Thermalization of pair plasma with baryonic loading

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Pair plasma in GRBs

Why e^+e^- pairs?

- Energy range: $10^{48} < E_0 < 10^{54}$ erg

(isotropic energy release, fraction of stellar mass)

- Size range: $10^6 < R_0 < 10^8$ cm

(time variability, NS-BH size)
Optical depth for pair production: $\tau = \sigma_T n_\gamma R \approx \sigma_T E_0 / R_0^2 \gg 1$.
Why baryons?

- Time duration of the whole burst, spectrum
- Progenitors of GRBs: massive stars, NS

Issues:

1. Microphysics: processes, baryonic loading, ...
2. Macrophysics: global dynamics, geometry, ...
3. Radiation: mechanisms, transparency, ...

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- **Plasma degeneracy** $\theta_F = \left[\left(\frac{\hbar}{mc}\right)^2 \left(3\pi^2 n_-\right)^{\frac{2}{3}} + 1\right]^{1/2} - 1$.

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- In our energy range $\epsilon \lesssim 10$ MeV the plasma is non-degenerate $\theta_F > \theta_{th}$.
- **Natural parameters for perturbative expansion are α and m/M.**

Baryonic loading

When admixture of protons and electrons is allowed it is characterized by a new parameter, the baryonic loading

$$\mathbf{B} = \frac{NMc^2}{\mathcal{E}_\gamma} = \frac{n_p Mc^2}{\rho_\gamma}. \tag{1}$$

In equilibrium, while e^+e^- are relativistic, $\epsilon_\pm \sim mc^2 \sim k_B T$, protons are not $Mv_p^2 \sim k_B T$, and thus

$$\frac{v_p}{c} \sim \sqrt{\frac{m}{M}}.$$

Also in equilibrium with $\epsilon_\pm \geq mc^2$ we have $\rho_\pm \approx n_\pm mc^2$ and thus

$$\frac{n_p}{n_\pm} \sim \frac{m}{M} B.$$

Interactions with pairs

Binary interactions	Radiative and pair producing variants
Møller and Bhabha scattering $e_1^\pm e_2^\pm \longrightarrow e_1^{\pm\prime} e_2^{\pm\prime}$ $e^\pm e^\mp \longrightarrow e^{\pm\prime} e^{\mp\prime}$	Bremsstrahlung $e_1^\pm e_2^\pm \longleftrightarrow e_1^{\pm\prime} e_2^{\pm\prime} \gamma$ $e^\pm e^\mp \longleftrightarrow e^{\pm\prime} e^{\mp\prime} \gamma$
Single Compton scattering $e^\pm \gamma \longrightarrow e^\pm \gamma'$	Double Compton scattering $e^\pm \gamma \longleftrightarrow e^{\pm\prime} \gamma' \gamma''$
Pair production and annihilation $\gamma\gamma' \longleftrightarrow e^\pm e^\mp$	Radiative pair production and three photon annihilation $\gamma\gamma' \longleftrightarrow e^\pm e^\mp \gamma''$ $e^\pm e^\mp \longleftrightarrow \gamma\gamma'\gamma''$
	$e^\pm \gamma \longleftrightarrow e^{\pm\prime} e^\mp e^{\pm\prime\prime}$

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Coulomb scattering $p_1 p_2 \longrightarrow p_1' p_2'$ $pe^\pm \longrightarrow p' e^{\pm\prime}$	Bremsstrahlung $p_1 p_2 \longleftrightarrow p_1' p_2' \gamma$ $pe^\pm \longleftrightarrow p' e^{\pm\prime} \gamma$
	$pe_1^\pm \longleftrightarrow p' e_1^{\pm\prime} e^\pm e^\mp$
Single Compton scattering $p\gamma \longrightarrow p'\gamma'$	Double Compton scattering and radiative pair production $p\gamma \longleftrightarrow p'\gamma'\gamma''$
	$p\gamma \longleftrightarrow p' e^\pm e^\mp$

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- **Proton Compton scattering:** $(n_p t_{\gamma p})^{-1} \approx \left(\frac{\epsilon}{Mc^2}\right)^2 (n_- t_{\gamma e})^{-1}$, $\epsilon \geq mc^2$.

