Numerical methods for relativistic plasma physics

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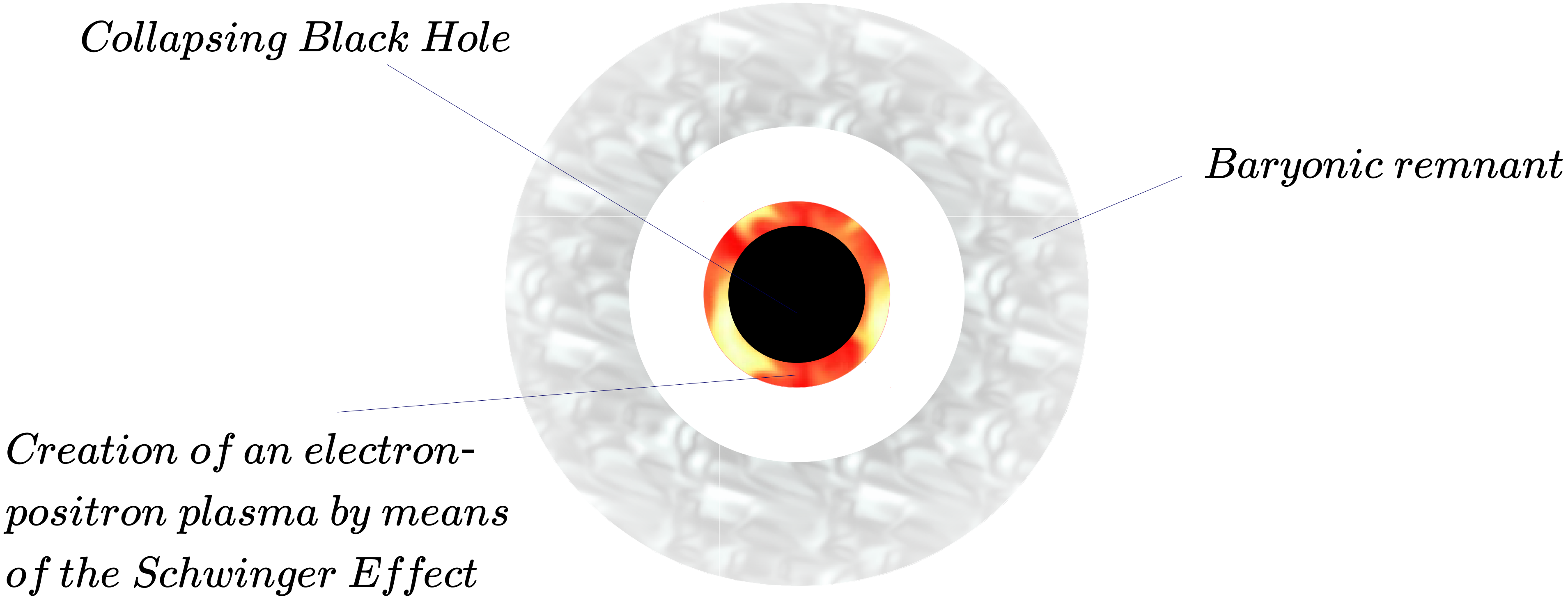
Adriatic Workshop 2016
Supernovae, Hypernovae and Binary Driven Hypernovae
ICRANet, Pescara

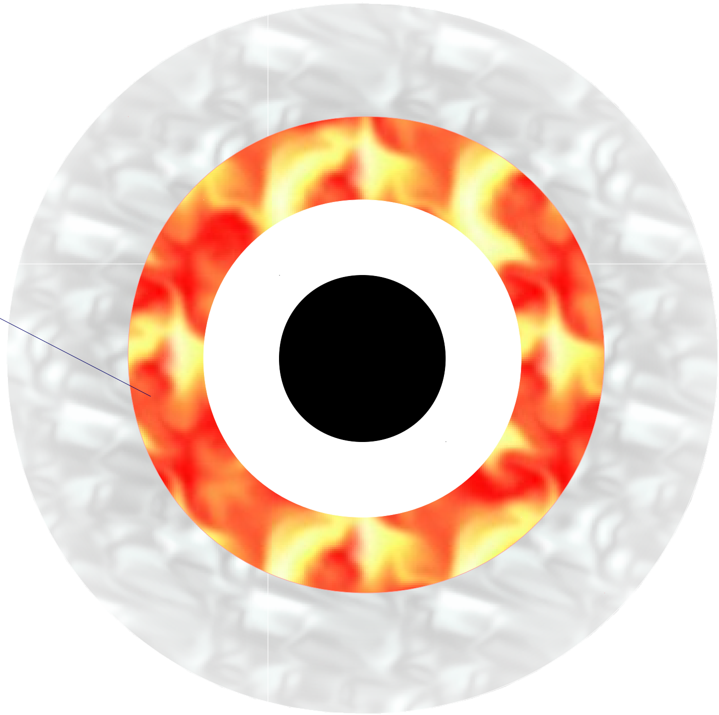
- *Outline of the physical problem*
- *Strategies followed so far*
- *Implementation of a hydrodinamical code*
- *Perspectives and conclusions*

Outline of the physical problem: *The EMBH model for GRBs*

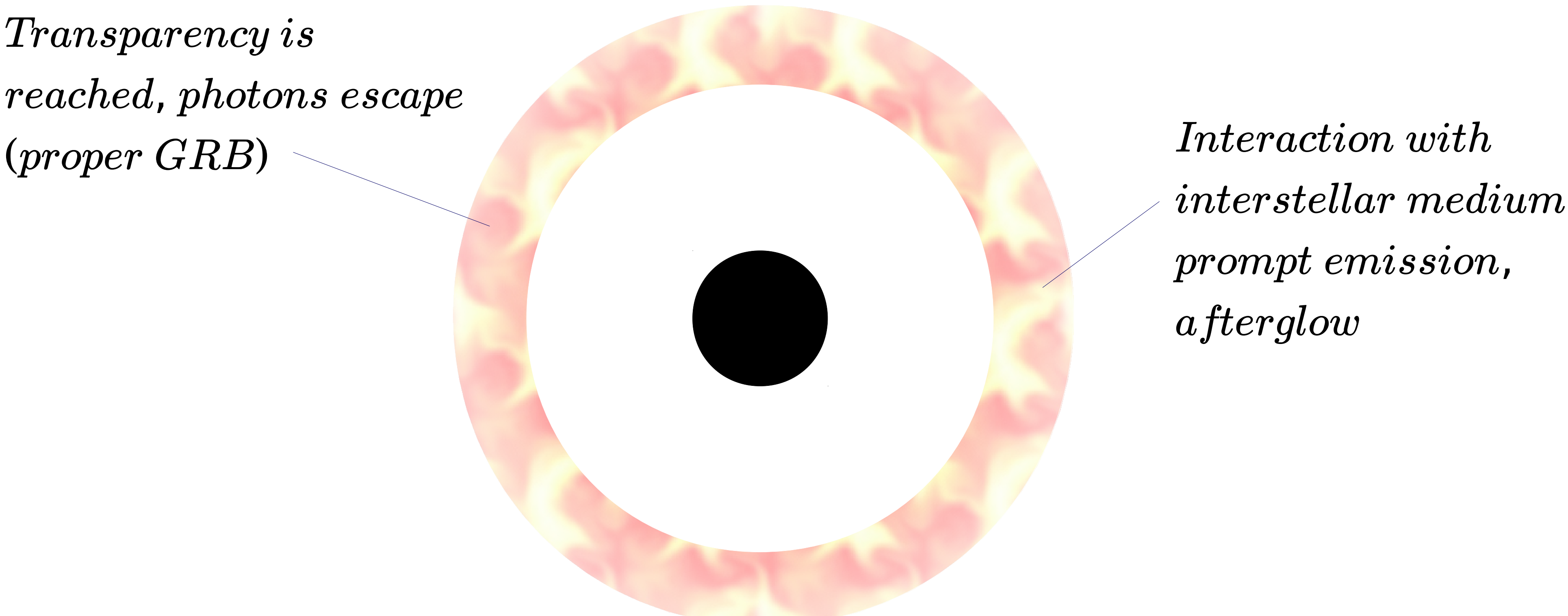


Outline of the physical problem: *The EMBH model for GRBs*

The (optically thick) plasma expands and accelerates, dragging with it matter from the remnant



Outline of the physical problem: *The EMBH model for GRBs*



Outline of the physical problem: *Equations of motion*

Spherical symmetry assumption → *Reissner-Nordstrom metric*:

$$ds^2 = -g_{tt}(r)dt^2 + g_{rr}(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \ ,$$

where $g_{tt}(r) = \left[1 - \frac{2GM}{c^2 r} + \frac{Q^2 G}{c^4 r^2}\right] \equiv \alpha(r)^2$ and $g_{rr}(r) = \alpha(r)^{-2}$.

Stress-energy tensor:

$$T^{\mu\nu} = pg^{\mu\nu} + (p+\rho)U^\mu U^\nu + \Delta T^{\mu\nu}$$

Dissipative effects
(*heat conduction, viscosity*)

Equation of state:

$$\Gamma(\rho, T) = 1 + \frac{p}{\epsilon}$$

(*For now, Γ = constant* = 4/3)

Outline of the physical problem: *Equations of motion*

Equations of motion (baryons+pairs)

Baryon number conservation:

$$(n_B U^\mu)_{;\mu} = 0$$

Energy-momentum conservation:

$$(T^{\mu\nu})_{;\nu} = 0$$

Outline of the physical problem: *Equations of motion*

Equations of motion (baryons+pairs)

Baryon number conservation:

$$(n_B U^\mu)_{;\mu} = 0$$

Energy-momentum conservation:

$$(T^{\mu\nu})_{;\nu} = 0$$

Some definitions: $\epsilon \equiv \rho - \rho_B$ COMOVING INTERNAL ENERGY DENSITY

$\rho_B \equiv n_B m_B c^2$ COMOVING BARYON MASS DENSITY

$\gamma \equiv \sqrt{1 + U^r U_r}, \qquad V^r \equiv \frac{U^r}{U^t}$ LORENTZ GAMMA FACTOR, RADIAL COORDINATE VELOCITY

Outline of the physical problem: *Equations of motion*

Final system of equations

$$E \equiv \epsilon\gamma, \qquad D \equiv \rho_B \gamma$$

Energy density Mass density

$$\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\alpha} D V^r\right)$$

$$\frac{\partial E}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\alpha} E V^r\right) - p\left[\frac{\partial \gamma}{\partial t} + \frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\alpha}\gamma V^r\right)\right]$$

$$S_r \equiv \alpha(p+\rho)U^t U_r = (D + \Gamma E)U_r$$

Radial momentum density

$$\frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\alpha} S_r V^r\right) - \alpha\frac{\partial p}{\partial r} - \frac{\alpha}{2}(p+\rho)\left[\frac{\partial g_{tt}}{\partial r}(U^t)^2 + \frac{\partial g_{rr}}{\partial r}(U^r)^2\right]$$

