth only comparable to the modifications of the linear gravitational theory of Newton introduced by the nonlinear general relativistic equations of Einstein. In the work of Euler, Oppenheimer and Debye, Born and his school it became clear that the existence of the Breit–Wheeler process was conceptually modifying the linearity of the Maxwell theory. In fact the creation of the electron–positron pair out of the two photons modifies the concept of superposition of the linear electromagnetic Maxwell equations and impose the necessity to transit to a nonlinear theory of electrodynamics. In a certain sense the Breit–Wheeler process was having for electrodynamics the same fundamental role of Gedankenexperiment that the equivalence principle had for gravitation. Two different attempts to study these nonlinearities in the electrodynamics were made: one by Born and Infeld Born (1933, 1934); Born and Infeld (1934) and one by Euler and Heisenberg Heisenberg and Euler (1936). These works prepared the even greater revolution of Quantum Electro-Dynamics by Tomonaga Tomonaga (1946), Feynman Feynman (1948, 1949b,a), Schwinger Schwinger (1948, 1949a,b) and Dyson Dyson (1949a,b).

In Section 3 in the following article we review the fundamental contributions to the electron–positron pair creation and annihilation and to the concept of the critical electric field. In Section 3.1 of the following article we review the Dirac derivation Dirac (1930) of the electron–positron annihilation process obtained within the perturbation theory in the framework of relativistic quantum mechanics and his derivation of the classical formula for the cross-section $\sigma^{\rm lab}_{e^+e^-}$ in the rest frame of the electron

$$\sigma^{\rm lab}_{e^+e^-} = \pi\left(\frac{\alpha\hbar}{m_e\,c}\right)^2 (\hat\gamma-1)^{-1}\left\{\frac{\hat\gamma^2+4\hat\gamma+1}{\hat\gamma^2-1}\ln[\hat\gamma+(\hat\gamma^2-1)^{1/2}]-\frac{\hat\gamma+3}{(\hat\gamma^2-1)^{1/2}}\right\},$$

where $\hat\gamma \equiv \mathcal{E}_+/m_e c^2 \geq 1$ is the energy of the positron and $\alpha = e^2/(\hbar c)$ is as usual the fine structure constant, and we recall the corresponding formula for the center of mass reference frame. In article Section 3.2 we recall the main steps in the classical Breit–Wheeler work Breit and Wheeler (1934) on the production of a real electron–positron pair in the collision of two photons, following the same method used by Dirac and leading to the evaluation of the total cross-section $\sigma_{\gamma\gamma}$ in the center of mass of the system

$$\sigma_{\gamma\gamma} = \frac{\pi}{2}\left(\frac{\alpha\hbar}{m_e\,c}\right)^2(1-\hat\beta^2)\Big[2\hat\beta(\hat\beta^2-2)+(3-\hat\beta^4)\ln\big(\frac{1+\hat\beta}{1-\hat\beta}\big)\Big],\quad \text{with}\quad \hat\beta=\frac{c|\mathbf{p}|}{\mathcal{E}},$$

where $\hat\beta$ is the reduced velocity of the electron or the positron. In Section 3.3 of the article we recall the basic higher order processes, compared to the Dirac and Breit–Wheeler ones, leading to pair creation. In Section 3.4 in the following review we recall the famous Klein paradox Klein (1929); Sauter (1931b) and the possible tunneling between the positive and negative energy states leading to the concept of level crossing and pair creation by analogy to the Gamow tunneling Gamow (1931) in the nuclear potential barrier. We then turn to the celebrated Sauter work Sauter (1931a) showing the possibility of creating a pair in a uniform electric field E. We recover in Section 3.5.1 of the review a JWKB approximation in order to reproduce and improve on the Sauter result by obtaining the classical Sauter exponential term as well as the prefactor

$$\frac{\Gamma_{\rm JWKB}}{V} \simeq D_s\frac{\alpha E^2}{2\pi^2\hbar}e^{-\pi E_c/E},$$

where $D_s = 2$ for a spin-1/2 particle and $D_s = 1$ for spin-0, V is the volume. Finally, in review Section 3.5.2 the case of a simultaneous presence of an electric and a magnetic field B is presented leading to the estimate of pair production rate

$$\frac{\Gamma_{\rm JWKB}}{V} \simeq \frac{\alpha\beta\varepsilon}{\pi\hbar} \coth\left(\frac{\pi\beta}{\varepsilon}\right) \exp\left(-\frac{\pi E_c}{\varepsilon}\right), \quad \text{spin} - 1/2 \ \text{particle}$$

and

$$\frac{\Gamma_{\rm JWKB}}{V} \simeq \frac{\alpha\beta\varepsilon}{2\pi\hbar} \sinh^{-1}\left(\frac{\pi\beta}{\varepsilon}\right) \exp\left(-\frac{\pi E_c}{\varepsilon}\right), \quad \text{spin} - 0 \ \text{particle},$$

where

$$\begin{aligned} \varepsilon &\equiv \sqrt{(S^2 + P^2)^{1/2} + S}, \\ \beta &\equiv \sqrt{(S^2 + P^2)^{1/2} - S}, \end{aligned}$$

where the scalar S and the pseudoscalar P are

$$S \equiv \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2); \quad P \equiv \frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} = \mathbf{E} \cdot \mathbf{B},$$

where $\tilde{F}^{\mu\nu} \equiv \epsilon^{\mu\nu\lambda\kappa} F_{\lambda\kappa}$ is the dual field tensor.

3.3. Nonlinear electrodynamics and rate of pair creation

In article Section 4 we first recall the seminal work of Hans Euler Euler (1936) pointing out for the first time the necessity of nonlinear character of electromagnetism introducing the classical Euler Lagrangian

$$\mathcal{L} = \frac{\mathbf{E}^2 - \mathbf{B}^2}{8\pi} + \frac{1}{\alpha} \frac{1}{E_0^2} \left[a_E \left(\mathbf{E}^2 - \mathbf{B}^2\right)^2 + b_E \left(\mathbf{E} \cdot \mathbf{B}\right)^2 \right],$$

where

$$a_E = -1/(360\pi^2), \quad b_E = -7/(360\pi^2),$$

a first order perturbation to the Maxwell Lagrangian. In review article Section 4.2 we review the alternative theoretical approach of nonlinear electrodynamics by Max Born Born (1934) and his collaborators, to the more ambitious attempt to obtain the correct nonlinear Lagrangian of Electro-Dynamics. The motivation of Born was to attempt a theory free of divergences in the observable properties of an elementary particle, what has become known as 'unitarian' standpoint versus the 'dualistic' standpoint in description of elementary particles and fields. We recall how the Born Lagrangian was formulated

$$\mathcal{L} = \sqrt{1 + 2S - P^2} - 1,$$

and one of the first solutions derived by Born and Infeld Born and Infeld (1934). We also recall one of the interesting aspects of the courageous approach of Born had been to formulate this Lagrangian within a unified theory of gravitation and electromagnetism following Einstein program. Indeed, we also recall the very interesting solution within the Born theory obtained by Hoffmann Hoffmann (1935); Hoffmann and Infeld (1937). Still in the work of Born Born (1934) the seminal idea of describing the nonlinear vacuum properties of this novel electrodynamics by an effective dielectric constant and magnetic permeability functions of the field arisen. We then review in Section 4.3.1 of the article the work of Heisenberg and Euler Heisenberg and Euler (1936) adopting the general approach of Born and generalizing to the presence of a real and imaginary part of the electric permittivity and magnetic permeability. They obtain an integral expression of the effective Lagrangian given by

$$\begin{aligned}\Delta\mathcal{L}_{\rm eff} &= \frac{e^2}{16\pi^2\hbar c}\int_0^\infty e^{-s}\frac{ds}{s^3}\Big[is^2\,\bar{E}\bar{B}\frac{\cos(s[\bar{E}^2-\bar{B}^2+2i(\bar{E}\bar{B})]^{1/2})+\text{c.c.}}{\cos(s[\bar{E}^2-\bar{B}^2+2i(\bar{E}\bar{B})]^{1/2})-\text{c.c.}}\\ &\quad+\left(\frac{m_e^2c^3}{e\hbar}\right)^2+\frac{s^2}{3}(|\bar{B}|^2-|\bar{E}|^2)\Big],\end{aligned}$$

where $\bar{E}, \bar{B}$ are the dimensionless reduced fields in the unit of the critical field E_c,

$$\bar{E} = \frac{|\mathbf{E}|}{E_c}, \quad \bar{B} = \frac{|\mathbf{B}|}{E_c}.$$

obtaining the real part and the crucial imaginary term which relates to the pair production in a given electric field. It is shown how these results give as a special case the previous result obtained by Euler (Eq. (4.1.3) in the review). In Section 4.3.2 of the following article the work by Weisskopf Weisskopf (1936) working on a spin-0 field fulfilling the Klein–Gordon equation, in contrast to the spin 1/2 field studied by Heisenberg and Euler, confirms the Euler-Heisenberg result. Weisskopf obtains explicit expression of pair creation in an arbitrary strong magnetic field and in an electric field described by $\bar{E}$ and $\bar{B}$ expansion.

For the first time Heisenberg and Euler provided a description of the vacuum properties by the characteristic scale of strong field E_c and the effective Lagrangian of nonlinear electromagnetic fields. In 1951, Schwinger Schwinger (1951, 1954a,b) made an elegant quantum field theoretic reformulation of this discovery in the QED framework. This played an important role in understanding the properties of the QED theory in strong electromagnetic fields.

The QED theory in strong coupling regime, i.e., in the regime of strong electromagnetic fields, is still a vast arena awaiting for experimental verification as well as of further theoretical understanding.

3.4. Pair production and annihilation in QED

In the review article in Section 5 after recalling some general properties of QED in Section 5.1 and some basic processes in Section 5.2 we proceed to the consideration of the Dirac and the Breit–Wheeler processes in QED in Secton 5.3. Then we discuss some higher order processes, namely double pair production in Section 5.4, electron-nucleus bremsstrahlung and pair production by a photon in the field of a nucleus in Section 5.5, and finally pair production by two ions in Section 5.6. In Section 5.7 the classical result for the vacuum to vacuum decay via pair creation in uniform electric field by Schwinger is recalled

$$\frac{\Gamma}{V} = \frac{\alpha E^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi E_c}{E}\right).$$

This formula generalizes and encompasses the previous results reviewed in our report: the JWKB results, discussed in Section 3.5, and the Sauter exponential factor (Eq. (3.5.11) in the review), and the Heisenberg-Euler imaginary part of the effective Lagrangian. We then recall the generalization of this formula to the case of a constant electromagnetic fields. Such results were further generalized to spatially nonuniform and time-dependent electromagnetic fields by Nikishov (1970), Vanyashin and Terent'ev (1965), Popov (1971a, 1972b, 2001), Narozhnyi and Nikishov (1970) and Batalin and Fradkin (1970). We then conclude this argument by giving the real and imaginary parts for the effective Lagrangian for arbitrary constant electromagnetic field recently published by Ruffini and Xue (2006). This result generalizes the previous result obtained by Weisskopf in strong fields. In weak field it gives the Euler-Heisenberg effective Lagrangian. As we will see in the Section 7.2 of the review much attention has been given experimentally to the creation of pairs in the rapidly changing electric fields. A fundamental contribution in this field studying pair production rates in an oscillating electric field was given by Brezin and Itzykson (1970) and we recover in review Section 5.8 their main results which apply both to the case of bosons and fermions. We recall how similar results were independently obtained two years later by Popov Popov (1972a). In Section 5.10 of the article we recall an alternative physical process considering the quantum theory of the interaction of free electron with the field of a strong electromagnetic waves: an ultrarelativistic electron absorbs multiple photons and emits only a single photon in the reaction Bula et al. (1996):

$$e + n\omega \rightarrow e' + \gamma.$$

This process appears to be of the great relevance as we will see in the next Section for the nonlinear effects originating from laser beam experiments. Particularly important appears to be the possibility outlined by Burke et al. (1997) that the high-energy photon γ created in the first process propagates through the laser field, it interacts with laser photons $n\omega$ to produce an electron–positron pair

$$\gamma + n\omega \rightarrow e^+ + e^-.$$

We also refer to the papers by Nikishov and Ritus (1964a,b, 1965, 1967, 1979); Narozhnyǐ et al. (1965) studying the dependence of this process on the status of the polarization of the photons.

We point out the great relevance of departing from the case of the uniform electromagnetic field originally considered by Sauter, Heisenberg and Euler, and Schwinger. We also recall some of the classical works of Brezin and Itzykson and Popov on time varying fields. The space variation of the field was also considered in the classical papers of Nikishov and Narozhny as well as in the work of Wang and Wong. Finally, we recall the work of Khriplovich Khriplovich (2000) studying the vacuum polarization around a Reissner–Nordström black hole. A more recent approach using the worldline formalism, sometimes called the string-inspired formalism, was advanced by Dunne and Schubert Schubert (2001); Dunne and Schubert (2005).

3.5. Semi-classical description of pair production in a general electric field

In review Section 6, after recalling studies of pair production in inhomogeneous electromagnetic fields in the literature by Dunne and Schubert (2005); Dunne et al. (2006); Dunne and Wang (2006); Kim and Page (2002, 2006, 2007), we present a brief review of our recent work Kleinert et al. (2008) where the general formulas for pair production rate as functions of either crossing energy level or classical turning point, and total production rate are obtained in external electromagnetic fields which vary either in one space direction $E(z)$ or in time $E(t)$. In Sections 6.1 and 6.2, these formulas are explicitly derived in the JWKB approximation and generalized to the case of three-dimensional electromagnetic configurations. We apply these formulas to several cases of such inhomogeneous electric field configurations, which are classified into two categories. In the first category, we study two cases: a semi-confined field $E(z) \neq 0$ for $z \lesssim \ell$ and the Sauter field

$$E(z) = E_0/\cosh^2(z/\ell), \quad V(z) = -\sigma_s\, m_e c^2 \tanh(z/\ell),$$

where ℓ is width in the z-direction, and

$$\sigma_s \equiv eE_0\ell/m_e c^2 = (\ell/\lambda_C)(E_0/E_c).$$

In these two cases the pairs produced are not confined by the electric potential and can reach an infinite distance. The resultant pair production rate varies as a function of space coordinate. The result we obtained is drastically different from the Schwinger rate in homogeneous electric fields without any boundary. We clearly show that the approximate application of the Schwinger rate to electric fields limited within finite size of space overestimates the total number of pairs produced, particularly when the finite size is comparable with the Compton wavelength λ_C, see article Figs. 6.2 and 6.3 where it is clearly shown how the rate of pair creation far from being constant goes to zero at both boundaries. The same situation is also found for the case of the semi-confined field $z(z) \neq 0$ for $|z| \lesssim \ell$, see Eq. (6.3.34). In the second category, we study a linearly rising electric field $E(z) \sim z$, corresponding to a harmonic potential $V(z) \sim z^2$, see Figs. 6.1. In this case the energy spectra of bound states are discrete and thus energy crossing levels for tunneling are discrete. To obtain the total number of pairs created, using the general formulas for pair production rate, we need to sum over all discrete energy crossing levels, see Eq. (6.4.11), provided these energy levels are not occupied. Otherwise, the pair production would stop due to the Pauli principle.

3.6. Phenomenology of electron-positron pair creation and annihilation

In Section 7 of the review we focus on the phenomenology of electron–positron pair creation and annihilation experiments. There are three different aspects which are examined: the verification of the process (3.0.1) initially studied by Dirac, the process (3.6.1) studied by Breit and Wheeler, and then the classical work of vacuum polarization process around a supercritical nucleus, following the Sauter, Euler, Heisenberg and Schwinger work. We first recall in Section 7.1 how the process (3.0.1) predicted by Dirac was almost immediately discovered by Klemperer Klemperer (1934). Following this discovery the electron–positron collisions have become possibly the most prolific field of research in the domain of particle physics. The crucial step experimentally was the creation of the first electron–positron collider the "Anello d'Accumulazione" (AdA) was built by the theoretical proposal of Bruno Touschek in Frascati (Rome) in 1960 Bernardini (2004). Following the success of AdA (luminosity $\sim 10^{25}/(\mathrm{cm}^2$ sec), beam energy $\sim$0.25GeV), it was decided to build in the Frascati National Laboratory a storage ring of the same kind, Adone. Electron-positron colliders have been built and proposed for this purpose all over the world (CERN, SLAC, INP, DESY, KEK and IHEP).

The aim here is just to recall the existence of this enormous field of research which appeared following the original Dirac idea. In the review the main cross-sections (7.1.1) and (7.1.2) are recalled and the diagram (Fig. 7.1) summarizing this very great success of particle physics is presented. While the Dirac process (3.0.1) has been by far one of the most prolific in physics, the Breit–Wheeler process (3.6.1) has been one of the most elusive for direct observations. In Earth-bound experiments the major effort today is directed to evidence this phenomenon in very strong and coherent electromagnetic field in lasers. In this process collision of many photons may lead in the future to pair creation. This topic is discussed in Section 7.2. Alternative evidence for the Breit–Wheeler process can come from optically thick electron–positron plasma which may be created either in the future in Earth-bound experiments, or currently observed in astrophysics, see Section 10. One additional way to probe the existence of the Breit–Wheeler process is by establishing in astrophysics an upper limits to observable high-energy photons, as a function of distance, propagating in the Universe as pioneered by Nikishov Nikishov (1961), see Section 7.4. We then recall in Section 7.3 how the crucial experimental breakthrough came from the idea of John Madey Deacon et al. (1977) of self-amplified spontaneous emission in an undulator, which results when charges interact with the synchrotron radiation they emit (Tremaine et al. (2002)). Such X-ray free electron lasers have been constructed among others at DESY and SLAC and focus energy onto a small spot hopefully with the size of the X-ray laser wavelength $\lambda \simeq O(0.1)$nm (Nuhn and Pellegrini (2000)), and obtain a very large electric field $E \sim 1/\lambda$, much larger than those obtainable with any optical laser of the same power. This technique can be used to achieve a very strong electric field near to its critical value for observable electron–positron pair production in vacuum. No pair can be created by a single laser beam. It is then assumed that each X-ray laser pulse is split into two equal parts and recombined to form a standing wave with a frequency ω. We then recall how for a laser pulse with wavelength λ about $1\mu m$ and the theoretical diffraction limit $\sigma_{\rm laser} \simeq \lambda$ being reached, the critical intensity laser beam would be

$$I^c_{\rm laser} = \frac{c}{4\pi}E_c^2 \simeq 4.6 \cdot 10^{29} W/\mathrm{cm}^2.$$

In review Section 7.2.1 we recall the theoretical formula for the probability of pair production in time-alternating electric field in two limiting cases of large frequency and small frequency. It is interesting that in the limit of large field and small frequency the production rate approach the one of the Sauter, Heisenberg, Euler and Schwinger, discussed in Section 5. In the following Section 7.2.2 we recall the actually reached experimental limits quoted by Ringwald Ringwald (2001) for a X-ray laser and give a reference to the relevant literature. In Section 7.2.3 we summarize some of the most recent theoretical estimates for pair production by a circularly polarized laser beam by Narozhny, Popov and their collaborators. In this case the field invariants (3.5.23) are not vanishing and pair creation can be achieved by a single laser beam. They computed the total number of electron–positron pairs produced as a function of intensity and focusing parameter of the laser. Particularly interesting is their analysis of the case of two counter-propagating focused laser pulses with circular polarizations, pair production becomes experimentally observable when the laser intensity $I_{\rm laser} \sim 10^{26} W/{\rm cm}^2$ for each beam, which is about $1 \sim 2$ orders of magnitude lower than for a single focused laser pulse, and more than 3 orders of magnitude lower than the critical intensity (7.2.4). Equally interesting are the considerations which first appear in treating this problem that the back reaction of the pairs created on the field has to be taken into due account. We give the essential references and we will see in Section 9 how indeed this feature becomes of paramount importance in the field of astrophysics. We finally review in Section 7.2.4 the technological situation attempting to increase both the frequency and the intensity of laser beams.

The difficulty of evidencing the Breit–Wheeler process even when the high-energy photon beams have a center of mass energy larger than the energy-threshold $2m_ec^2 = 1.02$ MeV was clearly recognized since the early days. We discuss the crucial role of the effective nonlinear terms originating in strong electromagnetic laser fields: the interaction needs not to be limited to initial states of two photons Reiss (1962, 1971). A collective state of many interacting laser photons occurs. We turn then in Section 7.3 of the review to an even more complex and interesting procedure: the interaction of an ultrarelativistic electron beam with a terawatt laser pulse, performed at SLAC Kotseroglou et al. (1996), when strong electromagnetic fields are involved. A first nonlinear Compton scattering process occurs in which the ultrarelativistic electrons absorb multiple photons from the laser field and emit a single photon via the process (5.9.1). The theory of this process has been given in Section 5.10. The second is a drastically improved Breit–Wheeler process (5.9.2) by which the high-energy photon γ, created in the first process, propagates through the laser field and interacts with laser photons $n\omega$ to produce an electron–positron pair Burke et al. (1997). In Section 7.3.1 we describe the status of this very exciting experiments which give the first evidence for the observation in the laboratory of the Breit–Wheeler process although in a somewhat indirect form. Having determined the theoretical basis as well as attempts to verify experimentally the Breit–Wheeler formula we turn in Section 7.4 to a most important application of the Breit–Wheeler process in the framework of cosmology. As pointed out by Nikishov Nikishov (1961) the existence of background photons in cosmology puts a stringent cutoff on the maximum trajectory of the high-energy photons in cosmology.

