Structure of relativistic, rapidly rotating Neutron Stars: interior and exterior spacetime

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Supernovae, Hypernovae and Binary Driven Hypernovae”

An Adriatic Workshop – 20 / 06 / 2016

SUMMARY

Relativistic Figures of Equilibrium: Neutron Stars

- Introduction to the problem of equilibrium in General Relativity (GR)
- Methods - RNS code
- Set of Equasions of State (EOS)
- Equilibrium Configurations
- Useful Fitting formulas
- Marginally Stable Circular Orbits
- Example of Application
- Conclusions and Future Perspectives

The problem of Equilibrium: Neutron Stars

- Determine the geometry of the spacetime interior and surrounding the studied object
- Astrophysical Motivations – **NSs** as cosmic laboratories
- Numerical simulations are needed to understand physical properties of NSs and other compact objects which populate the Universe:
  - EOS
  - Internal Structure
  - **Exterior Spacetime**

Methods

- HT[4] ➔ expantions to $O(\Omega^2)$ order ➔ slow rotation
- BI[5] ➔ Newton-Raphson ➔ ellyptic-type field equations + increment in angular velocity + space truncated at a finite distance
- KEH[6] ➔ 3 integral-type field equations + 1 first-order DE + increment in axis ratio + space truncated at a finite distance
- CST[7] ➔ modification of KEH ➔ new radial variable to map infinite region to a finite interval ➔ no truncation
- SF[8] ➔ **RNS** public code
- AKM[9] ➔ Multi-domain spectral method

[4] J. B. Hartle. ApJ, 150:1005, December 1967

[5] E. M. Butterworth and J. R. Ipser. ApJ, 204:200–223, February 1976

[6] H. Komatsu, Y. Eriguchi, and I. Hachisu. MNRAS, 237:355–379, March 1989

[7] G. B. Cook, S. L. Shapiro, and S. A. Teukolsky. ApJ, 422:227–242, February 1994

[8] N. Stergioulas and J. L. Friedman. ApJ, 444:306–311, May 1995

[9] R. Meinel, M. Ansorg, A. Kleinwächter, G. Neugebauer, and D. Petroff. Cambridge, UK: Cambridge University Press. September 2012

Geometry and Physics

Axisymmetric spacetime described by the following metric:

$$ds^2 = -e^{2\nu}dt^2 + r^2\sin^2(\theta)B^2e^{-2\nu}(d\phi - \omega dt)^2 + e^{2(\zeta-\nu)}(dr^2 + r^2 d\theta^2),$$

where ν, B, ω and ζ depend only on the so-called *quasi-isotropic* coordinates r and θ.

The stress-energy tensor is the one of a perfect fluid, with form:

$$T^{ab} = (\varepsilon + P)u^a u^b + P g^{ab},$$

being ε the energy density, P the pressure and u the fluid's 4-velocity.

Field Equations

$$\nabla\cdot(B\nabla\nu) = \frac{1}{2}r^2\sin^2\theta B^3 e^{-4\nu}\nabla\omega\cdot\nabla\omega + 4\pi B e^{2\zeta-2\nu}\left[\frac{(\varepsilon+P)(1+v^2)}{1-v^2}+2P\right],$$

$$\nabla\cdot\left(r^2\sin^2\theta B^3 e^{-4\nu}\nabla\omega\right) = -16\pi r\sin\theta B^2 \times e^{2\zeta-4\nu}\frac{(\varepsilon+P)v}{1-v^2},$$

$$\nabla\cdot(r\sin(\theta)\nabla B) = 16\pi r\sin\theta B e^{2\zeta-2\nu}P,$$

$$\begin{aligned}
\zeta_{,\mu} = &-\left\{\left(1-\mu^2\right)\left(1+r\frac{B_{,r}}{B}\right)^2+\left[\mu-\left(1-\mu^2\right)\frac{B_{,r}}{B}\right]^2\right\}^{-1}\left[\frac{1}{2}B^{-1}\left\{r^2B_{,rr}-\left[\left(1-\mu^2\right)B_{,\mu}\right]_{,\mu}-2\mu B_{,\mu}\right\}\right.\\
&\times\left\{-\mu+\left(1-\mu^2\right)\frac{B_{,\mu}}{B}\right\}+r\frac{B_{,r}}{B}\left[\frac{1}{2}\mu+\mu r\frac{B_{,r}}{B}+\frac{1}{2}\left(1-\mu^2\right)\frac{B_{,\mu}}{B}\right]+\frac{3}{2}\frac{B_{,\mu}}{B}\left[-\mu^2+\mu\left(1-\mu^2\right)\frac{B_{,\mu}}{B}\right]\\
&-\left(1-\mu^2\right)r\frac{B_{,\mu r}}{B}\left(1+r\frac{B_{,r}}{B}\right)-\mu r^2(\nu_{,r})^2-2\left(1-\mu^2\right)r\nu_{,\mu}\nu_{,r}+\mu\left(1-\mu^2\right)(\nu_{,\mu})^2-2\left(1-\mu^2\right)r^2B^{-1}B_{,r}\nu_{,\mu}\nu_{,r}\\
&+\left(1-\mu^2\right)B^{-1}B_{,\mu}\left[r^2(\nu_{,r})^2-\left(1-\mu^2\right)(\nu_{,\mu})^2\right]+\left(1-\mu^2\right)B^2e^{-4\nu}\left\{\frac{1}{4}\mu r^4(\omega_{,r})^2+\frac{1}{2}\left(1-\mu^2\right)r^3\omega_{,\mu}\omega_{,r}\right.\\
&\left.\left.-\frac{1}{4}\mu\left(1-\mu^2\right)r^2(\omega_{,\mu})^2+\frac{1}{2}\left(1-\mu^2\right)r^4B^{-1}B_{,r}\omega_{,\mu}\omega_{,r}-\frac{1}{4}\left(1-\mu^2\right)r^2B^{-1}B_{,\mu}\left[r^2(\omega_{,r})^2-\left(\mu^2\right)(\omega_{,\mu})^2\right]\right\}\right]
\end{aligned}$$

