Dark energy without dark energy

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DLW: **New J. Phys. 9 (2007) 377**

Phys. Rev. Lett. 99 (2007) 251101

arXiv:0712.3984

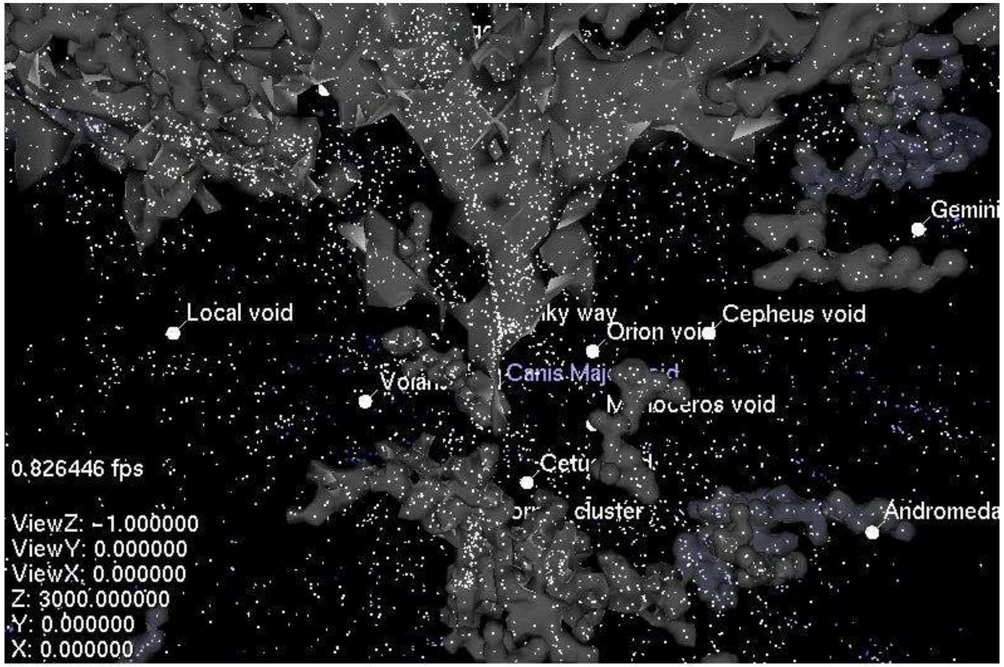
Phys. Rev. D78 (2008) in press

[arXiv:0809.1183];

new results, to appear

B.M. Leith, S.C.C. Ng and DLW:

ApJ 672 (2008) L91



What is "dark energy"?

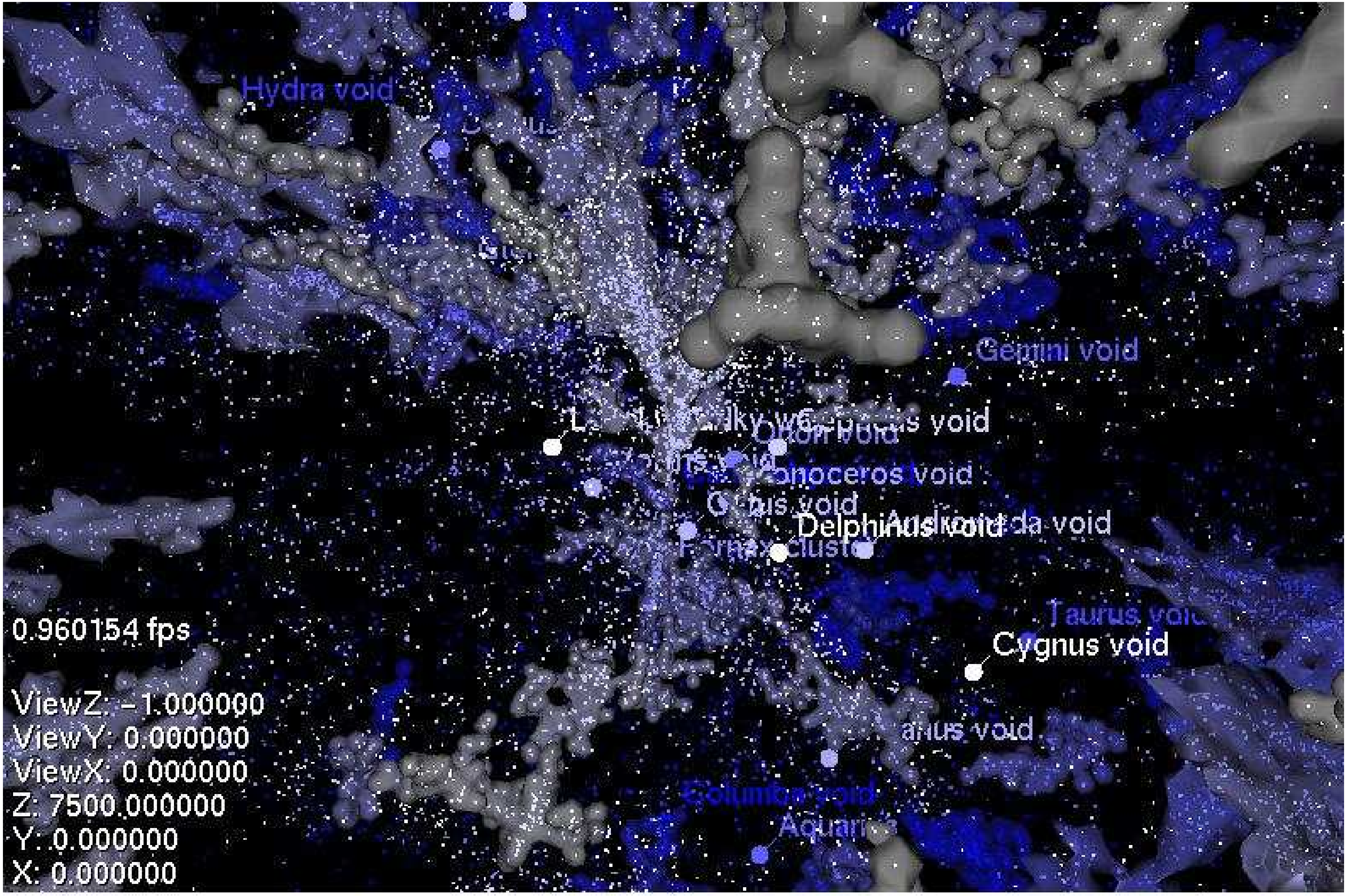
- Usual explanation: a homogeneous isotropic form of "stuff" which violates the strong energy condition. (Locally pressure $P = w\rho c^2$, $w < -\frac{1}{3}$; e.g., for cosmological constant, Λ, $w = -1$.)
- New explanation: in ordinary general relativity, a manifestation of global *variations* of those aspects of gravitational energy which by virtue of the equivalence principle cannot be localised – the *cosmological quasilocal gravitational energy* associated with *dynamical gradients* in spatial curvature generated by a universe as inhomogeneous as the one we observe. [Call this *dark energy* if you like. It involves *energy*; and "nothing" is dark.]

Overview

I will write down a viable model for the observed universe, with its almost isotropic Hubble flow but inhomogeneous matter distribution, considering

- the definition of gravitational energy;
- the decoupling bound systems from the expansion of space;
- operational issues associated with measurements and averaging in an inhomogeneous universe;
- resolving the Sandage-de Vaucouleurs paradox;
- understanding the problem of obtaining a void-dominated universe;
- how to realistically obtain apparent cosmological acceleration without exotic dark energy

6df: voids & bubble walls (A. Fairall, UCT)



From smooth to lumpy

- Universe was very smooth at time of last scattering; fluctuations in the fluid were tiny ($\delta\rho/\rho \sim 10^{-5}$ in photons and baryons; $\sim 10^{-3}$ in non–baryonic dark matter).
- FLRW approximation very good early on.
- Universe is very lumpy or inhomogeneous today.
- Recent surveys estimate that 40–50% of the volume of the universe is contained in voids of diameter $30h^{-1}$ Mpc. [Hubble constant $H_0 = 100h$ km sec^{-1} Mpc^{-1}] (Hoyle & Vogeley, ApJ 566 (2002) 641; 607 (2004) 751)
- Add some larger voids, and many smaller minivoids, and the universe is *void–dominated* at present epoch.
- Clusters of galaxies are strung in filaments and bubbles around these voids.

The Sandage-de Vaucouleurs paradox. . .

- Matter homogeneity only observed at $\gtrsim 200$ Mpc scales
- If "the coins on the balloon" are galaxies, their peculiar velocities should show great statistical scatter on scale much smaller than ~ 200 Mpc
- However, a nearly linear Hubble law flow begins at scales above 1.5–2 Mpc from barycentre of local group.
- Moreover, the local flow is statistically "quiet".
- Can we explain this as an effect of dark energy? Maybe. Peculiar velocities are isotropized in FLRW universes which expand forever (regardless of dark energy).
- Empirical results do not appear to match best-fit ΛCDM parameters (Axenides & Perivolaropoulos, PRD 65 (2002) 127301).

Inhomogeneous cosmology

- Need an averaging scheme to extract the average homogeneous geometry
- Only exact approaches dealing with *averages* of full non-linear Einstein equations considered here (NOT perturbation theory: Kolb et al. . . ; NOT LTB models etc)
- Still many approaches, with different assumptions
- Do we average tensors on curves of observers (Zalaletdinov 1992, 1993) . . . recent work Coley, Pelavas, and Zalaletdinov, PRL 95 (2005) 151102; Coley and Pelavas, PR D75 (2007) 043506
- Can we get away with averaging scalars (density, pressure, shear ...)? (Buchert 2000, 2001) . . . recent work Buchert CQG 23 (2006) 817; Astron. Astrophys. 454 (2006) 415; Gen. Rel. Grav. 40 (2008) 467 etc

Buchert's dust equations (2000)

For irrotational dust cosmologies, characterised by an energy density, $\rho(t,\mathbf{x})$, expansion, $\theta(t,\mathbf{x})$, and shear, $\sigma(t,\mathbf{x})$, on a compact domain, $\mathcal{D}$, of a suitably defined spatial hypersurface of constant average time, t, and spatial 3–metric, average cosmic evolution in Buchert's scheme is described by the exact equations

$$\begin{aligned} 3\frac{\dot{\bar{a}}^2}{\bar{a}^2} &= 8\pi G\langle\rho\rangle - \tfrac{1}{2}\langle\mathcal{R}\rangle - \tfrac{1}{2}\mathcal{Q} \\ 3\frac{\ddot{\bar{a}}}{\bar{a}} &= -4\pi G\langle\rho\rangle + \mathcal{Q} \\ \partial_t\langle\rho\rangle &+ 3\frac{\dot{\bar{a}}}{\bar{a}}\langle\rho\rangle = 0 \\ \mathcal{Q} &\equiv \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2\right) - 2\langle\sigma^2\rangle \end{aligned}$$

Back-reaction

Angle brackets denote the spatial volume average, e.g.,

$$\langle\mathcal{R}\rangle \equiv \left(\int_{\mathcal{D}} \mathrm{d}^3x\sqrt{\det {}^3\!g}\,\mathcal{R}(t,\mathbf{x})\right)/\mathcal{V}(t)$$

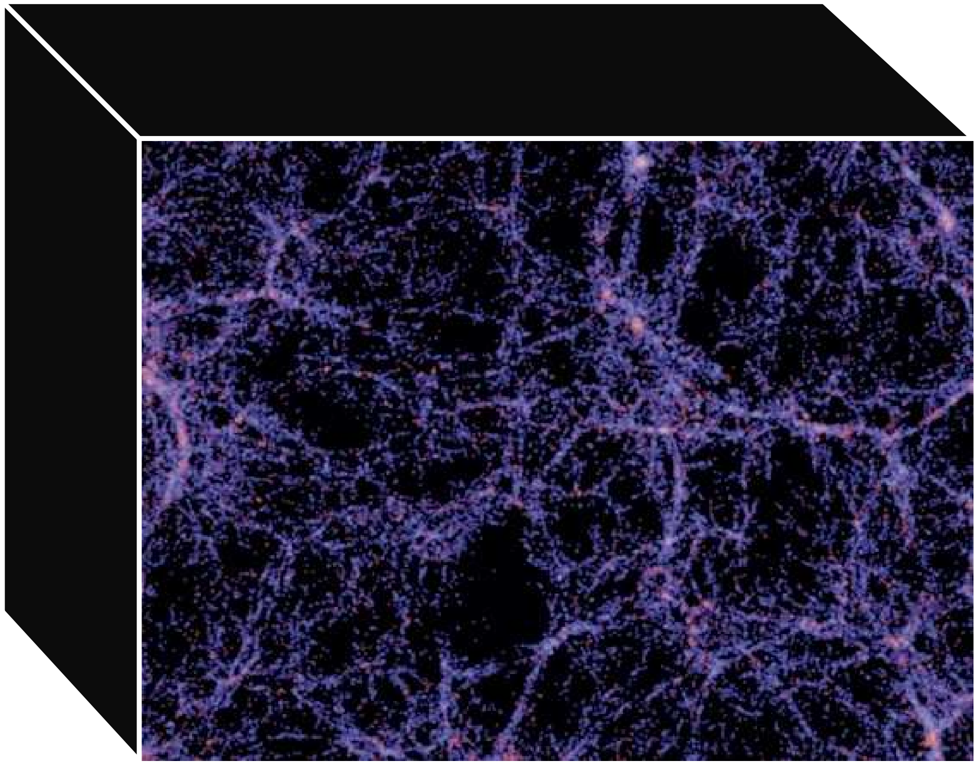
$$\langle\theta\rangle = 3\frac{\dot{\bar{a}}}{\bar{a}}$$

Generally for any scalar Ψ,

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\Psi\rangle - \langle\frac{\mathrm{d}\Psi}{\mathrm{d}t}\rangle = \langle\Psi\theta\rangle - \langle\theta\rangle\langle\Psi\rangle$$

- The extent to which the back–reaction, $\mathcal{Q}$, can lead to apparent cosmic acceleration or not has been the subject of much debate.