Having reviewed both the theoretical and observational evidence of the Dirac and Breit–Wheeler processes of creation and annihilation of electron–positron pairs we turn then to one of the most conspicuous field of theoretical and experimental physics dealing with the process of electron–positron pair creation by vacuum polarization in the field of a heavy nuclei. This topic has originated one of the vastest experimental and theoretical physics activities in the last forty years, especially by the process of collisions of heavy ions. We first review in Section 7.5 of the article the $Z = 137$ catastrophe, a collapse to the center, in semi-classical approach, following the Pomeranchuk work Pomeranchuk and Smorodinskii (1945) based on the imposing the quantum conditions on the classical treatment of the motion of two relativistic particles in circular orbits. We then proceed showing in Section 7.5.3 how the introduction of the finite size of the nucleus, following the classical work of Popov and Zeldovich Zeldovich and Popov (1971), leads to the critical charge of a nucleus of $Z_{cr} = 173$ above which a bare nucleus would lead to the level crossing between the bound state and negative energy states of electrons in the field of a bare nucleus. We then review in Section 7.5.5 the recent theoretical progress in analyzing the pair creation process in a Coulomb field, taking into account radial dependence and time variability of electric field. We finally recall in Section 7.6 the attempt to use heavy-ion collisions to form transient superheavy "quasimolecules": a long-lived metastable nuclear complex with $Z > Z_{cr}$. It was expected that the two heavy ions of charges respectively Z_1 and Z_2 with $Z_1 + Z_2 > Z_{cr}$ would reach small inter-nuclear distances well within the electron's orbiting radii. The electrons would not distinguish between the two nuclear centers and they would evolve as if they were bounded by nuclear "quasimolecules" with nuclear charge $Z_1 + Z_2$. Therefore, it was expected that electrons would evolve quasi-statically through a series of well defined nuclear "quasimolecules" states in the two-center field of the nuclei as the inter-nuclear separation decreases and then increases again. When heavy-ion collision occurs the two nuclei come into contact and some deep inelastic reaction occurs determining the duration Δt_s of this contact. Such "sticking time" is expected to depend on the nuclei involved in the reaction and on the beam energy. Theoretical attempts have been proposed to study the nuclear aspects of heavy-ion collisions at energies very close to the Coulomb barrier and search for conditions, which would serve as a trigger for prolonged nuclear reaction times, to enhance the amplitude of pair production. The sticking time Δt_s should be larger than $1 \sim 2 \cdot 10^{-21}$ sec Greiner and Reinhardt (1999) in order to have significant pair production. Up to now no success has been achieved in justifying theoretically such a long sticking time. In reality the characteristic sticking time has been found of the order of $\Delta t \sim 10^{-23}$ sec, hundred times shorter than the needed to activate the pair creation process. We finally recall in Section 7.6.2 of the review the Darmstadt-Brookhaven dialogue between the Orange and the Epos groups and the Apex group at Argonne in which the claim for discovery of electron–positron pair creation by vacuum polarization in heavy-ion collisions was finally retracted. Out of the three fundamental processes addressed in this report, the Dirac electron–positron annihilation and the Breit–Wheeler electron–positron creation from two photons have found complete theoretical descriptions within Quantum Electro-Dynamics. The first one is very likely the best tested process in physical science, while the second has finally obtained the first indirect experimental evidence. The third process, the one of the vacuum polarization studied by Sauter, Euler, Heisenberg and Schwinger, presents in Earth-bound experiments presents a situation "terra incognita".

3.6.1. The Breit-Wheeler cutoff in high-energy Gamma-rays

The Breit-Wheeler process for the photon-photon pair production is one of most relevant elementary processes in high energy astrophysics (see review Sec. 7.4). In addition to the importance of this process in dense radiation fields of compact objects (Bonometto and Rees, 1971), the essential role of this process in the context of intergalactic absorption of high-energy γ-rays was first pointed out by Nikishov (Nikishov, 1961; Gould and Schréder, 1967). The spectra of TeV radiation observed from distant ($d > 100$ Mpc) extragalactic objects suffer essential deformation during the passage through the intergalactic medium, caused by energy-dependent absorption of primary γ-rays at interactions with the diffuse extragalactic background radiation, for the optical depth $\tau_{\gamma\gamma}$ most likely significantly exceeding one (Gould and Schréder, 1967; Stecker et al., 1992; Vassiliev, 2000; Coppi and Aharonian, 1999). A relevant broad-band information about the cosmic background radiation (CBR) is important for the interpretation of the observed high-energy γ spectra (Aharonian et al., 2000; Kneiske et al., 2002; Dwek and Krennrich, 2005; Aharonian et al., 2006). For details see Hauser and Dwek (2001); Aharonian (2003). In this section, we are particularly interested in such absorption effect of high-energy γ-ray, originated from cosmological sources, interacting with the Cosmic Microwave Background (CMB) photons. Fazio and Stecker (Fazio and Stecker, 1970; Stecker et al., 1977) were the first who calculated the cutoff energy versus redshift for cosmological γ-rays. This calculation was applied to further study of the optical depth of the Universe to high-energy γ-rays (MacMinn and Primack, 1996; Kneiske et al., 2004; Stecker et al., 2006). With the Fermi telescope, such study turns out to be important to understand the spectrum of high-energy γ-ray originated from GRBs' sources at cosmological distance, we therefore offer the details of theoretical analysis as follow.

Breit-Wheeler cross-section in arbitrary frame

Breit and Wheeler (1934) studied the process

$$\gamma_1 + \gamma_2 \to e^+ + e^-, \tag{3.6.1}$$

in the center of mass of the system, the momenta of the electron and positron are equal and opposite $\mathbf{p}_1 = -\mathbf{p}_2$. The same thing holds for the momenta of the photons in the initial state: $\mathbf{k}_1 = -\mathbf{k}_2$. As a consequence, the energies of electron and positron are equal: $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}$, and so are the energies of the photons: $\hbar\omega_1 = \hbar\omega_2 = \mathcal{E}_\gamma = \mathcal{E}$. They found the total cross-section in the center of mass of the system:

$$\sigma_{\gamma\gamma} = \frac{\pi}{2}\left(\frac{\alpha\hbar}{m\,c}\right)^2 (1-\hat{\beta}^2)\left[2\hat{\beta}(\hat{\beta}^2-2)+(3-\hat{\beta}^4)\ln\left(\frac{1+\hat{\beta}}{1-\hat{\beta}}\right)\right], \quad \text{with} \quad \hat{\beta} = \frac{c|\mathbf{p}|}{\mathcal{E}}, \tag{3.6.2}$$

where $\mathbf{p}$ and $\hat{\beta}$ are respectively momentum and the reduced velocity of an electron or positron. The necessary kinematic condition in order for the process (3.6.1) taking place is that the energy of two colliding photons is larger than the energetic threshold $2m_ec^2$, i.e.,

$$\mathcal{E}_\gamma > m_ec^2. \tag{3.6.3}$$

The cross-section in line (3.6.2) can be easily generalized to an arbitrary reference frame $\mathcal{K}$, in which the two photons k_1 and k_2 are moving in opposite directions; for Lorentz invariance of $(k_1 \cdot k_2)$, one has $\omega_1\omega_2 = \mathcal{E}_\gamma^2$. Since

$$\mathcal{E}_\gamma = \mathcal{E} = m_ec^2/\sqrt{1-\hat{\beta}^2}, \tag{3.6.4}$$

to obtain the total cross-section in the arbitrary frame $\mathcal{K}$, we must therefore make the following substitution (Landau and Lifshitz, 1975),

$$\hat{\beta} \to \sqrt{1-m_e^2c^4/(\omega_1\omega_2)}, \tag{3.6.5}$$

in Eq. (3.6.2). For $\mathcal{E} \gg m_ec^2$, the total effective cross-section is approximately proportional to

$$\sigma_{\gamma\gamma} \simeq \pi\left(\frac{\alpha\hbar}{m_ec}\right)^2\left(\frac{m_ec^2}{\mathcal{E}}\right)^2 = \pi r_e^2\left(\frac{m_ec^2}{\mathcal{E}}\right)^2, \tag{3.6.6}$$

where $r_e = \left(\frac{\alpha\hbar}{m_ec}\right)$ is the electron classical radius and $\pi r_e^2 \simeq 2.5\cdot 10^{-25}\text{cm}^2$.

Opacity of high-energy GRB photons colliding with CMB photons

We study the Breit-Wheeler process (3.6.1) to the case that high-energy GRB photons ω_1, originated from GRBs sources at cosmological distance z, on their way traveling to us, collide with CMB photons ω_2 in the rest frame of CMB photons, leading to electron-positron pair production. We calculate the opacity and mean free-path of these high-energy GRB photons, find the energy-range of absorption as a function of the cosmological red-shift z.

In general, a high-energy GRB photon with a give energy ω_1, collides with background photons in all possible energies ω_2. We assume that i-type background photons have the spectrum distribution $f_i(\omega_2/T_i)$, where T_i is the characteristic energy scale of the distribution, the opacity is then given by

$$\tau^i_{\gamma\gamma}(\omega_1, z) = \int dr \int_{m_e^2c^4/\omega_1}^{\infty} \frac{\omega_2^2 d\omega_2}{\pi^2} f_i(\omega_2/T_i)\sigma_{\gamma\gamma}(\frac{\omega_1\omega_2}{m_e^2c^4}), \tag{3.6.7}$$

where $m_e^2c^4/\omega_1$ is the energy-threshold (3.6.3) above which the Breit-Wheeler process (3.6.1) can occurs and the cross-section $\sigma_{\gamma\gamma}(x)$ is given by Eqs. (3.6.2), depending only on $x = \frac{\omega_1\omega_2}{m_e^2c^4}$. The total opacity is then given by

$$\tau^{\rm total}_{\gamma\gamma}(\omega_1, z) = \sum_i \tau^i_{\gamma\gamma}(\omega_1, z), \tag{3.6.8}$$

which the sum is over all types of photon background in the Universe. The high-energy photons traveling path $\int dr$ is given by ,

$$\int_t^{t_0} \frac{dt'}{R(t')} = \int_0^{r(t)} \frac{dr}{(1-kr^2)^{1/2}} = \int_0^{r(t)} dr, \tag{3.6.9}$$

where $R(t)$ is the scalar factor, t_0 is the present time and t corresponds to epoch of the red-shift z for a flat $(k = 0)$ Freemann Universe. Using the relationship $z + 1 = R_0/R(t)$, we change integrand variable from t' to the red-shift z,

$$dt' = -\frac{dz}{(z'+1)H(z')}, \tag{3.6.10}$$

so that we have

$$\int_0^{r(t)} dr = \int_t^{t_0} \frac{dt'}{R(t')} = \frac{1}{R_0}\int_0^z \frac{dz}{H(z)}, \tag{3.6.11}$$

where $H(z) = \dot{R}(t)/R(t_0)$ is the Hubble function, obeyed the Friedmann equation

$$H(z) = H_0[\Omega_M(z+1)^3 + \Omega_\Lambda]^{1/2}, \quad \Omega_M + \Omega_\Lambda = 1, \tag{3.6.12}$$

$\Omega_M \simeq 0.3$ and $\Omega_M \simeq 0.7$.

In the case of CMB photons in a black-body distribution $1/(e^{\omega_2/T} - 1)$ with the temperature T, the opacity is given by

$$\tau_{\gamma\gamma}(\omega_1, z) = \int dr \int_{m_e^2c^4/\omega_1}^{\infty} \frac{d\omega_2}{\pi^2} \frac{\omega_2^2}{e^{\omega_2/T} - 1} \sigma_{\gamma\gamma}(\frac{\omega_1\omega_2}{m_e^2c^4}), \tag{3.6.13}$$

where the Boltzmann constant $k_B = 1$. To simply Eq. (3.6.13), we set $x = \frac{\omega_1\omega_2}{m_e^2c^4}$,

$$\tau_{\gamma\gamma}(\omega_1, z) = \int dr \left(\frac{m_e^2c^4}{\omega_1}\right)^3 \int_1^\infty \frac{dx}{\pi^2} \frac{x^2}{\exp\frac{xm_e^2c^4}{\omega_1 T} - 1}\sigma_{\gamma\gamma}(x). \tag{3.6.14}$$

In terms of CMB temperature and GRB-photons energy at the present time,

$$T = (z+1)T^0; \quad \omega_{1,2} = (z+1)\omega_{1,2}^0, \tag{3.6.15}$$

we obtain,

$$\tau_{\gamma\gamma}(\omega_1^0, z) = \frac{1}{R_0}\int_0^z \frac{dz'}{H(z')}\frac{1}{(z+1)^3}\left(\frac{m_e^2c^4}{\omega_1^0}\right)^3 \int_1^\infty \frac{dx}{\pi^2}\frac{x^2}{\exp(x/\theta)-1}\sigma_{\gamma\gamma}(x), \tag{3.6.16}$$

where

$$\theta = x^0(z+1)^2; \quad x^0 = \frac{\omega_1^0 T^0}{m_e^2c^4}, \tag{3.6.17}$$

and x^0 is the energy ω_1^0 in unit of $m_ec^2(m_ec^2/T^0) = 1.15 \cdot 10^{15}$eV. For the purpose of numerical calculations, we rewrite the expression,

$$\begin{aligned}
\tau_{\gamma\gamma}(x^0, z) = & \frac{\pi r_e^2}{R_0 H_0/c}\left(\frac{T^0}{x^0}\right)^3 \int_0^z \frac{dz'}{[\Omega_M(z'+1)^3 + \Omega_\Lambda]^{1/2}}\frac{1}{(z'+1)^3}\times \\
& \times \int_1^\infty \frac{dx}{2\pi^2}\frac{x^2 f_{\gamma\gamma}(x)}{\exp(x/\theta)-1} = \\
& = \frac{23.8}{R_0 h}\left(\frac{1}{x^0}\right)^3 \int_0^z \frac{dz'}{[\Omega_M(z'+1)^3 + \Omega_\Lambda]^{1/2}}\frac{1}{(z'+1)^3}\times \\
& \times \int_1^\infty \frac{dx}{2\pi^2}\frac{x^2 f_{\gamma\gamma}(x)}{\exp(x/\theta)-1},
\end{aligned} \tag{3.6.18}$$

where $R_0 = 1$, present Hubble constant $h = H_0/100\mathrm{km}/sec/\mathrm{Mpc}$ and

$$f_{\gamma\gamma}(x) \quad = \quad (1-\hat\beta^2)\Big[2\hat\beta(\hat\beta^2-2) + (3-\hat\beta^4)\ln\big(\frac{1+\hat\beta}{1-\hat\beta}\big)\Big], \quad \hat\beta = \sqrt{1-1/x}.$$

The $\tau_{\gamma\gamma}(\omega_1^0, z) = 1$ give the relationship $\omega_1^0 = \omega_1^0(z)$ that separates the absorbed regime $\tau_{\gamma\gamma}(\omega_1^0, z) > 1$ and transparent regime $\tau_{\gamma\gamma}(\omega_1^0, z) < 1$ in the $\omega_1^0 - z$ plane.

The numerical result is shown in Fig. 3.1. It clearly shows the following properties:

1. for the redshift z smaller than a critical value $z_c \simeq 0.1$ ($z < z_c$), the CMB photons are transparent $\tau_{\gamma\gamma}(\omega_1^0, z) < 1$ to GRB photons in any energy

bands, this indicates a minimal mean-free path of photons traveling in CMB photons background;

2. for the redshift z larger than the value ($z > z_c$), there are two branches of solutions for $\tau_{\gamma\gamma}(\omega_1^0, z) = 1$, respectively corresponding to the different energy-dependence of the cross-section (3.6.2): the cross-section increases with the center-mass-energy $x = \mathcal{E}_\gamma^2/(m_e c^2)^2$ from the energy-threshold $x = 1$ to $x \simeq 1.99$, and decreases (3.6.6) from $x \simeq 1.99$ to $x \to \infty$. The turn point ($z \simeq 0.1, \omega_1^0 \simeq 1.15 \cdot 10^{15}$eV) from one solution to another is determined by the maximal cross-section at $x \simeq 1.99$. Due to these two solutions, CMB photons are transparent to GRB photons of large and small energies, opaque to those GRB photons in an intermediate energy-range large for a given finite z-value;

3. CMB photons are transparent to very low-energy GRB photons $\omega_1^0 < 10^{12}$eV, i.e., $x^0 < 10^{-3}$, due to their energies are below the energetic threshold for the Breit-Wheeler process (3.6.1). In addition, CMB photons are transparent to very large-energy GRB photons $\omega_1^0 > 10^{18}$eV, i.e., $x^0 > 10^3$, due to the cross-section of Breit-Wheeler process (3.6.1) is very small for extremely high-energy photons. For very large $z \sim 10^3$, the Universe becomes completely opaque and photon distribution cannot be described by the black body spectrum, we disregard this regime.

Due to the fact that there are other radiation backgrounds (3.6.7), the background of CMB photons gives the lowest bound of opacity, absorption limit, to GRB photons with respect to the Breit-Wheeler process (3.6.1). Finally, we point out that Fazio and Stecker (Fazio and Stecker, 1970; Stecker et al., 1977) gave only asymptotic form of small-energy solution indicated in Fig. (3.1).

3.7. The extraction of blackholic energy from a black hole by vacuum polarization processes

We turn then to astrophysics, where, in the process of gravitational collapse to a black hole and in its outcomes these three processes will be for the first time verified on a much larger scale, involving particle numbers of the order of 10^{60}, seeing both the Dirac process and the Breit–Wheeler process at work in symbiotic form and electron–positron plasma created from the "blackholic energy" during the process of gravitational collapse. It is becoming more and more clear that the gravitational collapse process to a Kerr–Newman black hole is possibly the most complex problem ever addressed in physics and astrophysics. What is most important for this report is that it gives for the first time the opportunity to see the above three processes simultaneously at work under ultrarelativistic special and general relativistic regimes. The process of gravitational collapse is characterized by the

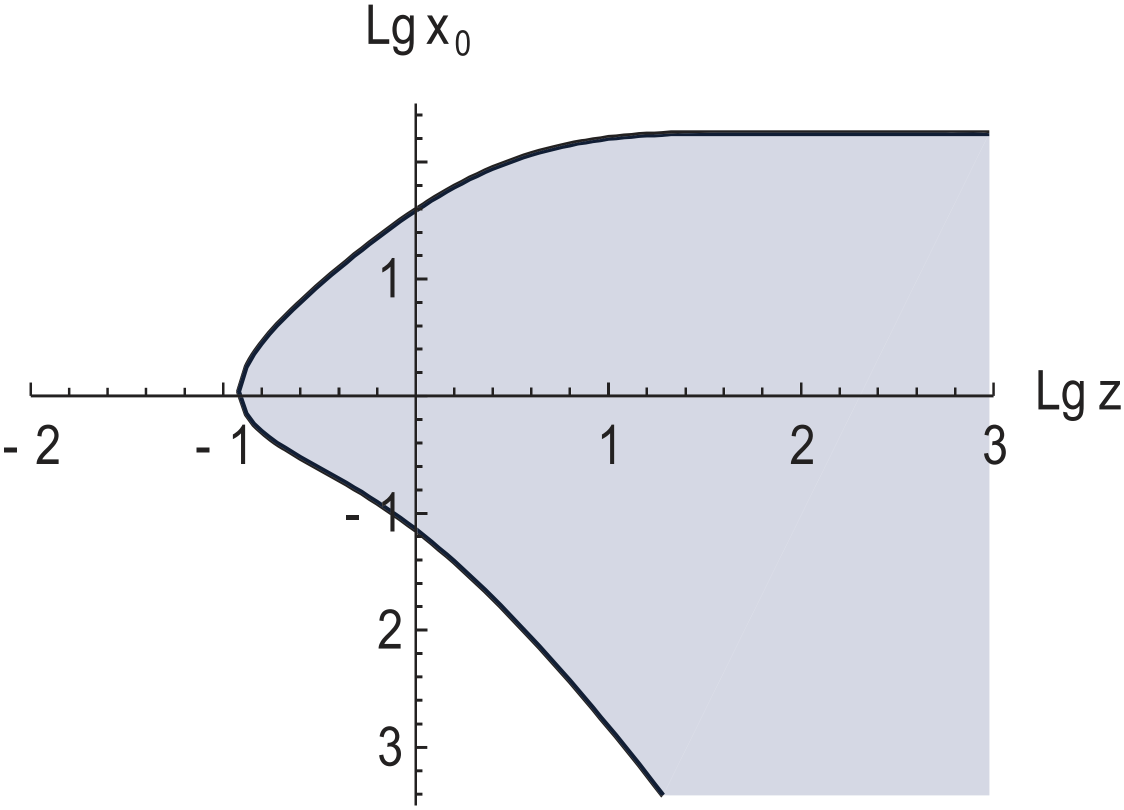

Figure 3.1.: This is a Log-Log plot for GRB photon energy x^0 (in unit of $1.11 \cdot 10^{15}$) vs redshift z. For $z > z_c \simeq 0.1$, the line that bounds shadow area indicates two solutions for the opacity $\tau_{\gamma\gamma} = 1$: (i) large-energy solution for $\omega_1^0 > 1.15 \cdot 10^{15}$eV; (ii) small-energy solution for $\omega_1^0 < 1.15 \cdot 10^{15}$eV, which separate the optically thick regime (shadow area) $\tau_{\gamma\gamma}(\omega_1^0, z) > 1$ and optically thin regime $\tau_{\gamma\gamma}(\omega_1^0, z) < 1$.