Outline of the physical problem: *Equations of motion*

Final system of equations

Transport terms

$$E \equiv \epsilon\gamma, \qquad D \equiv \rho_B \gamma$$

$$\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\alpha} D V^r\right)$$

$$\frac{\partial E}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\alpha} E V^r\right) - p\left[\frac{\partial \gamma}{\partial t} + \frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\alpha}\gamma V^r\right)\right]$$

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Outline of the physical problem: *Equations of motion*

Final system of equations

Transport

Expansion work (PdV)

$$E \equiv \epsilon\gamma, \qquad D \equiv \rho_B \gamma$$

$$\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\alpha} D V^r\right)$$

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Outline of the physical problem: *Equations of motion*

Final system of equations

$$E \equiv \epsilon\gamma, \qquad D \equiv \rho_B \gamma$$

Transport
Expansion work (PdV)
Pressure gradient
acceleration

$$\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\alpha} D V^r\right)$$

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$$S_r \equiv \alpha(p+\rho)U^t U_r = (D + \Gamma E)U_r$$

$$\frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\alpha} S_r V^r\right) - \alpha\frac{\partial p}{\partial r} - \frac{\alpha}{2}(p+\rho)\left[\frac{\partial g_{tt}}{\partial r}(U^t)^2 + \frac{\partial g_{rr}}{\partial r}(U^r)^2\right]$$

Outline of the physical problem: Equations of motion

Final system of equations

Transport
Expansion work (PdV)
Pressure gradient
acceleration
Metric acceleration

$$E \equiv \epsilon\gamma, \qquad D \equiv \rho_B \gamma$$

$$\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\alpha} D V^r\right)$$

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Strategies followed

- *Direct numerical solving of the GRHD equations*,
  Wilson and Salmonson, *Lawrence Livermore National Laboratory*, *University of California* (1999).

- *Approximate code using information from the Livermore code*,
  Ruffini, *Xue*, *Bianco*, *ICRANet* (1999-*present*):

  - *No gravitational interaction, special relativity*.

  - ***Pulse-like structure*** (*from the Livermore code*) *of* ***constant width in the coordinate frame*** *and uniform velocity*.

  - *Integration done until transparency is reached*.

Strategies followed: approximate code

$U_r(r) = U\frac{r}{\mathcal{R}}$

Constant coordinate width

Constant width in the comoving frame

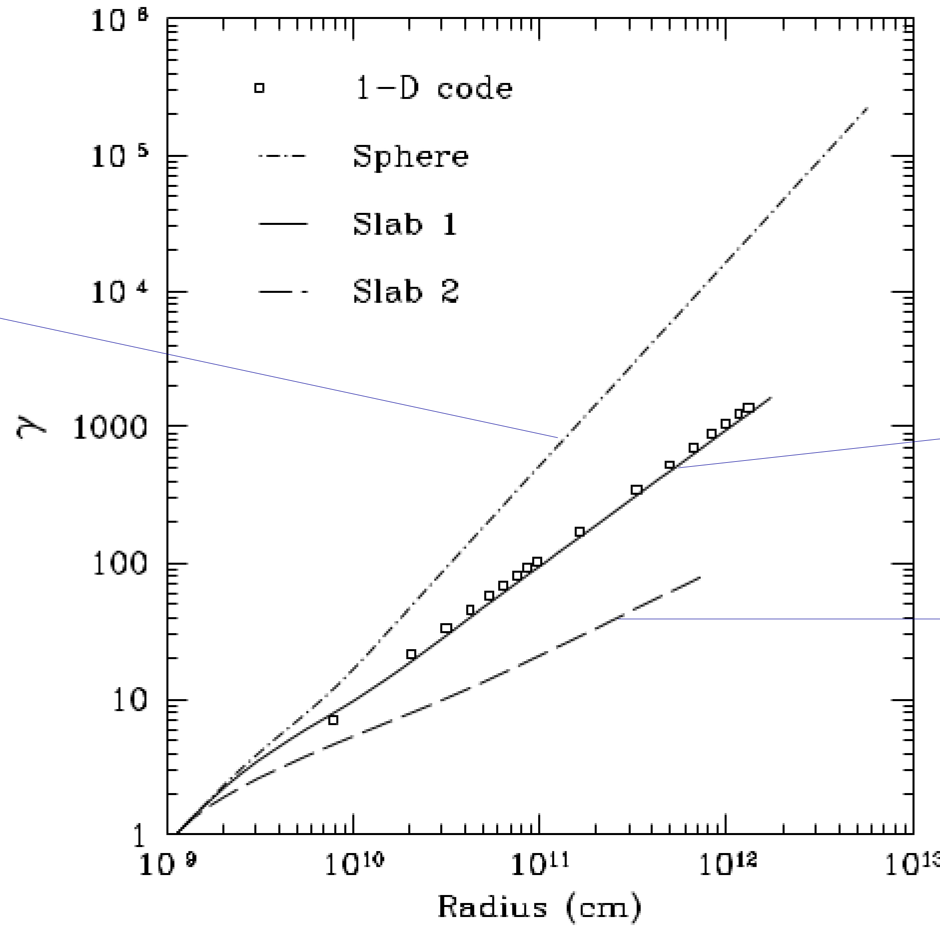

Fig. 3. Lorentz gamma factor γ as a function of radius. Three models for the expansion pattern of the PEM-pulse are compared with the results of the one dimensional hydrodynamic code for a $1000M_\odot$ black hole with charge to mass ratio $\xi = 0.1$. The 1-D code has an expansion pattern that strongly resembles that of a shell with constant coordinate thickness.

Strategies followed: *approximate code*, *interaction with baryons*

Assumptions:

- the PEM pulse does not change its geometry during the interaction;
- the collision between the PEM pulse and the baryonic matter is assumed to be inelastic,
- the baryonic matter reaches thermal equilibrium with the photons and pairs of the PEM pulse.

$$B = M_{\text{Baryons}}/E_{\text{Pulse}} \leq 10^{-2}$$

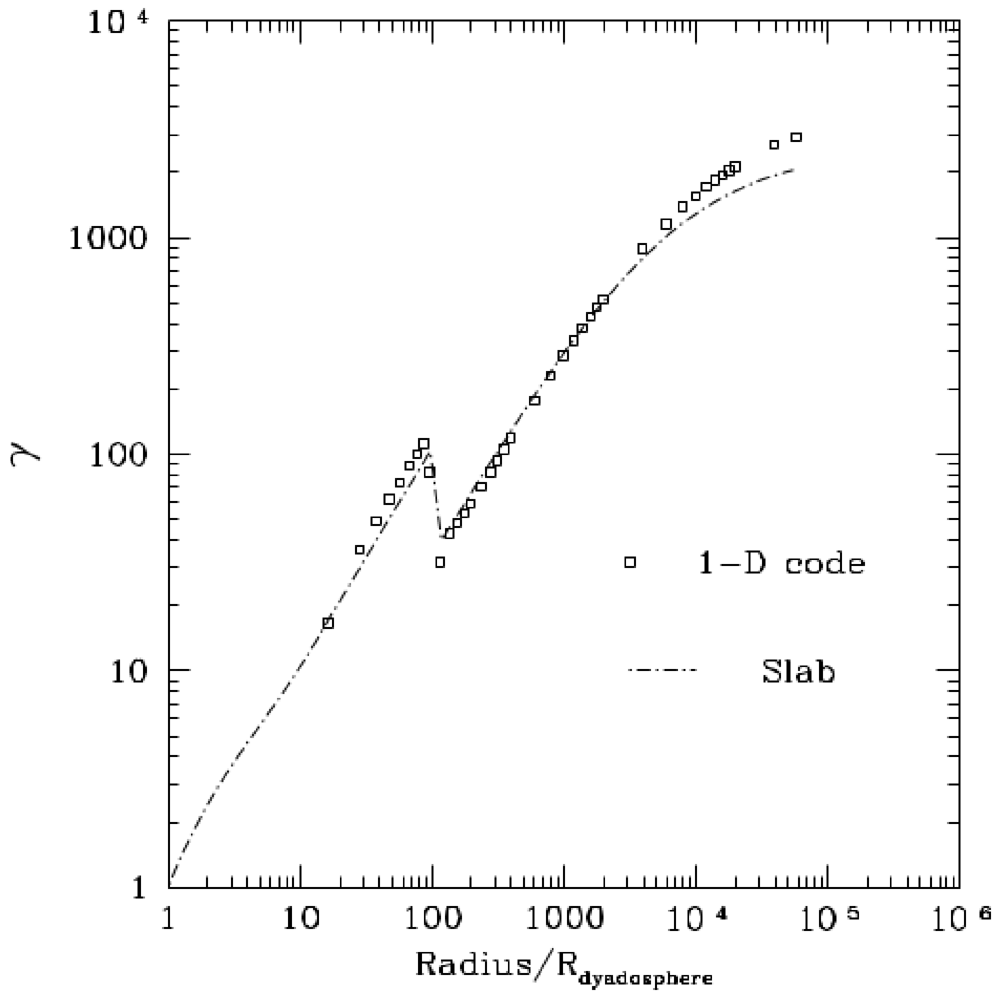

Fig. 7. Here we see a comparison of Lorentz factor γ for the one-dimensional (1-D) hydrodynamic calculations and slab calculations ($M_{\text{BH}} = 10^3 M_\odot, \xi = 0.1$ EMBH and $B \simeq 1.3 \cdot 10^{-4}$). The calculations show good agreement.