Within a statistically average cell



- Need to consider relative position of observers over scales of tens of Mpc over which $\delta\rho/\rho \sim -1$.
- GR is a local theory: gradients in spatial curvature and gravitational energy can lead to calibration differences between our rods and clocks and volume average ones

The Copernican principle

- Retain Copernican Principle - we are at an average position *for observers in a galaxy*
- Observers in bound systems are not at a volume average position in freely expanding space
- By Copernican principle other average observers should see an isotropic CMB
- BUT *nothing in theory, principle nor observation demands that such observers measure the same mean CMB temperature nor the same angular scales in the CMB anisotropies*
- Average mass environment (galaxy) can differ significantly from volume–average environment (void)

Dilemma of gravitational energy…

- In GR spacetime carries *energy* & *angular momentum*

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- On account of the strong equivalence principle, $T_{\mu\nu}$ contains localizable energy–momentum only
- Kinetic energy and energy associated with spatial curvature are in $G_{\mu\nu}$: variations are “quasilocal”!
- Newtonian version, $T - U = -V$, of Friedmann equation

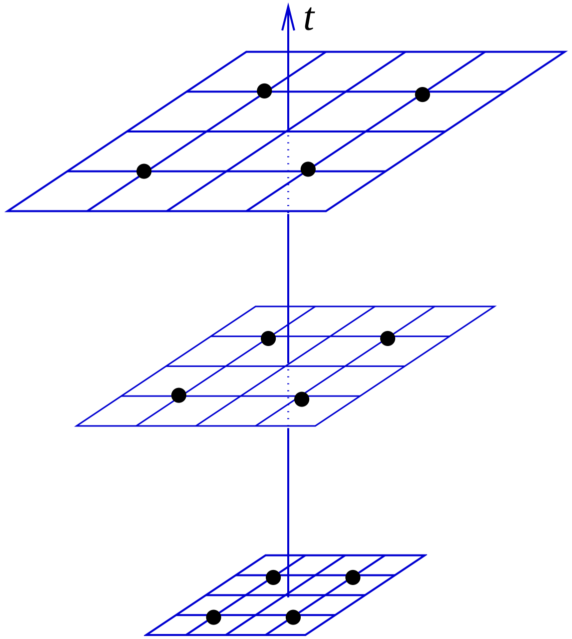
$$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G\rho}{3}$$

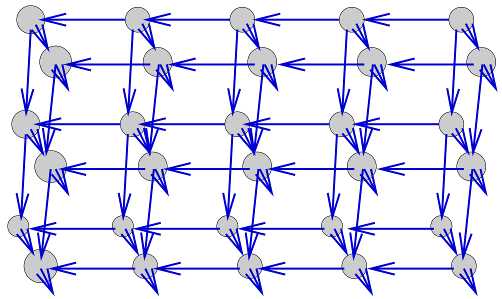
where $T = \frac{1}{2}m\dot{a}^2x^2$, $U = -\frac{1}{2}kmc^2x^2$, $V = -\frac{4}{3}\pi G\rho a^2x^2m$; $\mathbf{r} = a(t)\mathbf{x}$.

Ricci curvature and gravitational energy

- For Lemaître–Tolman–Bondi models constant spatial curvature replaced by energy function with $E(r) > 0$ in regions of negative spatial curvature.
- In quasilocal Hamiltonian approach of Chen, Nester and Liu (MPL A22 (2007) 2039) relative to a fiducial static Cartesian refencence frame a comoving observer in $k = -1$ FLRW universe sees negative quasilocal energy; or relative to the static frame the comoving observer has positive quasilocal energy.
- For perturbation theory I advocate “Machian gauge” of Bičak, Katz and Lynden–Bell (PR D76 (2007) 063501): uniform Hubble flow plus minimal shift distortion condition.

Cosmological Equivalence Principle





- Homogeneous isotropic volume expansion is locally indistinguishable from equivalent motion in static Minkowski space
- Extend to decelerating motion over long time intervals by Minkowski space analogue (semi-tethered lattice - indefinitely long tethers with one end fixed, one free end on spool, apply brakes syncronously at each site)

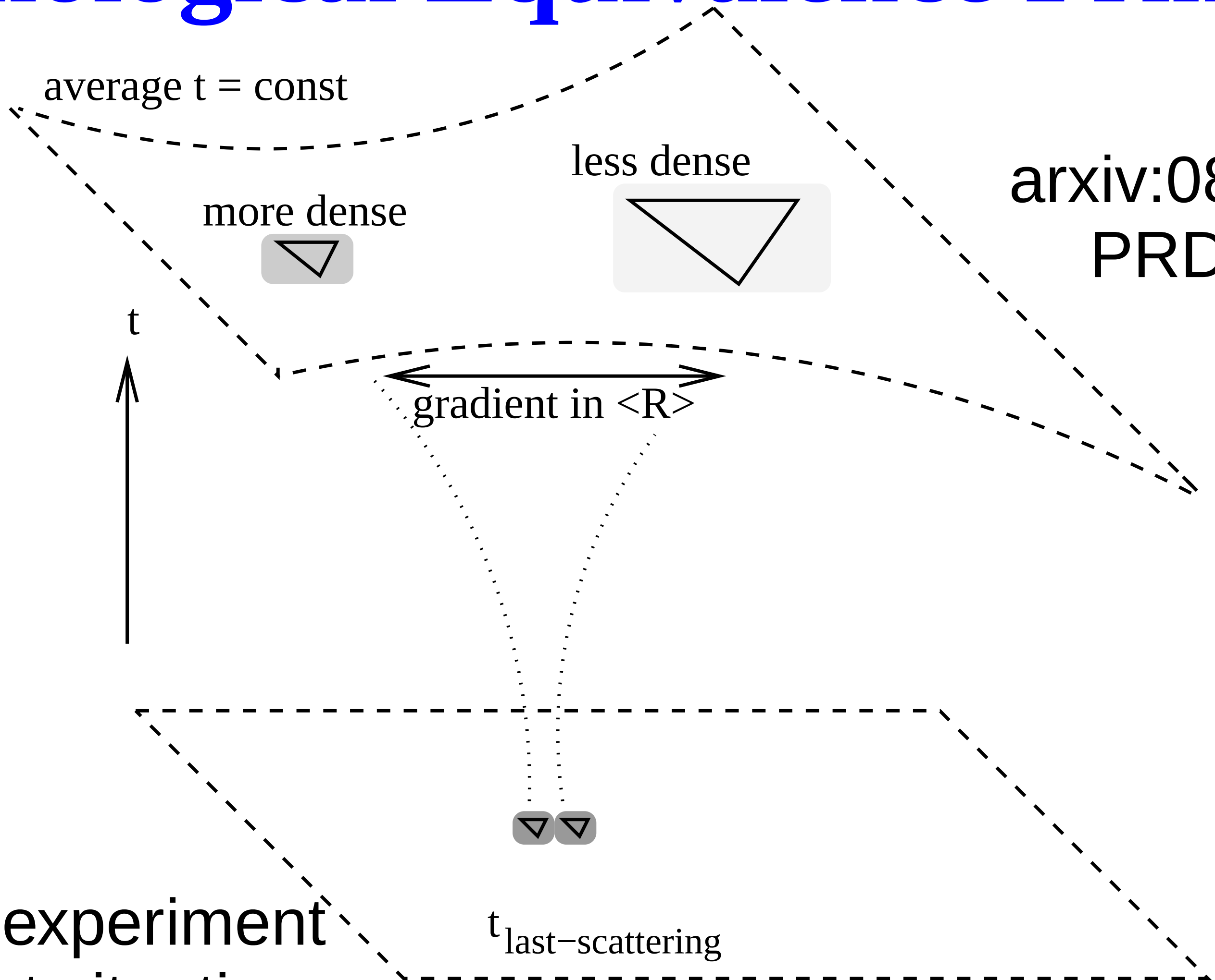
Cosmological Equivalence Principle

- *At any event, always and everywhere, it is possible to choose a suitably defined spacetime neighbourhood, the cosmological inertial frame, in which average motions (timelike and null) can be described by geodesics in a geometry which is Minkowski up to some time-dependent conformal transformation*,

$$\mathrm{d}s^2_{\mathrm{CIF}} = a^2(\eta)\left[-\mathrm{d}\eta^2 + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right],$$

- Defines Cosmological Inertial Frame (CIF)
- In semi-tethered lattice analogue there is local homogeneous isotropic deceleration: *no net force on any lattice observer*

Cosmological Equivalence Principle



arxiv:0809.1183
PRD in press

Thought experiment
equivalent situations:

- SR: observers volume decelerate at different rates
- GR: regions of different density have different volume deceleration (for same initial conditions)

Bound and unbound systems. . .

- Isotropic observers "at rest" within expanding space in voids may have clocks ticking at a rate $\mathrm{d}\tau_{\mathrm{v}} = \gamma(\tau_{\mathrm{w}}, \mathbf{x})\mathrm{d}\tau_{\mathrm{w}}$ with respect to static observers in bound systems. Volume average: $\mathrm{d}t = \bar{\gamma}_{\mathrm{w}}\mathrm{d}\tau_{\mathrm{w}}$, $\bar{\gamma}_{\mathrm{w}}(\tau_{\mathrm{w}}) = \langle -\xi^\mu n_\mu \rangle_{\mathcal{H}}$
- We are not restricted to $\gamma = 1 + \epsilon$, $\epsilon \ll 1$, as expected for typical variations of binding energy.
- Observable universe is assumed unbound.
- I find $\bar{\gamma} \simeq 1.38$ at present epoch from relative regional deceleration $\sim 10^{-10}$ms^{-2} integrated over age of universe. (N.B. Absolute upper bound: $\bar{\gamma} < 1.5$.)
- Where is infinity? In 1984 George Ellis suggested a notion of *finite infinity*: a region within which isolated systems, such as stars or galaxies, or galaxy clusters are approximately independent dynamical systems.

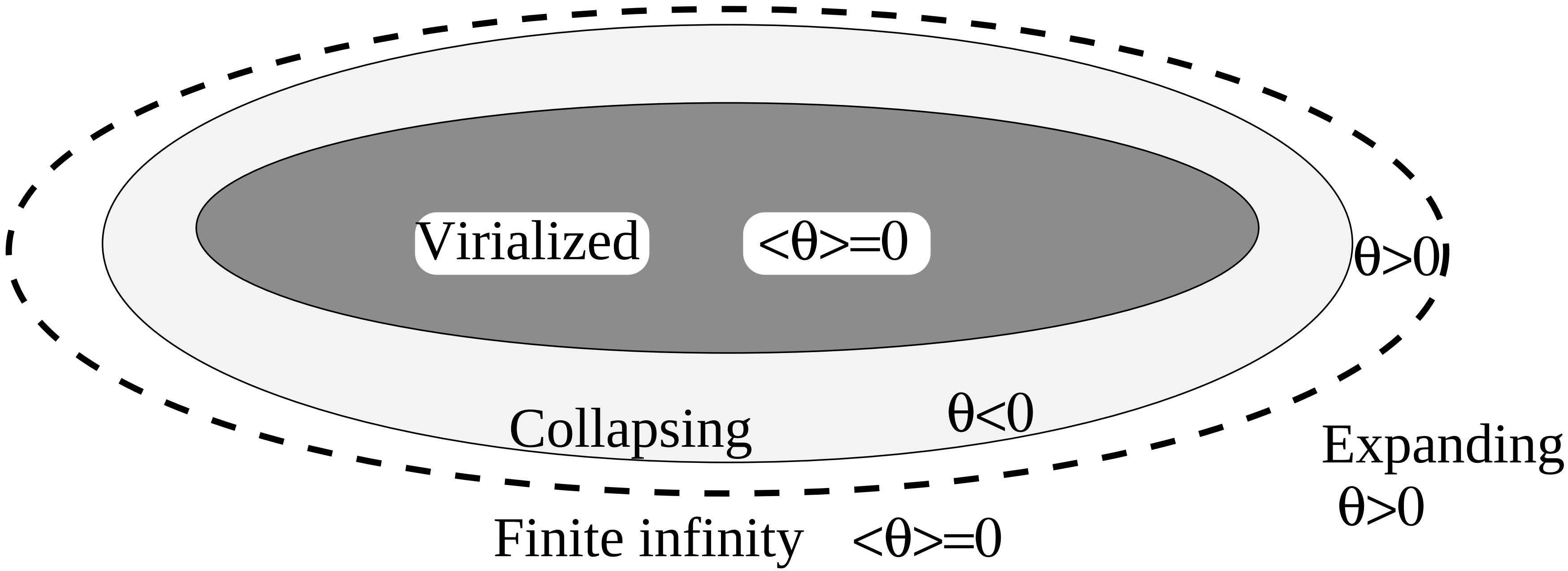
Where is infinity?