Boltzmann equation

Relativistic Boltzmann equations in spherically symmetric case

$$\frac{1}{c}\frac{\partial f_i}{\partial t} + \beta_i \left(\mu \frac{\partial f_i}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial f_i}{\partial \mu} \right) - \nabla U \frac{\partial f_i}{\partial \mathbf{p}} = \sum_q \left(\eta_i^q - \chi_i^q f_i \right), \qquad (2)$$

where $\mu = \cos\vartheta = \mathbf{r}\cdot\mathbf{p}$, U is a potential due to some external force, $\beta_i = v_i/c$, $f_i(\epsilon, t)$ are distribution functions, and η_i^q and χ_i^q are the emission and the absorption coefficients. This is a coupled system of partial-integro-differential equations.
For homogeneous and isotropic distribution functions of electrons, positrons and photons (2) reduces to

$$\frac{1}{c}\frac{\partial f_i}{\partial t} = \sum_q \left(\eta_i^q - \chi_i^q f_i \right). \qquad (3)$$

In (3) we also explicitely neglect the Vlasov term.

Collisional integrals 1: probability

Differential probability for all processes per unit time and unit volume ($\hbar = c = 1$)

$$dw = (2\pi)^4 \delta^{(4)}(P_f - P_i) \frac{|M_{fi}|^2}{\prod_b 2\epsilon_b} \prod_a \frac{d\mathbf{p}'_a}{(2\pi\hbar)^3},$$

where $\mathbf{p}'_a$ are momenta of outgoing particles, ϵ_b are energies of particles before and after interaction, M_{fi} are corresponding matrix elements, $\delta^{(4)}$ stands for energy-momentum conservation.
As example consider absorption coefficient for Compton scattering

$$\chi^{\gamma e^{\pm} \to \gamma' e^{\pm\prime}} f_\gamma = \int d\mathbf{k}' d\mathbf{p} d\mathbf{p}' w_{\mathbf{k}',\mathbf{p}';\mathbf{k},\mathbf{p}} f_\gamma(\mathbf{k}, t) f_\pm(\mathbf{p}, t),$$

where $\mathbf{p}$ and $\mathbf{k}$ are momenta of electron (positron) and photon respectively, $d\mathbf{p} = d\epsilon_\pm do \epsilon_\pm^2 \beta_\pm / c^3$, $d\mathbf{k}' = d\epsilon'_\gamma \epsilon'^2_\gamma do'_\gamma / c^3$.

Collisional integrals 2: integration over momentum

We can perform one integration over $d\mathbf{p}'$ as $\int d\mathbf{p}'\delta(d\mathbf{k}+d\mathbf{p}-d\mathbf{k}'-d\mathbf{p}')\rightarrow 1$, but it is necessary to take into account the momentum conservation in the next integration over $d\mathbf{k}'$,

$$\int d\epsilon'_\gamma \delta(\epsilon_\gamma+\epsilon_\pm-\epsilon'_\gamma-\epsilon'_\pm)=$$
$$=\int d(\epsilon'_\gamma+\epsilon'_\pm)\frac{1}{|\partial(\epsilon'_\gamma+\epsilon'_\pm)/\partial\epsilon'_\gamma|}\delta(\epsilon_\gamma+\epsilon_\pm-\epsilon'_\gamma-\epsilon'_\pm)\rightarrow$$
$$\rightarrow\frac{1}{|\partial(\epsilon'_\gamma+\epsilon'_\pm)/\partial\epsilon'_\gamma|}\equiv J_{\rm cs},$$

where the Jacobian of the transformation is

$$J_{\rm cs}=\frac{1}{1-\beta'_\pm\mathbf{b}'_\gamma\cdot\mathbf{b}'_\pm}, \tag{4}$$

where $\mathbf{b}_i=\mathbf{p}_i/p$, $\mathbf{b}'_i=\mathbf{p}'_i/p'$, $\mathbf{b}'_\pm=(\beta_\pm\epsilon_\pm\mathbf{b}_\pm+\epsilon_\gamma\mathbf{b}_\gamma-\epsilon'_\gamma\mathbf{b}'_\gamma)/(\beta'_\pm\epsilon'_\pm)$.