Strategies followed: Livermore code, operator splitting

$$\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}(\frac{r^2}{\alpha}DV^r)$$

$$\frac{\partial E}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}(\frac{r^2}{\alpha}EV^r) - p\left[\frac{\partial \gamma}{\partial t} + \frac{\alpha}{r^2}\frac{\partial}{\partial r}(\frac{r^2}{\alpha}\gamma V^r)\right]$$

$$S_r \equiv \alpha(p+\rho)U^t U_r = (D + \Gamma E)U_r$$

$$\frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}(\frac{r^2}{\alpha}S_r V^r) - \alpha\frac{\partial p}{\partial r} - \frac{\alpha}{2}(p+\rho)\left[\frac{\partial g_{tt}}{\partial r}(U^t)^2 + \frac{\partial g_{rr}}{\partial r}(U^r)^2\right]$$

Strategies followed: *Livermore code*, *operator splitting*

$$\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}(\frac{r^2}{\alpha} D V^r)$$

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Strategies followed: *Livermore code*, *operator splitting*

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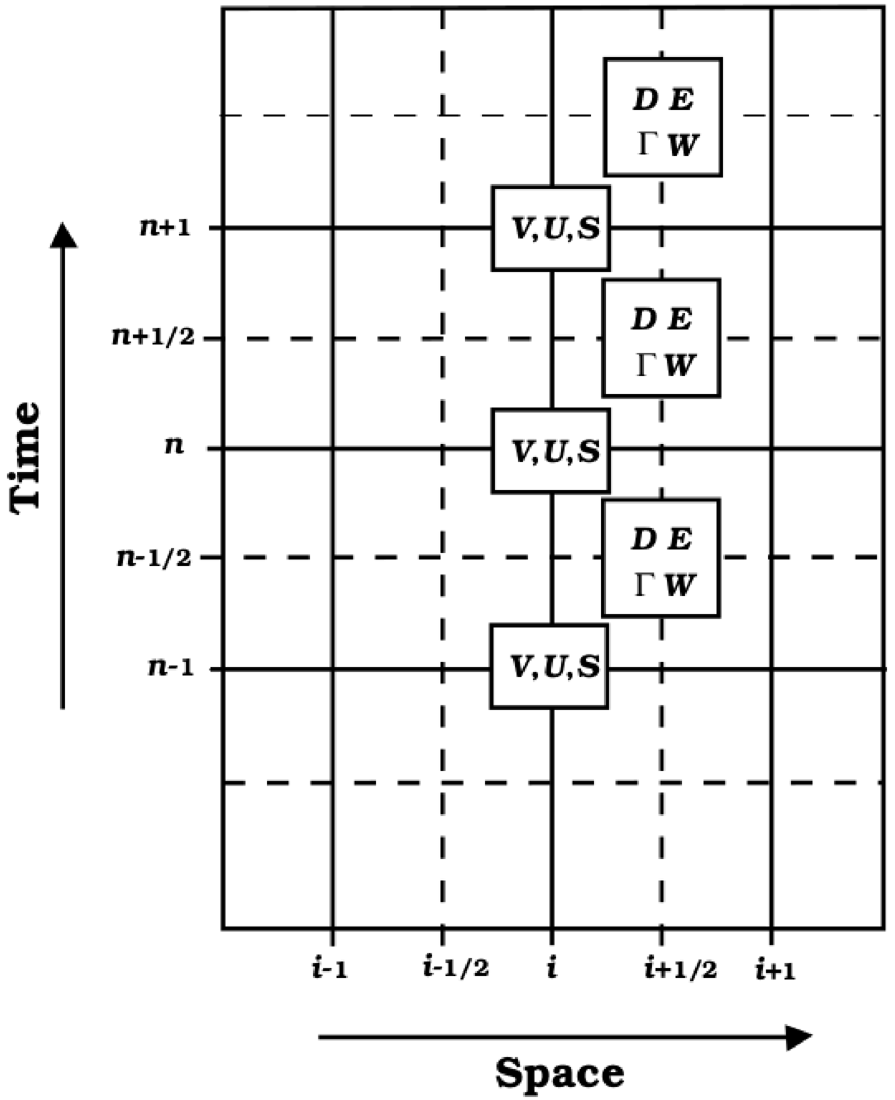
$$\frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}(\frac{r^2}{\alpha}S_r V^r) - \alpha\frac{\partial p}{\partial r} - \frac{\alpha}{2}(p+\rho)\left[\frac{\partial g_{tt}}{\partial r}(U^t)^2 + \frac{\partial g_{rr}}{\partial r}(U^r)^2\right]$$

Strategies followed: Livermore code, grid implementation

Leap-frog method



Strategies followed: *Livermore code*, *grid velocity*

$$\dot{D} + D\frac{\dot{\gamma}}{\gamma} + \frac{1}{\gamma}\frac{\partial}{\partial x^i}\left(\gamma D(V^i - V_g^i)\right) + \frac{D}{\gamma}\frac{\partial}{\partial x^i}\left(\gamma V_g^i\right) = 0,$$

$$\dot{S}_i + S_i\frac{\dot{\gamma}}{\gamma} - \frac{1}{\gamma}\frac{\partial}{\partial x^j}\left(S_i(V^j - V_g^i)\gamma\right) + \frac{S_i}{\gamma}\frac{\partial}{\partial x^i}\left(\gamma V_g^i\right) + \alpha\frac{\partial P}{\partial x^i}$$
$$- S_j\frac{\partial \beta^j}{\partial x^i} + (D + \Gamma E)\left(W\frac{\partial \alpha}{\partial x^i} + \frac{U_k U_j}{2W}\frac{\partial \gamma^{jk}}{\partial x^i}\right) = 0,$$

$$\dot{E} + \Gamma E\frac{\dot{\gamma}}{\gamma} + \frac{1}{\gamma}\frac{\partial}{\partial x^i}\left(E(V^i - V_g^i)\gamma\right) + \frac{\Gamma E}{\gamma}\frac{\partial}{\partial x^i}\left(\gamma V_g^i\right)$$
$$+ (\Gamma - 1)E\left[\frac{\dot{W}}{W} + \frac{1}{\gamma W}\frac{\partial}{\partial x^i}\left(W(V^i - V_g^i)\gamma\right)\right] = 0.$$

$$\det(g_{\alpha\beta}) = -\alpha^2\gamma^2$$

$$\gamma = r^2/\alpha(r)$$

$$W = \text{Lorentz gamma}$$

Strategies followed: *Livermore code*, *the advection part*

$$\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\alpha} D V^r\right)$$

$$\frac{\partial E}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\alpha} E V^r\right) - p\left[\frac{\partial \gamma}{\partial t} + \frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\alpha}\gamma V^r\right)\right]$$

$$\frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\alpha} S_r V^r\right) - \alpha\frac{\partial p}{\partial r} - \frac{\alpha}{2}(p+\rho)\left[\frac{\partial g_{tt}}{\partial r}(U^t)^2 + \frac{\partial g_{rr}}{\partial r}(U^r)^2\right]$$

Strategies followed: *Livermore code*, *the advection part*

$$\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}(\frac{r^2}{\alpha}DV^r)$$

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$$\hat{D} = \gamma D, \qquad \hat{E} = \gamma E, \qquad \hat{S} = \gamma S.$$

$\gamma = r^2/\alpha(r)$

$W =$ Lorentz gamma

Strategies followed: *Livermore code*, *the advection part*

$$\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}(\frac{r^2}{\alpha}DV^r)$$

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$$\frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2}\frac{\partial}{\partial r}(\frac{r^2}{\alpha}S_r V^r) - \alpha\frac{\partial p}{\partial r} - \frac{\alpha}{2}(p+\rho)\left[\frac{\partial g_{tt}}{\partial r}(U^t)^2 + \frac{\partial g_{rr}}{\partial r}(U^r)^2\right]$$

$$\hat{D} = \gamma D, \qquad \hat{E} = \gamma E, \qquad \hat{S} = \gamma S.$$

$$\gamma = r^2/\alpha(r)$$

W = Lorentz gamma

$$\frac{\partial \hat{D}}{\partial t} + \frac{\partial}{\partial x}\left(\hat{D}(V - V_g)\right) = 0,$$

$$\frac{\partial \hat{E}}{\partial t} + \frac{\partial}{\partial x}\left(\hat{E}(V - V_g)\right) = 0,$$

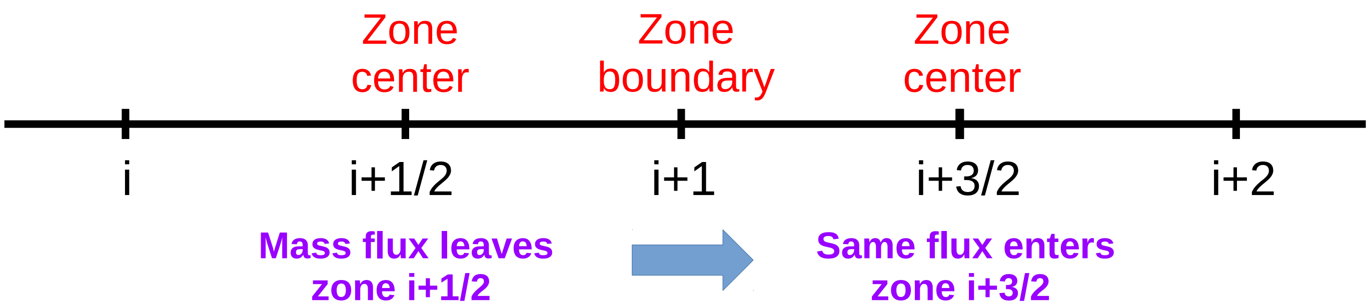
$$\frac{\partial \hat{S}}{\partial t} + \frac{\partial}{\partial x}\left(\hat{S}(V - V_g)\right) = 0.$$

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{v}) = 0$$

1-D conservation equations

Strategies followed: *Livermore code*, *the advection part*

Conservative scheme for advection:

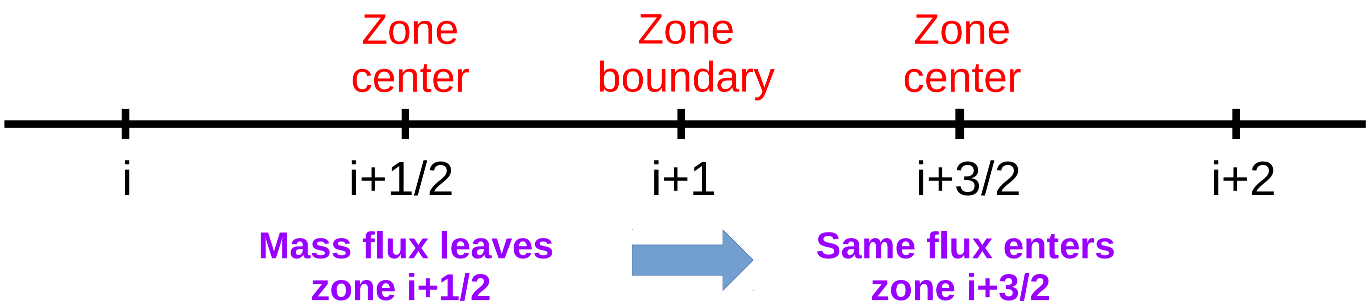


$$D^i(t+\delta t) = D^i(t) - (\Delta M_D^{i+1} - \Delta M_D^i)/\,Vol_b^i$$

$$\Delta M_D^i = \bar{D}_f^i A_a^i (V^i - V_g^i) dt$$

Strategies followed: *Livermore code*, *the advection part*

Conservative scheme for advection:



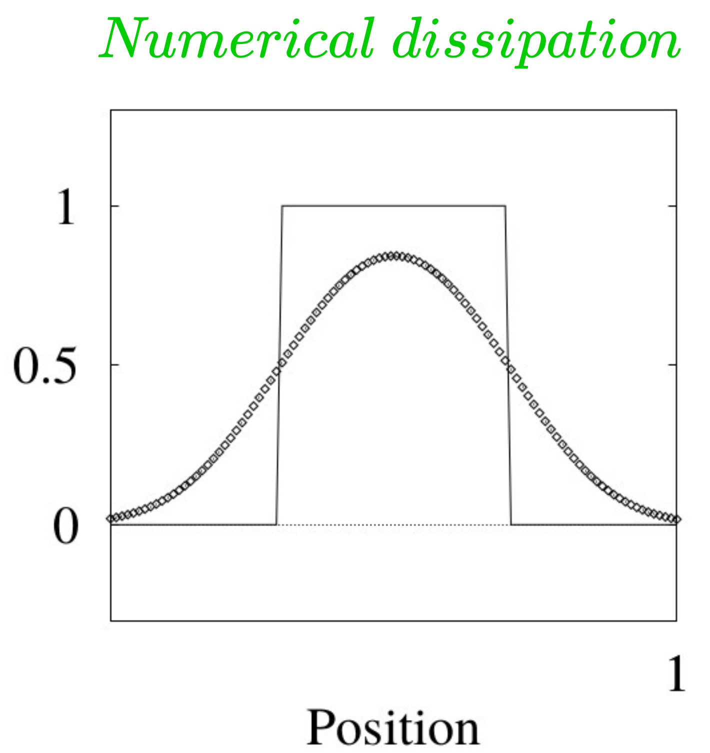
$$D^i(t+\delta t) = D^i(t) - (\Delta M_D^{i+1} - \Delta M_D^i)/\,Vol_b^i$$

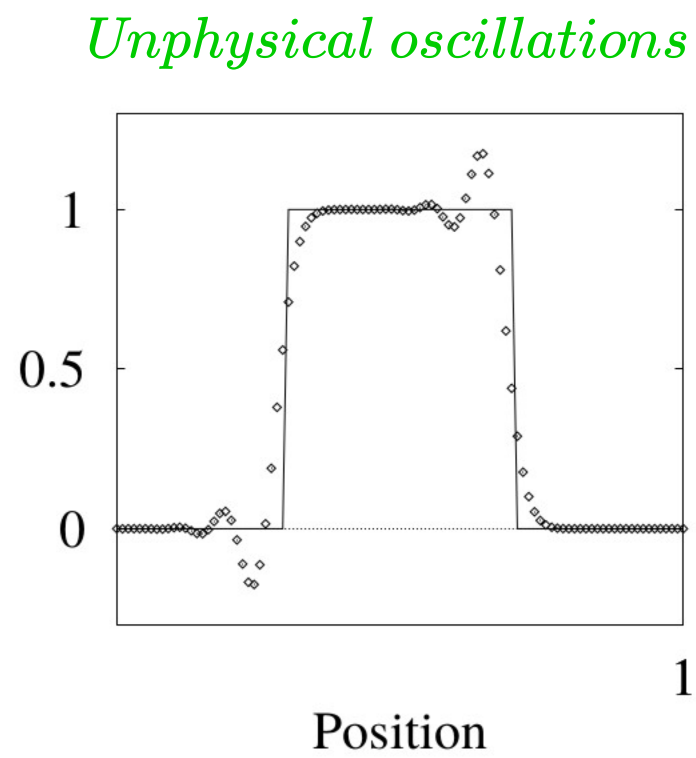
$$\Delta M_D^i = \bar{D}_f^i A_a^i (V^i - V_g^i) dt$$

$$\text{if} \qquad (V^i - V_g) > 0, \qquad \bar{D}_f^i = D^{i-1} + \frac{1}{2}\nabla \tilde{D}^{i-1}[dx_b^{i-1} - (V^i - V_g^i)dt]$$

$$\text{if} \qquad (V^i - V_g) < 0, \qquad \bar{D}_f^i = D^i - \frac{1}{2}\nabla \tilde{D}^i[dx_b^i + (V^i - V_g^i)dt]$$

Strategies followed: *Livermore code*, *some usual problems*





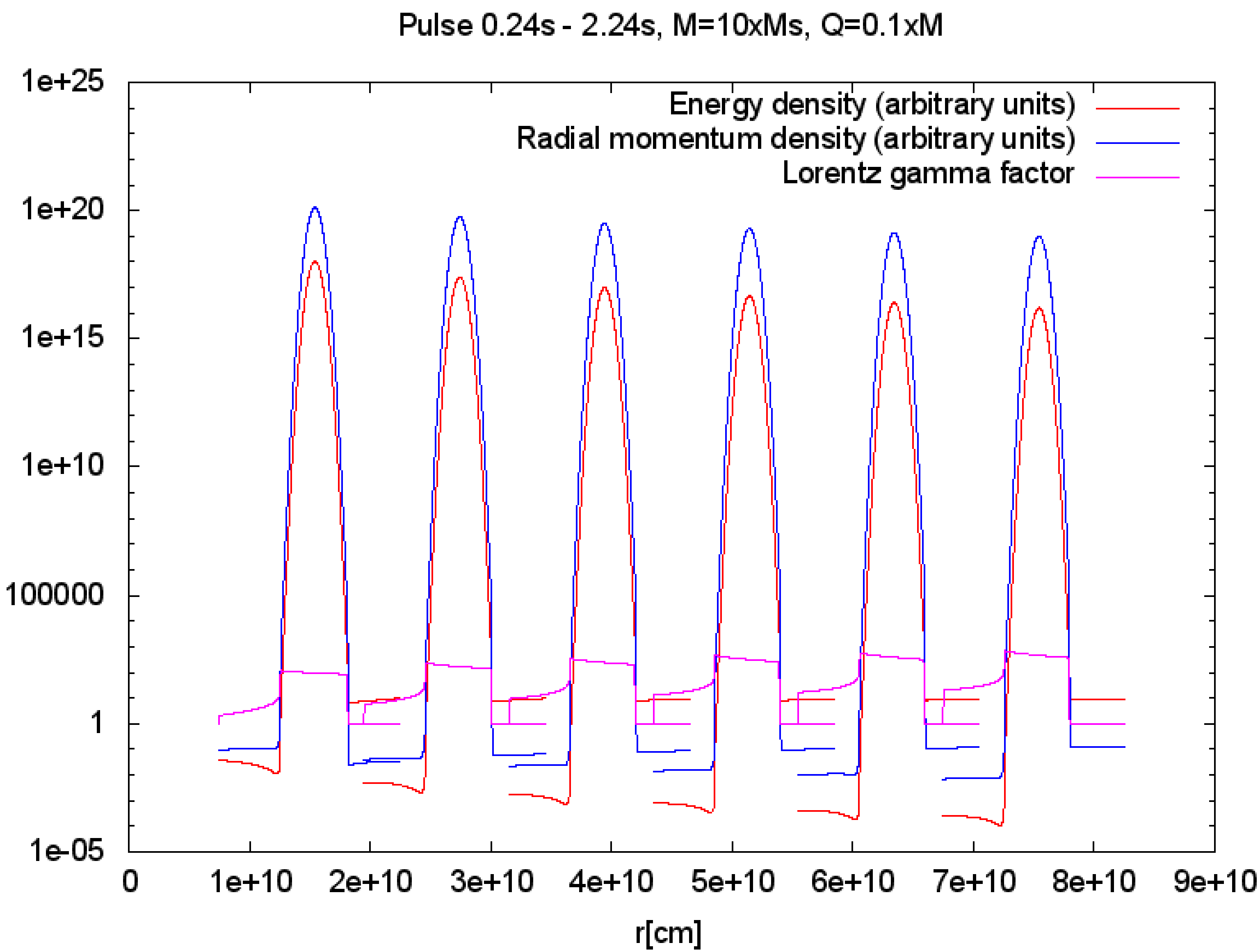
- Appropriate choice of interpolated boundary density for advection (second order).
- Inclusion of a grid velocity.
- The same, plus **artificial viscosity**.

Positivity: $S_r \equiv \alpha(p+\rho)U^t U_r = (D + \Gamma E)U_r$

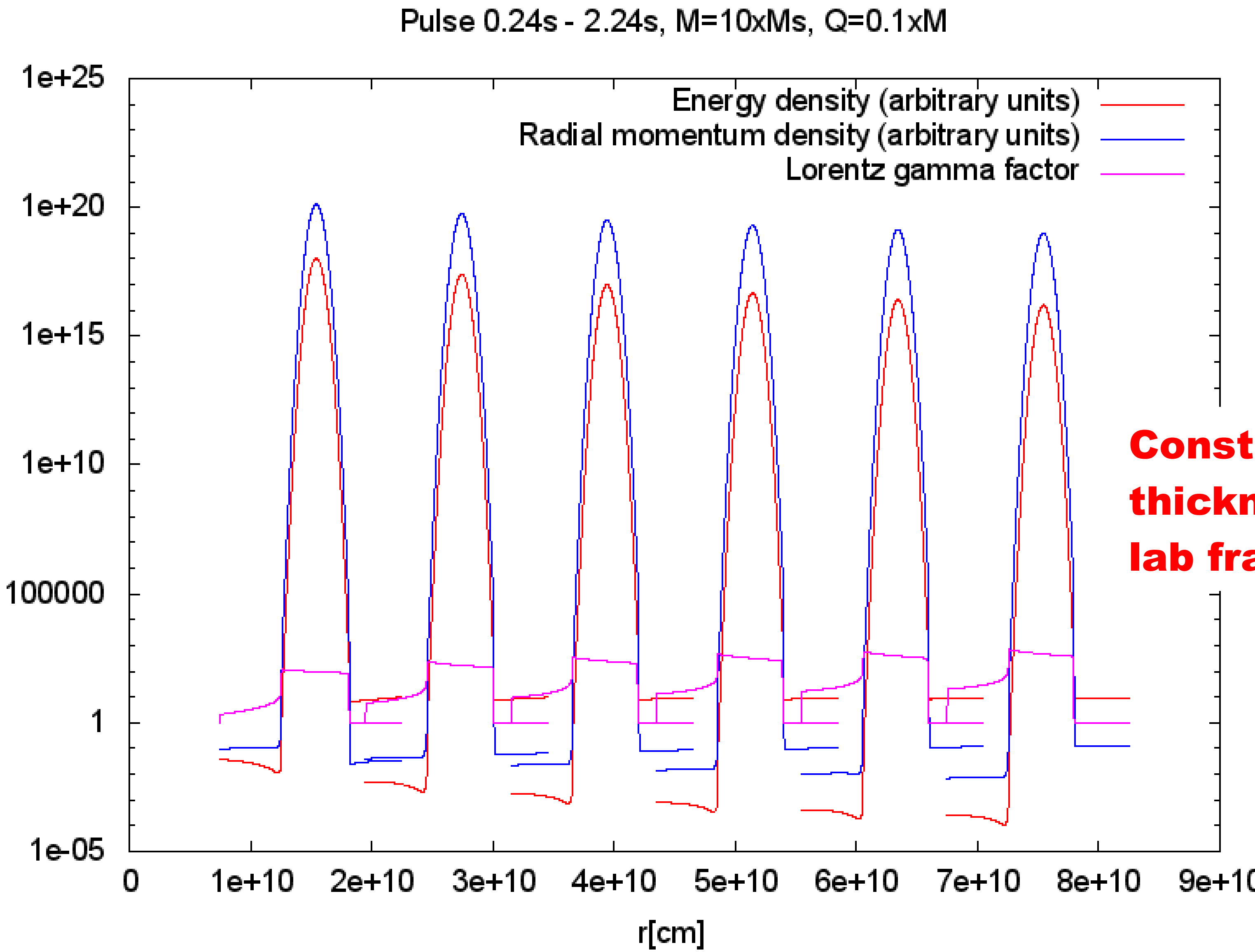
Strategies followed: *Livermore code*, *ordering prescription*