- Inflation provides us with boundary conditions.
- Initial smoothness at last–scattering ensures a uniform initial expansion rate. For gravity to overcome this a universal critical density exists.
  BUT if we assume a smooth average evolution we can overestimate the critical density today.

$$\rho_{\rm cr} \neq \frac{3H_{\rm av}^2}{8\pi G}$$

- Identify finite infinity relative to demarcation between bound and unbound systems, depending on the time evolution of the true critical density since last-scattering.
- Normalise *wall time*, $\tau_{\rm w}$, as the time at finite infinity, (close to galaxy clocks) by $\left\langle -\xi^\mu n_\mu \right\rangle_{\mathcal{F}_I} = \left\langle \gamma(\tau_{\rm w}, \mathbf{x}) \right\rangle_{\mathcal{F}_I} = 1$.

Finite infinity



- Define *finite infinity*, "*fi*" as boundary to minimal *connected* region within which *average expansion* vanishes $\langle\theta\rangle = 0$ or average curvature vanishes $\langle R\rangle = 0$.
- Shape of *fi* boundary irrelevant (minimal surface generally): could typically contain a galaxy cluster.

Cosmic rest frame

- Patch together CIFs for observers who see an isotropic CMB by taking surfaces of uniform volume expansion

$$\langle\frac{1}{\ell_r(\tau)}\frac{\mathrm{d}\ell_r(\tau)}{\mathrm{d}\tau}\rangle = \frac{1}{3}\langle\theta\rangle_1 = \frac{1}{3}\langle\theta\rangle_2 = \cdots = \bar{H}(\tau)$$

- Average over regions in which (i) spatial curvature is zero or negative; (ii) space is expanding at the boundaries, at least marginally.
- Solves the Sandage–de Vaucouleurs paradox implicitly.
- Voids appear to expand faster; but their local clocks tick faster, locally measured expansion can still be uniform.
- Global average $H_{\rm av}$ on large scales with respect to *any one set of clocks* will differ from $\bar{H}$

Two/three scale model

$$\bar{a}^3 = f_{\rm wi} a_{\rm w}{}^3 + f_{\rm vi} a_{\rm v}{}^3$$

- Splits into void fraction with scale factor $a_{\rm v}$ and "wall" fraction with scalar factor $a_{\rm w}$. Assume $\delta^2 H_w = \frac{1}{3}\langle\sigma^2\rangle_w$, $\delta^2 H_v = \frac{1}{3}\langle\sigma^2\rangle_v$.
- Buchert equations for volume averaged observer, with $f_{\rm v}(t) = f_{\rm vi} a_{\rm v}{}^3/\bar{a}^3$ (void volume fraction) and $k_{\rm v} < 0$

$$\frac{\dot{\bar{a}}^2}{\bar{a}^2} + \frac{\dot{f}_{\rm v}{}^2}{9 f_{\rm v}(1-f_{\rm v})} - \frac{\alpha^2 f_{\rm v}{}^{1/3}}{\bar{a}^2} = \frac{8\pi G}{3}\bar{\rho}_0 \frac{\bar{a}_0^3}{\bar{a}^3},$$

$$\ddot{f}_{\rm v} + \frac{\dot{f}_{\rm v}{}^2 (2f_{\rm v}-1)}{2f_{\rm v}(1-f_{\rm v})} + 3\frac{\dot{\bar{a}}}{\bar{a}}\dot{f}_{\rm v} - \frac{3\alpha^2 f_{\rm v}{}^{1/3}(1-f_{\rm v})}{2\bar{a}^2} = 0,$$

if $f_{\rm v}(t) \neq$ const; where $\alpha^2 = -k_{\rm v} f_{\rm vi}{}^{2/3}$.

Two/three scale model

- Universe starts as Einstein–de Sitter, from boundary conditions at last scattering consistent with CMB; almost no difference in clock rates initially.
- We must be careful to account for clock rate variations. Buchert's clocks are set at the *volume average* position, with a rate between wall clocks and void clock extreme.

$$\bar{H}(t) = \bar{\gamma}_{\rm w} H_{\rm w} = \bar{\gamma}_{\rm v} H_{\rm v}; \qquad H_{\rm w} \equiv \frac{1}{a_{\rm w}} \frac{{\rm d}a_{\rm w}}{{\rm d}t}, \quad H_{\rm v} \equiv \frac{1}{a_{\rm v}} \frac{{\rm d}a_{\rm v}}{{\rm d}t}$$

where $\bar{\gamma}_{\rm v} = \frac{{\rm d}t}{{\rm d}\tau_{\rm v}}$, $\bar{\gamma}_{\rm w} = \frac{{\rm d}t}{{\rm d}\tau_{\rm w}} = 1 + (1 - h_r) f_v / h_r$, $h_r = H_{\rm w}/H_{\rm v} < 1$.

- Need to be careful to obtain global $H_{\rm av}$ in terms of one set of isoptropic observer wall clocks, $\tau_{\rm w}$.

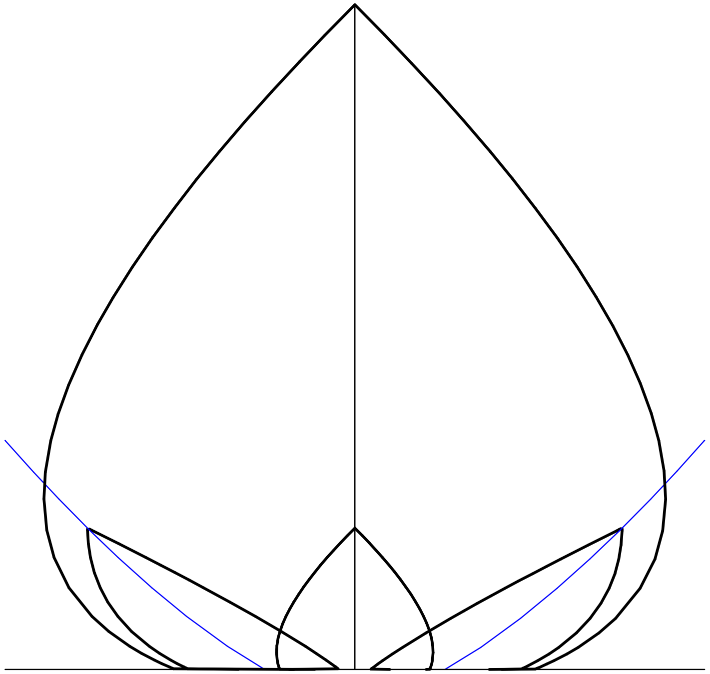
Bare cosmological parameters

- Different sets of cosmological parameters are possible
- Bare cosmological parameters are defined as fractions of the true critical density related to the bare Hubble rate

$$\begin{aligned}\bar{\Omega}_M &= \frac{8\pi G \bar{\rho}_{M0} \bar{a}_0^3}{3\bar{H}^2 \bar{a}^3}, \\ \bar{\Omega}_k &= \frac{\alpha^2 f_{\rm v}{}^{1/3}}{\bar{a}^2 \bar{H}^2}, \\ \bar{\Omega}_{\mathcal{Q}} &= \frac{-\dot{f}_{\rm v}{}^2}{9 f_{\rm v}(1-f_{\rm v})\bar{H}^2}.\end{aligned}$$

- These are the volume–average parameters, with first Buchert equation: $\bar{\Omega}_M + \bar{\Omega}_k + \bar{\Omega}_{\mathcal{Q}} = 1$.

Past light cone average



- Interpret Buchert solution by radial null cone average

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \bar{a}^2(t)\,\mathrm{d}\bar{\eta}^2 + A(\bar{\eta},t)\,\mathrm{d}\Omega^2,$$

- LTB metric but NOT an LTB solution
- Conformally match radial null geodesics to those of finite infinity geometry with uniform local Hubble flow.

Dressed cosmological parameters

- Conventional parameters for "wall observers" in galaxies: defined by assumption (no longer true) that others in entire observable universe have synchronous clocks and same local spatial curvature

$$
\begin{aligned}
\mathrm{d}s^2_{\mathcal{F}_I} &= -\mathrm{d}\tau_{\rm w}^2 + {a_{\rm w}}^2(\tau_{\rm w}) \left[\mathrm{d}\eta_{\rm w}^2 + \eta_{\rm w}^2 \mathrm{d}\Omega^2\right] \\
&= -\mathrm{d}\tau_{\rm w}^2 + \frac{\bar{a}^2}{\bar{\gamma}_{\rm w}^2} \left[\mathrm{d}\bar{\eta}^2 + r_{\rm w}^2(\bar{\eta}, \tau_{\rm w})\, \mathrm{d}\Omega^2\right]
\end{aligned}
$$

  where $r_{\rm w} \equiv \bar{\gamma}_{\rm w}\,(1 - f_{\rm v})^{1/3}\, {f_{\rm wi}}^{-1/3} \eta_{\rm w}(\bar{\eta}, \tau_{\rm w})$, and volume–average conformal time $\mathrm{d}\bar{\eta} = \mathrm{d}t/\bar{a} = \bar{\gamma}_{\rm w}\,\mathrm{d}\tau_{\rm w}/\bar{a}$.

- This leads to conventional dressed parameters *which do not sum to 1*, e.g.,

$$\Omega_M = \bar{\gamma}_{\rm w}^3 \bar{\Omega}_M \,.$$

Tracker solution PRL 99, 251101

- General exact solution possesses a “tracker limit”

$$\bar{a} = \frac{\bar{a}_0 (3\bar{H}_0 t)^{2/3}}{2 + f_{\rm v0}} \left[3 f_{\rm v0} \bar{H}_0 t + (1 - f_{\rm v0})(2 + f_{\rm v0}) \right]^{1/3}$$

$$f_{\rm v} = \frac{3 f_{\rm v0} \bar{H}_0 t}{3 f_{\rm v0} \bar{H}_0 t + (1 - f_{\rm v0})(2 + f_{\rm v0})} \,,$$

- Void fraction $f_{\rm v}(t)$ determines many parameters:

$$\bar{\gamma}_{\rm w} = 1 + \tfrac{1}{2} f_{\rm v} = \tfrac{3}{2} \bar{H} t$$

$$\tau_{\rm w} = \tfrac{2}{3} t + \frac{2(1 - f_{\rm v0})(2 + f_{\rm v0})}{27 f_{\rm v0} \bar{H}_0} \ln \left(1 + \frac{9 f_{\rm v0} \bar{H}_0 t}{2(1 - f_{\rm v0})(2 + f_{\rm v0})} \right)$$

$$\bar{\Omega}_M = \frac{4(1 - f_{\rm v})}{(2 + f_{\rm v})^2}$$

Apparent cosmic acceleration

- Volume average observer sees no apparent cosmic acceleration

$$\bar{q} = \frac{2\left(1 - f_{\rm v}\right)^2}{(2 + f_{\rm v})^2}\,.$$

As $t \to \infty$, $f_{\rm v} \to 1$ and $\bar{q} \to 0^+$.

