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Magnetohydrodynamical Effects of The Neutral Plasma Accretion into a Kerr Black hole

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Introduction

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- Study the accretion to the black hole also is important in the case of GRBs. Long and short GRBs have been divided into two sub-classes, depending whether or not a black hole (BH) is formed in the merger or in the hypercritical accretion process exceeding the NS critical mass. For intasnce for long bursts, when a BH is formed we have the sub-class of binary-driven hypernovae (BdHNe) Also for short bursts when a BH is formed, the authentic short GRBs (S-GRBs) occur.

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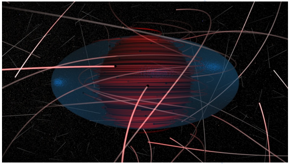
- In this work we use Ruffini-Wilson model of "the totally neutral fluid composed by positively and negatively charged plasma which is accreting to the Kerr black hole".



The Kerr space-time is stationary and axisymmetric, in the standard Boyer-Linquist type coordinates, it writes as

$$\begin{aligned} ds^2 &= -\left(1-\frac{2Mr}{\Sigma}\right)dt^2 \\ &\quad -\frac{2a\sin^2\theta}{\Sigma}(2Mr)\,dtd\phi+\frac{\Sigma}{\Delta}dr^2+\Sigma d\theta^2 \\ &\quad +\left[r^2+a^2+\frac{a^2\sin^2\theta}{\Sigma}(2Mr)\right]\sin^2\theta d\phi^2, \end{aligned} \tag{1}$$

which $\Sigma = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 - 2Mr + a^2$. M and a are the total mass and specific angular momentum respectively characterizing the spacetime. The (outer) event horizon is located at $r_+ = M + \sqrt{M^2 - a^2}$.

Assuming the negligible pressure, the motion of matter satisfies

$$U_{\mu;\nu}U^{\nu} = \frac{1}{\rho_m}F_{\mu\nu}J^{\nu} \tag{2}$$

which ρ_m is matter density and $F_{\mu\nu}$ is electromagnetic tensor of plasma and J^{ν} is net current density. The motion in first approximation, $\frac{1}{\rho_m}F_{\mu\nu}J^{\nu} = 0$, is geodesics.

$$V^r = \frac{U^r}{U^t} = \frac{-\Delta\left[-\Delta {U_\theta}^2 + 2Mr\left(r^2+a^2\right)\right]^{\frac{1}{2}}}{\Sigma\left(r^2+a^2\right) + 2Mra^2 sin^2\theta} \tag{3}$$

$$V^\theta = \frac{U^\theta}{U^t} = \frac{\Delta U_\theta}{\Sigma\left(r^2+a^2\right) + 2Mra^2 sin^2\theta} \tag{4}$$

$$V^\phi = \frac{U^\phi}{U^t} = \frac{2Mra}{\Sigma\left(r^2+a^2\right) + 2Mra^2 sin^2\theta} \tag{5}$$

U_ϕ and U_t are constants along the geodesics, by choosing $U_\phi = 0$ and $U_t = -1$, U_θ is also constant along the geodesics.

Let us consider the electromagnetic field associated with accreting plasma. Since we have considered stationary and axisymmetric configuration, the electromagnetic is described by component of A_{ϕ} and $F_{r\theta}$ which is related to magnetic field in the ϕ direction. So, the non-vanishing components of the electromagnetic tensor are

Electromagnetic Energy Momentum tensor component

$$F_{r\phi} = A_{\phi,r} \tag{6}$$

$$F_{\theta\phi} = A_{\phi,\theta} \tag{7}$$

$$F_{r\theta} \tag{8}$$

$F_{\mu\nu}U^{\nu}=0$ gives us other components of electromagnetic tensors.