timescale $\Delta t_g = GM/c^3 \simeq 5 \cdot 10^{-6} M/M_\odot$ sec and the energy involved are of the order of $\Delta E = 10^{54} M/M_\odot$ ergs. It is clear that this is one of the most energetic and most transient phenomena in physics and astrophysics and needs for its correct description such a highly time varying treatment. Our approach in Section 8 is to gain understanding of this process by separating the different components and describing 1) the basic energetic process of an already formed black hole, 2) the vacuum polarization process of an already formed black hole, 3) the basic formula of the gravitational collapse recovering the Tolman-Oppenheimer-Snyder solutions and evolving to the gravitational collapse of charged and uncharged shells. This will allow among others to obtain a better understanding of the role of irreducible mass of the black hole and the maximum blackholic energy extractable from the gravitational collapse. We will as well address some conceptual issues between general relativity and thermodynamics which have been of interest to theoretical physicists in the last forty years. Of course in these brief chapter we will be only recalling some of these essential themes and refer to the literature where in-depth analysis can be found. In Section 8.1 we recall the Kerr–Newman metric and the associated electromagnetic field. We then recall the classical work of Carter Carter (1968) integrating the Hamilton-Jacobi equations for charged particle motions in the above given metric and electromagnetic field. We then recall in Section 8.2 the introduction of the effective potential techniques in order to obtain explicit expression for the trajectory of a particle in a Kerr–Newman geometry, and especially the introduction of the reversible–irreversible transformations which lead then to the Christodoulou-Ruffini mass formula of the black hole

$$M^2c^4 = \left(M_{\rm ir}c^2 + \frac{c^2Q^2}{4GM_{\rm ir}}\right)^2 + \frac{L^2c^8}{4G^2M_{\rm ir}^2},$$

where $M_{\rm ir}$ is the irreducible mass of a black hole, Q and L are its charge and angular momentum. We then recall in article Section 8.3 the positive and negative root states of the Hamilton–Jacobi equations as well as their quantum limit. We finally introduce in Section 8.4 the vacuum polarization process in the Kerr–Newman geometry as derived by Damour and Ruffini Damour and Ruffini (1975) by using a spatially orthonormal tetrad which made the application of the Schwinger formalism in this general relativistic treatment almost straightforward. We then recall in Section 8.5 the definition of a dyadosphere in a Reissner–Nordström geometry, a region extending from the horizon radius

$$r_+ \quad = \quad 1.47 \cdot 10^5 \mu (1 + \sqrt{1 - \xi^2}) \text{ cm}$$

out to an outer radius

$$r^\star = \left(\frac{\hbar}{m_e c}\right)^{1/2} \left(\frac{GM}{c^2}\right)^{1/2} \left(\frac{m_\mathrm{p}}{m_e}\right)^{1/2} \left(\frac{e}{q_\mathrm{p}}\right)^{1/2} \left(\frac{Q}{\sqrt{G}M}\right)^{1/2} = \\ = 1.12 \cdot 10^8 \sqrt{\mu\xi}\ \mathrm{cm},$$

where the dimensionless mass and charge parameters $\mu = \frac{M}{M_\odot}$, $\xi = \frac{Q}{(M\sqrt{G})} \leq 1$. In Section 8.6 of the review the definition of a dyadotorus in a Kerr–Newman metric is recalled. We have focused on the theoretically well defined problem of pair creation in the electric field of an already formed black hole. Having set the background for the blackholic energy we recall some fundamental features of the dynamical process of the gravitational collapse. In Section 8.7 we address some specific issues on the dynamical formation of the black hole, recalling first the Oppenheimer-Snyder solution Oppenheimer and Snyder (1939) and then considering its generalization to the charged nonrotating case using the classical work of W. Israel and V. de la Cruz Israel (1966); De la Cruz and Israel (1967). In Section 8.7.1 we recover the classical Tolman-Oppenheimer-Snyder solution in a more transparent way than it is usually done in the literature. In the Section 8.7.2 we are studying using the Israel-de la Cruz formalism the collapse of a charged shell to a black hole for selected cases of a charged shell collapsing on itself or collapsing in an already formed Reissner–Nordström black hole. Such elegant and powerful formalism has allowed to obtain for the first time all the analytic equations for such large variety of possibilities of the process of the gravitational collapse. The theoretical analysis of the collapsing shell considered in the previous section allows to reach a deeper understanding of the mass formula of black holes at least in the case of a Reissner–Nordström black hole. This allows as well to give in Section 8.8 of the review an expression of the irreducible mass of the black hole only in terms of its kinetic energy of the initial rest mass undergoing gravitational collapse and its gravitational energy and kinetic energy T_+ at the crossing of the black hole horizon r_+

$$M_\mathrm{ir} = M_0 - \frac{M_0^2}{2r_+} + T_+.$$

Similarly strong, in view of their generality, are the considerations in Section 8.8.2 which indicate a sharp difference between the vacuum polarization process in an overcritical $E \gg E_c$ and undercritical $E \ll E_c$ black hole. For $E \gg E_c$ the electron–positron plasma created will be optically thick with average particle energy 10 MeV. For $E \ll E_c$ the process of the radiation will be optically thin and the characteristic energy will be of the order of 10^{21} eV. This argument will be further developed in a forthcoming report. In Section 8.9 we show how the expression of the irreducible mass obtained in the previous Section leads to a theorem establishing an upper limit to 50% of the total mass energy initially at rest at infinity which can be extracted from any process of gravitational collapse independent of the details. These results also lead to some general considerations which have been sometimes claimed in reconciling general relativity and thermodynamics.

3.8. Plasma oscillations in electric fields

The conditions encountered in the vacuum polarization process around black holes lead to a number of electron–positron pairs created of the order of 10^{60} confined in the dyadosphere volume, of the order of a few hundred times to the horizon of the black hole. Under these conditions the plasma is expected to be optically thick and is very different from the nuclear collisions and laser case where pairs are very few and therefore optically thin. We turn then in Section 9, to discuss a new phenomenon: the plasma oscillations, following the dynamical evolution of pair production in an external electric field close to the critical value. In particular, we will examine: (i) the back reaction of pair production on the external electric field; (ii) the screening effect of pairs on the electric field; (iii) the motion of pairs and their interactions with the created photon fields. In review Secs. 9.1 and 9.2, we review semi-classical and kinetic theories describing the plasma oscillations using respectively the Dirac-Maxwell equations and the Boltzmann-Vlasov equations. The electron–positron pairs, after they are created, coherently oscillate back and forth giving origin to an oscillating electric field. The oscillations last for at least a few hundred Compton times. We review the damping due to the quantum decoherence. The energy from collective motion of the classical electric field and pairs flows to the quantum fluctuations of these fields. This process is quantitatively discussed by using the quantum Boltzmann-Vlasov equation in Sections 9.4 and 9.5. The damping due to collision decoherence is quantitatively discussed in Sections 9.6 and 9.7 by using Boltzmann-Vlasov equation with particle collisions terms. This damping determines the energy flows from collective motion of the classical electric field and pairs to the kinetic energy of non-collective motion of particles of these fields due to collisions. In Section 9.7, we particularly address the study of the influence of the collision processes $e^+e^- \rightleftarrows \gamma\gamma$ on the plasma oscillations in supercritical electric field Ruffini et al. (2003b). It is shown that the plasma oscillation is mildly affected by a small number of photons creation in the early evolution during a few hundred Compton times (see Fig. 9.4 of the review). In the later evolution of 10^{3-4} Compton times, the oscillating electric field is damped to its critical value with a large number of photons created. An equipartition of number and energy between electron–positron pairs and photons is reached (see Fig. 9.4). In Section 9.8, we introduce an approach based on the following three equations: the number density continuity equation, the energy-momentum conservation equation and the Maxwell equations. We describe the plasma oscillation for both overcritical electric field $E > E_c$ and undercritical electric field $E < E_c$ Ruffini et al. (2007b). In additional of reviewing the result well known in the literature for $E > E_c$ we review some novel result for the case $E < E_c$. It was traditionally assumed that electron–positron pairs, created by the vacuum polarization process, move as charged particles in external uniform electric field reaching arbitrary large Lorentz factors. It is reviewed how recent computations show the existence of plasma oscillations of the electron–positron pairs also for $E \lesssim E_c$. For both cases we quote the maximum Lorentz factors $\gamma_{\max}$ reached by the electrons and positrons as well as the length of oscillations. Two specific cases are given. For $E_0 = 10E_c$ the length of oscillations $10\ \hbar/(m_e c)$, and $E_0 = 0.15E_c$ the length of oscillations $10^7\ \hbar/(m_e c)$. We also review the asymptotic behavior in time, $t \to \infty$, of the plasma oscillations by the phase portrait technique. Finally we review some recent results which differentiate the case $E > E_c$ from the one $E < E_c$ with respect to the creation of the rest mass of the pair versus their kinetic energy. For $E > E_c$ the vacuum polarization process transforms the electromagnetic energy of the field mainly in the rest mass of pairs, with moderate contribution to their kinetic energy.

We also study electron-positron pair oscillation in spatially inhomogeneous and bound electric fields by integrating the equations of energy-momentum and particle-number conservations and Maxwell equations. The space and time evolutions of the pair-induced electric field, electric charge- and current-densities are calculated. The results show non-vanishing electric charge-density and the propagation of pair-induced electric fields, that are different from the case of homogeneous and unbound electric fields. The space and time variations of pair-induced electric charges and currents emit an electromagnetic radiation. We obtain the narrow spectrum and intensity of this radiation, whose peak $\omega_{\rm peak}$ locates in the region around 4 keV for electric field strength $\sim E_c$. We discuss their relevances to both the laboratory experiments for electron and positron pair-productions and the astrophysical observations of compact stars with an electromagnetic structure.

The origin of electron-positron pairs being created strong electric field and their oscillations has been considered in Ruffini et al. (2007b). There it was shown that plasma oscillations occur not only for overcritical electric field, but also for undercritical electric field, provided the electric field is maintained on spatial distances larger than the distance of oscillations determined explicitly in Ruffini et al. (2007b).

In the paper by Han et al. (2010) the spectrum of electromagnetic radiation seen by far observer for initial phase of oscillations has been computed. It was shown there that the spectrum contain a narrow feature which corresponds to the frequency of plasma oscillations. We revisited the approach of Ruffini et al. (2007b) and showed that for the case of uniform external electric field it is possible to reduce the system of four first order ordinary differential equations governing the dynamics of particle number density, energy density, momentum and electric field to just one second order equation. Then we analysed the frequency of oscillations, and found that the frequency of oscillations coincides up to a factor close to unity with the plasma frequency, which is strongly time dependent due to pair creation process. Analytical arguments suggest that the frequency of oscillations should asymptotically reach the plasma frequency, and this fact has been demonstrated. The results of this work allow simple estimation of the frequency of plasma oscillations, and then of the spectrum of electromagnetic radiation generated by these oscillations.

3.9. Thermalization of the mildly relativistic pair plasma

We then turn in Section 10 of the review to the last physical process needed in ascertaining the reaching of equilibrium of an optically thick electron–positron plasma. The average energy of electrons and positrons we illustrate is $0.1 < \epsilon < 10$ MeV. These bounds are necessary from the one hand to have significant amount of electron–positron pairs to make the plasma optically thick, and from the other hand to avoid production of other particles such as muons. As we will see in the next report these are indeed the relevant parameters for the creation of ultrarelativistic regimes to be encountered in pair creation process during the formation phase of a black hole. We then review the problem of evolution of optically thick, nonequilibrium electron–positron plasma, towards an equilibrium state, following Aksenov et al. (2007, 2008). These results have been mainly obtained by two of us (RR and GV) in recent publications and all relevant previous results are also reviewed in this Section 10. We have integrated directly relativistic Boltzmann equations with all binary and triple interactions between electrons, positrons and photons two kinds of equilibrium are found: kinetic and thermal ones. Kinetic equilibrium is obtained on a timescale of few $(\sigma_T n_\pm c)^{-1}$, where σ_T and $n_\pm$ are Thomson's cross-section and electron–positron concentrations respectively, when detailed balance is established between all binary interactions in plasma. Thermal equilibrium is reached on a timescale of few $(\alpha\sigma_T n_\pm c)^{-1}$, when all binary and triple, direct and inverse interactions are balanced. In Section 10.1 basic plasma parameters are illustrated. The computational scheme as well as the discretization procedure are discussed in Section 10.2. Relevant conservation laws are given in Section 10.3. Details on binary interactions, consisting of Compton, Møller and Bhabha scatterings, Dirac pair annihilation and Breit–Wheeler pair creation processes, and triple interactions, consisting of relativistic bremsstrahlung, double Compton process, radiative pair production and three photon annihilation process, are presented in Section 10.5 and 10.6, respectively. In Section 10.5 collisional integrals with binary interactions are computed from first principles, using QED matrix elements. In Section 10.7 Coulomb scattering and the corresponding cutoff in collisional integrals are discussed. Numerical results are presented in Section 10.8 where the time dependence of energy and number densities as well as chemical potential and temperature of electron–positron-photon plasma is shown, together with particle spectra. The most interesting result of this analysis is to have differentiate the role of binary and triple interactions. The detailed balance in binary interactions following the classical work of Ehlers Ehlers (1973) leads to a distribution function of the form of the Fermi-Dirac for electron–positron pairs or of the Bose-Einstein for the photons. This is the reason we refer in the text to such conditions as the Ehlers equilibrium conditions. The crucial role of the direct and inverse three-body interactions is well summarized in fig. 10.1, panel A from which it is clear that the inverse three-body interactions are essential in reaching thermal equilibrium. If the latter are neglected, the system deflates to the creation of electron–positron pairs all the way down to the threshold of 0.5MeV. This last result which is referred as the Cavallo–Rees scenario Cavallo and Rees (1978) is simply due to improper neglection of the inverse triple reaction terms (see Appendix 10).

4. Publications (before 2005)

1. R. Ruffini and J. A. Wheeler, "Introducing the black hole ", Phys. Today, January (1971) 178.

   This article proved to be popular and was written with the intention of communicating some of the major processes made in understanding the final configurations of collapsed stars to the largest possible audience. In this article, the authors summarized the results of their students' work with particular emphasis on the work of D. Christodoulou (graduate student of R. Ruffini's at that time) together with some of their most significant new results. Moreover, it was emphasized that of all the procedures for identifying a collapsed object in space at a great distance, the most promising consisted of analyzing a close binary system in which one member is a normal star and the other a black hole. The X–ray emission associated with the transfer of material from the normal star to the collapsed object would then be of greatest importance in determining the properties of the collapsed object. This article has been reprinted many times and has been translated into many languages (Japanese, Russian, and Greek, among others). It has created much interest in the final configuration of stars after the endpoint of their thermonuclear evolution. The analysis of the possible processes leading to the formation of a black hole, via either a one–step process of a multistep process, was also presented for the first time in this article.

2. D. Christodoulou and R. Ruffini, "Reversible Transformations of a Charged Black Hole", Phys. Rev. D4 (1971) 3552.

   A formula is derived for the mass of a black hole as a function of its "irreducible mass," its angular momentum, and its charge. It is shown that 50% of the mass of an extreme charged black hole can be converted into energy as contrasted with 29% for an extreme rotating black hole.

3. T. Damour and R. Ruffini, "Quantum electrodynamical effects in Kerr-Newman geometries", Phys. Rev. Lett. 35 (1975) 463.

   Following the classical approach of Sauter, of Heisenberg and Euler and of Schwinger the process of vacuum polarization in the field of a "bare" Kerr-Newman geometry is studied. The value of the critical strength of the electromagnetic fields is given together with an analysis of the feedback of the discharge on the geometry. The relevance of this analysis for current astrophysical observations is mentioned.

4. J. Ferreirinho, R. Ruffini and L. Stella, "On the relativistic Thomas-Fermi model", Phys. Lett. B 91, (1980) 314. The relativistic generalization of the Thomas-Fermi model of the atom is derived. It approaches the usual nonrelativistic equation in the limit $Z \ll Z_{\rm crit}$, where Z is the total number of electrons of the atom and $Z_{\rm crit} = (3\pi/4)^{1/2}\alpha^{-3/2}$ and α is the fine structure constant. The new equation leads to the breakdown of scaling laws and to the appearance of a critical charge, purely as a consequence of relativistic effects. These results are compared and contrasted with those corresponding to N self-gravitating degenerate relativistic fermions, which for $N \approx N_{\rm crit} = (3\pi/4)^{1/2}(m/m_p)^3$ give rise to the concept of a critical mass against gravitational collapse. Here m is the mass of the fermion and $m_p = (\hbar c/G)^{1/2}$ is the Planck mass.

5. R. Ruffini and L. Stella,"Some comments on the relativistic Thomas-Fermi model and the Vallarta-Rosen equation", Phys. Lett. B 102 (1981) 442. Some basic differences between the screening of the nuclear charge due to a relativistic cloud of electrons in a neutral atom and the screening due to vacuum polarization effects induced by a superheavy ion are discussed.

6. G. Preparata, R. Ruffini and S.-S. Xue, "The dyadosphere of black holes and gamma-ray bursts", Astron. Astroph. Lett. 337 (1998) L3.

   The "dyadosphere" has been defined (Ruffini, Preparata et al.) as the region outside the horizon of a black hole endowed with an electromagnetic field (abbreviated to EMBH for "electromagnetic black hole") where the electromagnetic field exceeds the critical value, predicted by Heisenberg & Euler for e^+e^- pair production. In a very short time ($\sim O(\hbar/(mc^2))$), a very large number of pairs is created there. We here give limits on the EMBH parameters leading to a Dyadosphere for $10M_\odot$ and $10^5M_\odot$ EMBH's, and give as well the pair densities as functions of the radial coordinate. We here assume that the pairs reach thermodynamic equilibrium with a photon gas and estimate the average energy per pair as a function of the EMBH mass. These data give the initial conditions for the analysis of an enormous pair-electromagnetic-pulse or "P.E.M. pulse" which naturally leads to relativistic expansion. Basic energy requirements for gamma ray bursts (GRB), including GRB971214 recently observed at $z = 3.4$, can be accounted for by processes occurring in the dyadosphere. In this letter we do not address the problem of forming either the EMBH or the dyadosphere: we establish some inequalities which must be satisfied during their formation process.

7. R. Ruffini, "On the dyadosphere of black holes", at the XLIXth Yamada Conference on "Black Holes and High-Energy Astrophysics", H. Sato Ed., Univ. Acad. Press, Tokyo, 1998.

   The "dyadosphere" (from the Greek word "duas-duados" for pairs) is here defined as the region outside the horizon of a black hole endowed with an electromagnetic field (abbreviated to EMBH for "electromagnetic black hole") where the electromagnetic field exceeds the critical value, predicted by Heisenberg and Euler for electron-positron pair production. In a very short time, a very large number of pairs is created there. I give limits on the EMBH parameters leading to a Dyadosphere for 10 solar mass and 100000 solar mass EMBH's, and give as well the pair densities as functions of the radial coordinate. These data give the initial conditions for the analysis of an enormous pair-electromagnetic-pulse or "PEM-pulse" which naturally leads to relativistic expansion. Basic energy requirements for gamma ray bursts (GRB), including GRB971214 recently observed at z=3.4, can be accounted for by processes occurring in the dyadosphere.

8. R. Ruffini, J. D. Salmonson, J. R. Wilson and S.-S. Xue, "On the Pair Electromagnetic Pulse of a Black Hole with Electromagnetic Structure", Astron. Astroph. 350 (1999) 334.

   Starting from a nonequilibrium configuration we analyse the essential role of the direct and the inverse binary and triple interactions in reaching an asymptotic thermal equilibrium in a homogeneous isotropic electron-positron-photon plasma. We focus on energies in the range 0.1–10 MeV. We numerically integrate the integro-partial differential relativistic Boltzmann equation with the exact QED collisional integrals taking into account all binary and triple interactions in the plasma. We show that first, when detailed balance is reached for all binary interactions on a timescale $t_k \lesssim 10^{-14}$sec, photons and electron-positron pairs establish kinetic equilibrium. Successively, when triple interactions fulfill the detailed balance on a timescale $t_{eq} \lesssim 10^{-12}$sec, the plasma reaches thermal equilibrium. It is shown that neglecting the inverse triple interactions prevents reaching thermal equilibrium. Our results obtained in the theoretical physics domain also find application in astrophysics and cosmology.

9. R. Ruffini, J. D. Salmonson, J. R. Wilson and S.-S. Xue, "On Evolution of the Pair-Electromagnetic Pulse of a Charge Black Hole ", Astron. Astrophys. Suppl. Ser. 138 (1999) 511.

   Using hydrodynamic computer codes, we study the possible patterns of relativistic expansion of an enormous pair-electromagnetic-pulse (P.E.M. pulse); a hot, high density plasma composed of photons, electron-positron pairs and baryons deposited near a charged black hole (EMBH). On the bases of baryon-loading and energy conservation, we study the bulk Lorentz factor of expansion of the P.E.M. pulse by both numerical and analytical methods.

10. R. Ruffini, J. D. Salmonson, J. R. Wilson and S.-S. Xue, "On the pair-electromagnetic pulse from an electromagnetic Black Hole surrounded by a Baryonic Remnant ", Astron. Astrophys 359, 855-864 (2000).