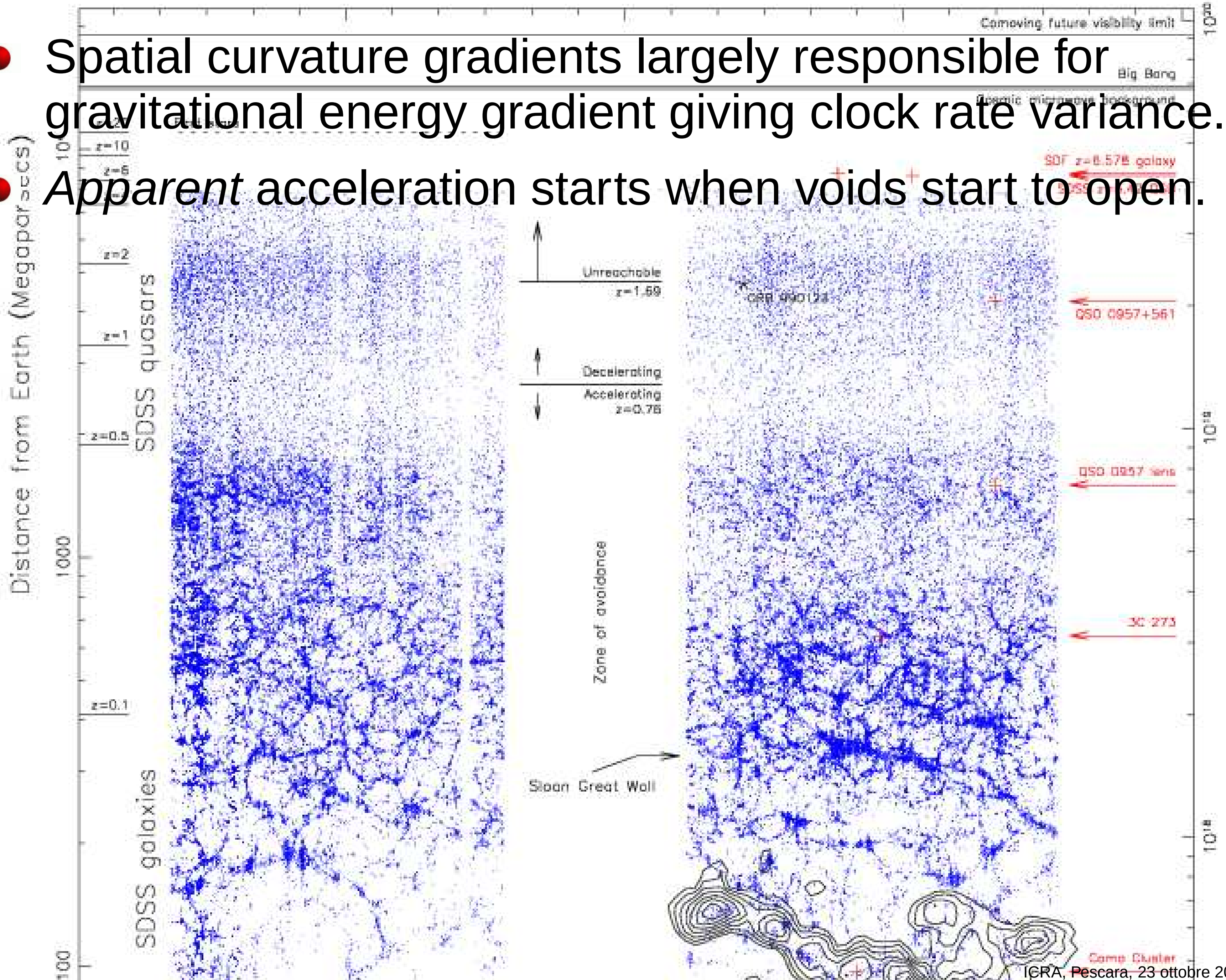
- A wall observer registers apparent cosmic acceleration

$$q = \frac{-\left(1 - f_{\rm v}\right)\left(8{f_{\rm v}}^3 + 39{f_{\rm v}}^2 - 12f_{\rm v} - 8\right)}{\left(4 + f_{\rm v} + 4{f_{\rm v}}^2\right)^2}\,,$$

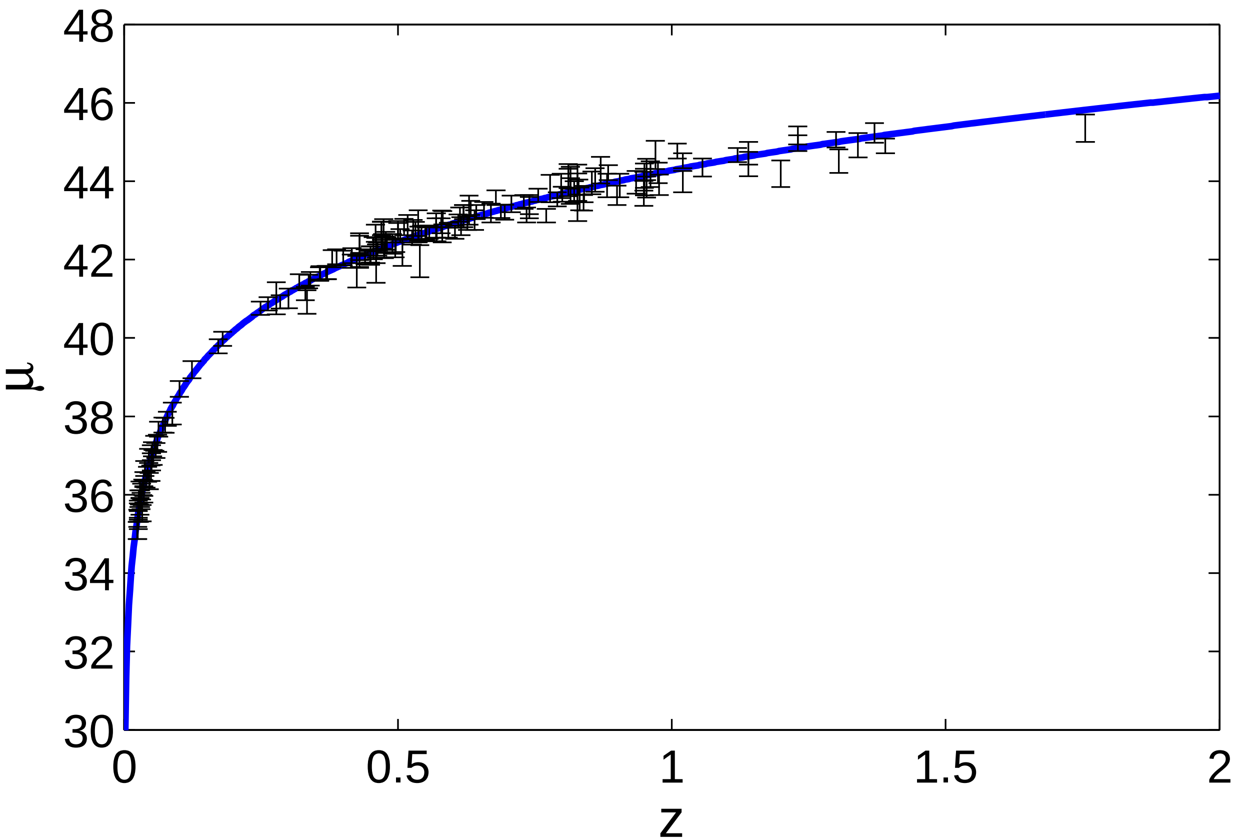
Effective deceleration parameter starts at $q \sim \frac{1}{2}$, for small $f_{\rm v}$; changes sign when $f_{\rm v} = 0.58670773\ldots$, and approaches $q \to 0^-$ at late times.

Cosmic coincidence problem solved

- Spatial curvature gradients largely responsible for gravitational energy gradient giving clock rate variance.
- *Apparent* acceleration starts when voids start to open.

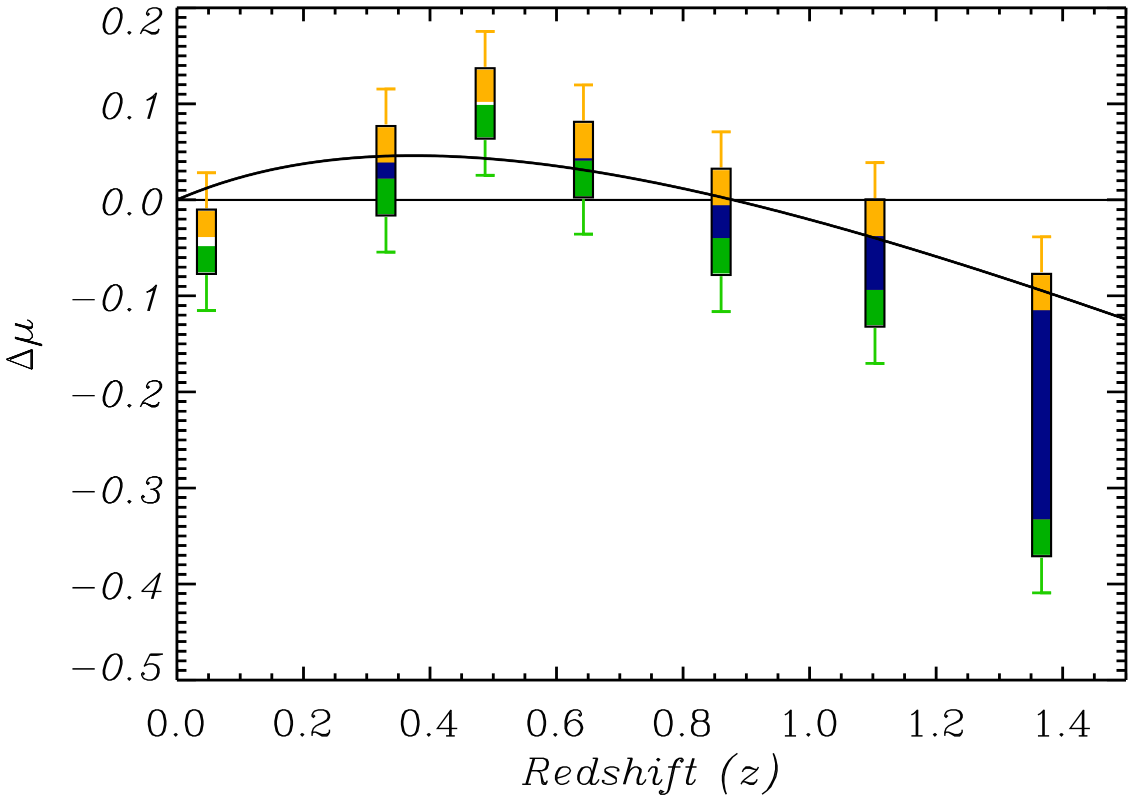


Test 1: SneIa luminosity distances



- Type Ia supernovae of Riess07 Gold data set fit with χ^2 per degree of freedom $= 0.9$
- With $55 \leq H_0 \leq 75$ km sec^{-1} Mpc^{-1}, $0.01 \leq \Omega_{M0} \leq 0.5$, find Bayes factor $\ln B = 0.27$ in favour or FB model (marginally): statistically indistinguishable from ΛCDM.

Test 1: SneIa luminosity distances

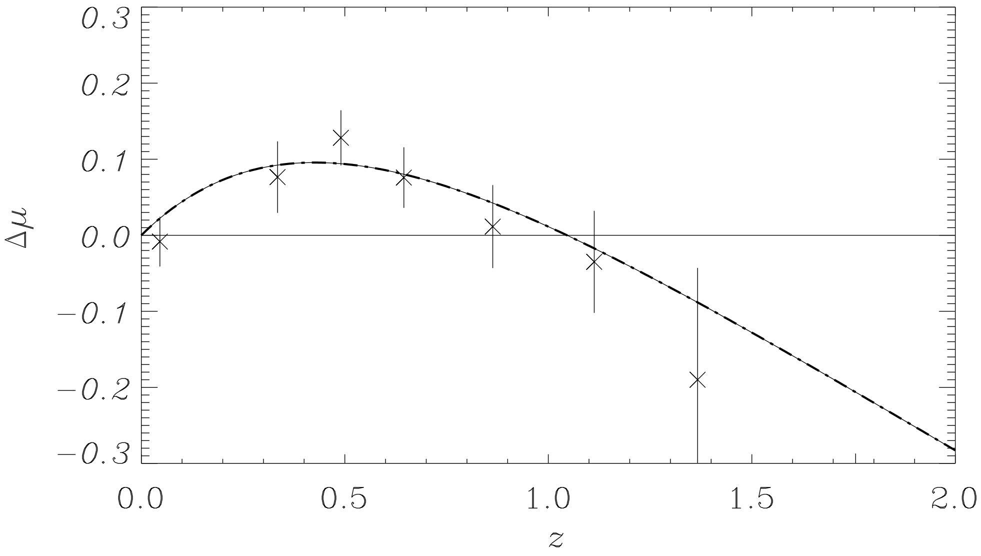


- Plot shows difference of model apparent magnitude and that of an empty Milne universe of same Hubble constant $H_0 = 61.73$ km sec^{-1} Mpc^{-1}. *Note*: residual depends on the expansion rate of the Milne universe subtracted (2σ limits on H_0 indicated by whiskers)