1. pressure acceleration
2. viscosity
3. velocities U, V and W
4. pressure PdV work on fluid
5. advection of state variables
6. velocities again
7. pressure PdV work again
8. time step dt calculation
9. grid update
10. output and post processing when appropriate.

Implementation: *shock evolution*

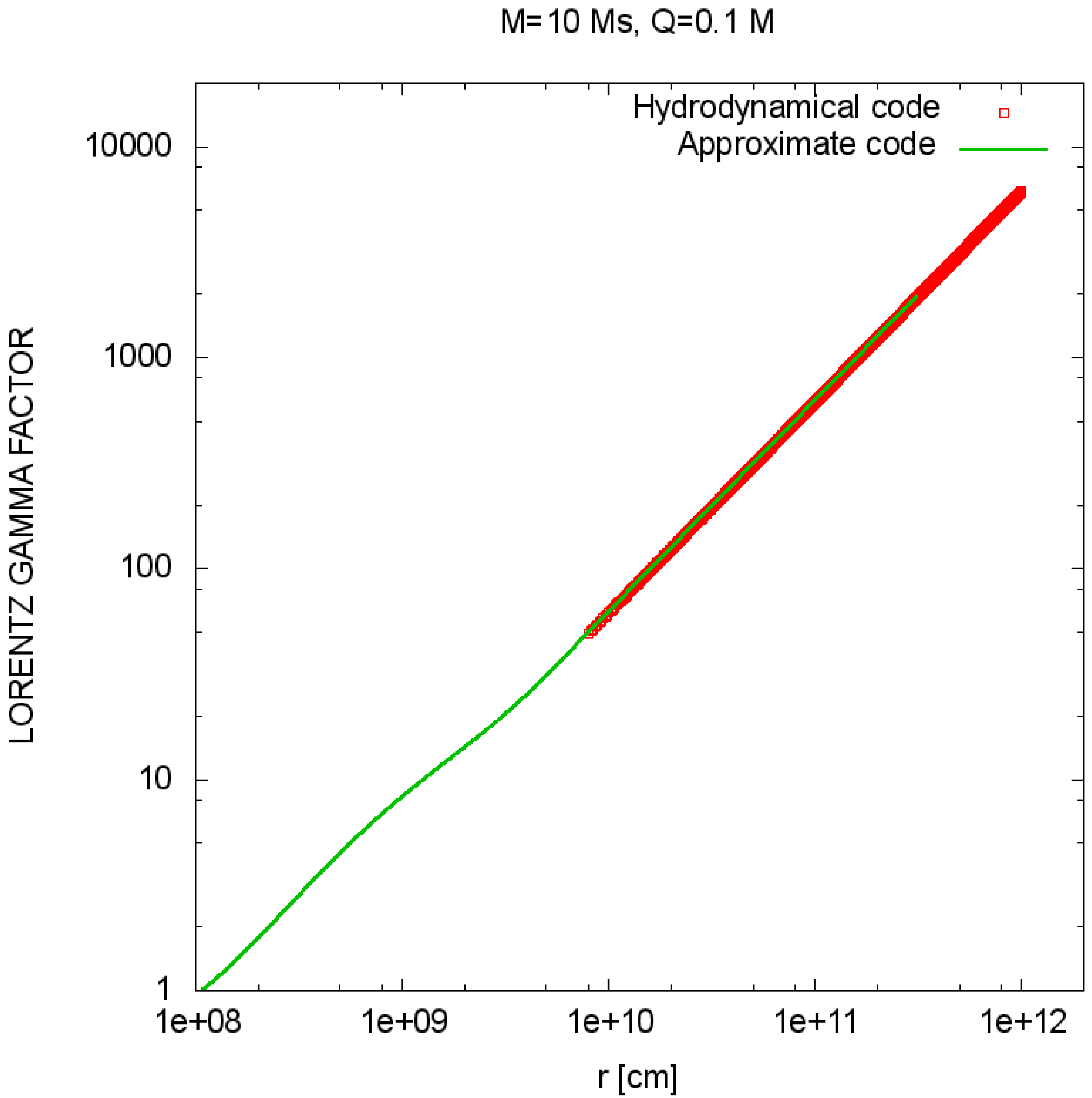


Implementation: *shock evolution*

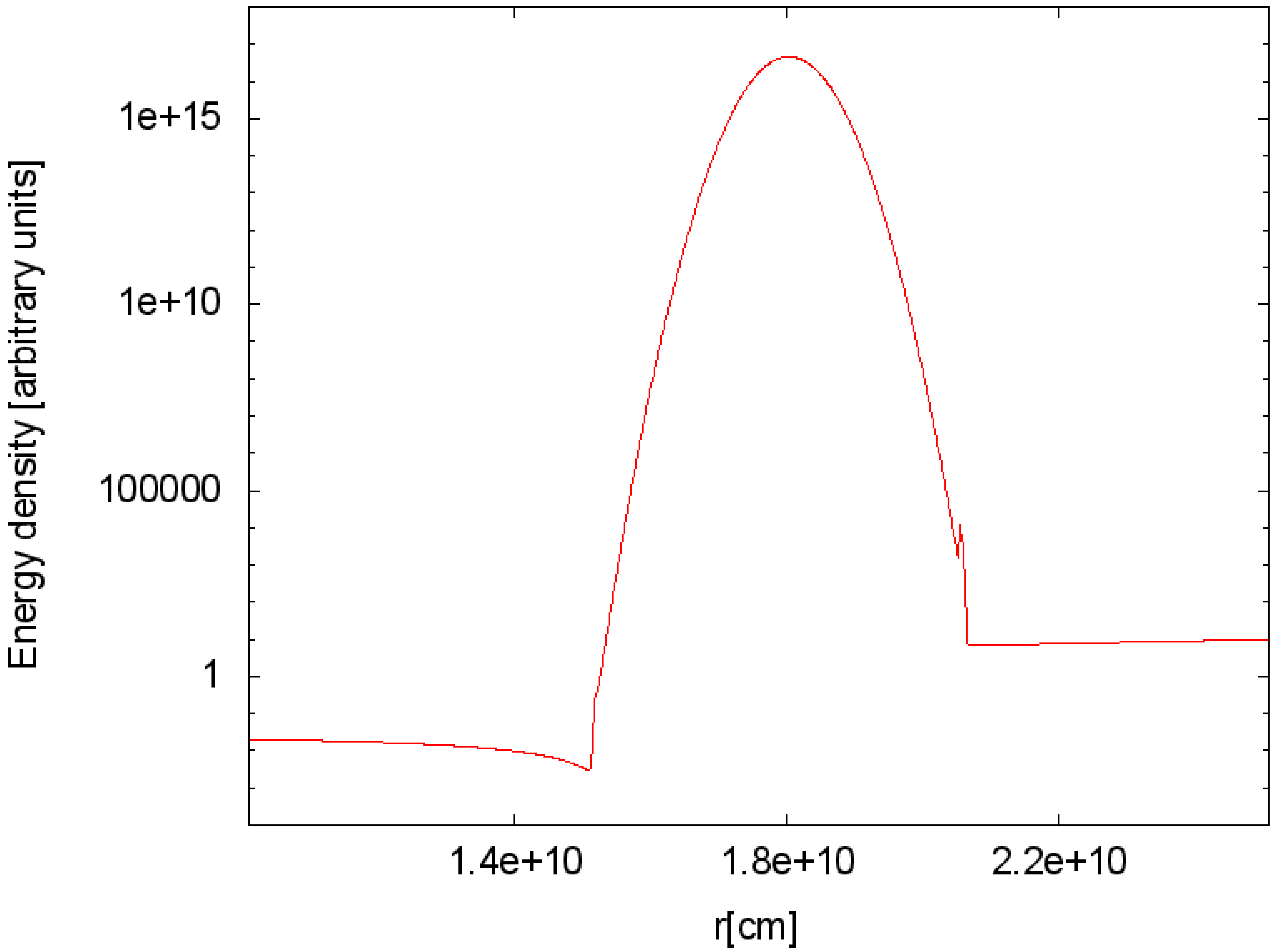


Constant thickness in lab frame

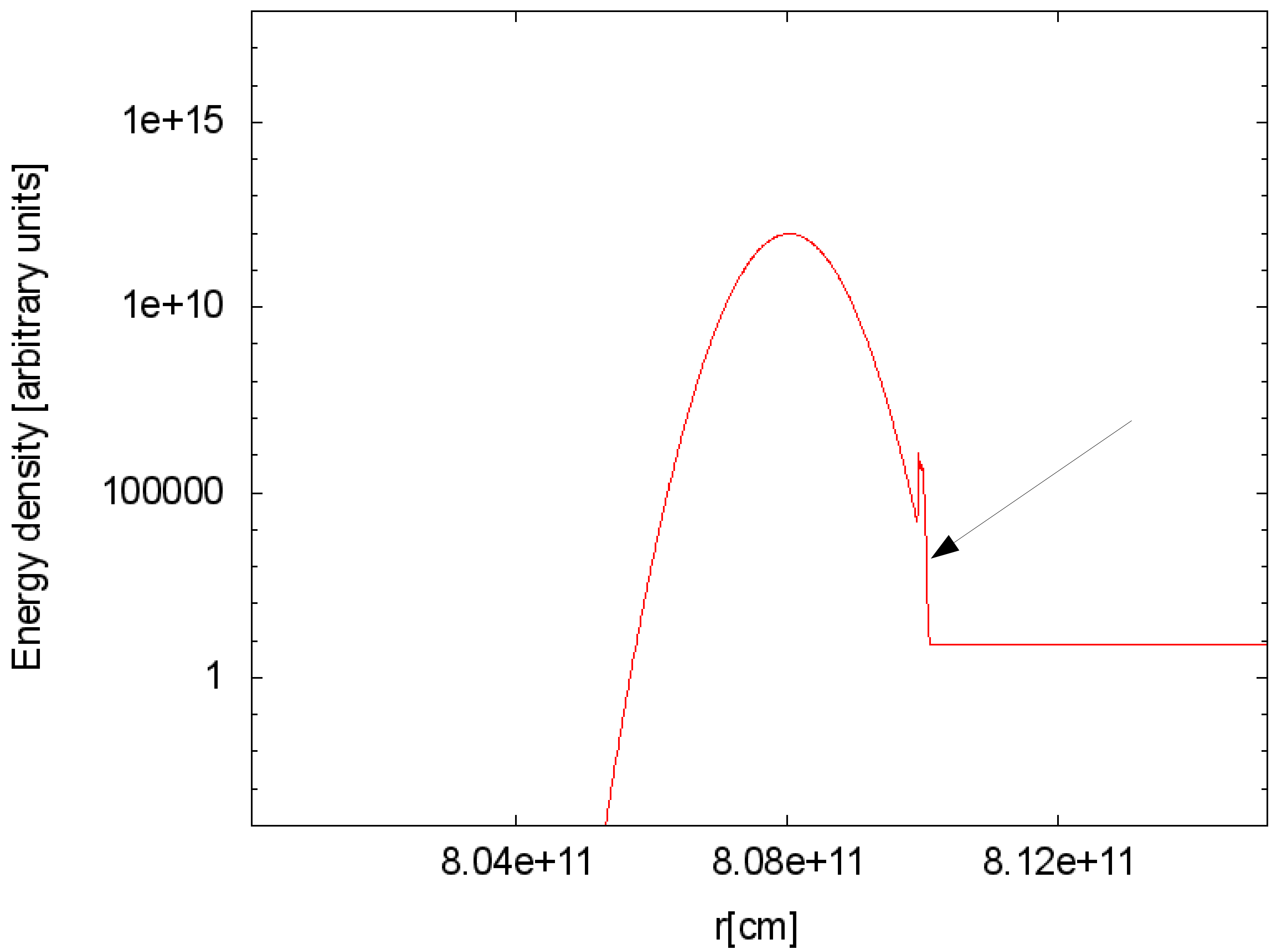
Implementation: *comparison with the approximate code*



$Implementation$: $shock\ profile$



Implementation: *shock profile*



Implementation: *interaction with baryons*

