A simple approach to GW150914

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- The inspiral has been analyzed as a system of two point particles and the second regime as a test-particle falling into a black hole.

- Using this simple approach we have obtaining values very close to the reported ones by LIGO-collaboration.

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$$-\frac{dE}{dt} = \mathcal{L}_{\mathrm{GW}}, \tag{1}$$

- In this adiabatic regime, the angular frequency ω evolves in time as:

$$\omega = 2\left(\frac{GM_c}{c^3}\right)^{-5/8}\left(\frac{5}{256}\frac{1}{\tau}\right)^{3/8} \tag{2}$$

where $M_c = \mu^{3/5}m^{2/5} = \nu^{3/5}m$, where μ is the reduced mass, m the total mass, and $\nu = \mu/m$ is the symmetric mass ratio; $t = t_c - t$ and t_c is the coalesence time.

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- The angular frequency at the ISCO is:

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- Tidal forces induce quadrupole moment $Q_1 = k_1 m_2 a_1^5/r^3$ and $Q_2 = k_2 m_2 a_2^5/r^3$

- Corrections to the orbital phase:

$$\varphi^{\text{size}} - \varphi_0 = -\frac{1}{8x^{5/2}}[1 + \text{const}\, k(x/K)^5], \tag{4}$$

where and $x = (Gm\omega/c^3)^{2/3}$ and K is the compactness.

Spectograms

- We have calculated the spectograms and made a best fit by supposing the inspiral of two compact objects.

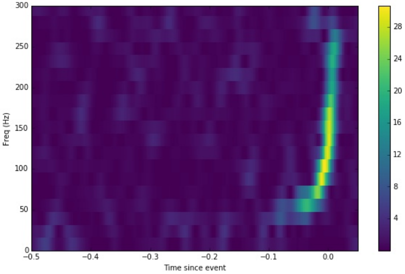

Figure: Normalized spectogram of H1 data

- The resulting chirp mass $M_c \approx 30.5\ M_\odot$
- The total mass of the system $m = M_c/\nu^{3/5}$, therefore $70.07 M_\odot \leq M_c$
- Energy radiated can be estimated as

$$\Delta E_{\text{inspiral}} = \left(1 - \sqrt{8/9}\right)\mu c^2 \leq 30.5 M_\odot c^2/(1+z). \tag{5}$$

Effecty One Body formalism

- Mapping from Post-Newtonian corrections of a two body problem to a effective one body problem which is a resummed version that contains non-perturbative effects.

- On the test-particle, approximation, $\nu \to 0$, the mass of the black hole corresponds to the total mass of the system.

- Test-particle approach does not take into account the tidal force and the reabsortion of gravitational waves.

Plunge-Merger-Ringdown (PMR)

- Test particle falling into a black-hole.

[1] M. Davis, R. Ruffini, W. H. Press, and R. H. Price, Phys. Rev. Lett. **27**, 1466 (1971); M. Davis, R. Ruffini and J. Tiomno, Phys, Rev. D **5**, 2932 (1972).

Plunge-Merger-Ringdown (PMR)

- Test particle falling into a black-hole.
- It was shown on Refs [1] the largest wave emission occurs from $r \approx 3Gm/c^2$, at the maximum of the effective potential:

$$V_l(r) = \left(1 - \frac{2m_{\rm BH}}{r}\right) \times \left[\frac{2\lambda^2(\lambda+1)r^3 + 6\lambda^2 m_{\rm BH} r^2 + 18\lambda m_{\rm BH}^2 r + 18 m_{\rm BH}^3}{r^3(\lambda r + 3m_{\rm BH})^2}\right] \quad (6)$$

.

- The total spectrum is peaked at:

$$\omega_{\rm peak} \approx \frac{c^3}{G}\frac{0.32}{m_{\rm BH}} \quad (7)$$

- Theorem $\omega_{\rm ISCO} < \omega_{\rm peak}$.