Hydrostationary Equilibrium equation

$$P_{,i} + (\varepsilon + P)\left[\nu_{,i} + \frac{1}{1-v^2}\left(-vv_{,i} + v^2\frac{\Omega_{,i}}{\Omega-\omega}\right)\right] = 0,$$

with v the fluid 3-velocity with respect to local ZAMO and Ω the angular velocity in the coordinate frame.

↓

One obtains models of self-gravitating,

uniformly rotating stars

(**RNS public code**)

http://www.gravity.phys.uwm.edu/rns/

What about *differential rotation*?

Differential rotation in a newly born NS could be soon braked by different physical mechanism, which favour uniform rotation[10].

After *the first year of formation*, adopting uniform rotation comports an error in the spacetime metric potentials of 10^{-12}.

Also **finite temperature** affects NS's structure during the first year ➔ temperature effects could be neglected for bulk properties.

[10] N. Stergioulas. Living Reviews in Relativity, 6:3, June 2003

<u>Realistic</u> <u>EOS</u> ➔ *Three* regions:

- the **CORE**: $\varepsilon > \varepsilon_{nuc} \approx 3*10^{14}\ g*cm^{-3}$ (the *nuclear saturation value*)

  - RMF models[11] ✚ TM1, GM1 and NL3 (*nuclear parametrizations*);

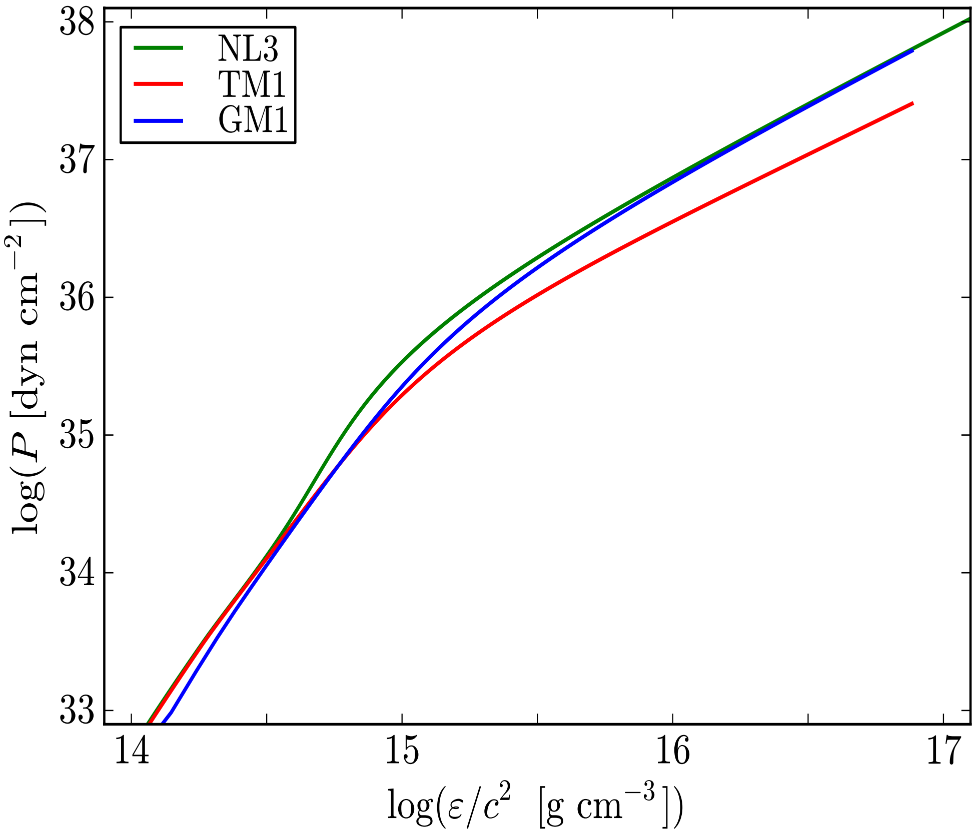
- the **CRUST**: $\varepsilon < \varepsilon_{drip} \approx 4.3*10^{11}\ g*cm^{-3}$("*neutron drip point*")