Electromagnetic Energy Momentum tensor components

Furthermore, we assume infinite conductivity which is

$$F_{\mu\nu}U^{\nu}=0 \tag{9}$$

Using ϕ component of infinite conductivity condition, one have

$$A_{\phi,r}dr+A_{\phi,\theta}d\theta=0 \tag{10}$$

So A_{ϕ} is constant along the trajectories and can be selected as an arbitrary function of θ at infinity θ_{∞}.

$$\theta_{\infty}=\theta-U_{\theta}\int_{r}^{\infty}\frac{1}{[-\Delta U_{\theta}^{2}+2Mr(r^{2}+a^{2})]^{\frac{1}{2}}}dr \tag{11}$$

From θ and ϕ components of infinite conductivity condition we have

$$F_{r\theta}=A_{\phi,\theta}\frac{U^{\phi}}{U^{r}},A_{\phi,r}=-\frac{U^{\theta}}{U^{r}}A_{\phi,\theta} \tag{12}$$

Ruffini-Wilson-Damour results

Charge separation, Current density and EM fields

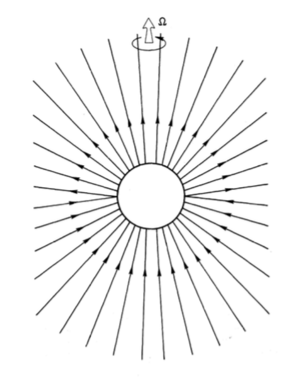


Figure: Magnetic lines of force in a plane ϕ = const. These lines are not exactly radial, then the charge will be induced on the horizon. This charge induction is due to the definition of $U_\theta \neq 0$.

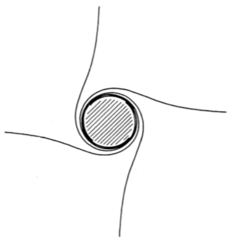


Figure: Magnetic lines of force in the equatorial plane of the black hole. The winding of the lines of force due to the dragging of the inertial frames is most clearly illustrated by this figure.

Figures from: R. Ruffini, J. R. Wilson [*Relativistic magnetohydro-dynamical effects of plasma accreting into a blackhole*]. PRD V. 12, N. 10, 1975

Damour: Torque & Momentum Transfer 117

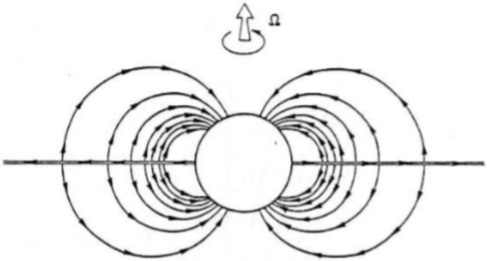


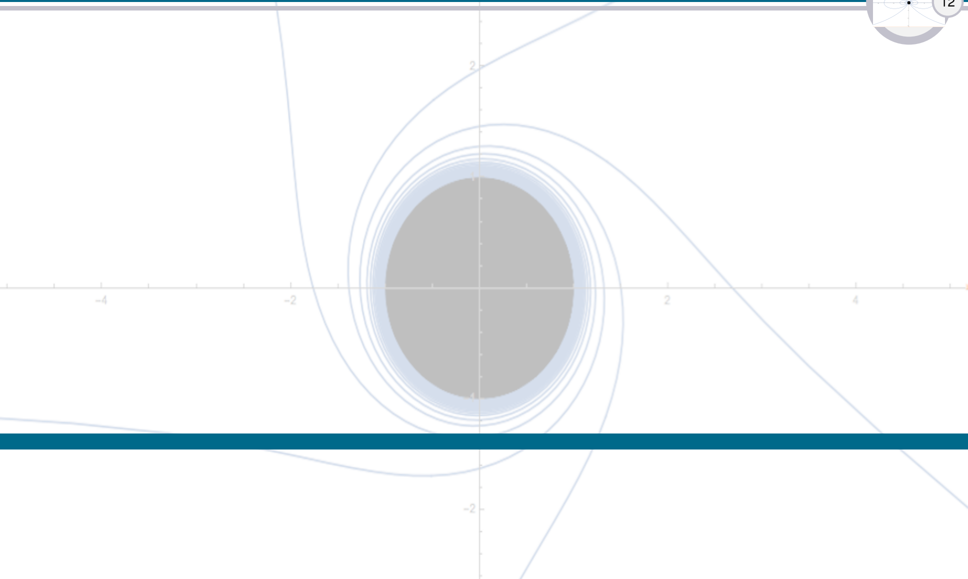
FIGURE 1. The lines of current calculated for a maximally rotating hole from Equations 19 and 29. The presence of an infinite sheet of current in the equatorial plane should be noticed.