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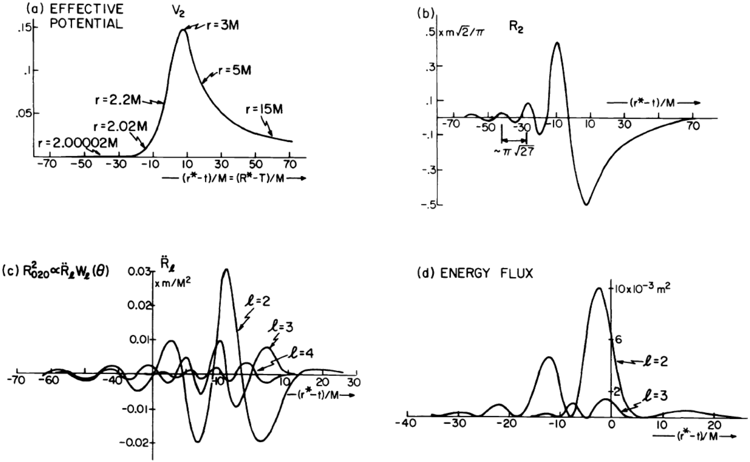
Plunge-Merger-Ringdown



Regime II

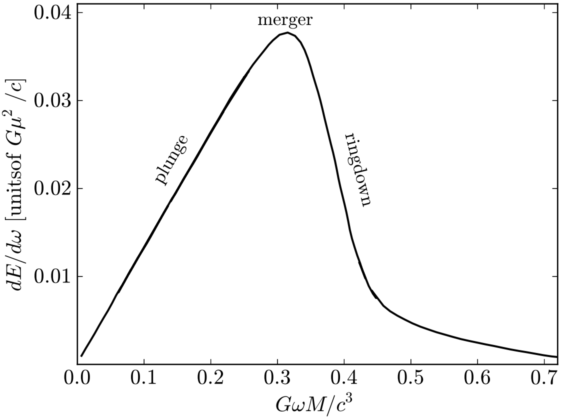


Figure: Spectrum of the GW during regime II

Ringdown Spectrum

- The spectrum during the plunge raises as:

$$\frac{dE}{d\omega} \approx 0.177 \frac{G\mu^2}{c} \left(\frac{2\omega G m}{c^3} \right)^{4/3}. \tag{8}$$

- Peaks and falls down exponentially following the empirical law:

$$\frac{dE}{d\omega} \propto \frac{G\mu^2}{c} \exp(-9.9 G m_{\rm BH} \omega / c^3) \tag{9}$$

- Simple approximation formulation to calculate the energy during regime II

$$\frac{dE}{d\omega} \approx \left[\left(\frac{dE}{d\omega} \right)^{-1} + \left(\frac{dE}{d\omega} \right)^{-1} \right]^{-1} \tag{10}$$

$$\Delta E_{\rm PMR} \approx 0.01 \frac{\mu^2 c^2}{m_{\rm BH}} \tag{11}$$

- The energy emitted during the regime II is affected by the rotation of the particle[2]
- Numerical results show that the position of ω_{peak} does not change, thus eq. (7) can be used.
- By fitting the numerical integration:

$$\Delta E_{\text{PMR}} \approx \Delta E_{\text{PMR}}^{J=0}[1 + 0.11 \exp(1.53j)], ; \qquad (12)$$

where $j = cJ/(G\mu m)$.

- Estimate the change on angular momentum:

$$\Delta J_{\text{PMR}} \approx \frac{2\Delta E_{\text{PMR}}}{\omega_{\text{ISCO}}} = 3.81 \frac{G\mu^2}{c} \qquad (13)$$

[2] S. L. Detweiler, ApJ, **225**, 687, (1978).

Angular momentum

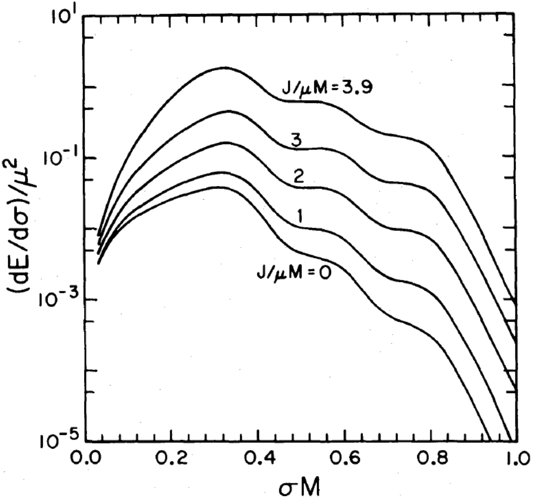


Figure: S. L. Detweiler, ApJ, **225**, 687, (1978).