Collisional integrals 3: three-particle interactions

Finally, for the absorption coefficient

$$\chi^{\gamma e^{\pm}\rightarrow\gamma' e^{\pm\prime}} f_\gamma = -\int do'_\gamma d\mathbf{p} \frac{c\epsilon'_\gamma |M_{fi}|^2}{16\epsilon_\pm \epsilon_\gamma \epsilon'_\pm c^3 (2\pi\hbar)^2} J_{\rm cs} f_\gamma(\mathbf{k},t) f_\pm(\mathbf{p},t),$$

As example of 3-particle reaction consider relativistic bremsstrahlung $e_1 + e_2 \leftrightarrow e'_1 + e'_2 + \gamma'$. For the time derivative, for instance, of the distribution function f_2 one has

$$\begin{aligned} \dot{f}_2 &= \int dp'_2 dk' (2\pi)^4 \delta^{(4)}(P_f - P_i) \frac{|M_{fi}|^2}{2^5 \epsilon_1 \epsilon_2 \epsilon'_1 \epsilon'_2 \epsilon'_\gamma} \times \\ &\times \left(\int \frac{d\mathbf{p}'_1 f'_k d\mathbf{p}_1 f'_1 f'_2}{(2\pi\hbar)^6} - \int \frac{d\mathbf{p}_1 d\mathbf{p}'_1 f_1 f_2}{(2\pi\hbar)^9} \right). \end{aligned}$$

In the case of kinetic equilibrium we have multipliers proportional to $\exp\frac{\varphi}{k_B T}$ in front of the integrals. The calculation is then reduced to the known thermal equilibrium case.

Conservation laws

Energy conservation

$$\frac{d}{dt}\sum_i \rho_i = 0, \quad \text{or} \quad \frac{d}{dt}\sum_{i,\omega} Y_{i,\omega} = 0, \quad \text{where} \quad Y_{i,\omega} = \int_{\epsilon_{i,\omega}-\Delta\epsilon_{i,\omega}/2}^{\epsilon_{i,\omega}+\Delta\epsilon_{i,\omega}/2} E_i d\epsilon.$$

Particle's conservation for binary reactions

$$\frac{d}{dt}\sum_i n_i = 0, \quad \text{or} \quad \frac{d}{dt}\sum_{i,\omega} \frac{Y_{i,\omega}}{\epsilon_{i,\omega}} = 0.$$

Baryonic number and charge conservation respectively

$$\frac{dn_p}{dt} = 0, \quad n_- = n_+ + n_p,$$

The condition for the chemical potentials

$$\varphi_+ + \varphi_- = 2\varphi_\gamma.$$

Discretization and computational scheme

We use a finite difference method with a computational grid in the phase space following (Aksenov, 2004). Our goal is to construct the scheme implementing given conservation laws. We use spectral energy densities $E_i(\epsilon_i) = \frac{4\pi\epsilon_i^3\beta_i f_i}{c^3}$,where $\beta_i = \sqrt{1-(m_i c^2/\epsilon_i)^2}$, in the phase space ϵ_i.
The Boltzmann equations then become

$$\frac{1}{c}\frac{\partial E_i}{\partial t} = \sum_q (\tilde{\eta}_i^q - \chi_i^q E_i), \tag{6}$$

where $\tilde{\eta}_i^q = (4\pi\epsilon_i^3\beta_i/c^3)\eta_i^q$.
We introduced the following computational grid $\{\epsilon_i, \mu, \phi\}$. The zone boundaries are $\epsilon_{i,\omega\mp 1/2}$, $\mu_{k\mp 1/2}$, $\phi_{l\mp 1/2}$ for $1 \le \omega \le \omega_{\max}$, $1 \le k \le k_{\max}$, $1 \le l \le l_{\max}$. The length of the i-th interval is $\Delta\epsilon_{i,\omega} \equiv \epsilon_{i,\omega+1/2} - \epsilon_{i,\omega-1/2}$. On the finite grid $E_{i,\omega} \equiv \frac{1}{\Delta\epsilon_{i,\omega}} \int_{\Delta\epsilon_{i,\omega}} d\epsilon E_i(\epsilon)$.