  - BPS EOS;

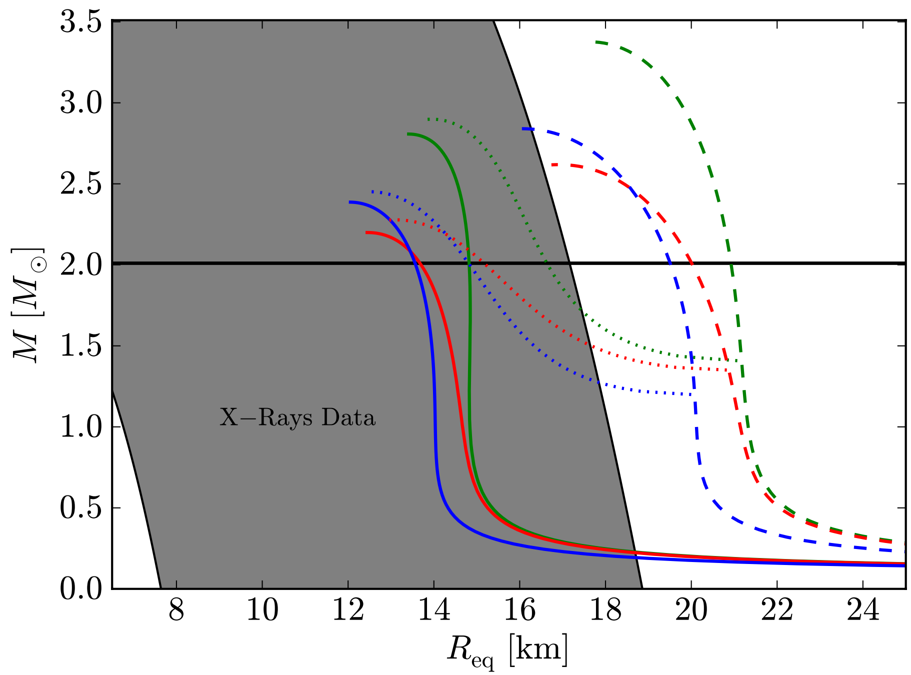
- "**Intermediate region**": $\varepsilon_{drip} < \varepsilon < \varepsilon_{nuc}$

  - <u>local charge neutrality</u>.

[11]Dutra et al, PRC (2014)



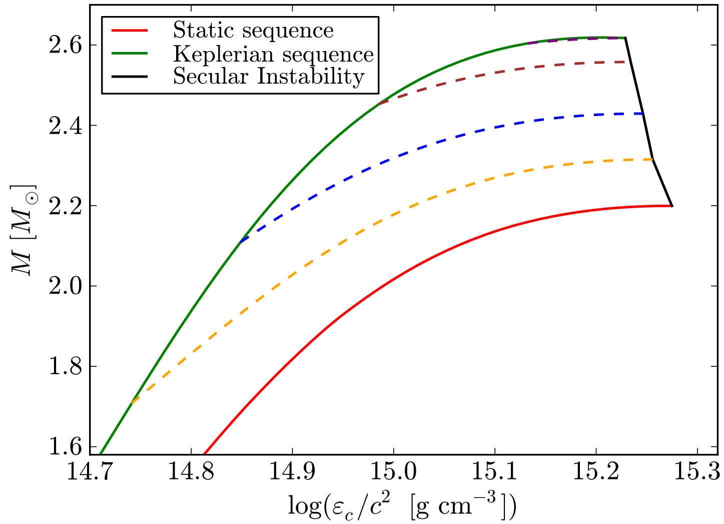
Solid = static sequence;
Dashed = Keplerian sequence;
Dotted = sequence rotating with f = 716 Hz



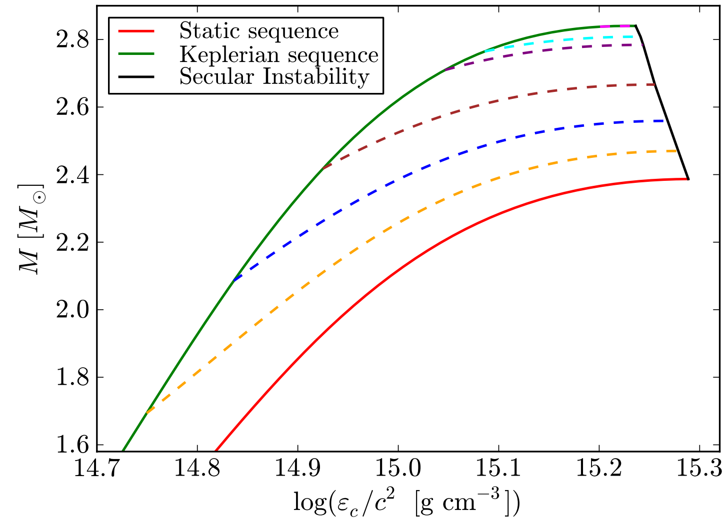
Ref.s: Demorest et al, Nature, (2010); Hessels et al, Science, (2006); Heinke el al, ApJ, (2006); Lattimer, Prakash, Eur Phys J (2014)