- Very different physical behaviour depending on whether we include a grid velocity or not.
- Artificial viscosity needed to prevent instabilities.
- Instabilities generated anyway, depending on the chosen initial conditions.

So, is our scheme reliable?

Implementation: *tests, Riemann shock tube* (1*D*)

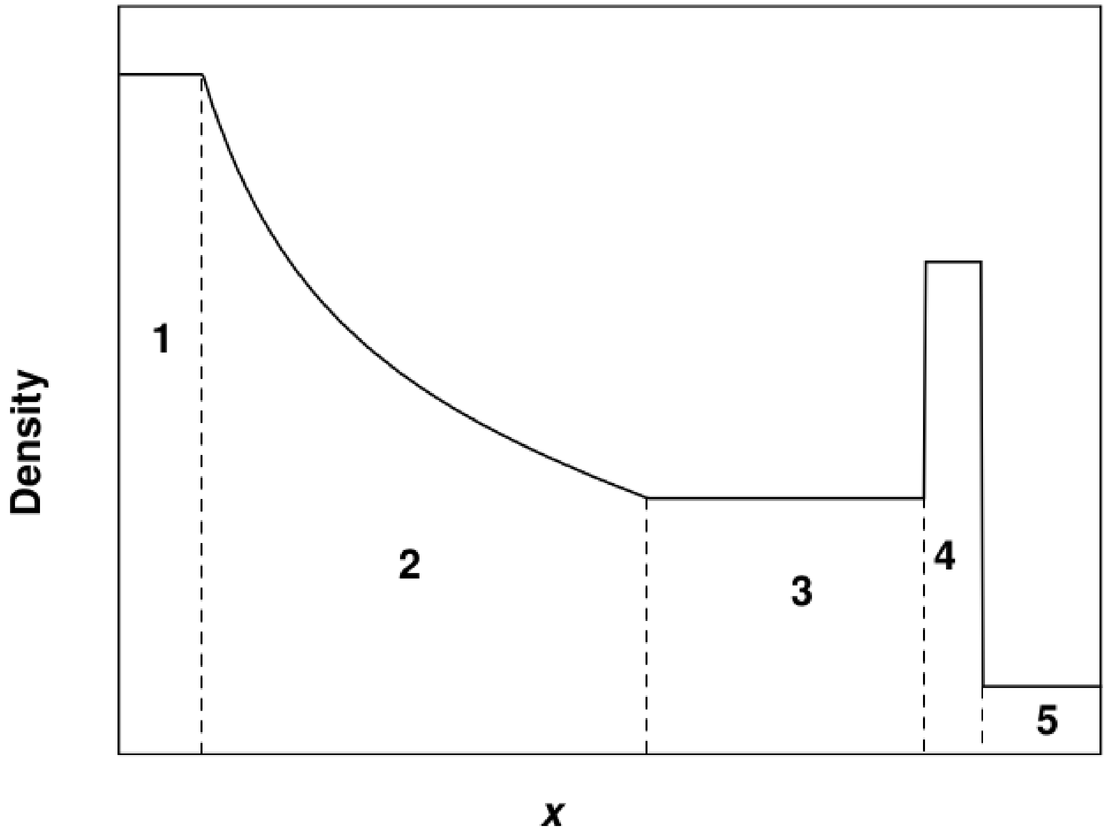
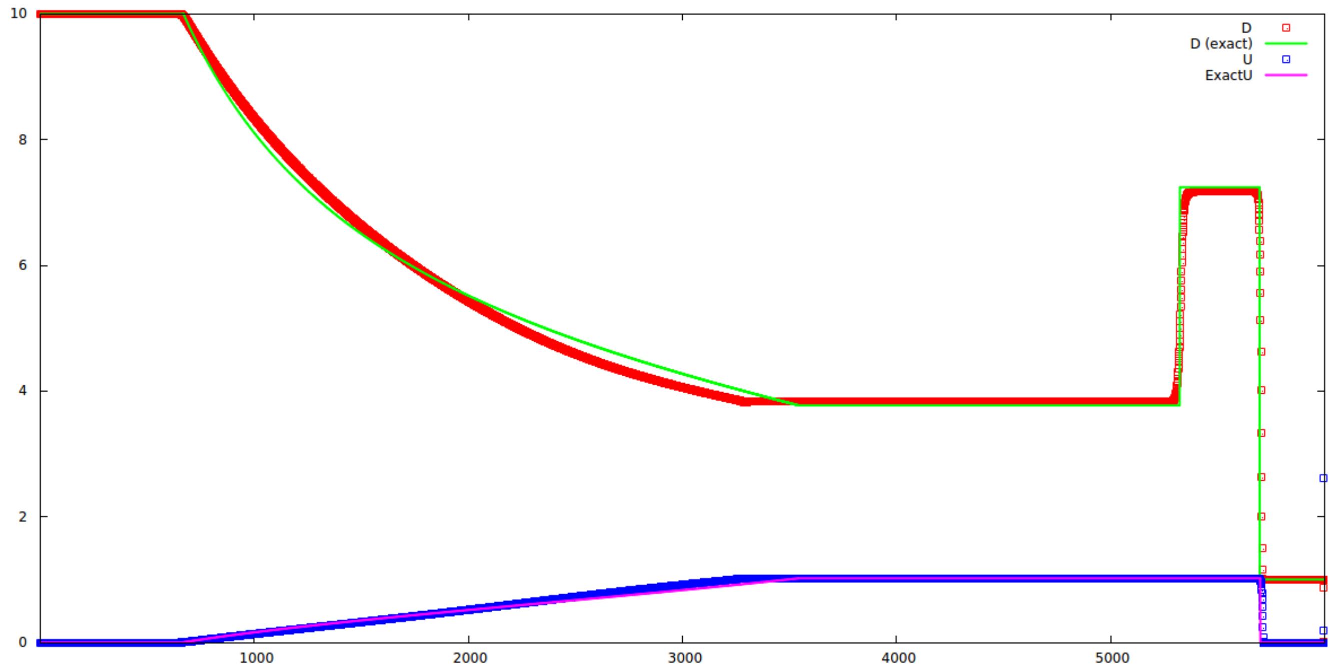


Fig. 2.4. Various regions in the shock tube problem. They are: (1) the undisturbed high density fluid; (2) the rarefaction wave; (3) a region of constant velocity and pressure which features a contact discontinuity separating regions of different density; (4) the shock itself; and (5) the undisturbed low density fluid.