Kinetic equilibrium 1

For photons we have

$$n_\gamma = \frac{1}{V_0}\exp\left(\frac{\nu_\gamma}{\theta}\right)2\theta^3, \qquad \frac{\rho_\gamma}{n_\gamma mc^2} = 3\theta, \qquad V_0 = \frac{1}{8\pi}\left(\frac{2\pi\hbar}{mc}\right)^3$$

for pairs

$$n_\pm = \frac{1}{V_0}\exp\left(\frac{\nu_\pm}{\theta}\right)j_1(\theta), \qquad \frac{\rho_\pm}{n_\pm mc^2} = j_2(\theta),$$

and for protons

$$n_p = \frac{1}{V_0}\sqrt{\frac{\pi}{2}}\left(\frac{M}{m}\right)^{3/2}\exp\left(\frac{\nu_p - M/m}{\theta}\right)\theta^{3/2}, \qquad \frac{\rho_p}{Mn_pc^2} = 1 + \frac{3}{2}\frac{m}{M}\theta,$$

$$\begin{aligned} j_1(\theta) &= \theta K_2(\theta^{-1}) \rightarrow \begin{cases} \sqrt{\frac{\pi}{2}}e^{-\frac{1}{\theta}}\theta^{3/2}, & \theta \rightarrow 0 \\ 2\theta^3, & \theta \rightarrow \infty \end{cases}, \\ j_2(\theta) &= \frac{3K_3(\theta^{-1}) + K_1(\theta^{-1})}{4K_2(\theta^{-1})} \rightarrow \begin{cases} 1 + \frac{3}{2}\theta, & \theta \rightarrow 0 \\ 3\theta, & \theta \rightarrow \infty \end{cases}. \end{aligned}$$

Kinetic equilibrium 2

Summing up energy densities

$$\sum_i \rho_i = \frac{mc^2}{V_0}\left\{6\theta^4 \exp\left(\frac{\nu_+}{\theta}\right)\left[1-\frac{n_p V_0}{j_1(\theta)}\exp\left(-\frac{\nu_+}{\theta}\right)\right]^{\frac{1}{2}} + \right.$$
$$\left. +j_2(\theta)\left[2j_1(\theta)\exp\left(\frac{\nu_+}{\theta}\right)-n_p V_0\right]+\frac{M}{m}\left(1+\frac{3}{2}\frac{m}{M}\theta\right)n_p V_0\right\},$$

and analogously for number densities

$$\sum_i n_i = \frac{1}{V_0}\left\{6\theta^4 \exp\left(\frac{\nu_+}{\theta}\right)\left[1-\frac{n_p V_0}{j_1(\theta)}\exp\left(-\frac{\nu_+}{\theta}\right)\right]^{\frac{1}{2}} + 2j_1(\theta)\exp\left(\frac{\nu_+}{\theta}\right)\right\},$$

so that two unknowns, ν_+ and θ can be found.

Kinetic equilibrium 3

$$\exp\left(\frac{\nu_-}{\theta}\right) = \exp\left(\frac{\nu_+}{\theta}\right) + \frac{n_p V_0}{j_1(\theta)},$$

$$\exp\left(\frac{\nu_\gamma}{\theta}\right) = \exp\left(\frac{\nu_+}{\theta}\right)\left[1 + \frac{n_p V_0}{j_1(\theta)}\exp\left(-\frac{\nu_+}{\theta}\right)\right]^{\frac{1}{2}},$$

$$\exp\left(\frac{\nu_p - M/m}{\theta}\right) = n_p V_0 \sqrt{\frac{2}{\pi}}\left(\frac{m}{M}\right)^{3/2}\theta^{-3/2}.$$

In thermal equilibrium $\nu_\gamma = 0$ and

$$\nu_\mp = \theta \ln\left[\sqrt{\left(\frac{n_p V_0}{2j_1(\theta)}\right)^2 - 1} \pm \frac{n_p V_0}{2j_1(\theta)}\right].$$

For $n_p > 0$ one always has $\nu_- > 0$ and $\nu_+ < 0$ in thermal equilibrium.