Choosen set EOSs

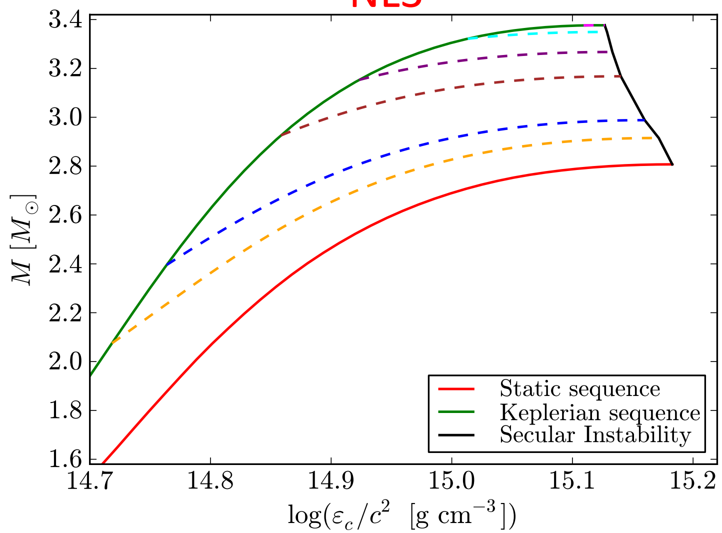
TM1



GM1



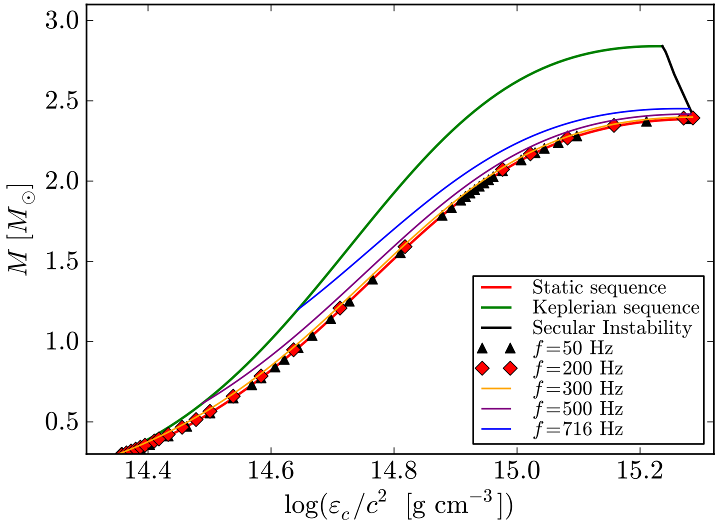
NL3



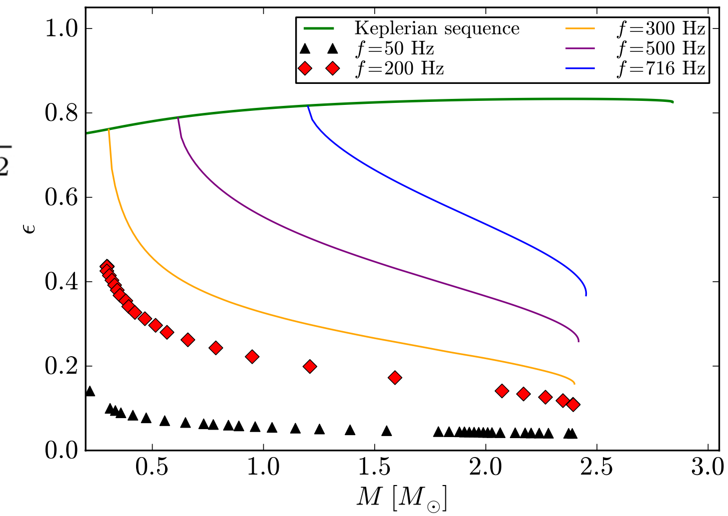
Dashed curves: constant angular momentum sequences

Ref.: Friedman, Ipser, Sorkin, ApJ (1988)