Comparison ΛCDM models

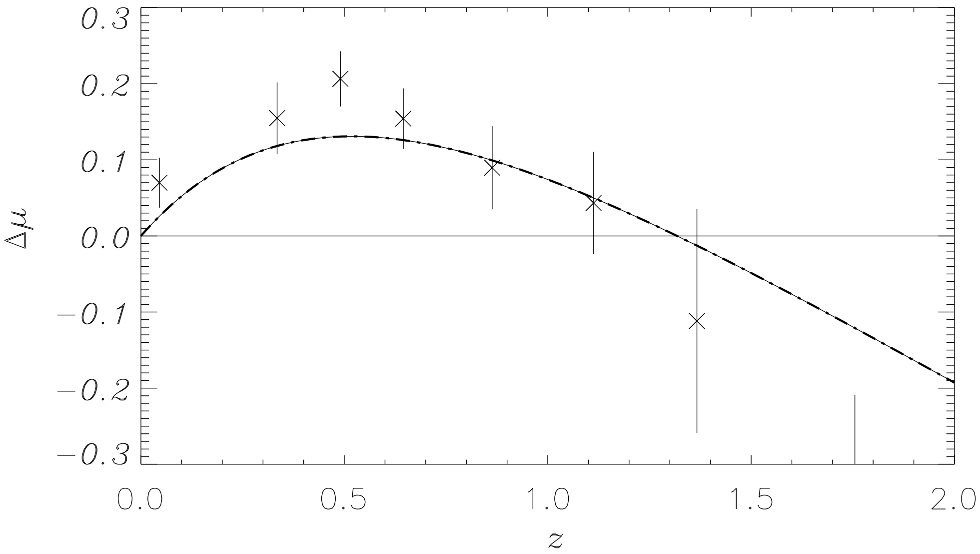
Best-fit spatially flat ΛCDM

$H_0 = 62.7$ km sec^{-1} Mpc^{-1},
$\Omega_{M0} = 0.34$, $\Omega_{\Lambda 0} = 0.66$

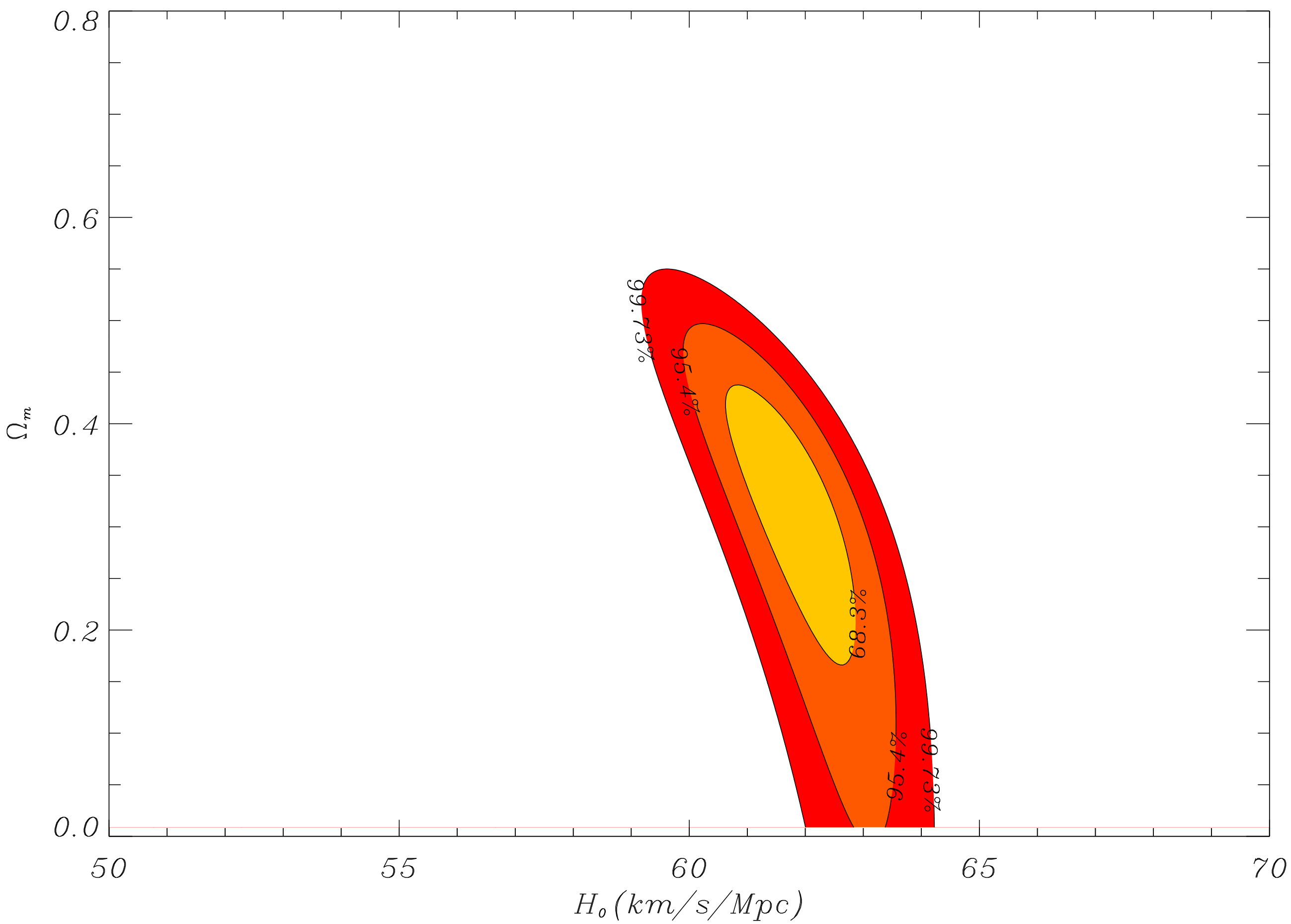


Riess astro-ph/0611572, p. 63

$H_0 = 65$ km sec^{-1} Mpc^{-1},
$\Omega_{M0} = 0.29$, $\Omega_{\Lambda 0} = 0.71$

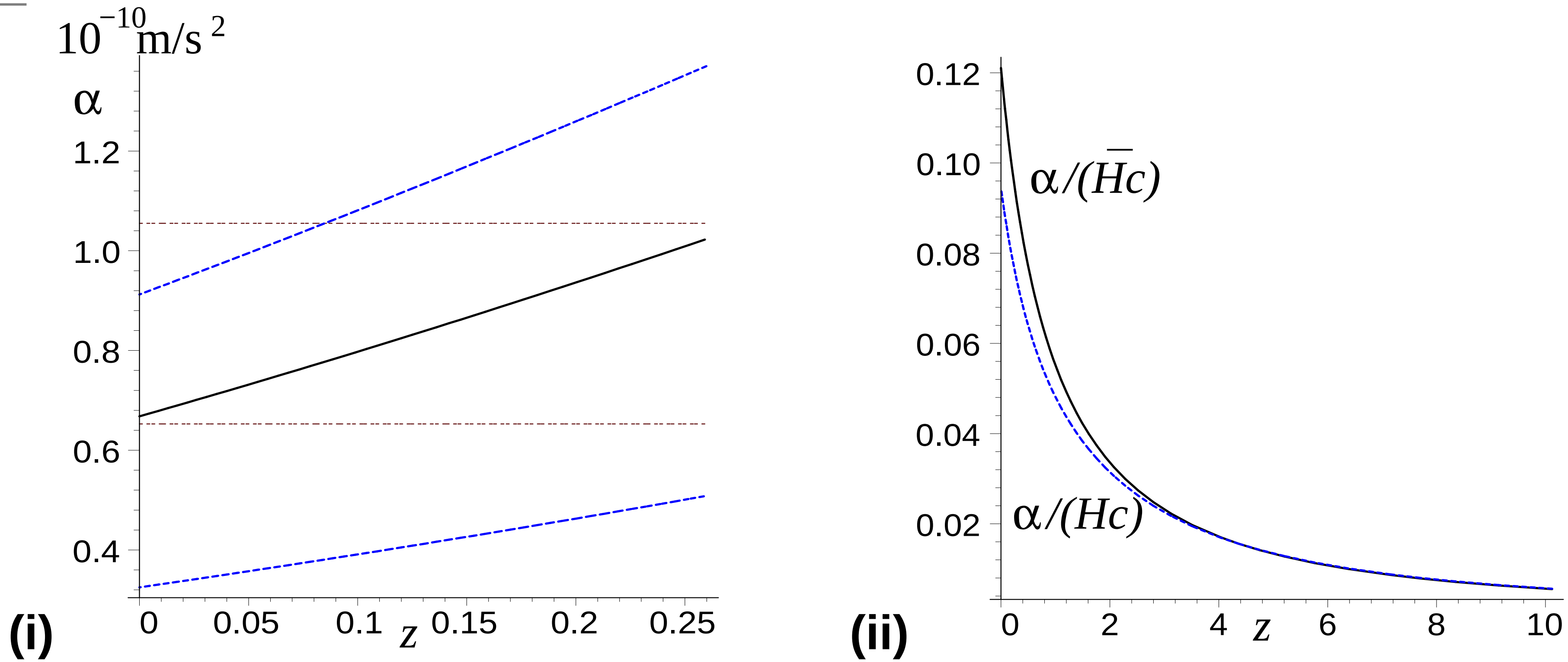


Test 1: SneIa luminosity distances



Best–fit H_0 agrees with HST key team, Sandage et al., $H_0 = 62.3 \pm 1.3$ (stat) $\pm\, 5.0$ (syst) km sec^{-1} Mpc^{-1} [ApJ 653 (2006) 843].

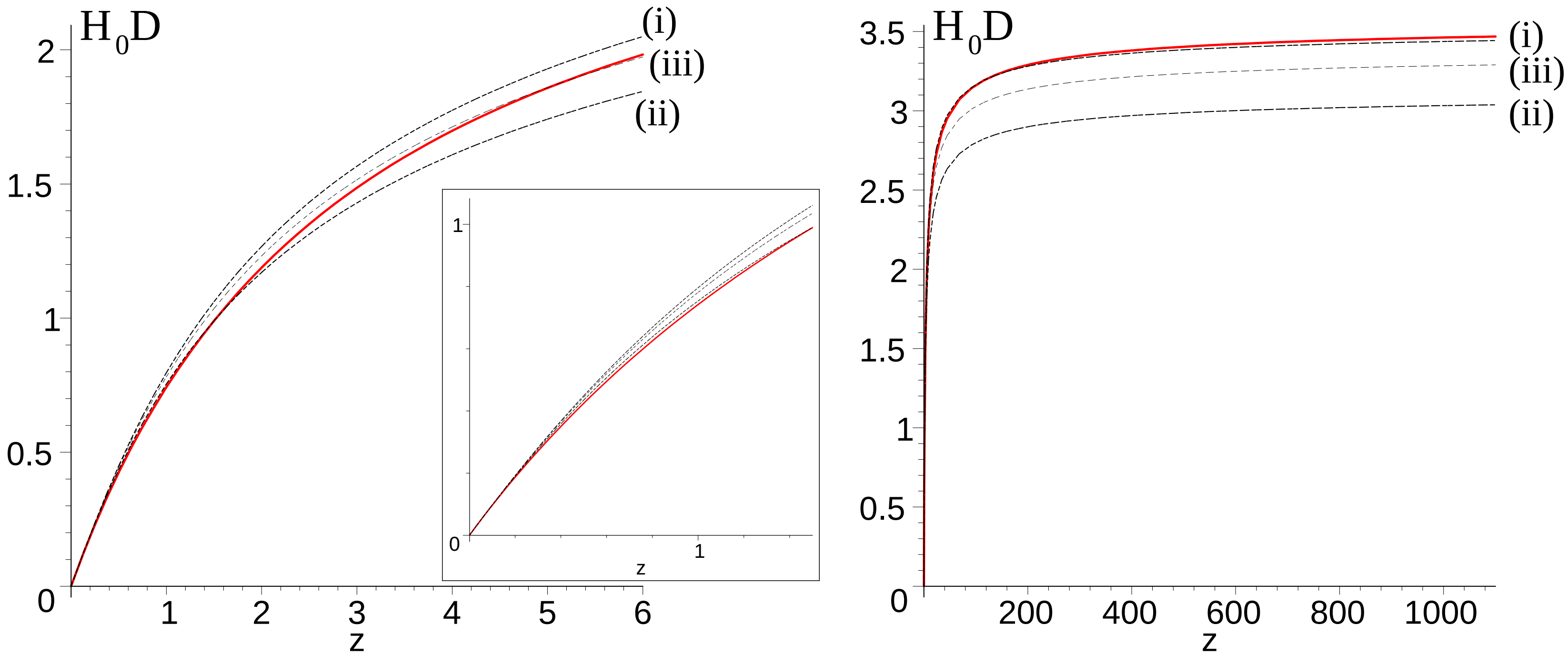
CEP relative acceleration scale



By equivalence principle the instantaneous relative deceleration of backgrounds gives an instantaneous 4-acceleration of magnitude $\alpha = H_0 c\bar{\gamma}\dot{\bar{\gamma}}/(\sqrt{\bar{\gamma}^2 - 1})$ beyond which *weak fi eld cosmological general relativity* will be changed from Newtonian expectations: **(i)** as absolute scale nearby; **(ii)** divided by Hubble parameter to large z.

- For $z \lesssim 0.25$, coincides with empirical MOND scale $\alpha_0 = 1.2^{+0.3}_{-0.2} \times 10^{-10}$ ms$^{-2}h^2_{75} = 8.1^{+2.5}_{-1.6} \times 10^{-11}$ms^{-2} for $H_0 = 61.7$ km sec^{-1} Mpc^{-1}.