Cutoff at small angles

Haug (1985) gives the minimal scattering angle in the center of mass system

$$\theta_{\min} = \frac{2\hbar}{\mathcal{M}cD}\frac{\gamma_r}{(\gamma_r+1)\sqrt{2(\gamma_r-1)}},$$

where the maximum impact parameter (neglecting the effect of protons) is

$$D = \frac{c^2}{\omega}\frac{p_0}{\epsilon_{10}},$$

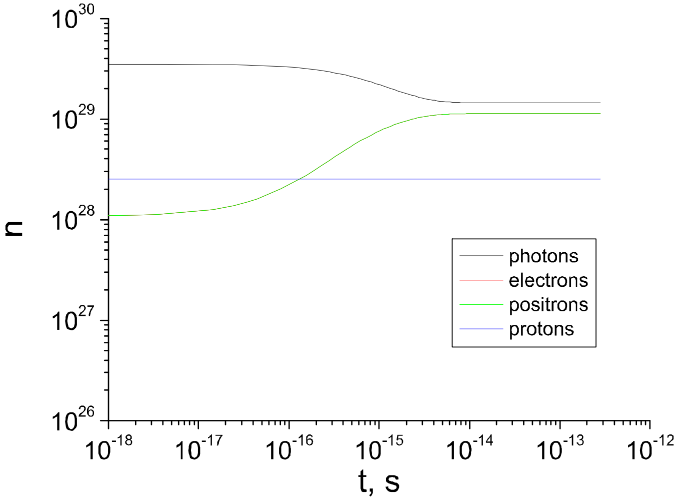
where p_0 and ϵ_{10} are CM quantities, and the invariant Lorentz factor of relative motion is

$$\gamma_r = \frac{1}{\sqrt{1-\left(\frac{v_r}{c}\right)^2}} = \frac{\epsilon_1\epsilon_2 - \mathbf{p}_1\mathbf{p}_2 c^2}{m_1 m_2 c^4}.$$