GM1



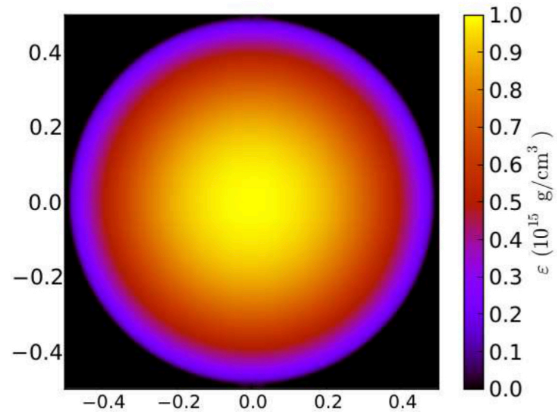
$$\epsilon = \sqrt{1 - \left(\frac{R_{\mathrm{pol}}}{R_{\mathrm{eq}}}\right)^2}$$



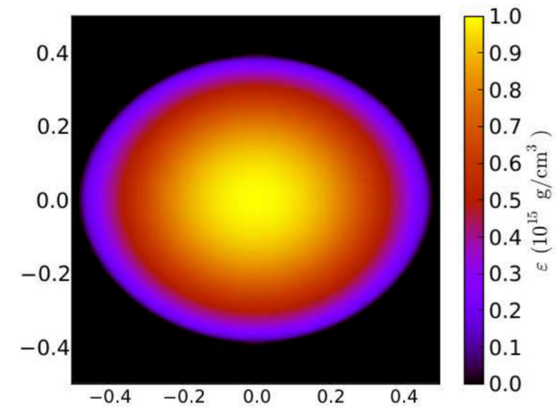
Slowly rotating config.s seem to slightly deviate from static ones both in mass and in shape.

SPHERICAL

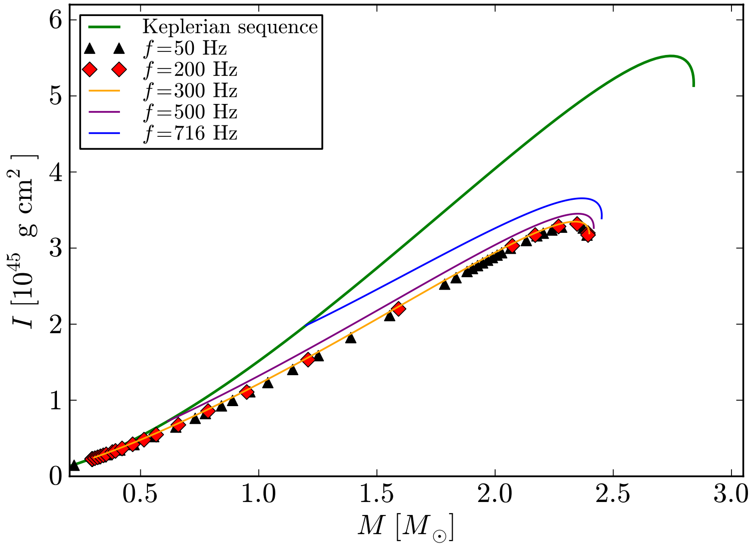
$$\varepsilon_c = 10^{15}\ \mathrm{g\ cm^{-3}}$$



$$j = 4$$

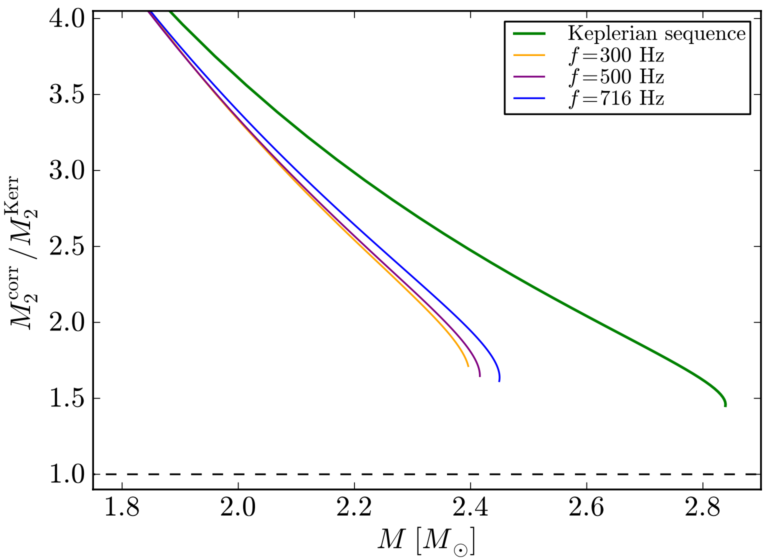


$$I = \frac{J}{\Omega}$$



For a fixed mass, the lower the frequency the more the moment of inertia approaches the non-rotating value.

$$M_2^{\rm Kerr} = \frac{J^2}{M}$$



- Pappas, Apostolatos, Phys. Rev. Lett. (2012)
- Yagi, Kyutoku, Pappas, Yunes, Apostolatos, PRD (2014)