    The interaction of an expanding Pair-Electromagnetic pulse (PEM pulse) with a shell of baryonic matter surrounding a Black Hole with electromagnetic structure (EMBH) is analyzed for selected values of the baryonic mass at selected distances well outside the dyadosphere of an EMBH. The dyadosphere, the region in which a super critical field exists for the creation of electron-positron pairs, is here considered in the special case of a Reissner-Nordstrom geometry. The interaction of the PEM pulse with the baryonic matter is described using a simplified model of a slab of constant thickness in the laboratory frame (constant-thickness approximation) as well as performing the integration of the general relativistic hydrodynamical equations. The validation of the constant-thickness approximation, already presented in a previous paper Ruffini, et al.(1999) for a PEM pulse in vacuum, is here generalized to the presence of baryonic matter. It is found that for a baryonic shell of mass-energy less than 1% of the total energy of the dyadosphere, the constant-thickness approximation is in excellent agreement with full general relativistic computations. The approximation breaks down for larger values of the baryonic shell mass, however such cases are of less interest for observed Gamma Ray Bursts (GRBs). On the basis of numerical computations of the slab model for PEM pulses, we describe (i) the properties of relativistic evolution of a PEM pulse colliding with a baryonic shell; (ii) the details of the expected emission energy and observed temperature of the associated GRBs for a given value of the EMBH mass; 10^3 solar masses, and for baryonic mass-energies in the range 10^{-8} to 10^{-2} the total energy of the dyadosphere.

11. G. Preparata, R. Ruffini and S.-S. Xue,"The role of the screen factor in GRBs ", Il Nuovo Cimento B115 (2000) 915.

    We derive the screen factor for the radiation flux from an optically thick plasma of electron-positron pairs and photons, created by vacuum polarization process around a black hole endowed with electromagnetic structure.

12. C. L. Bianco, R. Ruffini and S.-S. Xue, "The elementary spike produced by a pure e^+e^- pair-electromagnetic pulse from a Black Hole: The PEM Pulse ", Astron. Astrophys. 368 (2001) 377.

    In the framework of the model that uses black holes endowed with electromagnetic structure (EMBH) as the energy source, we study how an elementary spike appears to the detectors. We consider the simplest possible case of a pulse produced by a pure e^+e^- pair-electro-magnetic plasma, the PEM pulse, in the absence of any baryonic matter. The resulting time profiles show a *Fast-Rise-Exponential-Decay* shape, followed by a power-law tail. This is obtained without any special fitting procedure, but only by fixing the energetics of the process taking place in a given EMBH of selected mass, varying in the range from 10 to 10^3 $M_\odot$ and considering the relativistic effects to be expected in an electron-positron plasma gradually reaching transparency. Special attention is given to the contributions from all regimes with Lorentz γ factor varying from $\gamma = 1$ to $\gamma = 10^4$ in a few hundreds of the PEM pulse travel time. Although the main goal of this paper is to obtain the elementary spike intensity as a function of the arrival time, and its observed duration, some qualitative considerations are also presented regarding the expected spectrum and on its departure from the thermal one. The results of this paper will be comparable, when data will become available, with a subfamily of particularly short GRBs not followed by any afterglow. They can also be propedeutical to the study of longer bursts in presence of baryonic matter currently observed in GRBs.

13. R. Ruffini and L. Vitagliano, "Irreducible mass and energetics of an electromagnetic black hole ", Phys. Lett. B545 (2002) 233.

    The mass-energy formula for a black hole endowed with electromagnetic structure (EMBH) is clarified for the nonrotating case. The irreducible mass $M_{\rm irr}$ is found to be independent of the electromagnetic field and explicitly expressable as a function of the rest mass, the gravitational energy and the kinetic energy of the collapsing matter at the horizon. The electromagnetic energy is distributed throughout the entire region extending from the horizon of the EMBH to infinity. We discuss two conceptually different mechanisms of energy extraction occurring respectively in an EMBH with electromagnetic fields smaller and larger than the critical field for vacuum polarization. For a subcritical EMBH the energy extraction mechanism involves a sequence of discrete elementary processes implying the decay of a particle into two oppositely charged particles. For a supercritical EMBH an alternative mechanism is at work involving an electron-positron plasma created by vacuum polarization. The energetics of these mechanisms as well as the definition of the spatial regions in which thay can occur are given. The physical implementations of these ideas are outlined for ultrahigh energy cosmic rays UHECR) and gamma ray bursts (GRBs).

14. C. Cherubini, R. Ruffini and L. Vitagliano, "On the electromagnetic field of a charged collapsing spherical shell in general relativity ", Phys. Lett. B545 (2002) 226.

    A new exact solution of the Einstein-Maxwell equations for the gravitational collapse of a shell of matter in an already formed black hole is given. Both the shell and the black hole are endowed with electromagnetic structure and are assumed spherically symmetric. Implications for current research are outlined.

15. R. Ruffini, L. Vitagliano and S.-S. Xue, "On Plasma Oscillations in Strong Electric Fields ", Phys. Lett. B559 (2003) 12.

    We describe the creation and evolution of electron-positron pairs in a strong electric field as well as the pairs annihilation into photons. The formalism is based on generalized Vlasov equations, which are numerically integrated. We recover previous results about the oscillations of the charges, discuss the electric field screening and the relaxation of the system to a thermal equilibrium configuration. The timescale of the thermalization is estimated to be $\sim 10^3 - 10^4 \hbar / m_e c^2$.

16. R. Ruffini, L. Vitagliano and S.-S. Xue, "Electron-positron-photon plasma around a collapsing star ", (invited talk) in Proc. of the 28^{th} Joint ICFA Conference on Quantum Aspects of Beam Physics and Other Critical Issues of Beams in Physics and Astrophysics, January 7–11, 2003, Hiroshima University, Higashi–Hiroshima, Japan, Pisin Chen Ed., World Scientific, Singapore.

    We describe electron-positron pairs creation around an electrically charged star core collapsing to an electromagnetic black hole (EMBH), as well as pairs annihilation into photons. We use the kinetic Vlasov equation formalism for the pairs and photons and show that a regime of plasma oscillations is established around the core. As a byproduct of our analysis we can provide an estimate for the thermalization time scale.

17. G. Preparata, R. Ruffini and S.-S. Xue, "On the Dyadosphere of Black Hole", J. Korean Phys.Soc. 42 (2003) S99-S104 (astro-ph/0204080).

    Basic energy requirements of Gamma Ray Burst(GRB) sources can be easily accounted for by a pair creation process occurring in the "Dyadosphere" of a Black Hole endowed with an electromagnetic field (abbreviated to EMBH for "electromagnetic Black Hole"). This includes the recent observations of GRB971214 by Kulkarni et al. The "Dyadosphere" is defined as the region outside the horizon of an EMBH where the electromagnetic field exceeds the critical value for e^+e^- pair production. In a very short time $\sim O(\hbar mc^2)$, very large numbers of pairs are created there. Further evolution then leads naturally to a relativistically expanding pair-electromagnetic-pulse (PEM-pulse). Specific examples of Dyadosphere parameters are given for 10 and 10^5 solar mass EMBH's. This process does occur for EMBH with charge-to-mass ratio larger than 2.210^{-5} and strictly smaller than one. From a fundamental point of view, this process represents the first mechanism proved capable of extracting large amounts of energy from a Black Hole with an extremely high efficiency (close to 100%).

18. R. Ruffini and L. Vitagliano, "Energy Extraction From Gravitational Collapse to Static Black Holes ", Int. J. Mod. Phys. D12 (2003) 121.

    The mass–energy formula of black holes implies that up to 50% of the energy can be extracted from a static black hole. Such a result is reexamined using the recently established analytic formulas for the collapse of a shell and expression for the irreducible mass of a static black hole. It is shown that the efficiency of energy extraction process during the formation of the black hole is linked in an essential way to the gravitational binding energy, the formation of the horizon and the reduction of the kinetic energy of implosion. Here a maximum efficiency of 50% in the extraction of the mass energy is shown to be generally attainable in the collapse of a spherically symmetric shell: surprisingly this result holds as well in the two limiting cases of the Schwarzschild and extreme

Reissner-Nordström space-times. Moreover, the analytic expression recently found for the implosion of a spherical shell onto an already formed black hole leads to a new exact analytic expression for the energy extraction which results in an efficiency strictly less than 100% for any physical implementable process. There appears to be no incompatibility between General Relativity and Thermodynamics at this classical level.

5. Publications (2005-2010)

1. R. Ruffini, F. Fraschetti, L. Vitagliano and S.-S. Xue,"Observational signatures of an electromagnetic overcritical gravitational collapse ", Int. Journ. Mod. Phys. D14 (2005) 131.

   We present theoretical predictions for the spectral, temporal and intensity signatures of the electromagnetic radiation emitted during the process of the gravitational collapse of a stellar core to a black hole, during which electromagnetic field strengths rise over the critical value for e^+e^- pair creation. The last phases of this gravitational collapse are studied, leading to the formation of a black hole with a subcritical electromagnetic field, likely with zero charge, and an outgoing pulse of initially optically thick e^+e^--photon plasma. Such a pulse reaches transparency at Lorentz gamma factors of 10^2–10^4. We find a clear signature in the outgoing electromagnetic signal, drifting from a soft to a hard spectrum, on very precise time-scales and with a very specific intensity modulation. The relevance of these theoretical results for the understanding of short gamma-ray bursts is outlined.

2. Federico Fraschetti, Remo Ruffini, Luca Vitagliano, and She-Sheng Xue, "Theoretical predictions of spectral evolution of short GRBs ", in Venice (Italy), June 5-9, 2006, IL NUOVO CIMENTO Vol. 121 (2006) 1477.

   We present the properties of spectrum of radiation emitted during gravitational collapse in which electromagnetic field strengths rise over the critical value for e^+e^- pair creation. A drift from soft to a hard energy and a high energy cut off have been found; a comparison with a pure black body spectrum is outlined.

3. R. Ruffini and S.-S. Xue, "Effective Lagrangian of QED", Journal of the Korean physical society, Vol. 49, No. 2, august 2006, pp. 715.

   From the Euler-Heisenberg formula we calculate the exact real part of the one-loop effective Lagrangian of Quantum Electrodynamics in a constant electromagnetic field, and determine its strong-field limit.

4. R. Ruffini and S.-S. Xue, "Electron-positron pairs production in an electric potential of massive cores", submitted to Phys. Lett. B (2008).

   Classical and semi-classical energy states of relativistic electrons bounded by a massive and charged core with the charge-mass-radio Q/M and macroscopic radius R_c are discussed. We show that the energies of semi-classical (bound) states can be much smaller than the negative electron mass-energy ($-mc^2$), and energy-level crossing to negative energy continuum occurs. Electron-positron pair production takes place by quantum tunneling, if these bound states are not occupied. Electrons fill into these bound states and positrons go to infinity. We explicitly calculate the rate of pair-production, and compare it with the rates of electron-positron production by the Sauter-Euler-Heisenberg-Schwinger in a constant electric field. In addition, the pair-production rate for the electro-gravitational balance ratio $Q/M = 10^{-19}$ is much larger than the pair-production rate due to the Hawking processes. We point out that in neutral cores with equal proton and electron numbers, the configuration of relativistic electrons in these semi-classical (bound) states should be stabilized by photon emissions.

5. C. Cherubini, A. Geralico, J. Rueda and R. Ruffini, "On the "Dyadotorus" of Kerr-Newman space time ", Phys. Rev. D 79 124002 (2009).

   We present the geometrical properties of the region where vacuum polarization precess occur int he Kerr-Newman space time. We find that the shape of the region can be ellipsoid-like or torus-like depending on the charge of the black hole.

6. H. Kleinert, R. Ruffini and S.-S. Xue, "Electron-positron pair-production in nonuniform electric fields", Phys. Rev. D 78 (2008) 025011.

   Treating the production of electron and positron pairs in vacuum as quantum tunneling, at the semiclassical level $O(\hbar)$, we derive a general expression, both exponential and pre-exponential factors, of the pair-production rate in nonuniform electric fields varying only in one direction. In particularly we discuss the expression for the case when produced electrons (or positrons) fill into bound states of electric potentials with discrete spectra of energy-level crossings. This expression is applied to the examples of the confined field $E(z) \neq 0, |z| \lesssim \ell$, half-confined field $E(z) \neq 0, z \gtrsim 0$, and linear increasing field $E(z) \sim z$, as well as the Coulomb field $E(r) = eZ/r^2$ for a nucleus with finite size $r_{\rm n}$ and large $Z \gg 1$.

7. R. Ruffini, G. V. Vereshchagin and S.-S. Xue, "Vacuum polarization and plasma oscillations", Phys. Lett. A 371(2007) 399 (arXiv:0706.4363).

   We evidence the existence of plasma oscillations of electrons-positron pairs created by the vacuum polarization in an uniform electric field with $E < E_c$. Our general treatment, encompassing also the traditional, well studied case of $E > E_c$, shows the existence in both cases of a maximum Lorentz factor acquired by electrons and positrons and allows determination of the a maximal length of oscillation. We quantitatively estimate how plasma oscillations reduce the rate of pair creation and increase the time scale of the pair production. These results are particularly relevant in view of the experimental progress in approaching the field strengths $E < E_c$.

8. A. G. Aksenov, R. Ruffini, and G. V. Vereshchagin, "Thermalization of a nonequilibrium electron-positron-photon plasma ", Phys.Rev.Lett. 99 (2007) 125003 .

   Starting from a nonequilibrium configuration we analyse the essential role of the direct and the inverse binary and triple interactions in reaching an asymptotic thermal equilibrium in a homogeneous isotropic electron-positron-photon plasma. We focus on energies in the range 0.1–10 MeV. We numerically integrate the integro-partial differential relativistic Boltzmann equation with the exact QED collisional integrals taking into account all binary and triple interactions in the plasma. We show that first, when detailed balance is reached for all binary interactions on a timescale $t_k \lesssim 10^{-14}$sec, photons and electron-positron pairs establish kinetic equilibrium. Successively, when triple interactions fulfill the detailed balance on a timescale $t_{eq} \lesssim 10^{-12}$sec, the plasma reaches thermal equilibrium. It is shown that neglecting the inverse triple interactions prevents reaching thermal equilibrium. Our results obtained in the theoretical physics domain also find application in astrophysics and cosmology.

9. R. Ruffini, M. Rotondo and S.-S. Xue, "Electrodynamics for Nuclear Matter in Bulk ", Int. Journ. Mod. Phys. D Vol. 16, No. 1 (2007) 1-9.

   A general approach to analyze the electrodynamics of nuclear matter in bulk is presented using the relativistic Thomas-Fermi equation generalizing to the case of $N \simeq (m_{\rm Planck}/m_n)^3$ nucleons of mass m_n the approach well tested in very heavy nuclei ($Z \simeq 10^6$). Particular attention is given to implement the condition of charge neutrality globally on the entire configuration, versus the one usually adopted on a microscopic scale. As the limit $N \simeq (m_{\rm Planck}/m_n)^3$ is approached the penetration of electrons inside the core increases and a relatively small tail of electrons persists leading to a significant electron density outside the core. Within a region of 10^2 electron Compton wavelength near the core surface electric fields close to the critical value for pair creation by vacuum polarization effect develop. These results can have important consequences on the understanding of physical process in neutron stars structures as well as on the initial conditions leading to the process of gravitational collapse to a black hole.

10. V. Popov, M. Rotondo, R. Ruffini and S.-S. Xue, "Analytic treatment of the electrodynamics for nuclear matter in bulk", to be submitted to Phy. Rev. C.

    Using the relativistic Thomas-Fermi equation, we present an analytic treatment of the electrodynamic properties of nuclear matter in bulk. Following the works of Migdal and Popov we generalize to the case of a massive core with the mass number $A \sim 10^{57}$ the analytic approach well tested in very heavy nuclei with $A \sim 10^6$. Attention is given to implement the condition of charge neutrality globally on the entire configuration, versus the one usually adopted on a microscopic scale. It is confirmed that also in this limit A, an electric field develops near the core surface of magnitude close to the critical value of vacuum polarization. It is shown that such a configuration is energetically favorable with respect to the one which obeys local charge neutrality. These results can have important consequences on the understanding of the physical process in neutron stars as well as on the initial conditions leading to the process of gravitational collapse to a black hole.

11. R. Ruffini, M. Rotondo and S.-S. Xue, "Neutral nuclear core vs super charged one ", in Proceedings of the Eleventh Marcel Grossmann Meeting, R. Jantzen, H. Kleinert, R. Ruffini (eds.), (World Scientific, Singapore, 2008).

    Based on the Thomas-Fermi approach, we describe and distinguish the electron distributions around extended nuclear cores: (i) in the case that cores are neutral for electrons bound by protons inside cores and proton and electron numbers are the same; (ii) in the case that super charged cores are bare, electrons (positrons) produced by vacuum polarization are bound by (fly into) cores (infinity).

12. R. Ruffini and S.-S. Xue, "Dyadosphere formed in gravitational collapse ", AIP Conf. Proc. 1059 (2008) 72.

    We first recall the concept of Dyadosphere (electron-positron-photon plasma around a formed black holes) and its motivation, and recall on (i) the Dirac process: annihilation of electron-positron pairs to photons; (ii) the Breit-Wheeler process: production of electron-positron pairs by photons with the energy larger than electron-positron mass threshold; the Sauter-Euler-Heisenberg effective Lagrangian and rate for the process of electron-positron production in a constant electric field. We present a general formula for the pair-production rate in the semi-classical treatment of quantum mechanical tunneling. We also present in the *Quantum Electro-Dynamics* framework, the calculations of the Schwinger rate and effective Lagrangian for constant electromagnetic fields. We give a review on the electron-positron plasma oscillation in constant electric fields, and its interaction with photons leading to energy and number equipartition of photons, electrons and positrons. The possibility of creating an overcritical field in astrophysical condition is pointed out. We present the discussions and calculations on (i) energy extraction from gravitational collapse; (ii) the formation of Dyadosphere in gravitational collapsing process, and (iii) its hydrodynamical expansion in Reissner Nordström geometry. We calculate the spectrum and flux of photon radiation at the point of transparency, and make predictions for short Gamma-Ray Bursts.

13. Jorge A. Rueda, Remo Ruffini, and S.-S. Xue, "On the electrostatic structure of neutron stars", to be published in the proceedings of the 1st Zel-

dovich meeting, Minsk-Belarus (2009).

We consider neutron stars composed by, (1) a core of degenerate neutrons, protons, and electrons above nuclear density; (2) an inner crust of nuclei in a gas of neutrons and electrons; and (3) an outer crust of nuclei in a gas of electrons. We use for the strong interaction model for the baryonic matter in the core an equation of state based on the phenomenological Weizsacker mass formula, and to determine the properties of the inner and the outer crust below nuclear saturation density we adopt the well–known equation of state of Baym–Bethe–Pethick. The integration of the Einstein–Maxwell equations is carried out under the constraints of β–equilibrium and global charge neutrality. We obtain baryon densities that sharply go to zero at nuclear density and electron densities matching smoothly the electron component of the crust. We show that a family of equilibrium configurations exists fulfilling overall neutrality and characterized by a non–trivial electrodynamical structure at the interface between the core and the crust. We find that the electric field is overcritical and that the thickness of the transition surface–shell separating core and crust is of the order of the electron Compton wavelength.

14. Jorge A. Rueda H., B. Patricelli, M. Rotondo, R. Ruffini, and S. S. Xue, "The Extended Nuclear Matter Model with Smooth Transition Surface", to be published in the Proceedings of The 3rd Stueckelberg Workshop on Relativistic Field Theories, Pescara-Italy (2008).

    The existence of electric fields close to their critical value $E_c = m_e^2 c^3/(e\hbar)$ has been proved for massive cores of 10^7 up to 10^{57} nucleons using a proton distribution of constant density and a sharp step function at its boundary. We explore the modifications of this effect by considering a smoother density profile with a proton distribution fulfilling a Woods-Saxon dependence. The occurrence of a critical field has been confirmed. We discuss how the location of the maximum of the electric field as well as its magnitude is modified by the smoother distribution.

15. B. Patricelli, M. Rotondo and R. Ruffini, "On the Charge to Mass Ratio of Neutron Cores and Heavy Nuclei", AIP Conference Proceedings, Vol. 966 (2008), pp. 143-146.

    We determine theoretically the relation between the total number of protons N_p and the mass number A (the charge to mass ratio) of nuclei and neutron cores with the model recently proposed by Ruffini et al. (2007) and we compare it with other N_p versus A relations: the empirical one, related to the Periodic Table, and the semi-empirical relation, obtained by minimizing the Weizsäcker mass formula. We find that there is a very good agreement between all the relations for values of A typical of nuclei, with differences of the order of per cent. Our relation and the semi-empirical one are in agreement up to $A \approx 10^4$ for higher values, we find that the two relations differ. We interpret the different behavior of our theoretical relation as a result of the penetration of electrons (initially confined in an external shell) inside the core, that becomes more and more important by increasing A; these effects are not taken into account in the semi-empirical mass-formula.

16. M. Rotondo, R. Ruffini and S.-S Xue, "On the Electrodynamical properties of Nuclear matter in bulk", AIP Conference Proceedings, Vol. 966 (2008), pp. 147-152.

    We analyze the properties of solutions of the relativistic Thomas-Fermi equation for globally neutral cores with radius of the order of $R \approx 10$ Km, at constant densities around the nuclear density. By using numerical tecniques as well as well tested analytic procedures developed in the study of heavy ions, we confirm the existence of an electric field close to the critical value $E_c = m_e^2 c^3/e\hbar$ in a shell $\Delta R \approx 10^4 \hbar/m_\pi c$ near the core surface. For a core of ≈ 10 Km the difference in binding energy reaches 10^{49} ergs. These results can be of interest for the understanding of very heavy nuclei as well as physics of neutron stars, their formation processes and further gravitational collapse to a black hole.