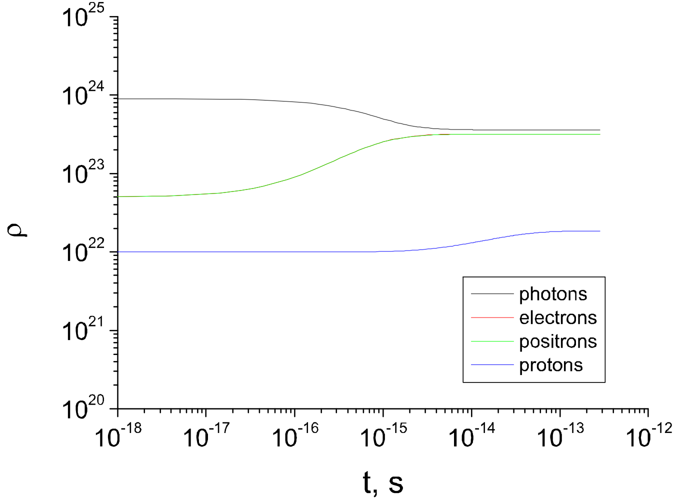
The Coulomb logarithm is

$$\ln\Lambda = \frac{1}{2} - \ln\left(\sqrt{2}\sin\theta_{\min}\right).$$

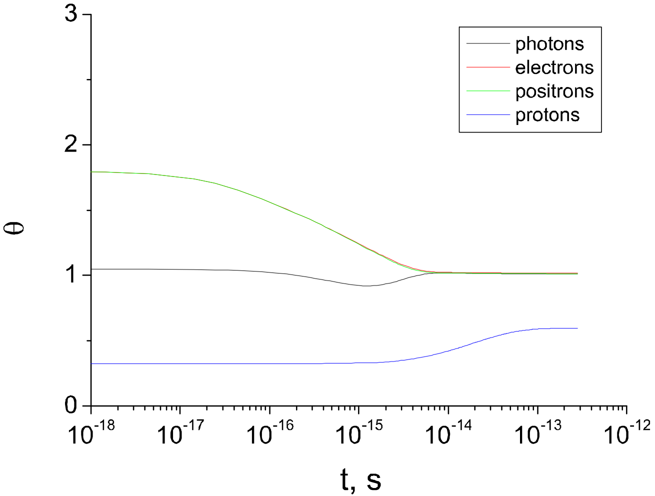
Preliminary results: concentrations



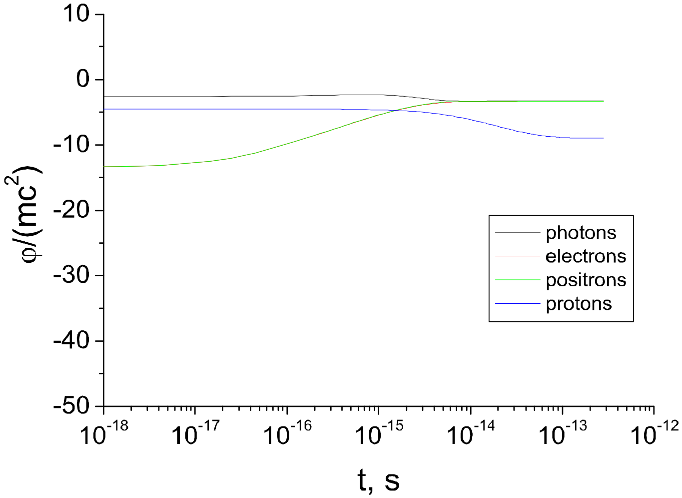
Preliminary results: energy density



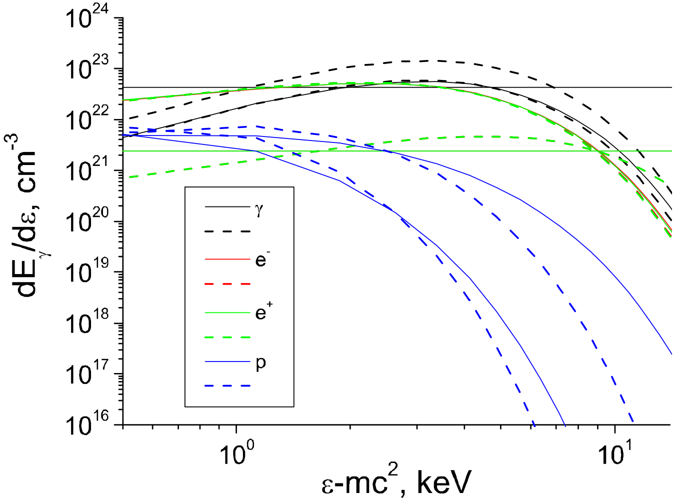
Preliminary results: temperature



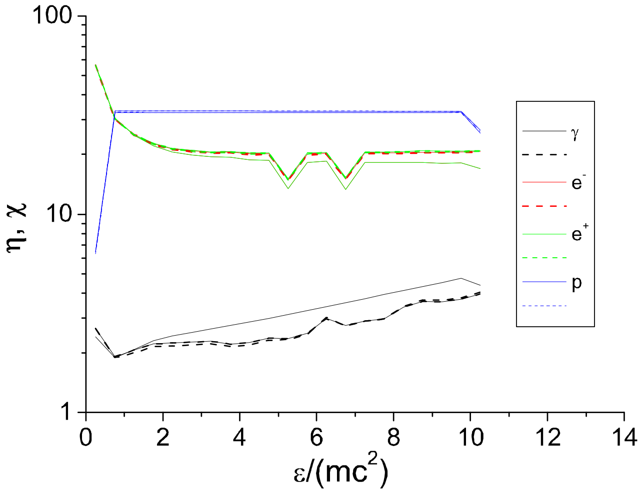
Preliminary results: chemical potential



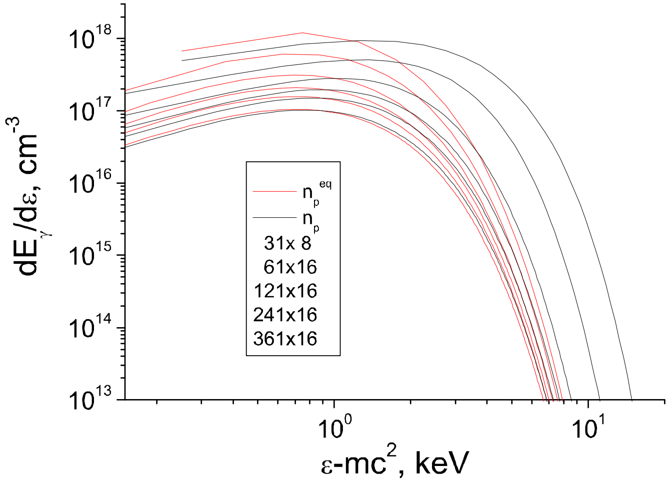
Preliminary results: spectra



Preliminary results: emission and absorption



Equilibrium of protons



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2. Kinetic equilibrium is obtained from the first principles.
3. Protons thermalize with other particles by proton-electron scatterings; proton-proton scatterings are inefficient.
4. **The timescale of thermalization is always shorter than the dynamical one.**