CORRECTION TERM

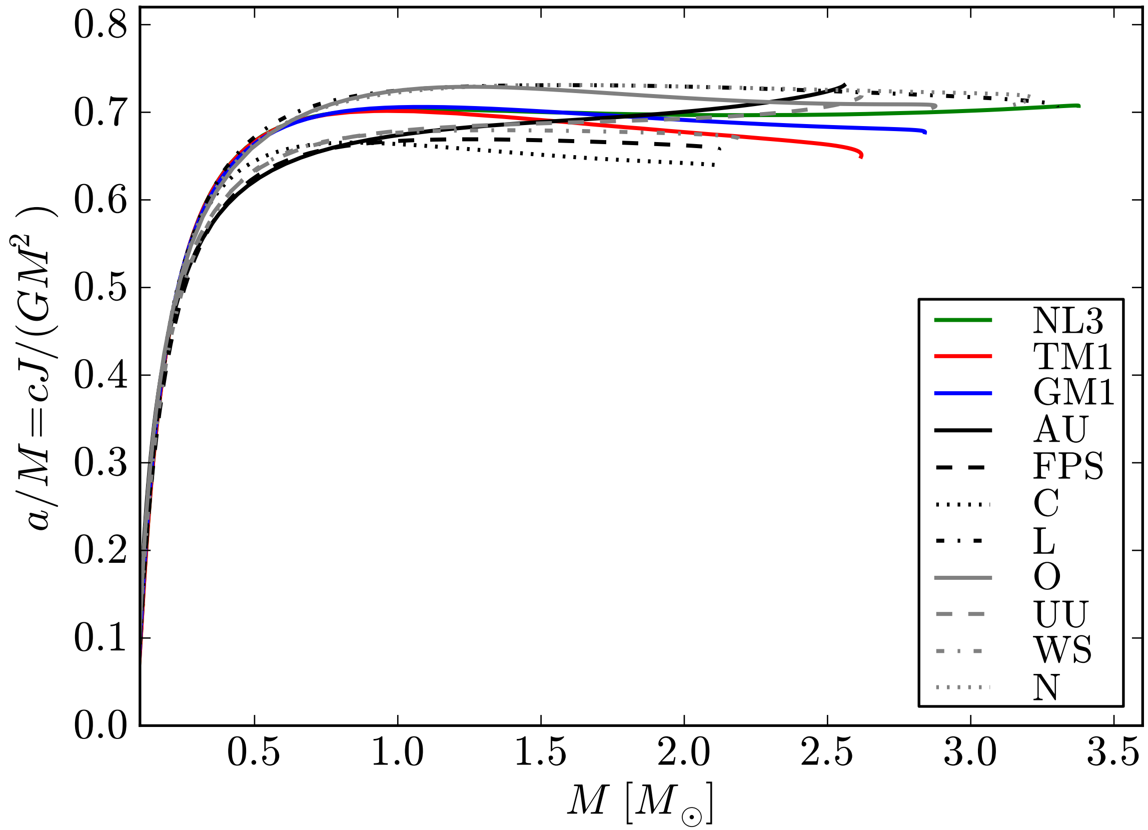
- Ryan, PRD (1995)

Maximum value for "Kerr parameter"

Defining the Kerr parameter as

$$a/M \equiv cJ/(GM^2),$$

We have noticed that the three EOSs implemented reach a common maximum of 0.7 (in the Keplerian regime) for this dimensionless angular momentum, also respected by other already known and used EOSs.



This universal behaviour has been already pointed out in literature (see e.g. Lo, Lin, ApJ (2011)).

Binding Energy

- Static: $$\frac{M_b}{M_\odot} \approx \frac{M}{M_\odot} + \frac{13}{200}\left(\frac{M}{M_\odot}\right)^2,$$ ➔ Maximum relative error around 1% (near the critical static values of configurations);
- Rotation: $$\frac{M_b}{M_\odot} = \frac{M}{M_\odot} + \frac{13}{200}\left(\frac{M}{M_\odot}\right)^2\left(1 - \frac{1}{130} j^{1.7}\right),$$ ➔ Maximum relative error of 2% for all EOSs.

$$j \equiv \frac{cJ}{GM_\odot^2},$$

Secular Axisymmetric Instability Sequence

$$M = M_{\max}^{J=0}(1 + kj^l),$$

Mass of configurations lying on the secular asisymmetric sequence

EOS	$M_{\max}^{J=0}$ $[M_\odot]$	$M_{\max}^{J\neq0}$ $[M_\odot]$	$f_{\max}$ [kHz]	k	l	Max. Rel. Err.
TM1	2.20	2.62	1.34	0.017	1.61	0.33%
GM1	2.39	2.84	1.49	0.011	1.69	0.44%
NL3	2.81	3.38	1.40	0.006	1.68	0.45%

Published: F. Cipolletta, C. Cherubini, S. Filippi, J. A. Rueda, R. Ruffini. PRD (2015)