Implementation: *tests, Riemann shock tube* (1*D*)

First attempt (Wilson's ordering prescription) not entirely successful:



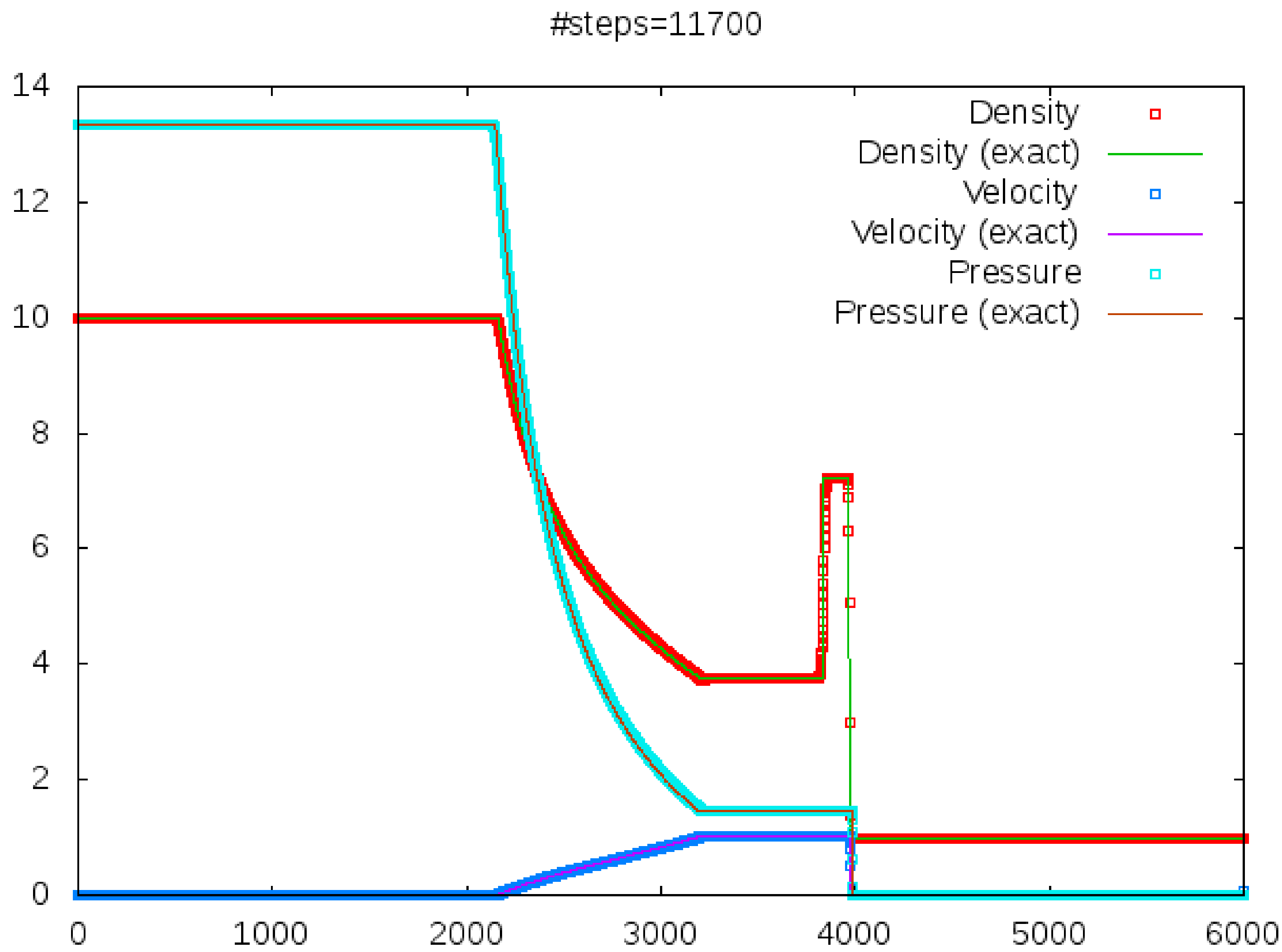
Anninos & Fragile (LLNL, 2003): optimal AV scheme, different ordering.

Artificial viscosity

The scalar viscosity Q_i is computed as a local quantity in a dimensionally split fashion, and active only in convergent flows for which $\nabla_i V^i < 0$

$$Q_i = (D + E + PW)\Delta l(\nabla_i V^i)\left[k_{q2}\Delta l(\nabla_i V^i)(1 - \phi^2) - k_{q1} C_s\right].$$

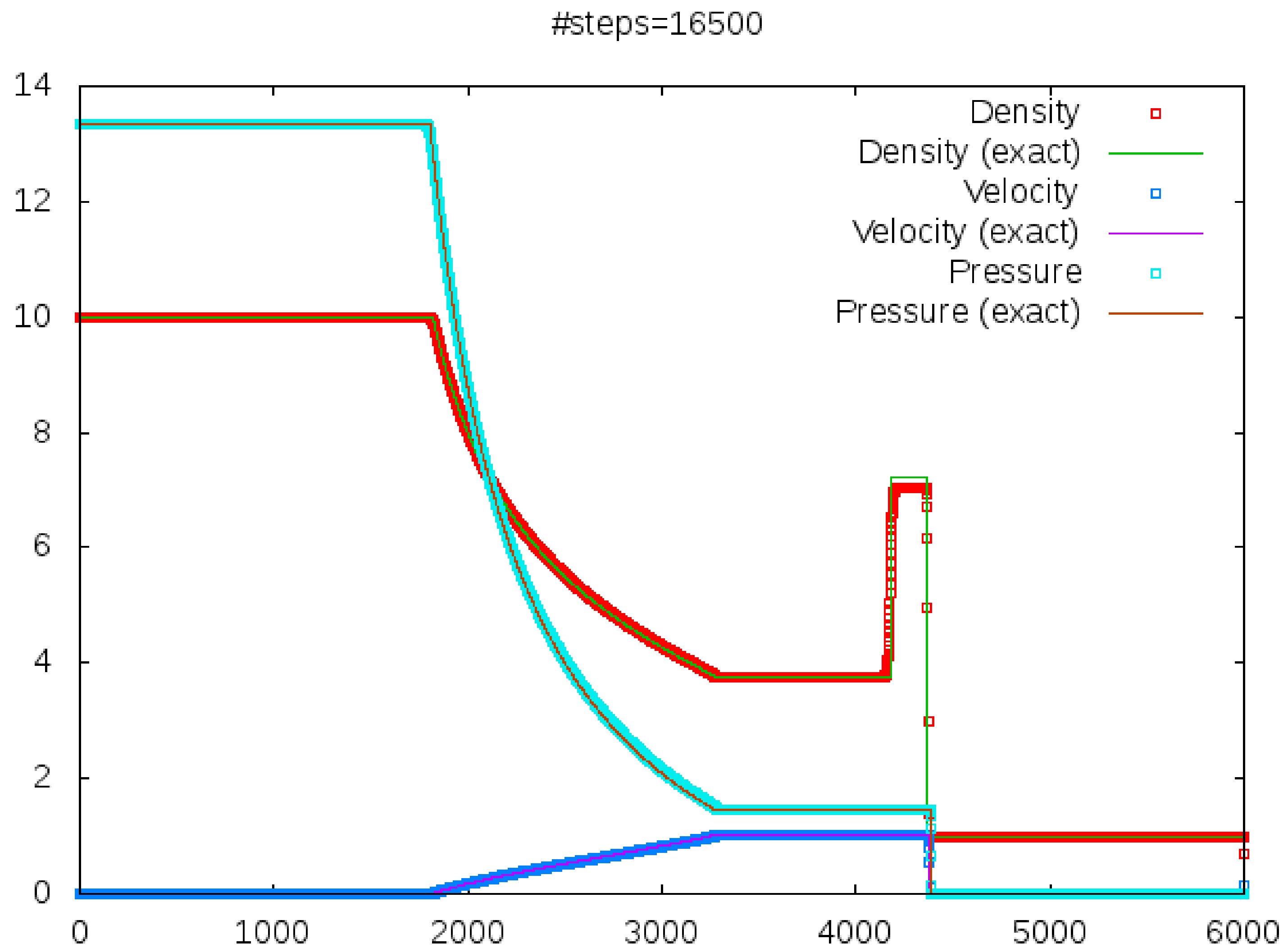
Implementation: *tests, Riemann shock tube, moderate boost*



Maximum boost factor=1.49

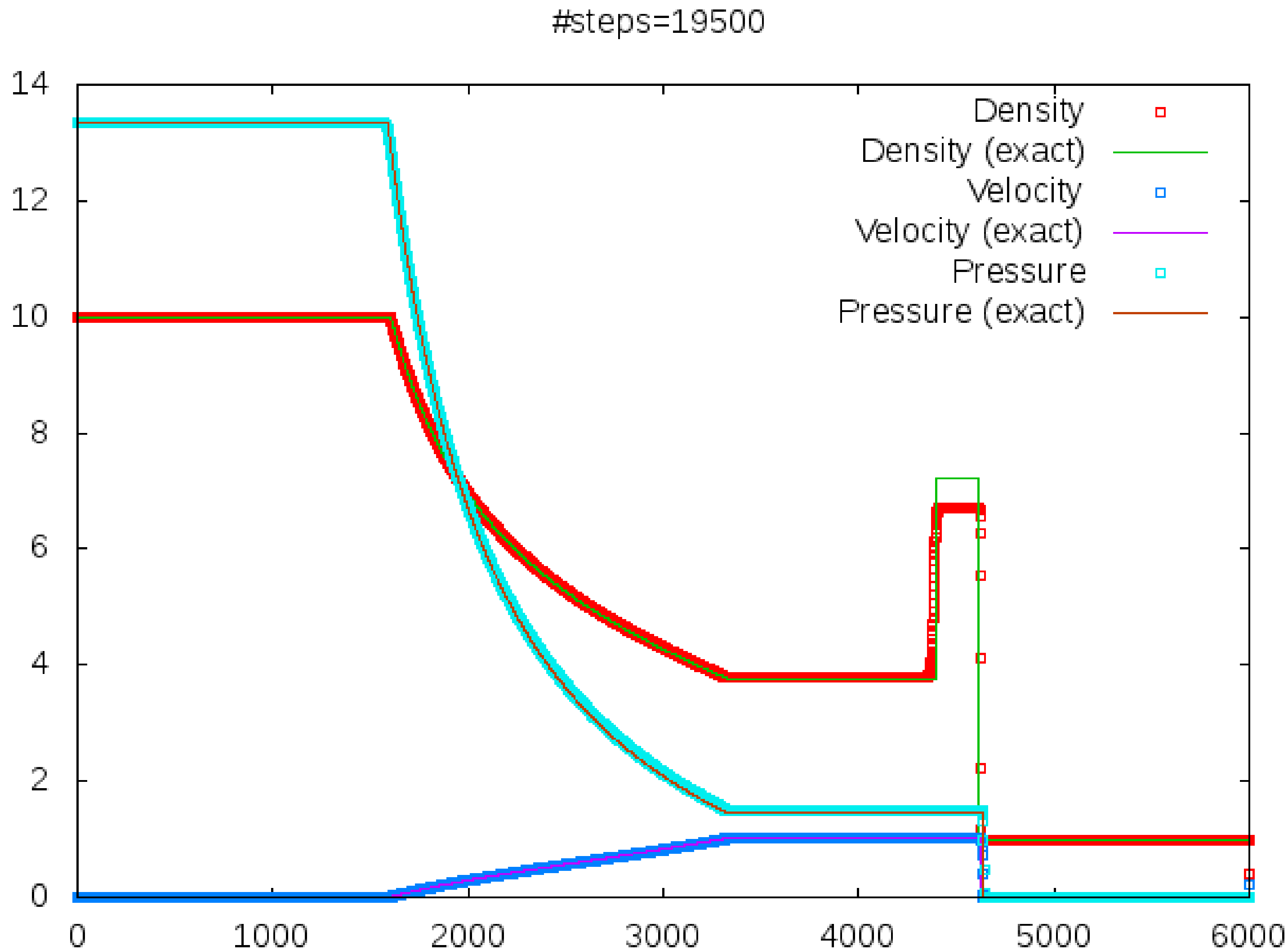
Best choice of AV parameters,
k1=0.32,k2=0.000005