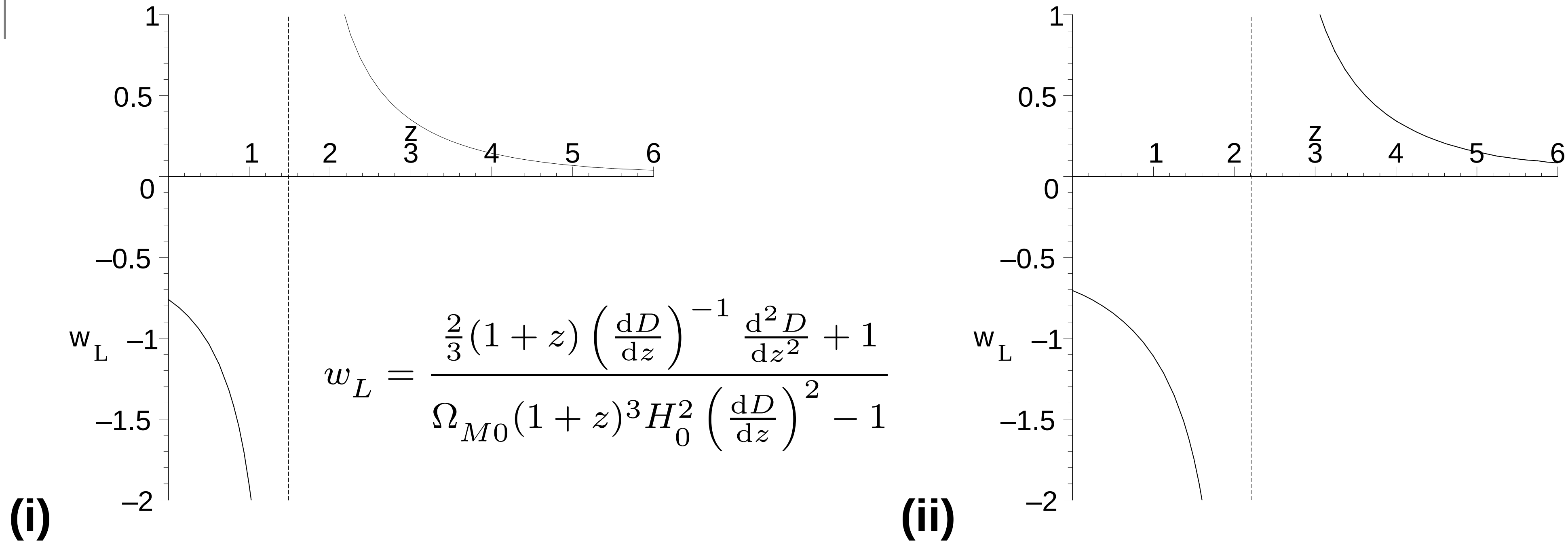
Dressed "comoving distance" $D(z)$



Best-fit FB model **(red line)** compared to 3 spatially flat ΛCDM models: **(i)** best–fit to WMAP5 only ($\Omega_\Lambda = 0.751$); **(ii)** best–fit to (Riess07) SneIa only ($\Omega_\Lambda = 0.66$); **(iii)** joint WMAP5 + BAO + SneIa fit ($\Omega_\Lambda = 0.721$)

- FB model closest to best–fit ΛCDM to *SneIa only* result ($\Omega_{M0} = 0.34$) at *low redshift*; and to *WMAP5 only* result ($\Omega_{M0} = 0.249$) at *high redshift*

Equivalent "equation of state"?

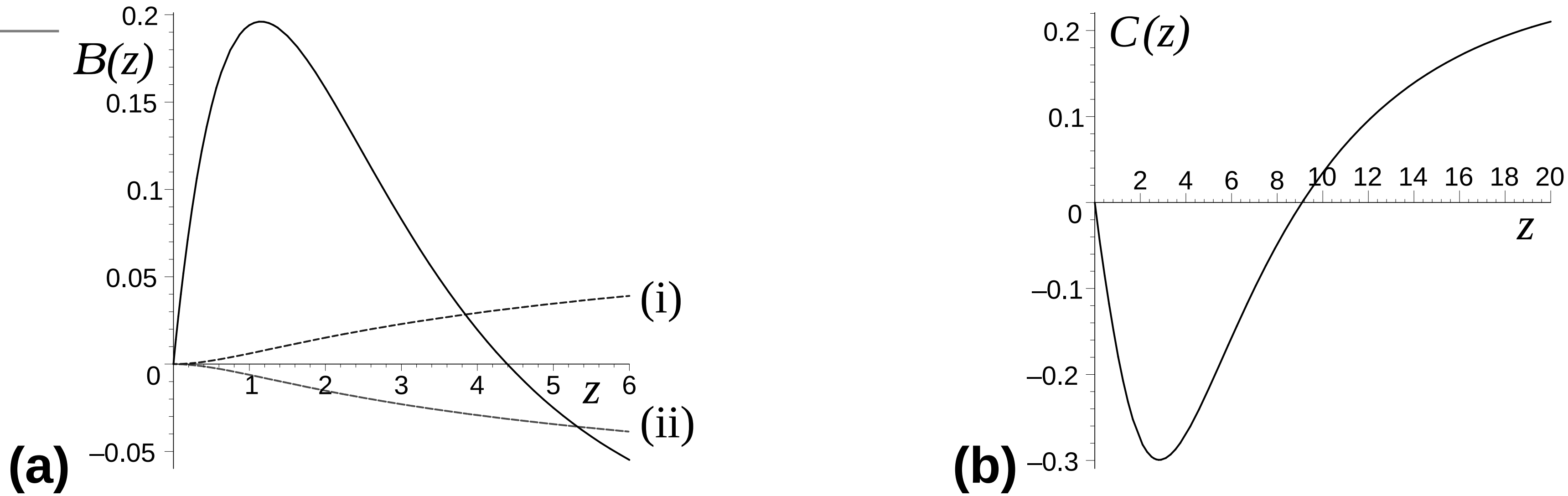


$$w_L = \frac{\frac{2}{3}(1+z)\left(\frac{\mathrm{d}D}{\mathrm{d}z}\right)^{-1}\frac{\mathrm{d}^2 D}{\mathrm{d}z^2}+1}{\Omega_{M0}(1+z)^3 H_0^2\left(\frac{\mathrm{d}D}{\mathrm{d}z}\right)^2-1}$$

A formal "dark energy equation of state" $w_L(z)$ for the best-fit FB model, $f_{v0} = 0.76$, calculated directly from $r_w(z)$: **(i)** $\Omega_{M0} = 0.33$; **(ii)** $\Omega_{M0} = 0.279$.

- Description by a "dark energy equation of state" makes no sense when there is no physics behind it; but average value $w_L \simeq -1$ for $z < 0.7$ makes empirical sense.

Clarkson et al homogeneity test



(a) $\mathcal{B} \equiv [H(z)D'(z)]^2 - 1$ for FB model (solid line) and two ΛCDM models (dashed lines): **(i)** $\Omega_{M0} = 0.28$, $\Omega_{\Lambda 0} = 0.71$, $\Omega_{k0} = 0.01$; **(ii)** $\Omega_{M0} = 0.28$, $\Omega_{\Lambda 0} = 0.73$, $\Omega_{k0} = -0.01$; **(b)** $\mathcal{C}(z)$.

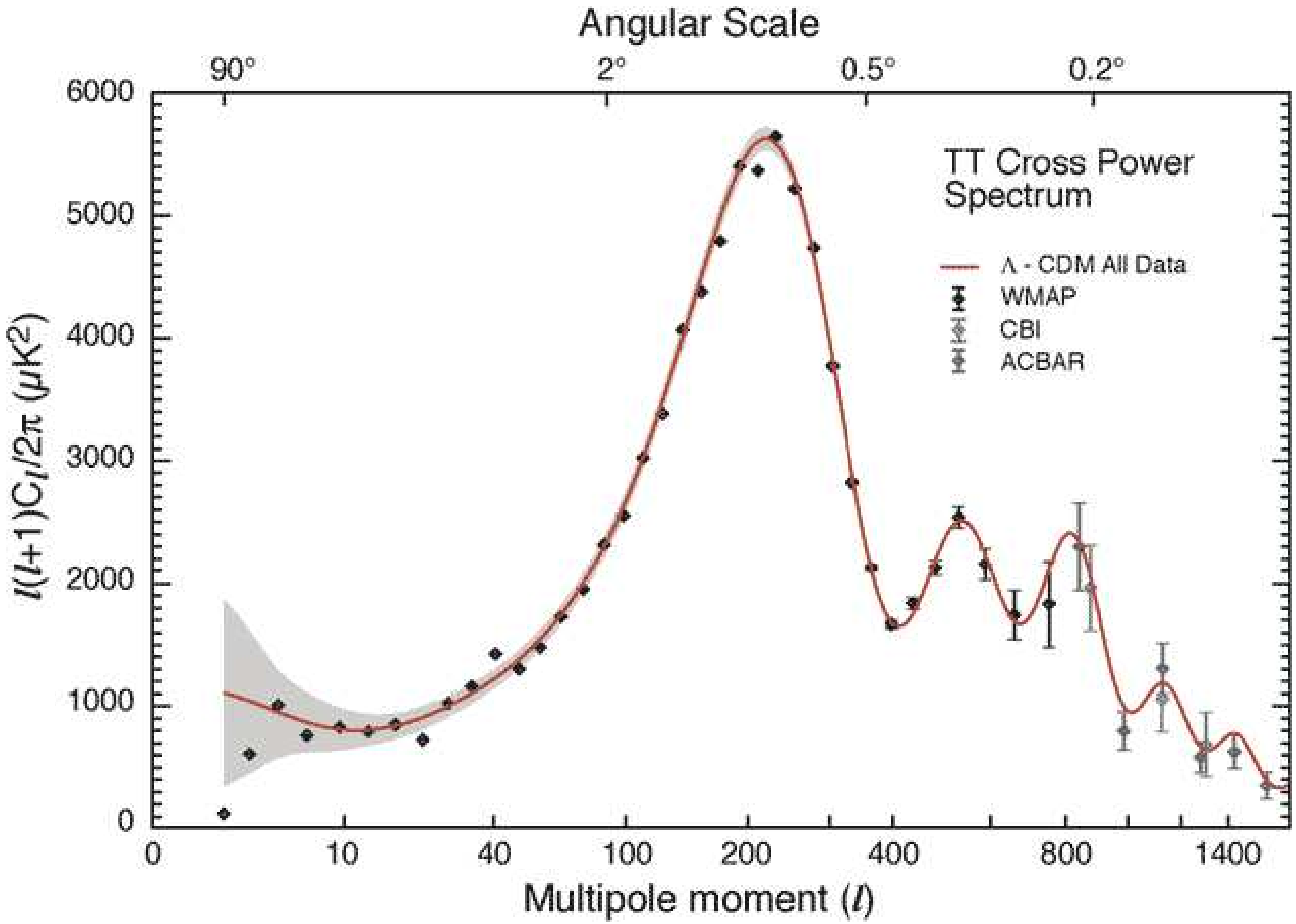
- For FLRW equations, irrespective of dark energy model

$$\Omega_{k0} = \left[(H(z)D'(z))^2 - 1\right] / [H_0 D(z)]^2 = \text{const}$$

$$\mathcal{C}(z) \equiv 1 + H^2(DD'' - D'^2) + HH'DD' = 0$$

- Will give a powerful test of FLRW assumption in future, with quantitative different prediction for FB model.