Marginally Stable Circular Orbits

$$V(r,\tilde{E},\tilde{L}) = e^{2\lambda+\gamma}\left(\frac{dr}{d\tau}\right)^2 = e^{-\rho}\left(\tilde{E}-\omega\tilde{L}\right)^2 - e^{\gamma} - \frac{e^{\rho}}{r^2}\tilde{L}^2,$$

To obtain circular orbits, one must impose

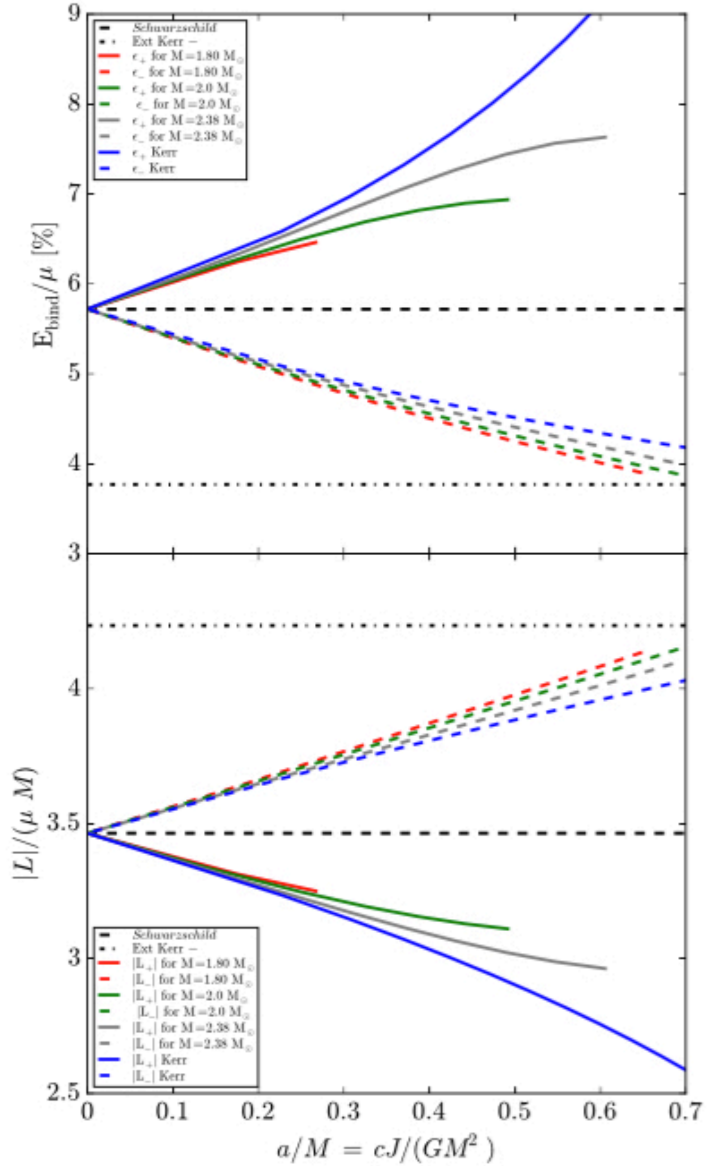
$$V = V_{,r} = 0,$$

And solving these equations give the radius for the orbit.

To find the minimum also this condition must be checked

$$V_{,rr} \geq 0,$$

GM1



Universal

$$\tilde{E} - \tilde{E}_0 = \mp 0.0132 \left(\frac{j}{M/M_\odot} \right)^{0.85} ,$$

$$|\tilde{L}| - \tilde{L}_0 = \mp 0.37 \left(\frac{j}{M/M_\odot} \right)^{0.85} ,$$

with respectively a maximum relative error of 1% and 0.3% where

$$\tilde{E}_0 = \sqrt{8/9}$$

$$\tilde{L}_0 = 2\sqrt{3}$$

(*Schwarzschild solution*)