Implementation: *tests, Riemann shock tube, moderate boost*



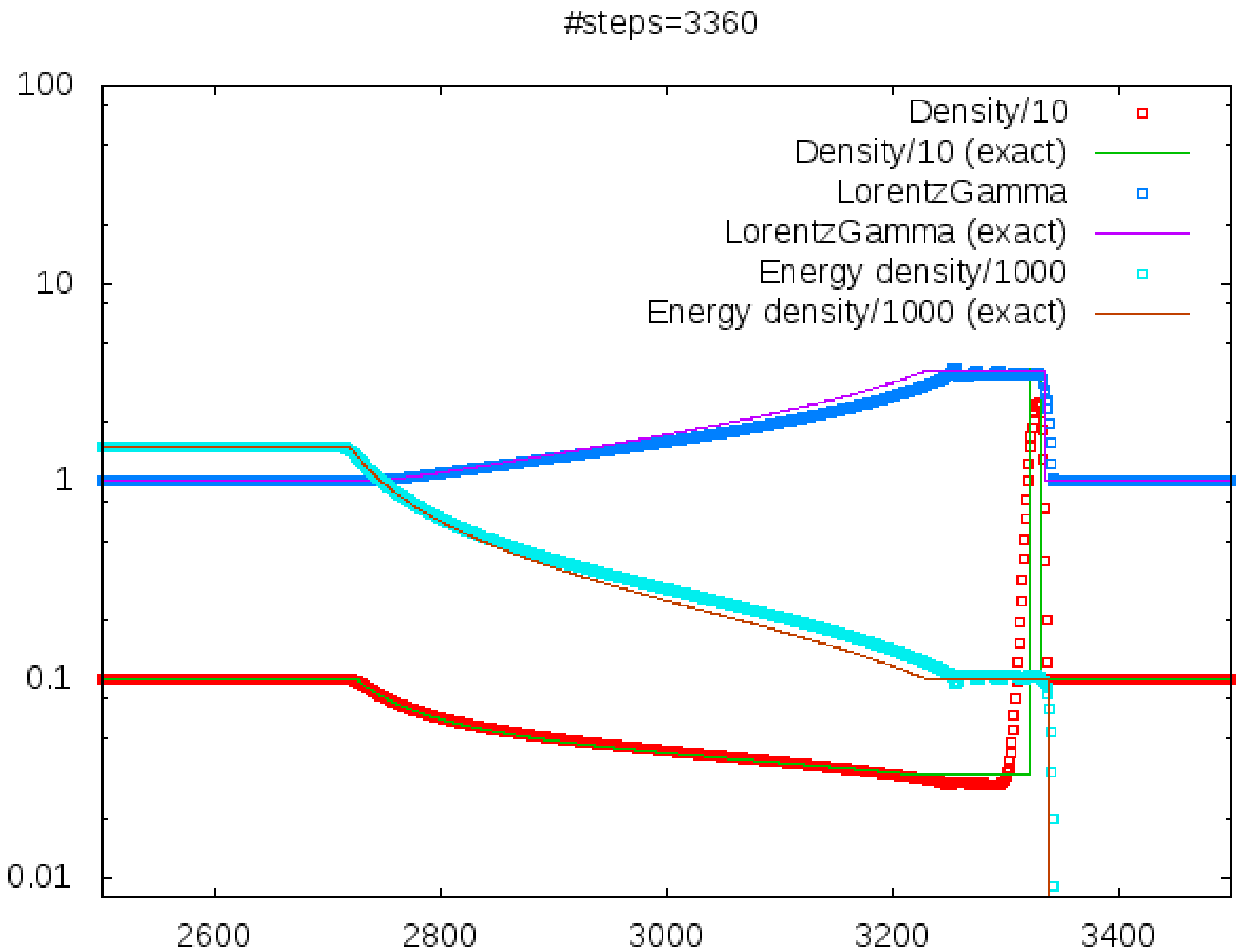
10% change in k1
k1=1.1x0.32,k2=0.000005

Implementation: *tests, Riemann shock tube, moderate boost*



30% change in k1
k1=1.3x0.32,k2=0.000005

Implementation: *tests, Riemann shock tube, high boost*

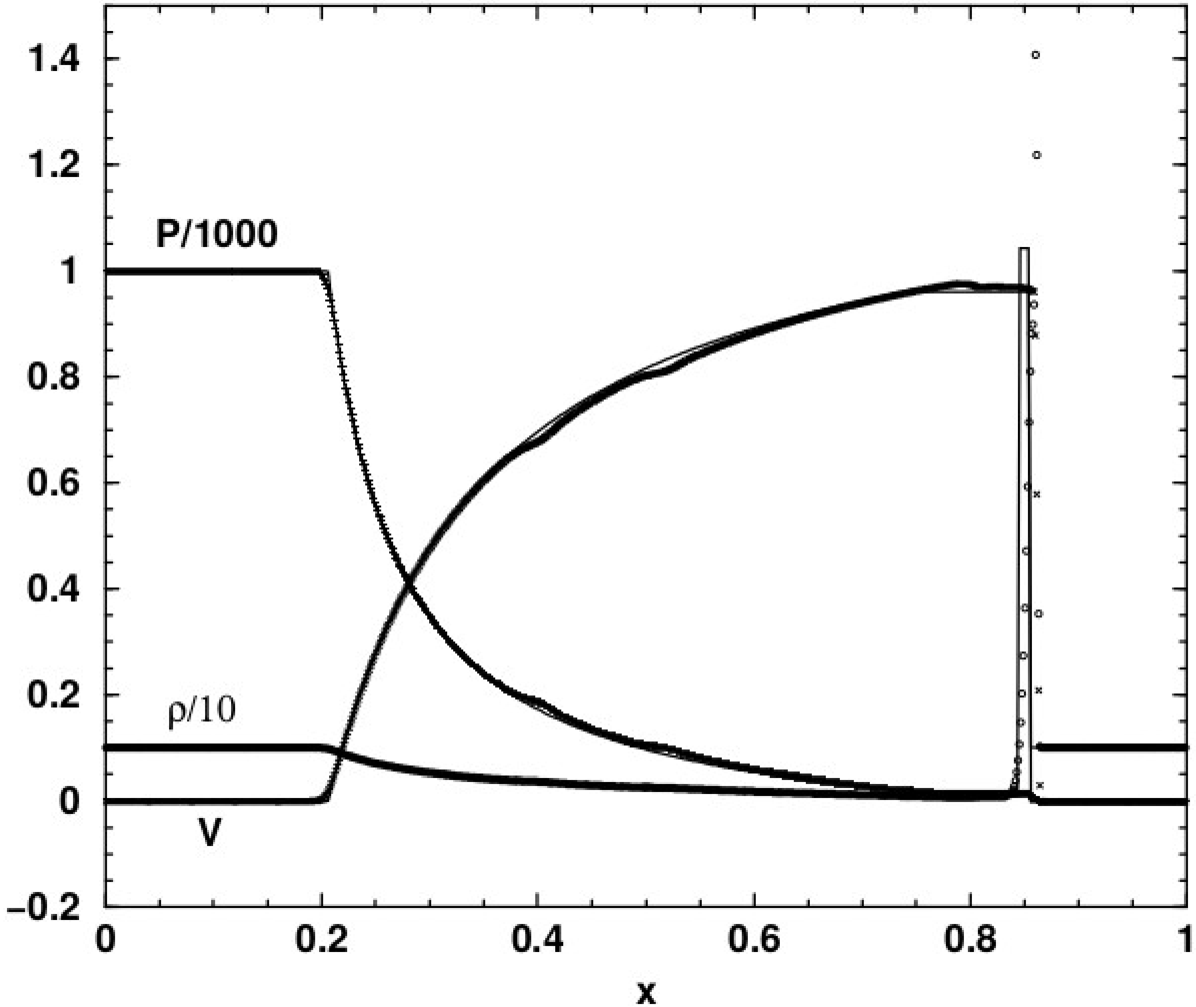


Maximum boost factor=3.59

Best choice in AV parameters, $k_1=0.05$, $k_2=1.2$

Implementation: *tests, Riemann shock tube*

Anninos & Fragile also need to change the AV parameters for high boosts. They obtain:



These AV schemes seem to fail dramatically for **gamma>3**!!

Perspectives

Method	Ultra-relativistic regime	Handling of discontinuities[a]	Extension to several spatial dimensions[b]	Extension to GRHD	RMHD
(c)AV-mono	×[c]	O, SE	√	√	√
cAV-implicit	√	√	×	×	×
RS-HRSC[d]	√	√	√[e]	√[f]	×[g]
rGlimm	√	√	×	×	×
Sym-HRSC	√	√	√	√[h]	√
van Putten	√[i]	D	√	×	√
FCT	√	O	√	×	×
SPH	√	D, O	√	√[j]	×[k]

[a] D: excessive dissipation; O: oscillations; SE: systematic errors.
[b] All finite difference methods are extended by directional splitting.

Martí, Müller (2003)

Conclusions

- We reproduced a hydrodynamical code similar to the one developed by Wilson and Salmonson (1999). In the absence of baryonic matter, **the thickness of the PEM pulse remains constant** during its evolution, which is in agreement with Wilson's results. Besides, **the gamma vs. r curve coincides with that obtained using the constant thickness approximation**.

- When the fluid velocities are high, this code leads to **excessive numerical dissipation** and does not reproduce shocks **if a grid velocity is not included**.

- When applied to the interaction of the plasma with a **baryonic remnant**, the code produces **results that depend on the implementation of the grid velocity**, and that may develop instabilities depending on that, the initial conditions, and the AV scheme.

- The Riemann Shock Tube test verifies that **AV schemes become unreliable for high fluid velocities (gamma>3)**, and that therefore our current case (gamma>100) should be treated using a different Eulerian scheme.

Thank you