17. B. Patricelli, M. Rotondo, J. A. Rueda H. and R. Ruffini, "The Electrodynamics of the Core and the Crust components in Neutron Stars", AIP Conference Proceedings, Vol. 1059 (2008), pp. 68-71.

    We study the possibility of having a strong electric field (E) in Neutron Stars. We consider a system composed by a core of degenerate relativistic electrons, protons and neutrons, surrounded by an oppositely charged leptonic component and show that at the core surface it is possible to have values of E of the order of the critical value for electron-positron pair creation, depending on the mass density of the system. We also describe Neutron Stars in general relativity, considering a system composed by the core and an additional component: a crust of white dwarf - like material. We study the characteristics of the crust, in particular we calculate its mass M_{crust}. We propose that, when the mass density of the star increases, the core undergoes the process of gravitational collapse to a black hole, leaving the crust as a remnant; we compare M_{crust} with the mass of the baryonic remnant considered in the fireshell model of GRBs and find that their values are compatible.

18. R. Ruffini, "The Role of Thomas-Fermi approach in Neutron Star Matter", to be published in the Proceedings of the 9^{th} International Conference "Path Integrals - New trends and perspectives", Max Planck Institute for the Physics of Complex Systems, Dresden, Germany, Semptember 23 - 28 2007, World Scientific 207 - 218 (2008), eds. W. Janke and A. Pelster

    The role of the Thomas-Fermi approach in Neutron Star matter cores is presented and discussed with special attention to solutions globally neutral and not fulfilling the traditional condition of local charge neutrality. A new stable and energetically favorable configuration is found. This new solution can be of relevance in understanding unsolved issues of the gravitational collapse processes and their energetics.

19. W.-B. Han, R. Ruffini and S.-S. Xue, "Electron-positron pair oscillation in spatially inhomogeneous electric fields and radiation", Physics Letters B, Vol. 691 (2010), pp. 99-104.

    It is known that strong electric fields produce electron and positron pairs from the vacuum, and due to the back-reaction these pairs oscillate back and forth coherently with the alternating electric fields in time. We study this phenomenon in spatially inhomogeneous and bound electric fields by integrating the equations of energy-momentum and particle-number conservations and Maxwell equations. The space and time evolutions of the pair-induced electric field, electric charge- and current-densities are calculated. The results show non-vanishing electric charge-density and the propagation of pair-induced electric fields, that are different from the case of homogeneous and unbound electric fields. The space and time variations of pair-induced electric charges and currents emit an electromagnetic radiation. We obtain the narrow spectrum and intensity of this radiation, whose peak $\omega_{\rm peak}$ locates in the region around 4 keV for electric field strength $\sim E_c$. We discuss their relevances to both the laboratory experiments for electron and positron pair-productions and the astrophysical observations of compact stars with an electromagnetic structure.

6. Invited talks in international conferences

1. Yamada conference “On the dyadosphere of black holes” in Kyoto Japan, April 1998 .
2. International workshop on Gamma Ray Bursts, Rome (1998) .
3. 19th Texas Symposium, Dec. 1998
4. “Exploring the Universe”, a Festschrift in honour of Riccardo Giacconi, (2000).
5. Fluctuating Paths and Fields - Dedicated to Hagen Kleinert on the Occasion of His 60th Birthday, Berlin 2001.
6. The ESO workshop on “Black Holes in Binaries and Galactic Nuclei”, in honour of Prof. R. Giacconi, (2000) .
7. Marcel Grossmann Meetings IX (Rome) (2000), X (Brazil) (2003) and XI Berlin (2006).
8. International conference in the quantum aspect of beam physics in Hiroshima Japan (2003)
9. “Frontiers in Astroparticle Physics and Cosmology”, 6th RESCEU International Symposium, Tokyo 2003.
10. International Conference “Analysis, manifolds and geometric structures in physics”, in Honour of Y. Choquet-Bruhat, Isola d’Elba June 24th-26th, 2004 .
11. Brazilian School of Cosmology and Gravitation X (2002), XI (2004) and XII (2006) (Portobello, Brazile).
12. Relativistic Astrophysics and Cosmology - Einstein’s Legacy meeting, November 7-11, 2005,
13. 35th COSPAR scientific assembly (Paris, 2004) and 36th COSPAR scientific assembly (Beijing , 2006).

14. 9th International Conference Path Integrals - New Trends and Perspectives, DRESDEN, Germany 23 - 28 September 2007
15. APS April meeting, April 12-15 2008, Saint Louis (USA).
16. V Italian-Sino Workshop, May 28- June 1 2008, Taipei (Taiwan).
17. III Stueckelberg Workshop, July 8-18 2008, Pescara (Italy).
18. XIII Brazilian School of Cosmology and Gravitation, July 20-August 2 2008, Rio de Janeiro (Brazil).
19. Path Integrals - New Trends and Perspectives, September 23 - 28 2007, Dresden (Germany)
20. APS April meeting, April 14-17 2007, Jacksonville (USA).
21. The first Sobral Meeting, May 26-29, 2009 Fortaleza (Cear) Brazi
22. Zeldovich Meeting, April 20-23, 2009 Minsk - (BELARUS).
23. XI Marcel Grossmann Meeting on General Relativity, July 23-29 2006, Berlin (Germany).
24. The first Galileo - Xu Guangqi Meeting October 26-30, 2009 - Shanghai - (CHINA).
25. 11th Italian-Korean Meeting November 2-4, 2009 - Seoul - (KOREA).
26. Christchurch Meeting December 16-18, 2009 - Christchurch - (New Zealand).
27. Annual Meeting of the Korean Physical Society October 19-22, 2010 - Seoul - (KOREA).

7. APPENDICES

A. Electron-positron pair oscillation in spatially inhomogeneous electric fields and radiation

Introduction. As reviewed in the recent report Ruffini et al. (2010), since the pioneer works by Sauter Sauter (1931a), Heisenberg and Euler Heisenberg and Euler (1936) in 1930's, then by Schwinger Schwinger (1951) in 1950's, it has been well known that positron-electron pairs are produced from the vacuum in external electric fields. In a constant electric field E_0 in dependent of space and time, the pair-creation rate per unit volume is given by Heisenberg and Euler (1936),

$$S \equiv \frac{dN}{dVdt} = \frac{m_e^4}{4\pi^3}\left(\frac{E_0}{E_c}\right)^2 \exp\left(-\pi\frac{E_c}{E_0}\right), \tag{A.0.1}$$

where the critical field $E_c \equiv m_e^2c^3/(e\hbar)$, the Plank's constant $\hbar$, the speed of light c, the electron mass m_e, the absolute value of electron charge e and the fine structure constant $\alpha = e^2/\hbar c$ (in this article we use the natural units $\hbar = c = 1$, unless otherwise specified). The pair-production rate (A.0.1) is significantly large for strong electric fields $E \gtrsim E_c \simeq 1.3 \cdot 10^{16}$V/cm. The critical field will probably be reached by recent advanced laser technologies in laboratory experiments Ringwald (2001); Tajima and Mourou (2002); Gordienko et al. (2005), X-ray free electron laser (XFEL) facilities XFE, optical high-intensity laser facilities such as Vulcan or ELI ELI, and SLAC E144 using nonlinear Compton scattering Burke et al. (1997). On the other hand, strong overcritical electric fields ($E \geq 10E_c$) can be created in astrophysical environments, for instance, quark stars Usov (1998); Usov et al. (2005) and neutron stars Ruffini et al. (2007a)-Popov et al. (2009).

The back-reaction and screening effects of electron and positron pairs on external electric fields lead to the phenomenon of plasma oscillations: electrons and positrons moving back and forth coherently with alternating electric fields. This means that external electric fields are not eliminated within the Compton time $\hbar/m_ec^2$ of pair-production process, rather oscillate collectively with the motion of pairs in a much longer timescale.

In a constant electric field E_0 (A.0.1), the phenomenon of plasma oscillations is studied in the two frameworks Ruffini et al. (2010): (1) the semi-classical QED with quantized Dirac field and classical electric field Kluger et al. (1991, 1992); Cooper and Mottola (1989); (2) the kinetic description using the Boltzmann-Vlasov and Maxwell equations Biro et al. (1984a); Gatoff et al. (1987); Cooper et al. (1993); Ruffini et al. (2003b, 2007b). In the second framework, the Boltzmann-Vlasov equation is used to obtain the equations for the continuity and energy-momentum conservations Gatoff et al. (1987).

Ref. Ruffini et al. (2007b) shows the evidence of plasma oscillation in undercritical field ($E < E_c$) and the relation between the kinetic energy and numbers of oscillating pairs in a given electric field strength E_0. Taking into account the creation and annihilation process $e^+ + e^- \Leftrightarrow \gamma + \gamma$, it is shown Ruffini et al. (2003b) that the plasma oscillation in an overcritical field is led to a plasma of photons, electrons and positions with the equipartition of their number- and energy-densities. The phenomenon of plasma oscillations is studied in connection with pair creation in heavy ions collisions Biro et al. (1984a)-Cooper et al. (1993), the laser field Ringwald (2001)-Hebenstreit et al. (2008), and gravitational collapse Ruffini et al. (2003a). It is worthwhile to emphasize that the plasma oscillation occurs not only at overcritical field-strengths $E_0 \gtrsim E_c$ (see for instance Refs. Kluger et al. (1991, 1992); Ruffini et al. (2003b)), but also undercritical field-strengths $E_0 \lesssim E_c$ (see Ref. Ruffini et al. (2007b)), and plasma oscillation frequency is related to field-strength E_0, while the number of oscillating pairs depends on the pair-production rate (A.0.1). More details can be found in the recent review article Ruffini et al. (2010).

The realistic ultra-strong electric fields are not only vary with space and time, but also confined in a finite region. In this letter, studying the plasma oscillations in spatially inhomogeneous electric field, we present the evidence of electric fields propagation, leading to electromagnetic radiation with a peculiar narrow spectrum in the keV-region, which should be distinctive and experimentally observable.

In the kinetic description for the plasma fluids of positrons ($+$) or electrons ($-$), whose single-particle spectrum $p^0_\pm = (\mathbf{p}^2_\pm + m_e^2)^{1/2}$, we define the number-densities $n_\pm(t, \mathbf{x})$ and "averaged" velocities $\mathbf{v}_\pm(t, \mathbf{x})$ of the fluids:

$$n_\pm(t, \mathbf{x}) \equiv \int \frac{d^3\mathbf{p}_\pm}{(2\pi)^3} f_\pm(t, \mathbf{p}_\pm, \mathbf{x}), \tag{A.0.2}$$

$$\mathbf{v}_\pm(t, \mathbf{x}) \equiv \frac{1}{n_\pm} \int \frac{d^3\mathbf{p}_\pm}{(2\pi)^3} \left(\frac{\mathbf{p}_\pm}{p^0_\pm}\right) f_\pm(t, \mathbf{p}_\pm, \mathbf{x}), \tag{A.0.3}$$

where $f_\pm(t, \mathbf{p}_\pm, \mathbf{x})$ is the distribution function in the phase space. The four-velocities of the electron and positron fluids $U^\mu_\pm = \gamma_\pm(1, \mathbf{v}_\pm)$, the Lorentz factor $\gamma_\pm = (1 - |\mathbf{v}_\pm|^2)^{-1/2}$, and the comoving number-densities $\bar{n}_\pm = n_\pm(\gamma_\pm)^{-1}$, where we choose the laboratory frame where pairs are created at rest. The collision-less plasma fluid of electrons and positrons coupling to electromagnetic fields is governed by the continuity, energy-momentum conservation and Maxwell equations:

$$\frac{\partial\left(\bar{n}_\pm U^\mu_\pm\right)}{\partial x^\mu} = S, \tag{A.0.4}$$

$$\frac{\partial T^{\mu\nu}_\pm}{\partial x^\nu} = -F^\mu_\sigma(J^\sigma_\pm + J^\sigma_{\pm\text{pola}}), \tag{A.0.5}$$

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = -4\pi(J^\mu_{\text{cond}} + J^\mu_{\text{pola}} + J^\mu_{\text{ext}}), \tag{A.0.6}$$

where is the pair-production rate, $J^\mu_\pm = \pm e\bar{n}_\pm U^\mu_\pm$ electric currents and the energy-momentum tensors Weinberg (1972)

$$T^{\mu\nu}_\pm = \bar{p}_\pm g^{\mu\nu} + (\bar{p}_\pm + \bar{\epsilon}_\pm)U^\mu_\pm U^\nu_\pm, \tag{A.0.7}$$

and the pressure $\bar{p}_\pm$ and comoving energy-density $\bar{\epsilon}_\pm$ is related by the equation of state, in general $0 \leq \bar{p}_\pm \leq \bar{\epsilon}_\pm/3$. In the laboratory frame, the fluid energy-density $\epsilon_\pm \equiv T^{00}$ and momentum-density $p^i_\pm \equiv T^{i0}$ are given by

$$\epsilon_\pm = (\bar{\epsilon}_\pm + \bar{p}_\pm \mathbf{v}^2_\pm)\gamma^2_\pm, \quad \mathbf{p}_\pm = (\bar{\epsilon}_\pm + \bar{p}_\pm)\gamma^2_\pm \mathbf{v}_\pm. \tag{A.0.8}$$

In Eqs. (A.0.5,B.0.5) F^μ_σ is the tensor of electromagnetic fields ($\mathbf{E}, \mathbf{B}$), the conducting four-current density

$$J^\mu_{\text{cond}} \equiv e(\bar{n}_+ U^\mu_+ - \bar{n}_- U^\mu_-), \quad \partial_\mu J^\mu_{\text{cond}} = 0, \tag{A.0.9}$$

and polarized four-current density $J^\mu_{\text{pola}} = \sum_\pm J^\mu_{\pm\text{pola}}$ and $J^\mu_{\pm\text{pola}} = \left(\rho^\pm_{\text{pola}}, \mathbf{J}^\pm_{\text{pola}}\right)$ Gatoff et al. (1987); Kajantie and Matsui (1985)

$$F^\nu_\mu J^\mu_{\pm\text{pola}} = \Sigma^\nu_\pm, \quad \Sigma^\nu_\pm \equiv \int \frac{d^3\mathbf{p}_\pm}{(2\pi)^3 p^0_\pm} p^\nu_\pm \mathcal{S}, \tag{A.0.10}$$

and $S = \int d^3\mathbf{p}_\pm/[(2\pi)^3 p^0_\pm]\mathcal{S}$. Using "averaged" velocities (A.0.3) of the fluids, we approximately have

$$\mathbf{J}^\pm_{\text{pola}} \simeq \frac{m_e\gamma_\pm S}{|\mathbf{E}|}\hat{\mathbf{E}}, \quad \rho^\pm_{\text{pola}} \simeq \pm\frac{m_e\gamma_\pm|\mathbf{v}_\pm|S}{|\mathbf{E}|}, \tag{A.0.11}$$

where the magnetic field $\mathbf{B} = 0$. In Eq. (B.0.5), $J^\mu_{\text{ext}} = (\rho_{\text{ext}}, \mathbf{J}_{\text{ext}})$ is an external electric current.

Basic equations of motion. For simplicity to start with, we consider the electric field $\mathbf{E}_{\text{ext}}$ created by a capacitor made of two parallel plates, one carries an external charge $+Q$ and another $-Q$. The sizes of two parallel plats are L_x and

L_y, which are much larger than their separation ℓ in the $\hat{\mathbf{z}}$-direction, i.e., $L_x \gg \ell$ and $L_y \gg \ell$. For $|z| \sim \mathcal{O}(\ell)$, the system has an approximate translation symmetry in the (x, y) plane. As results the electric field $\mathbf{E}_{\rm ext}(x, y, z) \approx E_{\rm ext}(z)\hat{\mathbf{z}}$ and $\mathbf{B}_{\rm ext}(x, y, z) \approx 0$, is approximately homogeneous in the (x, y) plane and confined within the capacitor. In addition, $\partial \mathbf{E}_{\rm ext}/\partial t \approx 0$, namely, this electric field is assumed to be continuously supplied by an external source $(+Q, -Q)$ or slowly varying.

In order to do calculations we model this electric field as the one-dimensional Sauter electric field in the $\hat{\mathbf{z}}$-direction

$$E_{\rm ext}(z) = E_0/\cosh^2(z/\ell), \quad \sigma \equiv eE_0\ell/m_e c^2 = (\ell/\lambda_C)(E_0/E_c), \tag{A.0.12}$$

where the λ_C is Compton wavelength, the external electric charge is given by $\partial E_{\rm ext}(z)/\partial z = 4\pi\rho_{\rm ext}$ and the external electric current vanishes $J_{\rm ext} = 0$ for the field being static $\partial E_{\rm ext}/\partial t = 0$. In the electric field configuration (A.0.12) and $\mathbf{B} \approx 0$, the "averaged" velocities $v_\pm$ of electrons and positrons fluids are in the $\hat{\mathbf{z}}$-direction,

$$U^\mu_\pm = \gamma_\pm\,(1, 0, 0, \pm v_\pm)\,, \tag{A.0.13}$$

and the total fluid current- and charge-densities (B.0.5) $J^\mu = (\rho, \mathbf{J})$ are

$$J_z = en_+v_+ + en_-v_- + \frac{m_e(\gamma_+ + \gamma_-)S}{E}, \tag{A.0.14}$$

$$\rho = e\,(n_+ - n_-) + \frac{m_e(\gamma_+v_+ - \gamma_-v_-)S}{E}. \tag{A.0.15}$$

The system can be approximately treated as a $1+1$ dimensional system in terms of space-time variables (z, t), and Eqs. (A.0.4-B.0.5) become for zero pressure [1],

$$\frac{\partial n_\pm}{\partial t} \pm \frac{\partial n_\pm v_\pm}{\partial z} = S, \tag{A.0.16}$$

$$\frac{\partial \epsilon_\pm}{\partial t} \pm \frac{\partial p_\pm}{\partial z} = en_\pm v_\pm E + m_e\gamma_\pm S, \tag{A.0.17}$$

$$\frac{\partial p_\pm}{\partial t} \pm \frac{\partial p_\pm v_\pm}{\partial z} = en_\pm E + m_e\gamma_\pm v_\pm S, \tag{A.0.18}$$

$$\frac{\partial E}{\partial t} = -4\pi J_z, \tag{A.0.19}$$

$$\frac{\partial E}{\partial z} = 4\pi(\rho + \rho_{\rm ext}). \tag{A.0.20}$$

The total electric field $E(z,t)$ in Eqs. (A.0.14-A.0.20) is the superposition of two components:

$$E(z,t) = E_{\rm ext}(z) + E_{\rm ind}(z,t), \tag{A.0.21}$$

where the space- and time-dependent $E_{\rm ind}(z,t)$ is the electric field created by electron and positron pairs. We call $J_z(z,t)$ (A.0.14), $\rho(z,t)$ (A.0.15) and $E_{\rm ind}(z,t)$ pair-induced electric current, charge and field.

As for the pair-production rate S in Eqs. (A.0.16-A.0.19), instead of the pair-production rate (A.0.1) for a constant field E_0, we adopt the following z-dependent formula for the pair-production rate in the Sauter field (A.0.12), obtained by using the WKB-method to calculate the probability of quantum-mechanical tunneling Kleinert et al. (2008),

$$S(z) = \frac{m_e^4}{4\pi^3}\frac{E_0E(z)}{E_c^2\tilde{G}[0,\mathcal{E}]}e^{-\pi G[0,\mathcal{E}]E_c/E_0}, \tag{A.0.22}$$

where $G(0,\mathcal{E})$ and $\tilde{G}(0,\mathcal{E})$ are functions of the energy-level crossings $\mathcal{E}(z)$ and we approximately adopt $E(z) \approx E_0/G(0,\mathcal{E}) \approx E_0/\tilde{G}(0,\mathcal{E})$ in Eq. (A.0.22) in order to do feasible numerical calculations. As shown by the Fig. 2 in Ref. Kleinert et al. (2008), the deviation of the pair-production rate (A.0.22) due to this approximation is small. The formula (A.0.22) is derived for the static Sauter field (A.0.12). However, analogously to the discussions for

[1] For an electric field $E \sim E_c$, the number-density of electron-positron pairs is small and the pressure of pairs can be neglected. While for an over electric field $E \gg E_c$, the number-density of pairs is large and the collisions and annihilation of pairs into photons are important, leading to the energy equipartition of electron, positrons and photons. In this case, the pressure, effective temperature and equation of state have to be considered.For an electric field $E \sim E_c$, the number-density of electron-positron pairs is small and the pressure of pairs can be neglected. While for an over electric field $E \gg E_c$, the number-density of pairs is large and the collisions and annihilation of pairs into photons are important, leading to the energy equipartition of electron, positrons and photons. In this case, the pressure, effective temperature and equation of state have to be considered.

the plasma oscillations in spatially homogeneous fields Cooper et al. (1993)-Ruffini et al. (2007b), it can be approximately used for a time-varying electric field $E(z,t)$ (A.0.21), provided the time-dependent component $E_{\rm ind}(z,t)$, created by electron-positron pair-oscillations, varies much slowly compared with the rate of electron-positron pair-productions $\mathcal{O}(m_e c^2/\hbar)$. This can be justified by the inverse adiabaticity parameter Greiner et al. (1985)-Popov (1973),

$$\eta = \frac{m_e}{\omega}\frac{E_0}{E_c} \gg 1, \tag{A.0.23}$$

where ω is the frequency of pair-oscillations.