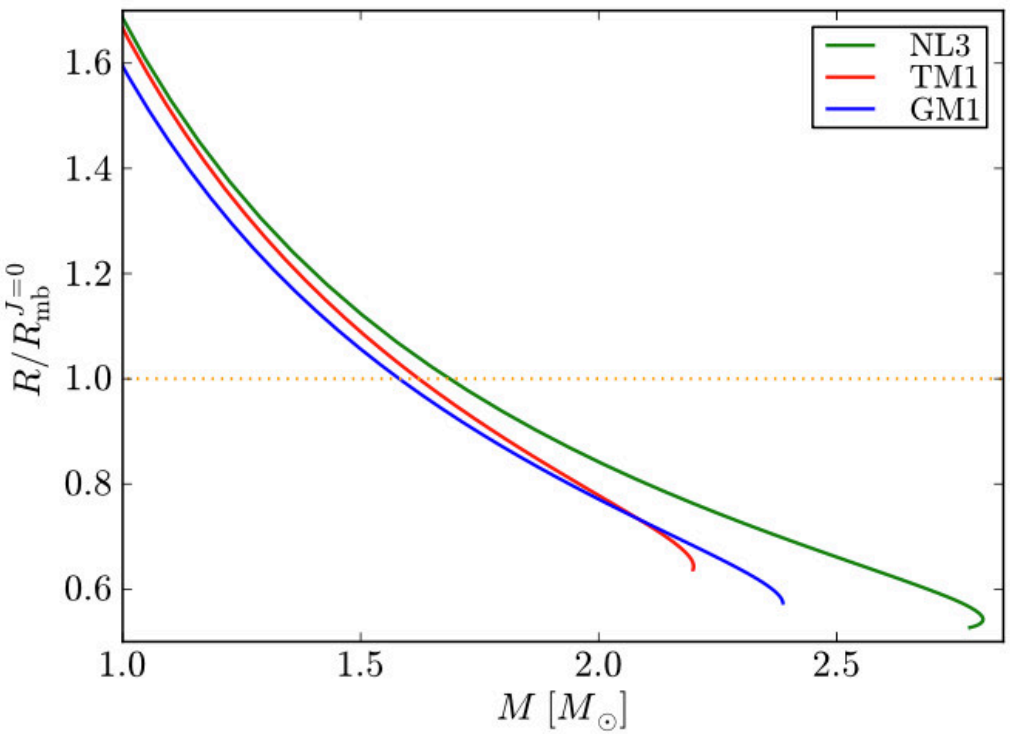
Both co- and counter-rotating particles.

Considering a fixed angular momentum, it is possible to see that one obtains a minimum mass for which the ISCO/MBO is external to the configuration.

⬇

Fixing a mass there is a maximum angular momentum.

$$J = 0 \rightarrow R_{\rm mb}^{J=0} = 6GM/c^2$$



TM1 ➔ 1.78

GM1 ➔ 1.61

NL3 ➔ 1.67

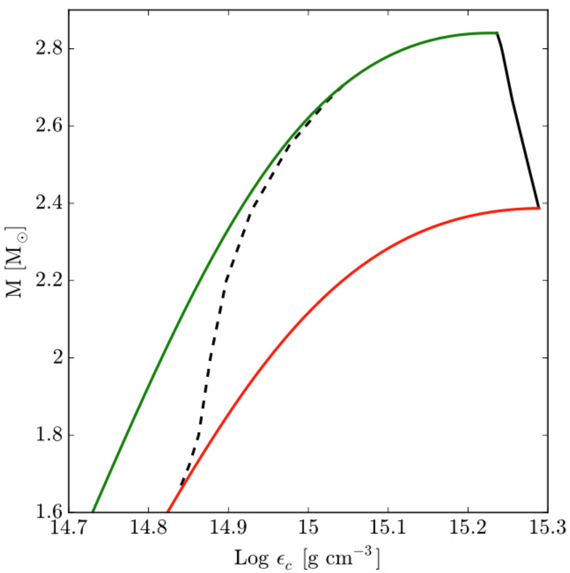
R = radius of non-rotating star

EOS-dependent

$$\frac{M_{\rm min}}{M_\odot} = \frac{M_{\rm min}^{j=0}}{M_\odot} + c_1 j^{c_2},$$

EOS	$M_{\rm min}^{j=0}$	c_1	c_2	Max rel err (%)	$M_{\rm Max\,rel\,err}$
NL3	1.78	0.125	1.235	0.97	2.00
TM1	1.71	0.130	1.30	0.65	1.90
GM1	1.67	0.130	1.30	0.71	1.80

GM1



➔ *Fixing j, one obtain a **minimum** mass, M_{min}, for which the ISCO/MBO is external*

In Preparation : F. Cipolletta, J. A. Rueda, R. Ruffini. *On the mostly bound circular orbit around rapidly rotating neutron stars.*

Example of Application

An example of application is within an *hypercritical accretion scenario of a binary-driven Hypernova*. Consider a CO core-NS binary system[12], where it has been shown that GRB explosion and BH formation can be obtained:

- CO core explode in a SN, forming a NS (material is ejected)
- Relation for baryonic mass as function of NS mass and angular momentum (binding energy)
- Minimum mass of the NS which allows an external ISCO/MBO
- Ejected materials have enough angular momentum to circularize around NS, before collapse on its surface
- Angular momentum conservation ➔ NS spin-up
- If the mass of NS is enough, the accretion process will lead to secular instability ➔ Collapse
- Conditions to have the formation of a BH after a GRB explosion

[12] **Published:** L. Becerra, F. Cipolletta, C. L. Fryer, J. A. Rueda, and R. Ruffini. ApJ, 812:100, October 2015

CONCLUSIONS AND PERSPECTIVES – RELATIVISTIC

- Structure of realistic NSs config.s has been reconstructed
- Already known NS constraints and results have been confirmed with our set of EOSs
- Fitting formulas (both universal and EOS-dependent) have been obtained
- **PERSPECTIVES:**
    - Implement new realistic EOS
    - Global charge neutrality (instead of local)
    - Astrophysical applications

Thank you