Test 2: Angular scale of CMB Doppler peaks

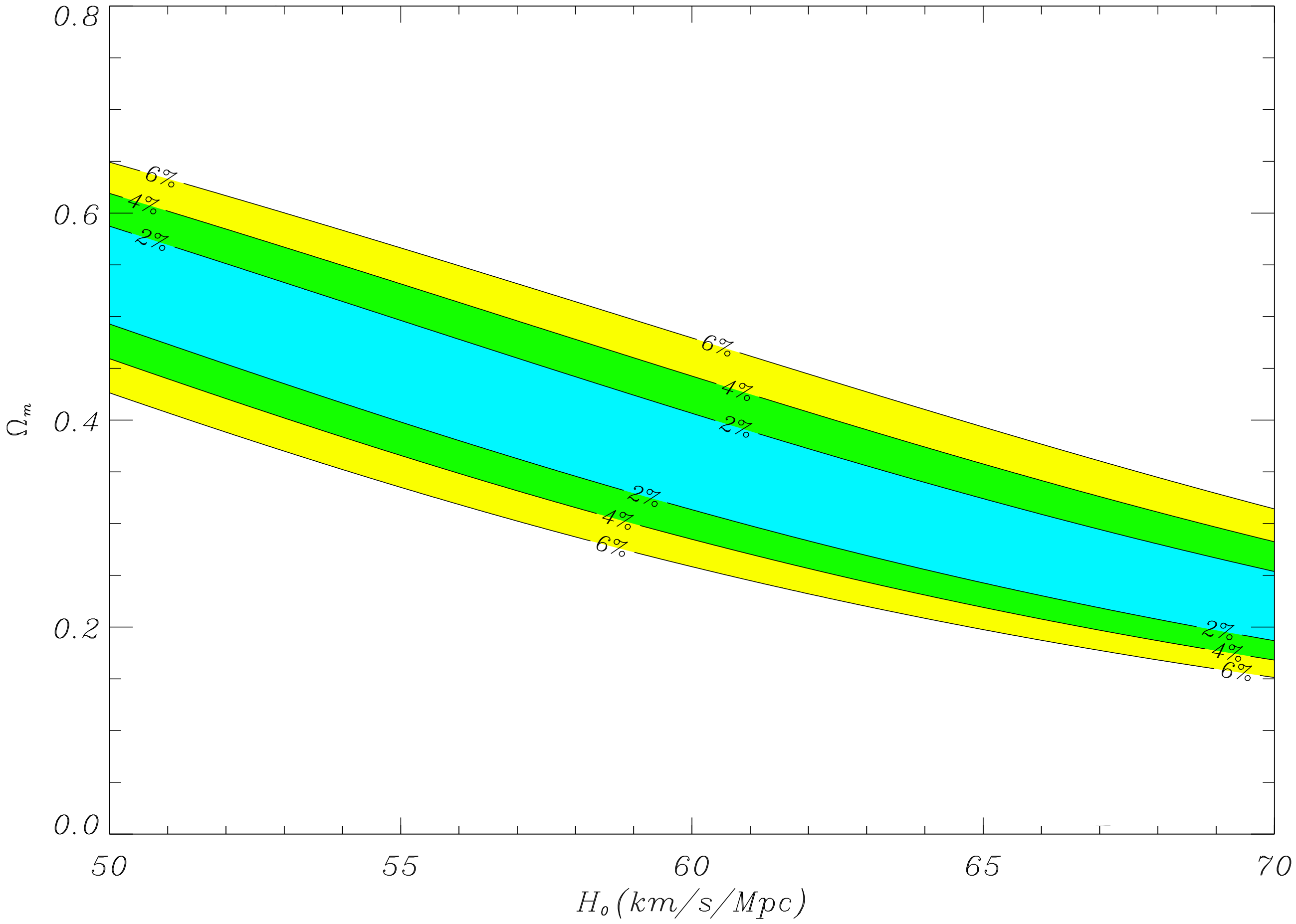


Power in CMB temperature anisotropies versus angular size of fluctuation on sky

Test 2: Angular scale of CMB Doppler peaks

- Angular scale is related to spatial curvature of FLRW models
- Relies on the simplifying assumption that spatial curvature is same everywhere
- In new approach spatial curvature is not the same everywhere
- Volume–average observer measures lower mean CMB temperature ($\bar{T}_0 \sim 1.98$ K, c.f. $T_0 \sim 2.73$ K in walls) and a smaller angular anisotropy scale
- Relative focussing between voids and walls
- Integrated Sachs–Wolfe effect needs recomputation
- Here just calculate angular–diameter distance of sound horizon

Test 2: Angular scale of CMB Doppler peaks

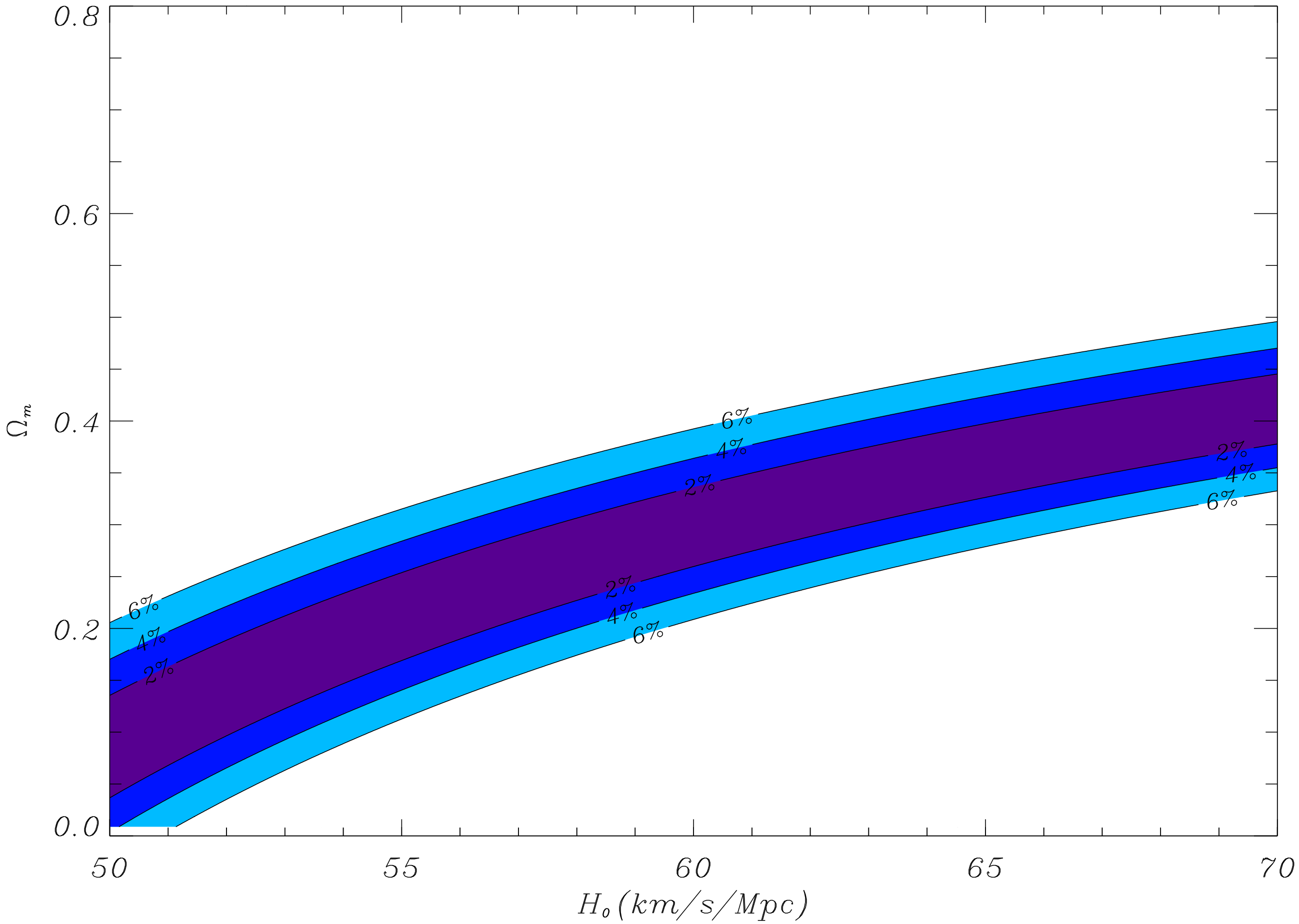


Parameters within the (Ω_m,H_0) plane which fit the angular scale of the sound horizon $\delta = 0.01$ rad deduced for WMAP, to within 2%, 4% and 6%.

Test 3: Baryon acoustic oscillation scale

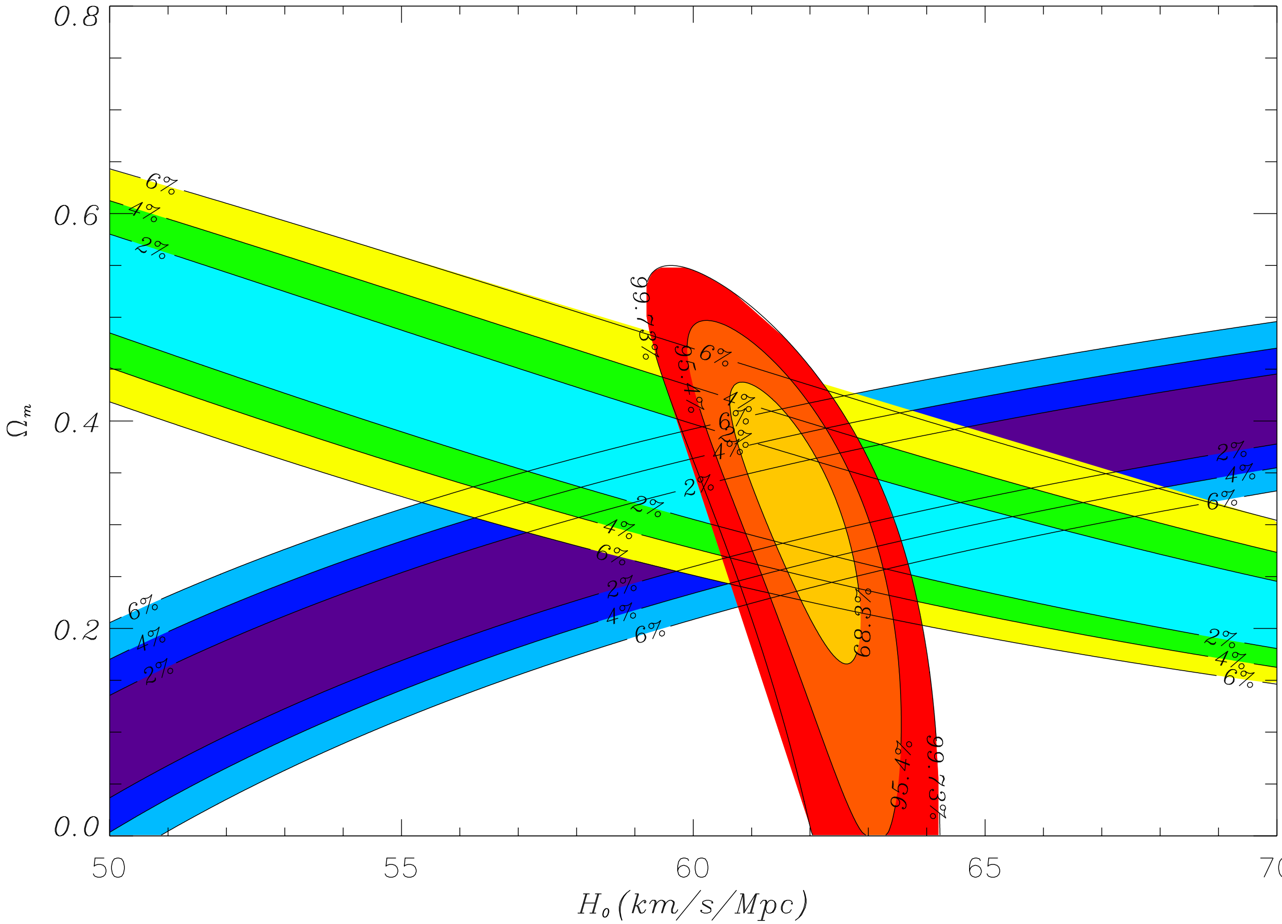
- In 2005 Cole et al. (2dF), and Eisenstein et al. (SDSS) detected the signature of the comoving baryon acoustic oscillation in galaxy clustering statistics
- Powerful independent probe of “dark energy”
- Here the effective dressed geometry should give an equivalent scale

Test 3: Baryon acoustic oscillation scale



Parameters within the (Ω_m,H_0) plane which fit the effective comoving baryon acoustic oscillation scale of $104h^{-1}$ Mpc, as seen in 2dF and SDSS.