Eqs. (A.0.16,A.0.17,A.0.18) describe the motion of electron-positron plasma coupling to the electric field E and source S of pair-productions. The Maxwell equations (A.0.19,A.0.20) describe the motion of the electric field (A.0.21) coupled to the current- and charge-densities (A.0.15), leading to the wave equation of the propagating electric field $E_{\rm ind}(z,t)$ Jackson (1998),

$$\frac{\partial^2 E_{\rm ind}}{\partial t^2} - \frac{1}{c^2}\frac{\partial^2 E_{\rm ind}}{\partial z^2} = 4\pi\left(\frac{\partial \rho}{\partial z} + \frac{1}{c^2}\frac{\partial J_z}{\partial t}\right), \tag{A.0.24}$$

where we use $\partial E_{\rm ext}/\partial z = 4\pi\rho_{\rm ext}$ and $\partial E_{\rm ext}/\partial t = 0$. This wave equation shows the propagating electric field $E_{\rm ind}(z,t)$ in the region $\mathcal{R}$ where the non-vanishing current J_z and charge ρ are, and both the propagation and polarization of the electric field are in the $\hat{\mathbf{z}}$-direction. This implies a wave transportation of electromagnetic energies inside the region $\mathcal{R}$. Since the current- and charge-densities (ρ, J_z) are functions of the field $E(t,z)$ (A.0.21), the wave equation is highly nonlinear, the dispersion relation of the field is very complex and the velocity of field-propagation is not the speed of light.

Numerical integrations. Given the parameters $E_0 = E_c$ and $\ell = 10^5\lambda_C$ of the Sauter field (A.0.12) as an initial electric field $E_{\rm ext}$, we numerically integrate Eqs.(A.0.16-A.0.19) in the spatial region $\mathcal{R}$: $-\ell/2 \le z \le \ell/2$ and time interval $\mathcal{T}$: $0 \le t \le 3500\tau_C$, where τ_C is the Compton time. The value $\mathcal{T} \le 3500\tau_C$ is chosen so that the adiabatic condition (A.0.23) is satisfied, and the spatial range $\mathcal{R}$ is determined by the capacity of computer for numerical calculations. The electric field strength E_0 is chosen around the critical value E_c, so that the semiclassical pair-production rate (A.0.22) can be approximately used. Actually, E_0, ℓ and $\mathcal{T}$ are attributed to the characteristics of external ultra-strong electric fields $E_{\rm ext}$ established by either experimental setups or astrophysical conditions.

In Figs. A.1 and A.2, we respectively plot the time- and space-evolution of the total electric fields $E(z,t)$ (A.0.21) as functions of t and z at three different spatial points and times. As discussed in Figure captions, numerical results show the properties of the electric field wave $E_{\rm ind}(z,t)$ propagating in the plasma of oscillating electron-positron pairs, as described by the wave equation (A.0.24). This electric field wave propagates along the directions in which external electric field-strength decreases. The wave propagation is rather complex, depending on the space and time variations of the net charge density $\rho(z,t)$ and current density $j_z(z,t)$, as shown in Figs. A.4-A.5. The net charge density ρ oscillates (see Figs. A.3 and A.4) proportionally to the field-gradient (A.0.20) and at the center $z = 0$ the charge density and field-gradient are zero independent of time evolution (see Fig. A.4). However, the total charge of pairs $Q = \int_{\mathcal{R}} d^3x\rho$ must be zero at any time, as required by the neutrality. The electric current $j_z(z,t)$ alternating in space and time follows the space and time evolution of the electric field $E(z,t)$ see Eq. (A.0.19), as shown in Figs. A.5 and A.6.

We recall the discussions of the plasma oscillations in the case of spatially homogeneous electric field E_0 without boundary Ruffini et al. (2003b, 2007b). Due to the spatial homogeneity of electric fields and pair-production rate S (A.0.1), the number-densities $n_\pm(t,\mathbf{x}) = n(t)$ (A.0.2), "averaged" velocities $|\mathbf{v}_\pm(t,\mathbf{x})| = v(t)$ (A.0.3) and energy-momenta $\epsilon_\pm(t,\mathbf{x}) = \epsilon(t), |\mathbf{p}_\pm(t,\mathbf{x})| = p(t)$ (A.0.8) are spatially homogeneous so that the charge density (A.0.15) $\rho \equiv 0$ identically vanishes and current (A.0.14) $J_z = J_z(t)$. All spatial derivative terms in Eqs. (A.0.16-A.0.18) and Eq. (A.0.24) vanish and Eq. (A.0.20) becomes irrelevant. As results, the plasma oscillations described is the oscillations of electric fields and currents with respect time at each spatial point, and the electric field has no any spatial correlation and does not propagate.

In contrary to the plasma oscillation in homogeneous fields, the presence of such field-propagation in inhomogeneous fields is due to: (i) non-vanishing field-gradient $\partial_z E$ (A.0.20) and net charge-density ρ (A.0.15), as shown in Figs. A.3 and A.4, give the spatial correlations of the fields at neighboring points; (ii) the stronger field-strength, the larger field-oscillation frequency is, as shown in Fig. A.1; (iii) at the center $z = 0$ the field-strength is largest and the field-oscillation is most rapid, and the field-oscillations at points $|z| > 0$ are slower and in retard phases, as shown in Fig. A.2. The point (i) is essential, the charge density ρ oscillates (see Figs. A.3 and A.4) proportionally to the field-gradient Eq. (A.0.20) and at the center $z = 0$ the charge density and field-gradient are zero independent of time evolution (see Fig. A.4). Such field-propagation is reminiscent of the drift motion of particles driven by a field-gradient ("ponderomotive") force, which is a cycle-averaged force on a charged particle in a spatially inhomogeneous oscillating electromagnetic field Boot and R.-S.-Harvie (1957); Kibble (1966); Hopf et al. (1976).

Radiation fields. As numerically shown in Fig. A.1-A.6, the propagation of the electric field wave $E_{\text{ind}}(z,t)$ inside the region $\mathcal{R}$ is rather complex, due to th high non-linearity of wave equation (A.0.24). Nevertheless, the electromagnetic radiation fields $\mathbf{E}_{\text{rad}}$ and $\mathbf{B}_{\text{rad}}$ far away from the region $\mathcal{R}$ are completely determined and could be experimentally observable. At the space-time point $(t,\mathbf{x})$ of an observer, the electromagnetic radiation fields $\mathbf{E}_{\text{rad}}(z,t)$ and $\mathbf{B}_{\text{rad}}(z,t)$, emitted by the variations of electric charge density $\rho(\mathbf{x}',t')$ and current-density $\mathbf{J}(\mathbf{x}',t')$ in the region $\mathcal{R}$ ($\mathbf{x}' \in \mathcal{R}$) and time t' ($t' \in \mathcal{T}$), are given

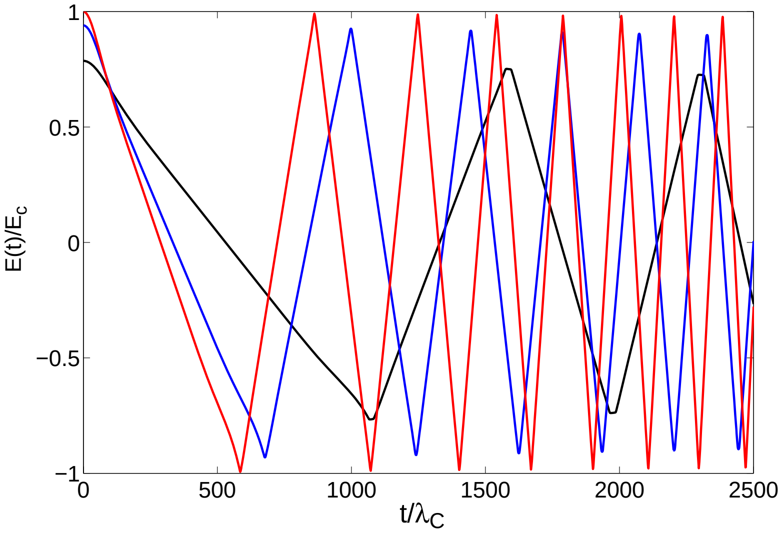

Figure A.1.: Electric fields $E(z,t)$ are plotted as functions of t at three different points: $z = 0$ (red), $z = \ell/4$ (blue) and $z = \ell/2$ (black). Analogously to the plasma oscillation in homogeneous fields, the stronger initial field-strength, the larger field-oscillation frequency is, i.e., $\omega(z=0) > \omega(z=\ell/4) > \omega(z=\ell/2)$, where $\omega(z)$ is the field oscillating frequency at the spatial point z.

by Jackson (1998)

$$\mathbf{E}_{\rm rad}(t,\mathbf{x}) = -\int_{\mathcal{R}} d^3\mathbf{x}'\Big\{\frac{\hat{\mathbf{R}}}{R^2}\left[\rho(t',\mathbf{x}')\right]_{\rm ret} + \frac{\hat{\mathbf{R}}}{cR}\left[\frac{\partial\rho(t',\mathbf{x}')}{\partial t'}\right]_{\rm ret} + \frac{1}{c^2R}\left[\frac{\partial\mathbf{J}(t',\mathbf{x}')}{\partial t'}\right]_{\rm ret}\Big\}, \tag{A.0.25}$$

$$\mathbf{B}_{\rm rad}(t,\mathbf{x}) = \int_{\mathcal{R}} d^3\mathbf{x}'\left\{\left[\mathbf{J}(t',\mathbf{x}')\right]_{\rm ret}\times\frac{\hat{\mathbf{R}}}{cR^2} + \left[\frac{\partial\mathbf{J}(t',\mathbf{x}')}{\partial t'}\right]_{\rm ret}\times\frac{\hat{\mathbf{R}}}{c^2R}\right\}. \tag{A.0.26}$$

where the subscript "ret" indicates $t' = t - R/c$, $R = |\mathbf{x}-\mathbf{x}'|$. In the radiation zone $|\mathbf{x}| \gg |\mathbf{x}'|$ and $R \approx |\mathbf{x}|$, where is far away from the plasma oscillation region $\mathcal{R}$, the radiation fields (A.0.25,A.0.26) approximately are

$$\mathbf{E}_{\rm rad}(t,\mathbf{x}) \approx -\frac{1}{c^2|\mathbf{x}|}\int d^3\mathbf{x}'\left[\frac{\partial\mathbf{J}(t',\mathbf{x}')}{\partial t'}\right]_{\rm ret}, \tag{A.0.27}$$

$$\mathbf{B}_{\rm rad}(t,\mathbf{x}) \approx \hat{\mathbf{R}}\times\mathbf{E}_{\rm rad}(t,\mathbf{x}), \tag{A.0.28}$$

where we use the charge conservation (A.0.9) and total neutrality condition of pairs $\int_{\mathcal{R}} d^3\mathbf{x}'\rho(t',\mathbf{x}') = 0$. The first terms in Eqs. (A.0.25,A.0.26) are the Coulomb-type fields decaying away as $\mathcal{O}(1/|\mathbf{x}|^2)$. The Fourier transforms of

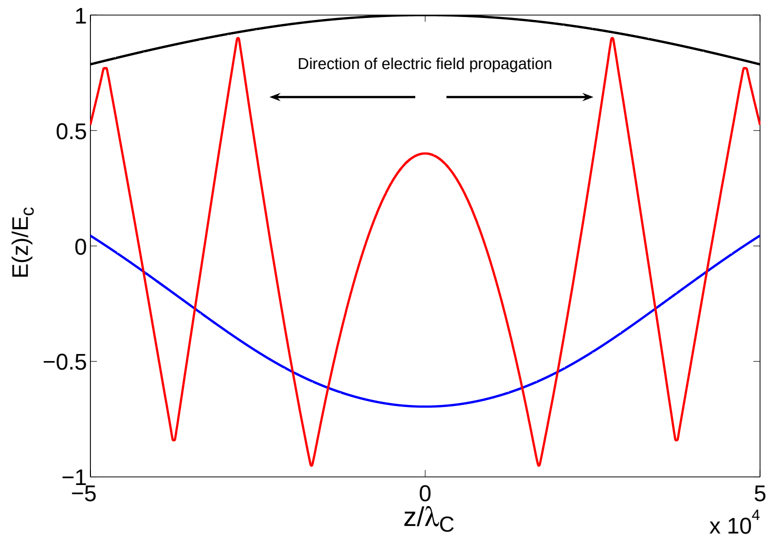

Figure A.2.: Electric fields $E(z,t)$ are plotted as functions of z at three different times in the Compton unit: $t = 1$(black), $t = 500$ (blue) and $t = 1500$ (red). As shown in Fig. A.1, the electric field $E(z,t)$ oscillation at the center ($z = 0$) is most rapid, and gets slower and slower at spatial points ($|z| > 0$) further away from the center. This implies the electric field wave propagating in the space, and the directions of propagations are indicated.

Eqs. (A.0.27) and (A.0.28) are

$$\tilde{\mathbf{E}}_{\rm rad}(\omega,\mathbf{x}) \approx -\frac{e^{-ik|\mathbf{x}|}}{c^2|\mathbf{x}|}\tilde{\mathbf{D}}(\omega), \quad \tilde{\mathbf{B}}_{\rm rad}(\omega,\mathbf{x}) \approx \hat{\mathbf{R}} \times \tilde{\mathbf{E}}_{\rm rad}(\omega,\mathbf{x}) \tag{A.0.29}$$

$$\tilde{\mathbf{D}}(\omega) \equiv \int_{\mathcal{R}} d^3\mathbf{x}' \int_{\mathcal{T}} dt' e^{i\omega t'} \left[\frac{\partial \mathbf{J}(t',\mathbf{x}')}{\partial t'}\right], \tag{A.0.30}$$

where the wave number $k = \omega/c$ and the numerical integration (A.0.30) is carried out overall the space-time evolution of the electric current $\mathbf{J}(\mathbf{x}',t')$ (see Figs. A.6 and A.5). For definiteness we thinks of the oscillation currents occurring for some finite interval of time $\mathcal{T}$ or at least falling off for remote past and future times, so that the total energy radiated is finite, thus the energy radiated per unit solid angle per frequency interval is given by Jackson (1998)

$$\frac{d^2 I}{d\omega d\Omega} = 2|\tilde{\mathbf{D}}(\omega)|^2. \tag{A.0.31}$$

The squared amplitude $|\tilde{\mathbf{D}}(\omega)|^2$ as a function of ω gives the spectrum of the radiation (see Fig. A.7), which is very narrow as expected with a peak locating at $\omega_{\rm peak} \approx 0.08 m_e = 4$keV for $E_0 = E_c$, consistently with the plasma

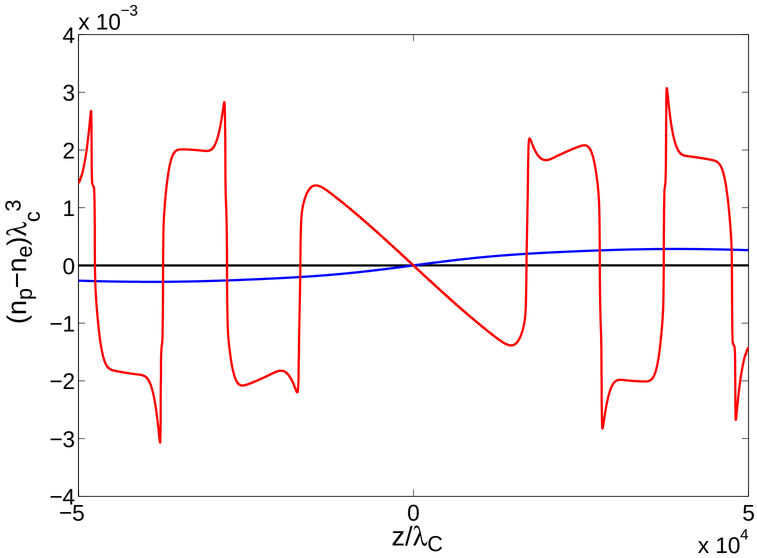

Figure A.3.: The net charge density $\rho(z,t)$ [see Eq. (A.0.15)] as a function of z at three different times: $t = 1$ (black, nearly zero), $t = 500$ (blue) and $t = 1500$ (red). It is shown that the net charged density value $|\rho(z,t)|$ is zero at the center where the initial electric field gradient vanishes [see Eq. (A.0.20)], whereas it increases as the initial electric field gradient increases for $|z| > 0$.

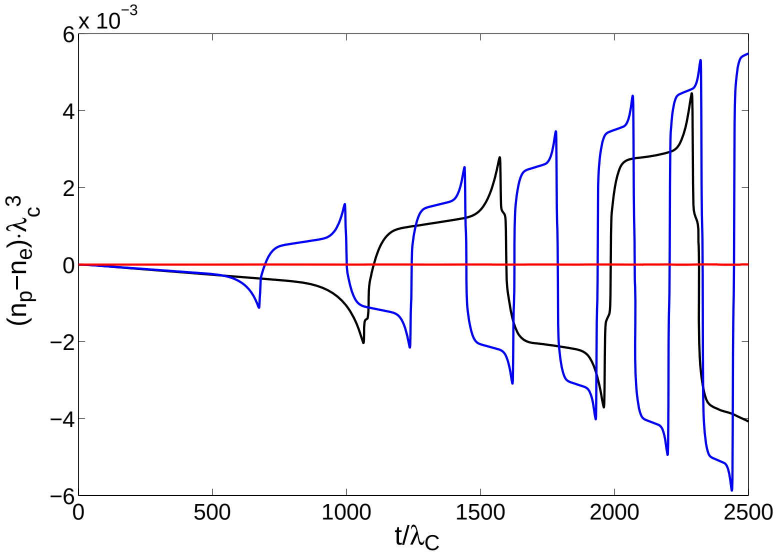

Figure A.4.: The net electric charge density $\rho(z,t)$ [see Eq. (A.0.15)] as a function of t at three different points: $z = 0$ (red, nearly zero) , $z = \ell/4$ (blue) and $z = \ell/2$ (black). It is shown that the net electric charge density $\rho(z,t)$ (except the center $z = 0$) increases as time.

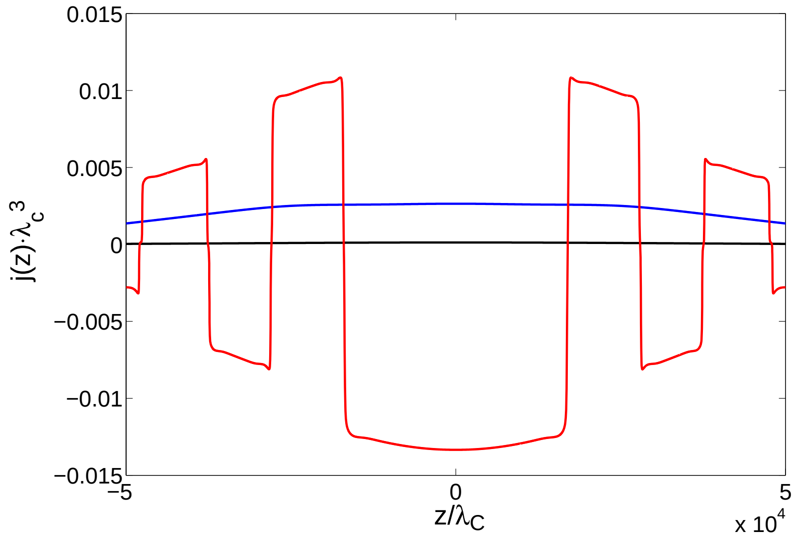

Figure A.5.: Electric current densities $j_z(z,t)$ [see Eq. (A.0.14)] as functions of z at three different times: $t = 1$ (black), $t = 500$ (blue) and $t = 1500$ (red). Following Eq. (A.0.19), the electric current alternates following the alternating electric field (see Fig. A.1), the plateaus indicate the current saturation for $v \sim c$ and its spatial distribution is determined by the initial electric field $E_{\rm ext}(z)$.

oscillation frequency (see Fig. A.1). The energy-spectrum and its peak are shifted to high-energies as the initial electric field-strength increases, and the relation between the spectrum peak location and the electric field-strength is shown in Fig. A.8. In addition, the energy-spectrum and its peak are also shifted to high-energies as the temporary duration $\mathcal{T}$ of plasma oscillations increases (see Fig. A.1). In calculations, the temporary duration $\mathcal{T} = 3500\tau_C$ is chosen, not only to satisfy the adiabaticity condition Eq. (A.0.23) [2], but also to be in the time duration when the oscillatory behavior is distinctive (see Figs. A.1,A.4,A.6), since the oscillations of pair-induced currents damp and pairs annihilate into photons Ruffini et al. (2003b). The radiation intensity (A.0.31) depends on the strength, spatial dimension and temporal duration of strong external electric fields, created by either experimental setups or astrophysical conditions.

Conclusions and remarks. We show the space and time evolutions of pair-induced electric charges, currents and fields in strong external electric fields bounded within a spatial region. These results imply the wave propagation

[2] We check the two cases $E_0 = E_c$ and $E_0 = 10E_c$, and find for the first oscillation $\eta = 865$ and $\eta = 487$ respectively. As can be seen for the Fig. A.1 the frequencies ω of pair-oscillations increase with time which means the parameter η becoming smaller. Eventually it may reach unity so the formula (A.0.22) becomes inapplicable.