They assume that the system is globally neutral then

$$Q_{TOT} = Q_{BH} + Q_{Magnetosphere} = 0 \tag{13}$$

Furthermore, by assuming $A_\phi = A_0(1 - |cos\theta|)$ and $U_\theta(\theta) = -U_\theta(\pi - \theta)$ finally

$$Q_{BH} = -Q_{Magnetosphere} = \frac{\pi a A_0 U_\theta}{16M^2} \tag{14}$$



Present Studies

Charge

The charge at time $t = t_0$ and inside a $r = r_0$ surface is

$$Q_{(t=t_0, r \leq r_0)} = \frac{1}{4\pi} \int_{\partial\Sigma} F^{tr} \sqrt{-g} d\theta d\phi \tag{15}$$

Then we have

$$Q_{(t=t_0, r \leq r_0)} = \frac{1}{4\pi} \int_{\partial\Sigma} \frac{2Mar \sin\theta}{\Sigma} \frac{u_\theta \partial_\theta A_\phi}{[-\Delta u_\theta^2 + 2Mr(r^2 + a^2)]^{1/2}} d\theta d\phi \tag{16}$$

When $r \to \infty$, Eq. (16) becomes

$$Q_{(t=t_0, r \to \infty)} \propto r^{-5/2} \to 0 \tag{17}$$

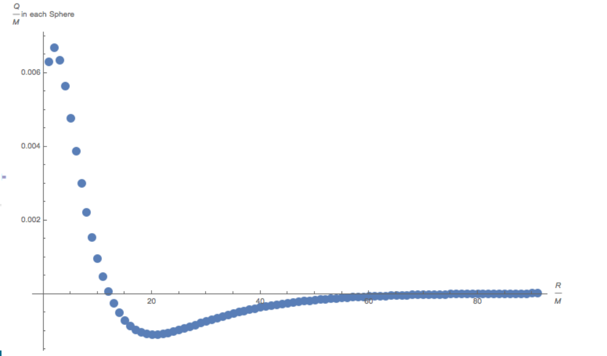


Figure: Charge in different radii, in fact each point in this figure represents the total charge inside the Gauss surface of constant radius.

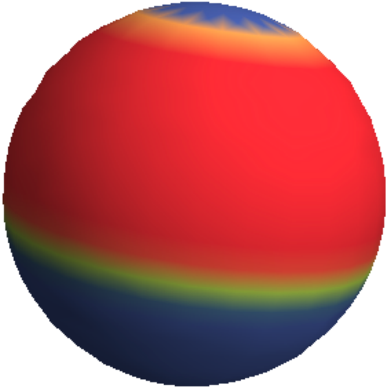


Figure: surface charge density on the Horizon.

The Maxwell equations satisfy

$$F^{\mu\nu}{}_{;\nu} = 4\pi J^{\mu} \tag{18}$$

$$J^{r} = \frac{1}{4\pi\sqrt{-g}}(\sqrt{-g}F^{r\mu})_{,\mu} = \frac{1}{4\pi\sqrt{-g}}(\sqrt{-g}F^{r\theta})_{,\theta} \tag{19}$$

$$J^{\theta} = \frac{1}{4\pi\sqrt{-g}}(\sqrt{-g}F^{\theta\mu})_{,\mu} = \frac{1}{4\pi\sqrt{-g}}(\sqrt{-g}F^{\theta r})_{,r} \tag{20}$$