- From the spectogram we have found that $f_{\rm peak}^{\rm obs} = 144 \pm 4$ Hz.
- From (7) we estimate the total mass:

$$m_{\rm obs} = 72 \pm 2\ M_{\odot} \tag{14}$$

- The symmetric mass radio is:

$$\nu = \frac{\mu}{m} = \left(\frac{M_c}{m}\right)^{5/3} \approx 0.24 \pm 0.01. \tag{15}$$

- Mass ratio:

$$q = \frac{m_1}{m_2} = \frac{4\nu}{\left(1 + \sqrt{1 - 4\nu}\right)^2} \approx 0.07 \tag{16}$$

Masses of the objects

- The observed masses of the objects are:

$$m_1^{\rm obs} = \frac{m}{(1+q)} \approx 43.1^{+4.3}_{-7.9}\ M_\odot, \qquad (17)$$

$$m_2^{\rm obs} = \frac{q\,m}{(1+q)} \approx 28.9^{+6.3}_{-5.9}\ M_\odot. \qquad (18)$$

- The energy radiated during the regimes it is obtained from eqs. (5) and (11):

$$\Delta E_{\rm inspiral} \approx \Delta E_{\rm PMR} \approx 1\ M_\odot \Longrightarrow \Delta E_{\rm total} \approx 2\ M_\odot. \qquad (19)$$

Mass and spin of the final black hole

- Using angular momentum conservation:

$$J_{\rm BH} = J_{\rm LSO} - \Delta J_{\rm PMR}, \tag{20}$$

- The dimensionless angular momentum of the newborn black-hole is

$$\alpha \equiv \frac{c J_{\rm BH}}{G m_{\rm BH}^2} \approx \frac{2\sqrt{3}\nu - 3.81\nu^2}{\beta(\nu)^2}. \tag{21}$$

where,

$$\beta(\nu) \equiv \left[1 - \left(1 - 2\sqrt{2}/3\right)\nu M - 0.24\nu^2\right]. \tag{22}$$

- From energy conservation the mass of the newborn black-hole is given by

$$m_{\rm BH}^{\rm final} \approx m\beta(\nu), \tag{23}$$

- The resulting parameters are:

$$m_{\rm BH}^{\rm final} \approx 70.0^{+2.0}_{-2.0}\ M_\odot, \quad \alpha \approx 0.65^{+0.02}_{-0.02} \tag{24}$$

Summary

	Current approach	Reported by LIGO
$M_c/M_\odot$	30.5	$30.2^{+2.5}_{-1.9}$
$m/M_\odot$	$72.0^{+2.0}_{-2.0}$	$70.3^{+5.3}_{-4.8}$
$m_1^{\text{obs}}/M_\odot$	$43.1^{+4.3}_{-7.9}$	$39.4^{+5.5}_{-4.9}$
$m_2^{\text{obs}}/M_\odot$	$28.9^{+6.3}_{-5.9}$	$30.9^{+4.8}_{-4.4}$
$m_{\text{BH}}^{\text{obs}}/M_\odot$	$70.0^{+2.0}_{-2.0}$	$62^{+4.0}_{-4.4}$
α	$0.65^{+0.02}_{-0.02}$	$0.67^{+0.05}_{-0.07}$

Table: Comparison of the two approaches

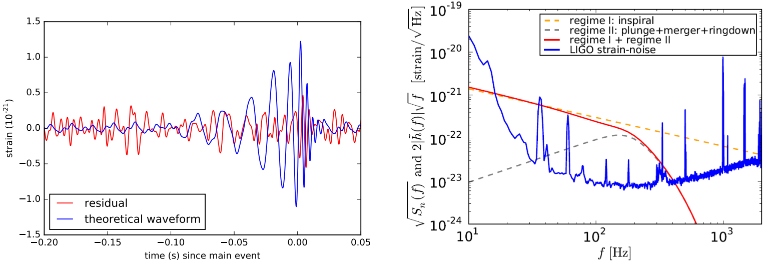


Figure: Sensitivity of LIGO

Discussion

- There are two different regimes, characterized by different parameters, all needed to determine the astrophysical nature of the source.

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- Regarding to the second detection, since the signal is deep in the noise, the peak of the signal can not be determined easily as the present approach.

- For systems which are not very compact, the dimensions of the objects become important and the inspiral regime is followed directly by the merger.