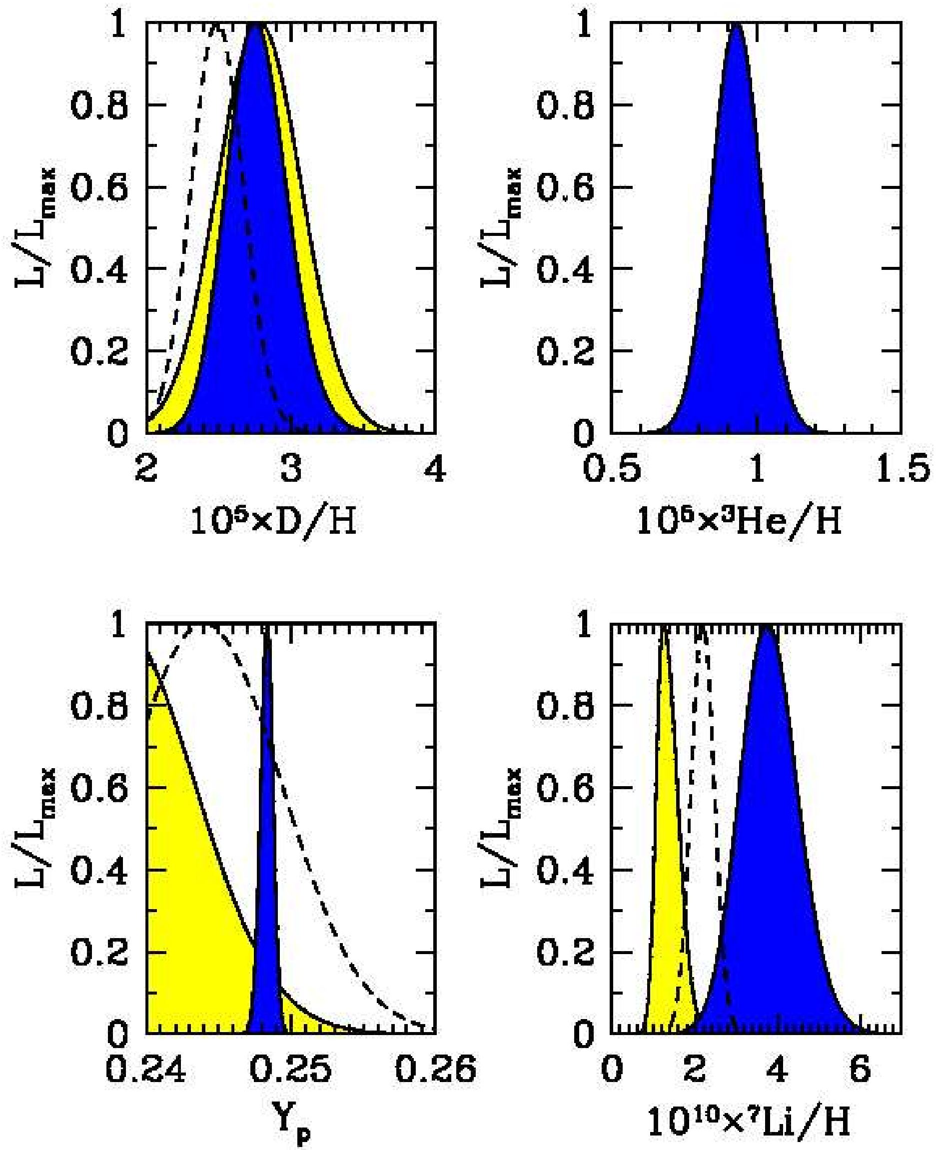
Agreement of independent tests



Best–fit parameters: $H_0 = 61.7^{+1.2}_{-1.1}$ km sec^{-1} Mpc^{-1}, $\Omega_m = 0.33^{+0.11}_{-0.16}$ (1σ errors for SneIa only) [Leith, Ng & Wiltshire, ApJ 672 (2008) L91]

Li abundance anomaly

Big-bang nucleosynthesis, light element abundances and WMAP with ΛCDM cosmology.



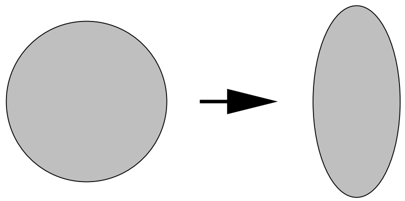
Resolution of Li abundance anomaly?

- Tests 2 & 3 shown earlier use the baryon–to–photon ratio $\eta_{B\gamma} = 4.6\text{–}5.6 \times 10^{-10}$ admitting concordance with lithium abundances favoured prior to WMAP in 2003
- Conventional dressed parameter $\Omega_{M0} = 0.33$ for wall observer means $\bar{\Omega}_{M0} = 0.127$ for the volume–average.
- Conventional theory predicts the *volume–average baryon fraction*. With old BBN favoured $\eta_{B\gamma}$:

  $\bar{\Omega}_{B0} \simeq 0.027\text{–}0.033$; but this translates to a conventional dressed baryon fraction parameter $\Omega_{B0} \simeq 0.072\text{–}0.088$
- The mass ratio of baryonic matter to non–baryonic dark matter is typically increased to 1:3
- Enough baryon drag to fit peak heights ratio

Spatial curvature: ellipticity anomaly

- Negative spatial curvature should manifest itself in other ways than angular–diameter distance of sound horizon
- Indeed it does: greater geodesic mixing from negative spatial curvature registers ellipticity in the CMB anisotropy spectrum



- Ellipticity has been detected since COBE, and statistical significance increases with each data release (Gurzadyan et al., Phys. Lett. **A 363** (2007) 121; Mod. Phys. Lett. **A 20** (2005) 813,. . .)
- For FLRW models this is an anomaly; here it is expected; but still needs quantitative analysis

Alleviation of age problem

- Old structures seen at large redshifts are a problem for ΛCDM.
- Problem alleviated here; expansion age is increased, by an increasingly larger relative fraction at larger redshifts, e.g., for best–fit values
  ΛCDM $\tau = 0.85$ Gyr at $z = 6.42$, $\tau = 0.365$ Gyr at $z = 11$
  FB $\tau = 1.14$ Gyr at $z = 6.42$, $\tau = 0.563$ Gyr at $z = 11$
- Present age of universe for best-fit is $\tau_0 \simeq 14.7$ Gyr for wall observer; $t_0 \simeq 18.6$ Gyr for volume–average observer.
- Suggests problems of under–emptiness of voids in Newtonian N-body simulations may be an issue of using volume–average time?? The simulations need to carefully reconsidered.

Variance of Hubble flow

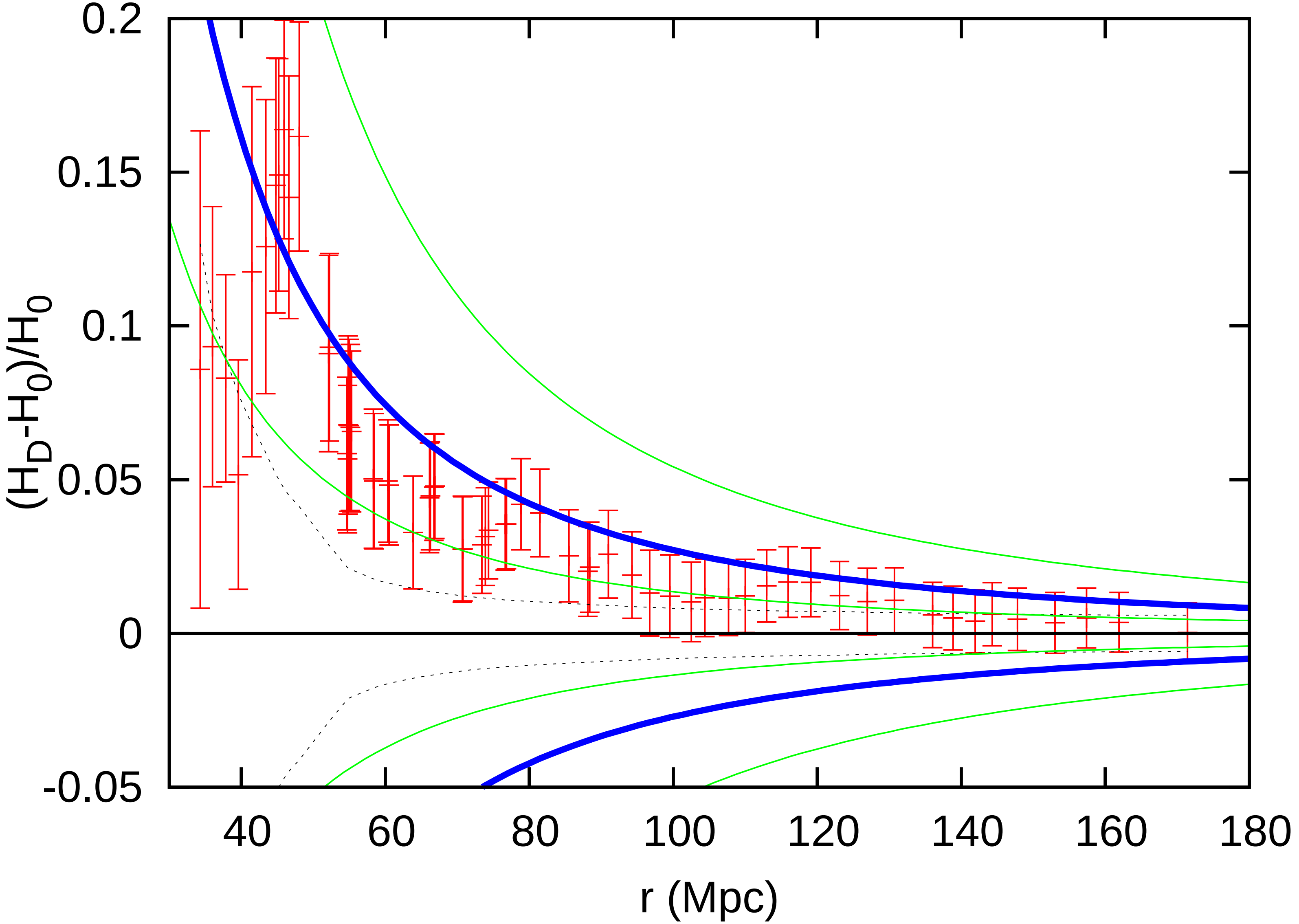
- Relative to “wall clocks” the global average Hubble parameter $H_{\rm av} > \bar{H}$
- $\bar{H}$ is nonetheless also the locally measurable Hubble parameter within walls
- TESTABLE PREDICTION:

$$H_{\rm av} = \bar{\gamma}_{\rm w} \bar{H} - \bar{\gamma}_{\rm w}^{-1} \bar{\gamma}_{\rm w}'$$

- With $H_0 = 62$ km sec^{-1} Mpc^{-1}, expect according to our measurements:
  $\bar{H}_0 = 48$ km sec^{-1} Mpc^{-1} within ideal walls (e.g., around Virgo cluster?); and
  $\bar{H}_{\rm v0} = 76$ km sec^{-1} Mpc^{-1} across local voids (scale $\sim$ 45 Mpc)

Explanation for Hubble bubble

- As voids occupy largest volume of space expect to measure higher average Hubble constant locally until the global average relative volumes of walls and voids are sampled at scale of homogeneity; thus expect maximum H_0 value for isotropic average on scale of dominant void diameter, $30h^{-1}$Mpc, then decreasing til levelling out by $100h^{-1}$Mpc.
- Consistent with observed Hubble bubble feature (Jha, Riess, Kirshner ApJ 659, 122 (2007)), which is unexplained (and problem for) ΛCDM.
- Intrinsic variance in apparent Hubble flow exposes a local scale dependence which may partly explain difficulties astronomers have had in converging on a value for H_0.



N. Li and D. Schwarz, arxiv:0710.5073v1–2

Best fit parameters

- Hubble constant $H_0 = 61.7^{+1.2}_{-1.1}$ km sec^{-1} Mpc^{-1}
- present void volume fraction $f_{\mathrm{v0}} = 0.76^{+0.12}_{-0.09}$
- bare density parameter $\bar{\Omega}_{M0} = 0.125^{+0.060}_{-0.069}$
- dressed density parameter $\Omega_{M0} = 0.33^{+0.11}_{-0.16}$
- non–baryonic dark matter / baryonic matter mass ratio $(\bar{\Omega}_{M0} - \bar{\Omega}_{B0})/\bar{\Omega}_{B0} = 3.1^{+2.5}_{-2.4}$
- bare Hubble constant $\bar{H}_0 = 48.2^{+2.0}_{-2.4}$ km sec^{-1} Mpc^{-1}
- mean lapse function $\bar{\gamma}_0 = 1.381^{+0.061}_{-0.046}$
- deceleration parameter $q_0 = -0.0428^{+0.0120}_{-0.0002}$
- wall age universe $\tau_0 = 14.7^{+0.7}_{-0.5}$ Gyr

Model comparison

	ΛCDM	FB scenario
SneIa luminosity distances	Yes	Yes
BAO scale (clustering)	Yes	Yes
Sound horizon scale (CMB)	Yes	Yes
Doppler peak fine structure	Yes	[still to calculate]
Integrated Sachs–Wolfe effect	Yes	[still to calculate]
Primordial ^{7}Li abundances	No	Yes?
CMB ellipticity	No	[Maybe]
CMB low multipole anomalies	No	[Foreground void: Rees–Sciama dipole]
Hubble bubble	No	Yes
Nucleochronology dates of old globular clusters	Tension	Yes
X-ray cluster abundances	Marginal	Yes
Emptiness of voids	No	[Maybe]
Sandage-de Vaucouleurs paradox	No	Yes
Coincidence problem	No	Yes

Conclusion

- Apparent cosmic acceleration can be understood purely within general relativity; by (i) treating geometry of universe more realistically; (ii) understanding fundamental aspects of general relativity which have not been fully explored – *quasi–local gravitational energy*, of *gradients* in spatial curvature etc.
- The "fractal bubble" model passes three major independent tests which support ΛCDM and may resolve significant puzzles and anomalies.
- Every cosmological parameter requires subtle recalibration, but no "new" physics beyond dark matter: no Λ, no exotic scalars, no modifications to gravity.
- Questions raised – otherwise unanswered – should be addressed irrespective of phenomenological success.