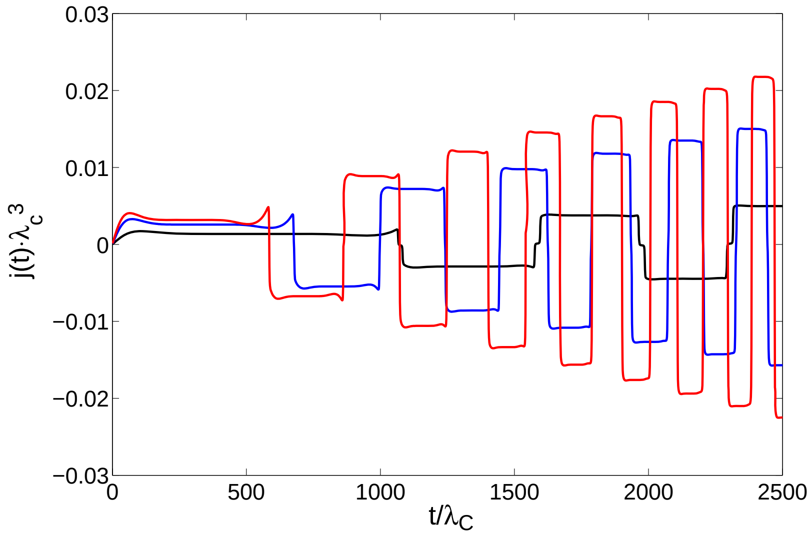

Figure A.6.: Electric current densities $j_z(z,t)$ [see Eq. (A.0.14)] as functions of t at three different points: $z = \ell/2$ (black), $z = \ell/4$ (blue) and $z = 0$ (red). The plateaus (see also Fig. A.6) for the current saturation values increases as time, mainly due to the number-densities $n_\pm$ of electron-positron pairs increase with time. In addition, they are maximal at the center $z = 0$ where the initial electric field is maximal, and decrease as the initial electric field $E_{\rm ext}(z)$ decreasing for $|z| > 0$.

of the pair-induced electric field and wave-transportation of the electromagnetic energy in the strong field region. Analogously to the electromagnetic radiation emitted from an alternating electric current, the space and time variations of pair-induced electric currents and charges emit an electromagnetic radiation. We show that this radiation has a the peculiar energy-spectrum (see Fig. A.7) that is clearly distinguishable from the energy-spectra of the bremsstrahlung radiation, electron-positron annihilation and other possible background events. This possibly provides a distinctive way to detect the radiative signatures for the production and oscillation of electron-positron pairs in ultra-strong electric fields that can be realized in either ground laboratories or astrophysical environments.

As mentioned in introduction, the critical electric field E_c will be reached soon in ground laboratories and sensible methods to detect signatures of pair-productions become important. Recently, the momentum signatures of pair-production is found Hebenstreit et al. (2009) in a time-varying electric field $E(t)$ with sub-cycle structure. On the other hand, space-based telescopes the Swift-BAT NASA (2004), NuSTAR caltech (2010) and Astro-H japan (2010) focusing high-energy X-ray missions, will also give possibilities of detecting X-ray radiation signature, discussed in this paper, from compact stars with

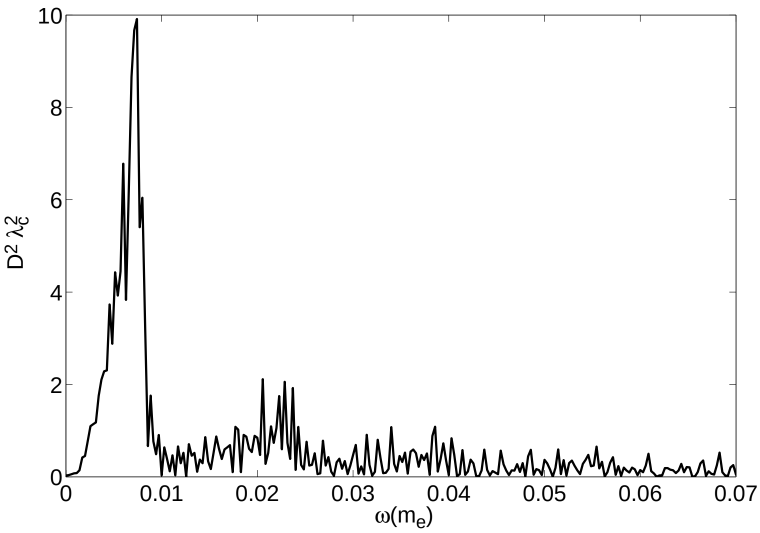

Figure A.7.: In the Compton unit, normalizing $\tilde{D}(\omega)$ [see Eq. (A.0.30)] by the volume $\mathcal{V} \equiv \int d^3\mathbf{x}'$ of the radiation source $\mathbf{J}(t', \mathbf{x}')$, we plot $|\tilde{D}(\omega)|^2$ [see Eq. (A.0.31)] representing the narrow energy-spectrum of the radiation field $\mathbf{E}_{\rm rad}$ and peak locates at the frequency $\omega_{\rm peak} \approx 0.08 m_e$.

electromagnetic structure.

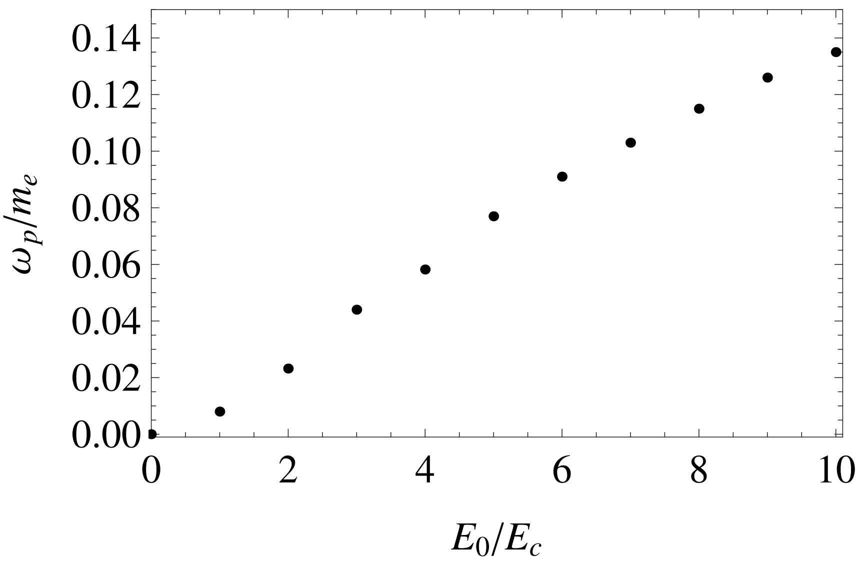

Figure A.8.: The peak frequency $\omega_{\rm peak}$ of the radiation approximately varies from 4keV to 70 keV as the initial electric field strength E_0 varies from E_c to $10E_c$. The values for very large field-strengths $E_0/E_c > 1$ possibly receive corrections, since the semiclassical pair-production rate (A.0.22) is approximately adopted and the pressure term (see footnote on page 875) is not properly taken into account.

B. On the frequency of oscillations in the pair plasma generated by a strong electric field

The plasma oscillation phenomenon of the electron-positron pairs created by vacuum polarization for $E > E_c \equiv m^2c^3/(e\hbar)$ ($\hbar$ is the Planck's constant, c is the speed of light, m and e are the electron mass and the absolute value of electron charge respectively) represents one of the most popular topics in relativistic field theory today. In particular the Vlasov-Boltzmann equation has been used within QCD in Białas and Czyż (1984); Białas et al. (1988), and with the semiclassical field equations in Kluger et al. (1992) within QED. This approach was shown in Kluger et al. (1992) to be in excellent agreement with quantum field theory calculations Cooper and Mottola (1989). Applications of these studies range from heavy ion collisions Biro et al. (1984b); Cooper et al. (1993), Gatoff et al. (1987) to lasers Ringwald (2001).

We have introduced collisional terms in the Vlasov-Boltzmann equation for such a system in Ruffini et al. (2003b). Our results have been considered of interest in the studies of pair production in free electron lasers Ringwald (2003),Narozhny et al. (2004), Bulanov et al. (2006) in optical lasers Blaschke et al. (2006), of millicharged fermions in extensions of the standard model of particle physics Gies et al. (2006), electromagnetic wave propagation in a plasma Bulanov et al. (2005), as well in astrophysics Ruffini et al. (2003a).

In this Letter we explore the case of undercritical electric field, already considered in Ruffini et al. (2007b), in which it has been proved that for $E < E_c$ back reaction of the created electrons and positrons on the external electric field can not be neglected.

We apply an approach based on continuity, energy-momentum conservation and Maxwell equations Ruffini et al. (2007b) in order to account for the back reaction of the created pairs. By this treatment we can analyse the new case of undecritical field, $E < E_c$, and recover the old results for overcritical field, $E > E_c$. In particular, we are focusing on the range $E_c < E < 10E_c$

It is generally assumed that electrons and positrons are created at rest in pairs, due to vacuum polarization in uniform electric field with strength E (Sauter (1931a); Heisenberg and Euler (1936); Schwinger (1951); Narozhnyi and Nikishov (1970); Greiner et al. (1985); Grib et al. (1980)), with the average rate per unit volume and per unit time[1]

$$S \equiv \frac{dN}{dVdt} = \frac{m^4}{4\pi^3}\left(\frac{E}{E_c}\right)^2 \exp\left(-\pi\frac{E_c}{E}\right), \quad E = \sqrt{-\frac{1}{2}F_{\mu\nu}F^{\mu\nu}} \tag{B.0.1}$$

where the magnitude of the electric field can be written in a covariant form taking into account that we can always choose a local reference frame in which the magnetic field is zero.

This formula is derived for uniform constant in time electric field. However, it still can be used for slowly time-varying electric field provided the inverse adiabaticity parameter (Greiner et al. (1985); Grib et al. (1980); Brezin and Itzykson (1970); Popov (1971b, 1973)) is much larger than one,

$$\eta = \frac{m}{\omega}\frac{E_{peak}}{E_c} = \tilde{T}\tilde{E}_{peak} \gg 1, \tag{B.0.2}$$

where ω is the frequency of oscillations, $\tilde{T} = m/\omega$ is dimensionless period of oscillations. Equation (B.0.2) implies that time variation of the electric field is much slower than the rate of pair production. In two specific cases considered in this paper, $E = 10E_c$ and $E = E_c$ we find for the first oscillation $\eta = 334$ and $\eta = 3.1 \times 10^6$ respectively. This demonstrates applicability of the formula (B.0.1) in our case.

Following Ruffini et al. (2007b) the conservation laws and Maxwell equations written for electrons, positrons and electromagnetic field are

$$\frac{\partial\left(\bar{n}U^\mu\right)}{\partial x^\mu} = S, \tag{B.0.3}$$

$$\frac{\partial T^{\mu\nu}}{\partial x^\nu} = -F^{\mu\nu}J_\nu, \tag{B.0.4}$$

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = -4\pi J^\mu, \tag{B.0.5}$$

where $\bar{n}$ is the comoving number density of electrons, $T^{\mu\nu}$ is energy-momentum tensor of electrons and positrons

$$T^{\mu\nu} = m\bar{n}\left(U^\mu_{(+)}U^\nu_{(+)} + U^\mu_{(-)}U^\nu_{(-)}\right), \tag{B.0.6}$$

$F^{\mu\nu}$ is electromagnetic field tensor, $J^\mu = J^\mu_{cond} + J^\mu_{pol}$ is the total four-current density U^μ is four velocity respectively of positrons and electrons. Electrons and positrons move along the electric field lines in opposite directions.

Using the same procedure pursued in Ruffini et al. (2007b) we choose a coordinate frame where pairs are created at rest and introduce coordinate

[1] We use in the following the system of units where $\hbar = c = 1$, $e = \sqrt{\alpha} \approx \sqrt{1/137}$, α being the fine structure constant.

number density $n = \bar{n}\gamma$.

Defining energy density of positrons

$$\rho = \frac{1}{2}T^{00} = mn\gamma, \tag{B.0.7}$$

we find from (B.0.4) its time derivative

$$\dot{\rho} = envE + m\gamma S. \tag{B.0.8}$$

Due to homogeneity of the electric field and plasma, electrons and positrons have the same energy and absolute value of the momentum density p, but their momenta have opposite directions. Our definitions also imply

$$\rho^2 = p^2 + m^2 n^2, \tag{B.0.9}$$

which is relativistic relation between the energy, momentum and mass densities of particles.

It has been shown in Ruffini et al. (2007b) that in a uniform electric field, from the system (B.0.3)-(B.0.5), the following system of four coupled ordinary differential equations may be obtained

$$\frac{d\tilde{n}}{d\tilde{t}} = \tilde{S}, \tag{B.0.10}$$
$$\frac{d\tilde{\rho}}{d\tilde{t}} = \tilde{n}\tilde{E}\tilde{v} + \tilde{\gamma}\tilde{S}, \tag{B.0.11}$$
$$\frac{d\tilde{p}}{d\tilde{t}} = \tilde{n}\tilde{E} + \tilde{\gamma}\tilde{v}\tilde{S}, \tag{B.0.12}$$
$$\frac{d\tilde{E}}{d\tilde{t}} = -8\pi\alpha\left(\tilde{n}\tilde{v} + \frac{\tilde{\gamma}\tilde{S}}{\tilde{E}}\right), \tag{B.0.13}$$

Where, using the dimensionless variables $n = m^3\tilde{n}$, $\rho = m^4\tilde{\rho}$, $p = m^4\tilde{p}$, $E = E_c\tilde{E}$, $t = m^{-1}\tilde{t}$, we get $\tilde{S} = \frac{1}{4\pi^3}\tilde{E}^2\exp\left(-\frac{\pi}{\tilde{E}}\right)$, $\tilde{v} = \frac{\tilde{p}}{\tilde{\rho}}$ and $\tilde{\gamma} = \left(1-\tilde{v}^2\right)^{-1/2}$, $\alpha = e^2/(\hbar c)$. For the new system (B.0.10)-(B.0.13) there exist two conservation laws

$$\tilde{\rho}^2 = \tilde{p}^2 + m^2\tilde{n}^2 \tag{B.0.14}$$
$$16\pi\tilde{\rho} = \tilde{E}_0^2 - \tilde{E}^2, \tag{B.0.15}$$

where $\tilde{E}_0$ is the constant of integration, so the particle energy density vanishes for initial value of the electric field $\tilde{E}_0$. Combining together the previous two equations, we get that the maximum number density of pairs we can achieve is

$$\tilde{n}_{max} = \frac{\tilde{E_0}^2}{16\pi(1+\tilde{u}^2)^{1/2}}. \tag{B.0.16}$$

In Ruffini et al. (2007b) the system was reduced to two equations and analyzed on the phase plane $(\tilde{E}, \tilde{v})$.

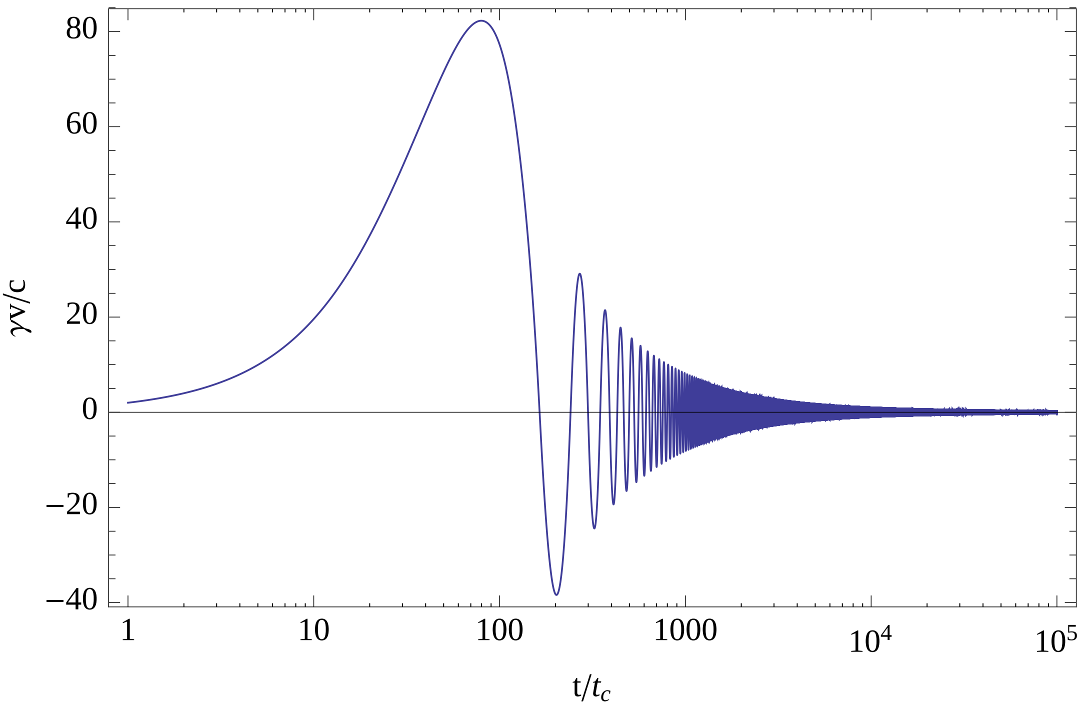

Figure B.1.: Evolution in time of the velocity $\tilde{u}$ for $E_0 = 2E_c$. This plot shows damped oscillations with a frequency rapidly increasing in time as it has been predicted reasoning on eq. (B.0.18).

Notice that eq. (B.0.4) allows to get the equation of motion

$$\dot{\tilde{u}} = \tilde{E} \tag{B.0.17}$$

where we have introduced another variable defined as $\tilde{u} = \tilde{\gamma}\tilde{v} = \gamma v/c$. Then eq. (B.0.13) and the previous equation can be combined to obtain a second order equation for $\tilde{u}$

$$\ddot{\tilde{u}} + \frac{1}{2}\frac{\tilde{u}}{1+\tilde{u}^2}\left(\tilde{E}_0^2 - \dot{\tilde{u}}^2\right) = -\frac{2\alpha}{\pi^2}\dot{\tilde{u}}\left(1+\tilde{u}^2\right)^{1/2}\exp\left(-\frac{\pi}{|\dot{\tilde{u}}|}\right), \tag{B.0.18}$$

where we have used $\dot{\tilde{u}} = d\tilde{u}/d\tilde{t}$ for convenience. In the limit non relativistic limit ($\tilde{u} \simeq \tilde{v} \ll 1$) we recover the formula

$$\ddot{\tilde{u}} + \tilde{\omega}^2\,\tilde{u} + k\,\dot{\tilde{u}} = 0 \tag{B.0.19}$$

which gives rise to harmonic damped oscillations. Knowing that $\tilde{S}(t \to \infty) \to 0$, we expect to get constant frequency $\omega \to \omega_p$ and constant amplitude for $\tilde{u}$.

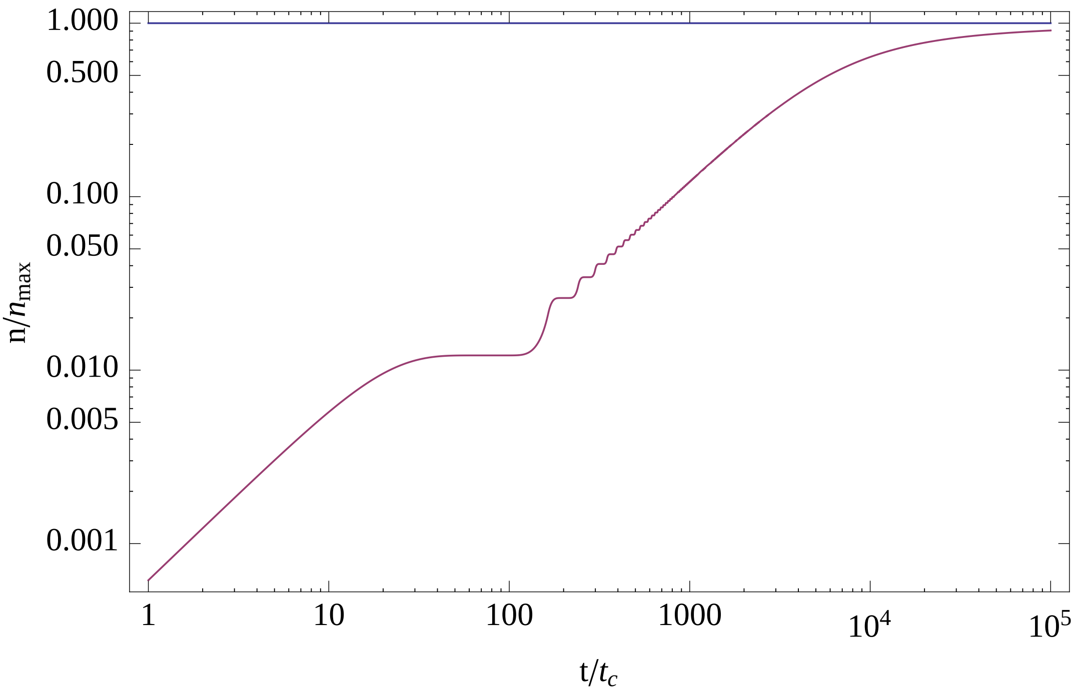

Figure B.2.: Ratio between number density and maximum achievable number density for $E_0 = 2E_c$; it becomes closer to unity after $10^5\,t_c$.