$$J^{t} = \frac{1}{4\pi\sqrt{-g}}(\sqrt{-g}F^{t\mu})_{,\mu} = \frac{1}{4\pi\sqrt{-g}}[(\sqrt{-g}F^{tr})_{,r} + (\sqrt{-g}F^{t\theta})_{,\theta}] \tag{21}$$

and

$$J^{\phi} = \frac{1}{4\pi\sqrt{-g}}(\sqrt{-g}F^{\phi\mu})_{,\mu} = \frac{1}{4\pi\sqrt{-g}}[(\sqrt{-g}F^{\phi r})_{,r} + (\sqrt{-g}F^{\phi\theta})_{,\theta}] \tag{22}$$

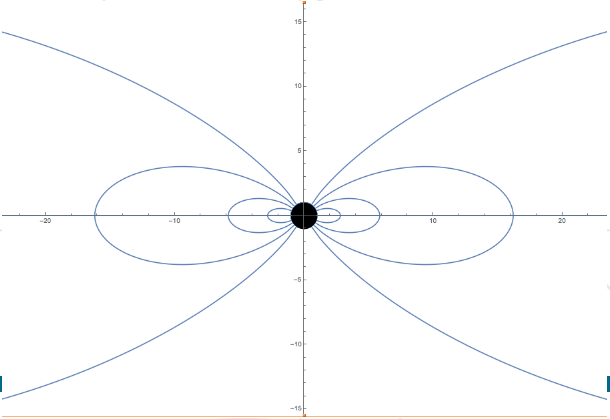


Figure: Current density lines in r-θ plane, this is $\phi =$constant plane, so we do not see the J^{ϕ}, for see the twisting of J^{ϕ} we need another plot, see Fig. 6.

Charge and Current Density

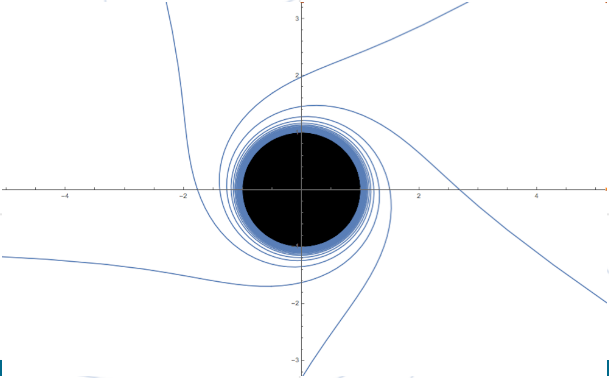
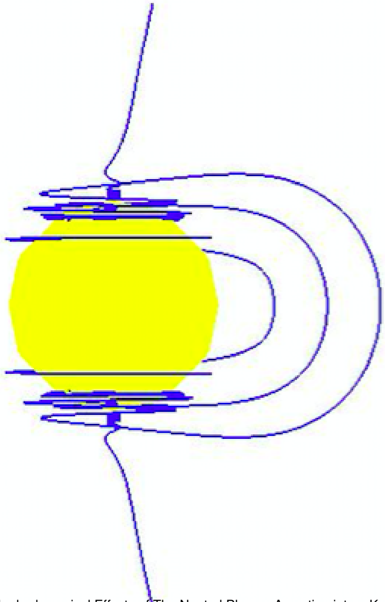


Figure: Current density lines in r-ϕ plane, for $\theta = \pi/4$, we can see the twisting of current density lines.

Charge and Current Density



Charge and Current Density

Using the EM current one can obtain electric charge density

$$J^{\mu} = \frac{\rho_e}{\sqrt{g_{tt}}} \frac{dX^{\mu}}{dX^0} \tag{23}$$

So, the component j^t of current four-vector, multiplied by $\sqrt{g_{tt}}$, is the spatial density of charge.

$$\rho_e = J^t \sqrt{g_{tt}} \tag{24}$$

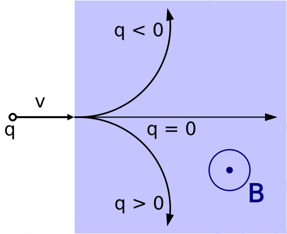
What is the physics behind current density?

We assume that the plasma has two kinds of fluids, one fluid has positively charged particles with mass m_+ and electric charge e, and the other has negatively charged particles with mass m_- and electric charge $-e$. One can define average current density of plasma as follows

$$J^\mu = e(n_+ u_+^\mu - n_- u_-^\mu) \tag{25}$$

$u_\pm^\mu$ is 4-velocity of particles, n_+ and n_- are the proper particle number densities of positively and negatively charged particles, respectively.
As it is obvious from Eq. (25) current comes from the different velocities of two positively and negatively charged particles, so in Eq. (23) the term $\frac{dX^\mu}{dX^0}$ comes from this different velocities of two positively and negatively charged particles and is not velocity of free falling particles in Carter solution. Actually for calculating the ρ_e in Eq. (24) we are lucky, because we are working with $\frac{dX^0}{dX^0}$ which is 1.

Charge and Current Density



Therefore, in presence of the magnetic field, the Lorentz forces ($q\vec{V} \times \vec{B}$) acting on the positive and negative charges are different and opposite, so there is charge separation and generated electric field is perpendicular (smaller) to B field. This effect cannot be seen by considering geodesics solution of Carter only. Actually if all particles were following the geodesics there were not any electromagnetic current at all. In fact assuming the infinity conductivity condition $F_{\mu\nu}U^{\nu} = 0$ represented the properties of $E \perp B$ and $E < B$

A convenient frame for computing the electric and magnetic field is the following orthogonal tetrad

$$\omega^{(\hat{t})} = (\Delta/\Sigma)^{1/2}(dt - a\sin^2\theta d\phi), \tag{26}$$

$$\omega^{(\hat{r})} = (\Sigma/\Delta)^{1/2}dr, \tag{27}$$

$$\omega^{(\hat{\theta})} = \Sigma^{1/2}d\theta, \tag{28}$$

$$\omega^{(\hat{\phi})} = \sin\theta\Sigma^{-1/2}((r^2 + a^2)d\phi - adt). \tag{29}$$

In the so fixed Lorentz frame, the components of EM tensor are components of magnetic and electric fields, transformation of EM tensor is

$$F_{\hat{\mu}\hat{\nu}} = \frac{\partial x^{\mu}}{\partial x^{\hat{\mu}}} \frac{\partial x^{\nu}}{\partial x^{\hat{\nu}}} F_{\mu\nu} \tag{30}$$

and electric and magnetic fields in this frame are

$$E_{\hat{i}} = F_{\hat{t}\hat{i}}, \tag{31}$$

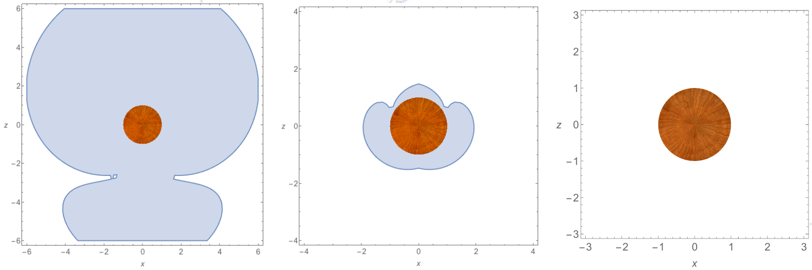
$$B_{\hat{i}} = \frac{1}{2}\epsilon^{\hat{i}\hat{j}\hat{k}}F_{\hat{j}\hat{k}} \tag{32}$$

which $\hat{i}, \hat{j}$ and $\hat{k}$ denote the spatial coordinates only. By assuming

$$A_\phi = A_0|cos(\theta_\infty)| \tag{33}$$

and using different values of magnetic field at $R = 100M$ electric field can be calculated. Fig.[24] show the magnitude of the electric field around the black hole. In all these cases $U_\theta = 3M$.