Since

$$\tilde{n} = \frac{\tilde{E_0}^2 - \dot{\tilde{u}}^2}{16\pi(1+\tilde{u}^2)^{1/2}}, \tag{B.0.20}$$

in the limit $\tilde{S} \to 0$ we identify the plasma frequency in eq. (B.0.18) as

$$\tilde{\omega}_p = \sqrt{\frac{4\pi\,\tilde{n}}{(1+\tilde{u}^2)^{1/2}}}. \tag{B.0.21}$$

We solve numerically equation (B.0.18) with the initial conditions $\dot{\tilde{u}}(0) = \tilde{E}_0$ and $\tilde{u}(0) = 0$ for the cases $\tilde{E}_0 = 1, 2, 5, 10$. Once this equation has been solved, we have the solution for the number density from eq. (B.0.20) and for the plasma frequency by means of eq. (B.0.21). In fig. (B.1) we plot the

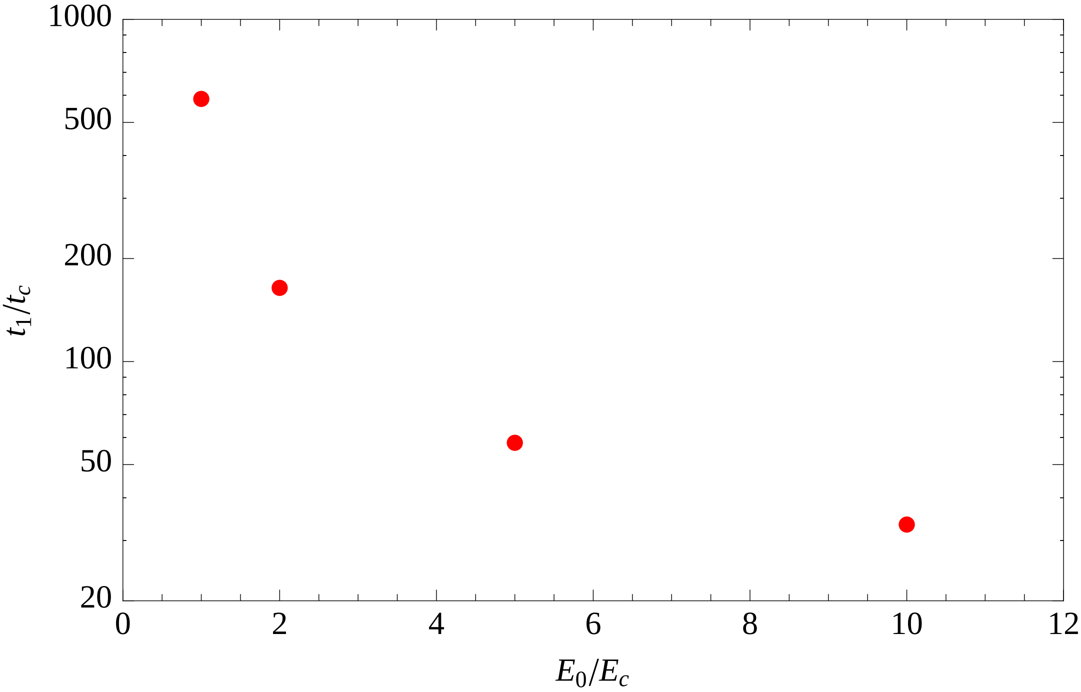

Figure B.3.: In this figure are shown, for all the cases under interest, the half period of the first oscillation t_1 in Compton times.

evolution in time of $\tilde{u}$ where the amplitude of the oscillations decreases in time exponentially as expected.

Once we have the solution for $\tilde{u}$ and its time derivative, eq. (B.0.20) provides us the number density of the electron-positron pairs. We plotted $\tilde{n}$ in fig. (B.2) for the case $E_0 = 2\,E_c$ as a fraction of the maximum achievable $\tilde{n}_0$. In all the cases under interest, this number is asymptotically achieved indicating that the final result of the process will be the complete conversion of the electromagnetic energy density into the rest mass of the pairs.

Moreover, looking at fig. (B.2) we recover the result obtained in Ruffini et al. (2007b); in fact we can see that after the first oscillation, higher is E_0 larger is $\tilde{n}$. This means that the first oscillation gives the leading contribution to the process in which the electromagnetic energy of the field is converted in the rest mass of pairs, with moderate contribution to their kinetic energy.

Because of the presence of $\tilde{u}^2$ inside eq. (B.0.21), $\tilde{\omega}_p$ shows amplitude oscillations; besides their frequency increases in time. For this reason, we calculated the average of $\tilde{\omega}_p$ in order to get a smooth function $\tilde{\omega}_p^{av}$ around which $\tilde{\omega}_p$ oscillates. We use this new function for the plasma frequency to make a comparison with the frequency of the pairs oscillations. In fig. (B.6) it is shown, for the case $E_0 = 2\,E_c$, that $\tilde{\omega}_p^{av}$ is achieved asymptotically as we expect from eq. (B.0.18). The same behavior occurs for each case we have taken into account.

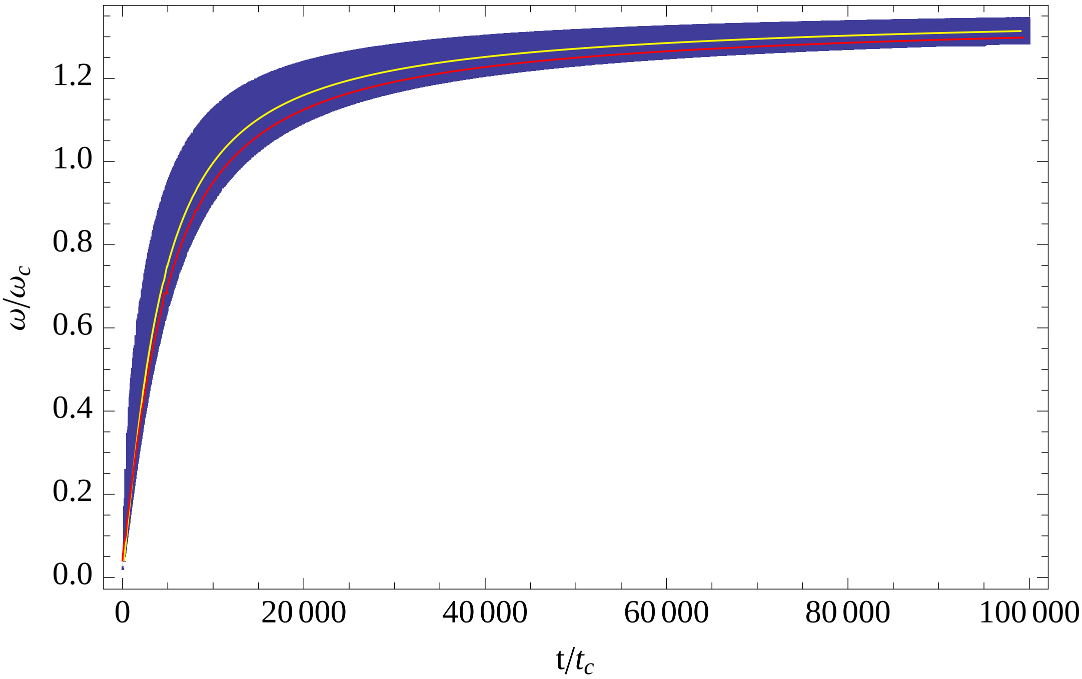

Figure B.4.: In this plot the blue area represents the plasma frequency as defined in eq. (B.0.21); it appears like a continuum because of the fast oscillations. The yellow curve is his average in time $\tilde{\omega}_p^{av}$ which can be compared with the pairs oscillation frequency $\tilde{\omega}$ given by the red curve. This plot corresponds to the case $E_0 = 2E_c$.

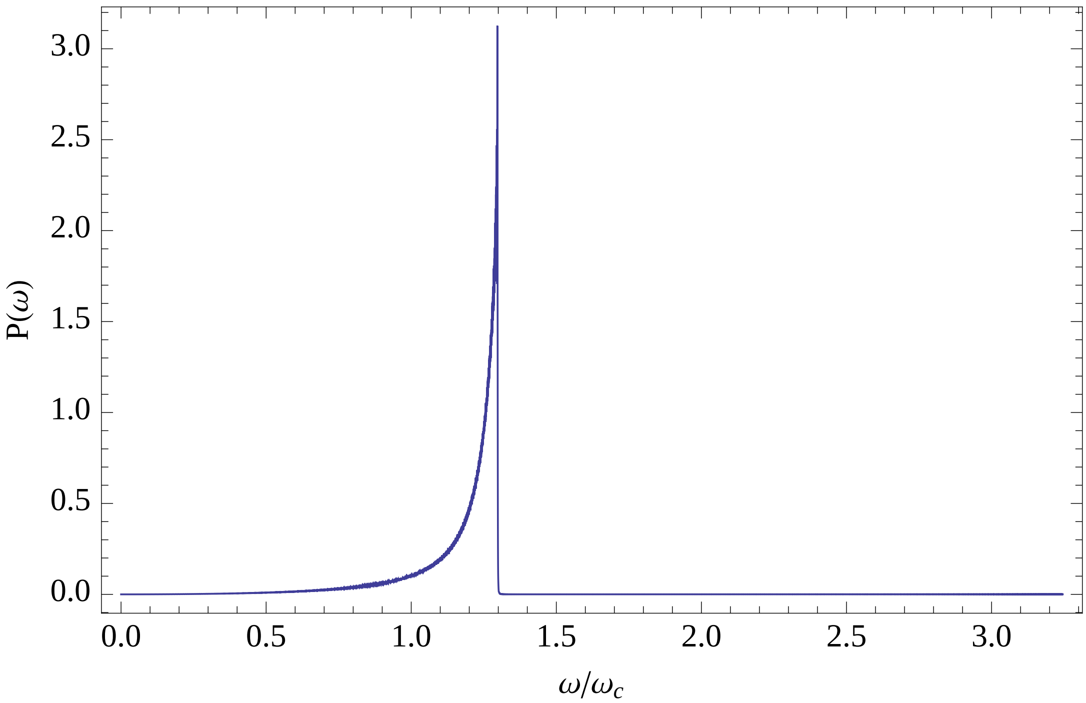

Figure B.5.: The power spectrum in arbitrary units has been obtained using eq. (B.0.22). The peak almost corresponds to the plasma frequency; indeed the main contribution is given by the oscillations which last for a long time, namely when the asymptotic limit of the plasma frequency is attained.

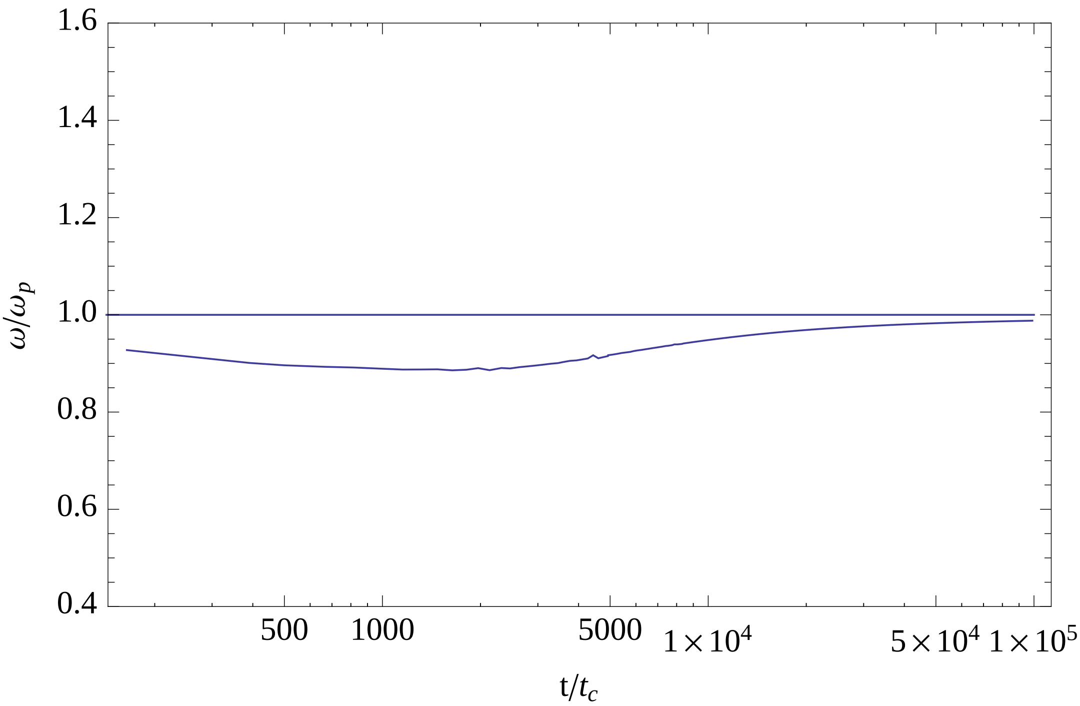

Figure B.6.: Behavior of the ratio $\tilde{\omega}/\tilde{\omega}_p^{av}$ in time for $E_0 = 2E_c$. The plasma frequency is attained asymptotically because of the limit $\tilde{S}(\tilde{t} \to \infty) \to 0$.

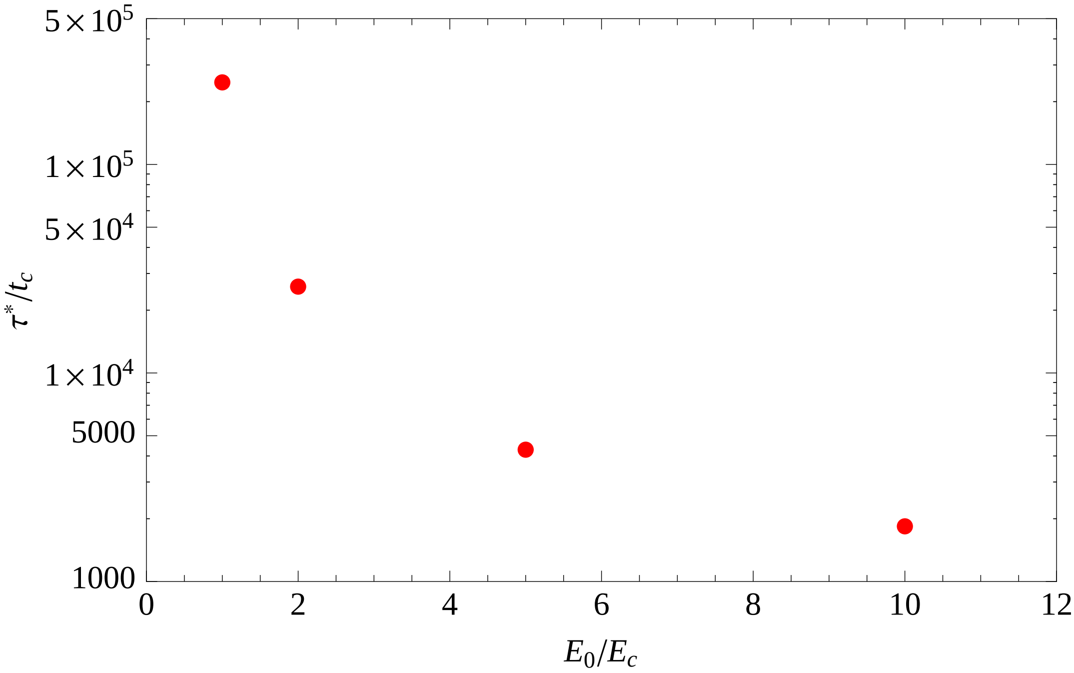

Figure B.7.: Characteristic time-scales τ^* needed to the pairs oscillation frequency to reach the plasma frequency.

We worked out the frequency of the i-th oscillation as $\tilde{\omega}^i = \pi/\tilde{T}^i_{1/2}$, where $\tilde{T}^i_{1/2}$ is the corresponding half period, calculated considering the time interval between the i-th and (i+1)-th subsequent roots of $\tilde{u}$.

The power spectrum is given by

$$\mathbf{P}(\omega) = 2\,|\tilde{\mathbf{D}}(\tilde{\omega})|^2, \tag{B.0.22}$$

where the amplitude $\tilde{\mathbf{D}}(\tilde{\omega})$ is proportional to the Fourier transform of the electric current time derivative Han et al. (2010)

$$\tilde{\mathbf{D}}(\tilde{\omega}) \propto \int_{\tilde{\mathfrak{T}}} d\tilde{t}\, e^{i\tilde{\omega}\tilde{t}} \left[\frac{\partial \mathbf{J}(\tilde{t})}{\partial \tilde{t}}\right]. \tag{B.0.23}$$

The electric current is simply related to the new variables by the following expression

$$\tilde{J} = 2\,\tilde{n}\,\frac{\tilde{u}}{(1+\tilde{u}^2)^{1/2}}. \tag{B.0.24}$$

From the fig. (B.5) it is clear that the main contribution is given by the final and fastest oscillations which last for a longer time. Therefore, the power spectrum shows a peak close to the plasma frequency reached asymptotically.

Once we know the frequency of the pairs oscillations and the plasma frequency, we easily obtain their ratio as it is shown in fig. (B.6). Besides, we use $\tilde{\omega}/\tilde{\omega}_p^{av}$ in order to compute the characteristic time-scale τ^* needed to the pairs oscillation frequency to reach the plasma frequency. For all the considered cases, this quantity is plotted in fig. (B.7) and from which we understand that the general trend is that larger is the initial electric field, larger will be the starting oscillation frequency. In fig. (B.8) we also show the ratio τ^*/t_1 which is not constant for different initial electric fields.

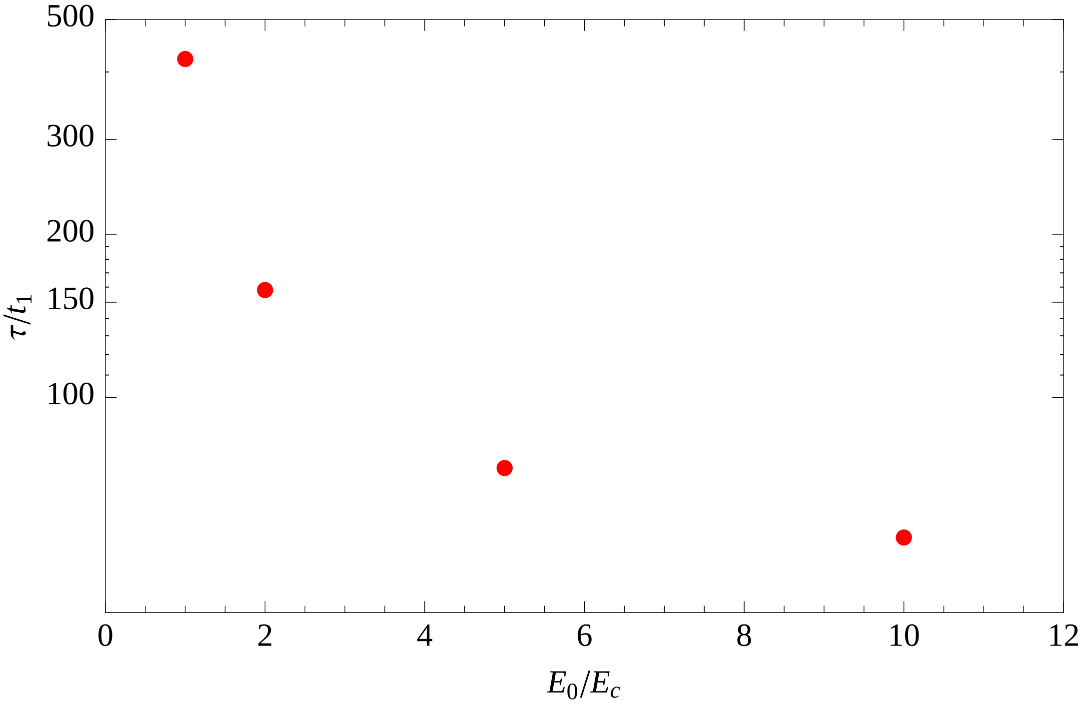

Figure B.8.: Ratio ratio τ^*/t_1 between the characteristic time scale for the approach to the plasma frequency and the half period of the first oscillation: also this parameter is not constant for the considered cases.

We can therefore reach the following conclusions:

- The study of the oscillation due to the vacuum polarization in uniform electric field can be reduced to the analysis of a second order ordinary differential equation for $\tilde{u} = \gamma v/c$. All the other physical quantities of interest can be obtained easily from the solution of eq. (B.0.18). This procedure allows us to study the evolution of the system for a long evolution time for the system.

- As expected, the plasma frequency is reached asymptotically for all the considered cases $E_c \leq E_0 \leq 10E_c$. The difference between them rely on the time scale the system needs to approach the plasma frequency as it is established in fig. (B.7). In particular, larger is the initial electric field, shorter will be the time scale to get the plasma frequency.

- Surprisingly we find that, for all the cases we have taken into account, $\tilde{\omega} \simeq \tilde{\omega}_p^{av}$ even at the beginning when we are far from the limit case $\tilde{S}(\tilde{t} \to \infty) \to 0$.

- The power spectrum has been obtained showing that the system exhibits a peak close, but always below, to the plasma frequency. The left tail in fig. (B.5) is due to the first oscillations with frequencies smaller than $\tilde{\omega}_p^{av}$, while the main contribution is due to the final evolution when the pairs oscillate almost with the same frequency close to $\tilde{\omega}_p^{av}$.

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