Charge and Current Density



Left: Magnitude of electric field of a 10 solar masses blackhole with $a = 1$ in X-Z plane. In this case magnetic field at $R = 100M$ is 10^{12} G. Middle: Magnitude of electric field of a 10 solar masses blackhole with $a = 1$ in X-Z plane. In this case magnetic field at $R = 100M$ is 10^{10} G. Right:Magnitude of electric field of a 10 solar masses blackhole with $a = 1$ in X-Z plane, in case of magnetic field at $R = 100M$ is 10^{8} G. Inside the blue line, the magnitude of electric field is bigger than 10^{13} G.

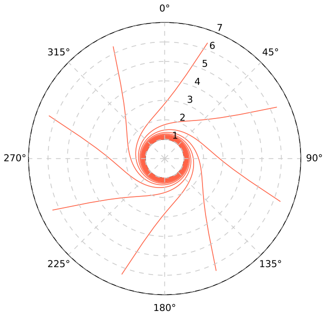


Figure: magnetic line for a unit mass black hole with $a = 1$ in a $r - \phi$ plane where $\theta = \pi/4$. Magnetic line bends when approaching the horizon, clearly evidencing the effect of frame dragging.

Electromagnetic Energy

The total electromagnetic energy distributed in a stationary spacetime is defined as follow

$$E(\zeta) = \int_{\Sigma} T_{\mu\nu}^{(em)} \zeta^{\mu} d\Sigma^{\nu} \tag{34}$$

where ζ is timelike killing vector, $d\Sigma^{\nu}$=$n^{\nu} d\Sigma$ is the surface element vector with n the unit timelike normal to the spacelike hyper- surface Σ and

$$T_{\mu\nu}^{(em)} = \frac{1}{4\pi}(F_{\mu\alpha}F_{\nu}^{\alpha} - \frac{1}{4}g_{\mu\nu}F_{\alpha\gamma}F^{\alpha\gamma}) \tag{35}$$

is the electromagnetic tensor.

Table: charge and electromagnetic energy of different initial conditions of B

B at $R = 100M$	Q/M	Electromagnetic Energy
10^{12} G	6×10^{-2}	5.9×10^{54} *ergs*
$10^{1}0$ G	6×10^{-4}	5.7×10^{50} *ergs*
10^{8} G	6×10^{-6}	5.4×10^{46} *ergs*

matter energy momentum tensor is,

$$T^{(M)}_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} \tag{36}$$

which u_μ is four velocity of fluid, ρ is proper density and p is pressure of fluid which here we assume is equal to zero. Total energy momentum is

$$T^{\mu\nu}_{(T)} = T^{\mu\nu}_{(EM)} + T^{\mu\nu}_{(M)} \tag{37}$$

energy is

$$E^{\mu} = \zeta_t{}^{\nu} T_{(T)\nu}{}^{\mu} \tag{38}$$

which ζ_t is timelike Killing vector. The total energy momentum is conserved, $T^{(T)\mu\nu}_{;\nu} = 0$ or $E^{\mu}_{;\mu} = 0$ Finally we have

$$\int_S T_t^{(M)r} \sqrt{-g} d\theta d\phi = - \int_S T_t^{(EM)r} \sqrt{-g} d\theta d\phi \tag{39}$$

as we know from matter energy momentum tensor $T_t^{(M)r} = \rho u_t u^r = -\rho u^r$. So, by using the Eq. (39) and this fact that we know the electromagnetic energy momentum tensor components, one can find the mass accretion rate.

$$\dot{M} = \int_S \rho u^r \sqrt{-g} d\theta d\phi \tag{40}$$

Thank you for your attention!