Extended bodies in General Relativity (applications to BH spacetimes)

Donato Bini

IAC-CNR, Roma

Collaborations

C. Cherubini (Roma)

F. De Felice (Padova)

A. Geralico (Roma)

R.T. Jantzen (USA)

B. Mashhoon (USA)

P. Fortini (Ferrara)

A. Ortolan (Padova)

Open problems
(motivations for work)

Do we have a complete and general relativistically correct description of the motion of extended bodies?

Is the motion the same for macroscopic and microscopic particles?

Does the spin (or in general the inner structure) of the body play a special role, in the sense that certain kind of couplings are preferred with respect to others?

No experiments→ not a definite answer!

What has been already done: a short review

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Class. Quantum Grav. **21** (2004) 5427–5439 PII: S0264-9381(04)84061-9

Spinning test particles and clock effect in Schwarzschild spacetime

Donato Bini[1,2,3], **Fernando de Felice**[4] **and Andrea Geralico**[2,5]

[1] Istituto per le Applicazioni del Calcolo 'M Picone', CNR I-00161 Rome, Italy
[2] International Center for Relativistic Astrophysics, University of Rome, I-00185 Rome, Italy
[3] INFN—Sezione di Firenze, Polo Scientifico, Via Sansone 1, I-50019, Sesto Fiorentino (FI), Italy
[4] Dipartimento di Fisica, Università di Padova, and INFN, Sezione di Padova, Via Marzolo 8, I-35131 Padova, Italy
[5] Dipartimento di Fisica, Università di Lecce, and INFN—Sezione di Lecce, Via Arnesano, CP 193, I-73100 Lecce, Italy

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Spinning test particles and clock effect in Kerr spacetime

Donato Bini[1,2,3], Fernando de Felice[4] and Andrea Geralico[2,5]

[1] Istituto per le Applicazioni del Calcolo 'M Picone', CNR, I-00161 Rome, Italy
[2] International Center for Relativistic Astrophysics, University of Rome, I-00185 Rome, Italy
[3] INFN—Sezione di Firenze, Polo Scientifico, Via Sansone 1, I-50019 Sesto Fiorentino (FI), Italy
[4] Dipartimento di Fisica, Università di Padova, and INFN, Sezione di Padova, Via Marzolo 8, I-35131 Padova, Italy
[5] Dipartimento di Fisica, Università di Lecce, and INFN—Sezione di Lecce, Via Arnesano, CP 193, I-73100 Lecce, Italy

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Spinning particles in the vacuum C metric

Donato Bini[1,2,3], Christian Cherubini[2,4], Andrea Geralico[2,5,6] and Bahram Mashhoon[7]

[1] Istituto per le Applicazioni del Calcolo 'M Picone', CNR I-00161 Rome, Italy
[2] International Center for Relativistic Astrophysics, University of Rome, I-00185 Rome, Italy
[3] INFN—Sezione di Firenze, Polo Scientifico, Via Sansone 1, I-50019, Sesto Fiorentino (FI), Italy
[4] Facoltà di Ingegneria, Università 'Campus Biomedico', Via E Longoni 47, I-00155 Rome, Italy
[5] Dipartimento di Fisica, Università di Lecce, Lecce, Italy
[6] INFN—Sezione di Lecce, Via Arnesano, CP 193, I-73100 Lecce, Italy
[7] Department of Physics and Astronomy, University of Missouri–Columbia, Columbia, MO 65211, USA

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Spin precession in the Schwarzschild spacetime: circular orbits

Donato Bini[1,2,3], Fernando de Felice[4], Andrea Geralico[2,5] and Robert T Jantzen[2,6]

[1] Istituto per le Applicazioni del Calcolo 'M Picone', CNR, I-00161 Rome, Italy
[2] International Center for Relativistic Astrophysics, University of Rome, I-00185 Rome, Italy
[3] INFN—Sezione di Firenze, Polo Scientifico, Via Sansone 1, I-50019 Sesto Fiorentino (FI), Italy
[4] Dipartimento di Fisica, Università di Padova and INFN, Sezione di Padova, Via Marzolo 8, I-35131 Padova, Italy
[5] Dipartimento di Fisica, Università di Lecce and INFN—Sezione di Lecce, Via Arnesano, CP 193, I-73100 Lecce, Italy
[6] Department of Mathematical Sciences, Villanova University, Villanova, PA 19085, USA

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Spin precession along circular orbits in the Kerr spacetime: the Frenet–Serret description

Donato Bini[1,2,3], Fernando de Felice[4], Andrea Geralico[2] and Robert T Jantzen[2,5]

[1] Istituto per le Applicazioni del Calcolo 'M. Picone', CNR I-00161 Rome, Italy
[2] ICRA, University of Rome, I-00185 Rome, Italy
[3] INFN sezione di Firenze, I-00185 Sesto Fiorentino (FI), Italy
[4] Dipartimento di Fisica, Università di Padova, I-35131 Padova, Italy
[5] Department of Mathematical Sciences, Villanova University, Villanova, PA 19085, USA

spin

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CHARGED SPINNING PARTICLES ON CIRCULAR ORBITS IN THE REISSNER–NORDSTRÖM SPACE–TIME

DONATO BINI*,‡,§ and ANDREA GERALICO†,§,¶

‡*Istituto per le Applicazioni del Calcolo "M. Picone", CNR I-00161 Rome, Italy*

§*International Center for Relativistic Astrophysics – I.C.R.A., University of Rome "La Sapienza", I-00185 Rome, Italy*

and

¶*Dipartimento di Fisica, Università di Lecce and INFN, Sezione di Lecce, Via Arnesano, CP 193, I-73100 Lecce, Italy*

**binid@icra.it*

†*geralico@icra.it*

FERNANDO DE FELICE

Dipartimento di Fisica, Università di Padova and INFN, Sezione di Padova, Via Marzolo 8, I-35131 Padova, Italy

fernando.defelice@pd.infn.it

spin and charge

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Quadrupole effects on the motion of extended bodies in Schwarzschild spacetime

Donato Bini[1,2,3], **Pierluigi Fortini**[4], **Andrea Geralico**[2,5] **and Antonello Ortolan**[6]

[1] Istituto per le Applicazioni del Calcolo 'M Picone', CNR I-00161 Rome, Italy
[2] ICRA, University of Rome 'La Sapienza', I-00185 Rome, Italy
[3] INFN—Sezione di Firenze, Polo Scientifico, Via Sansone 1, I-50019, Sesto Fiorentino (FI), Italy
[4] Department of Physics, University of Ferrara and INFN Sezione di Ferrara, I-44100 Ferrara, Italy
[5] Physics Department, University of Rome 'La Sapienza', I-00185 Rome, Italy
[6] INFN—National Laboratories of Legnaro, I-35020 Legnaro (PD), Italy

Quadrupole

GR model for a test particle

A test particle in the spacetime is represented by a single world line, with a “label”: the mass of the particle.

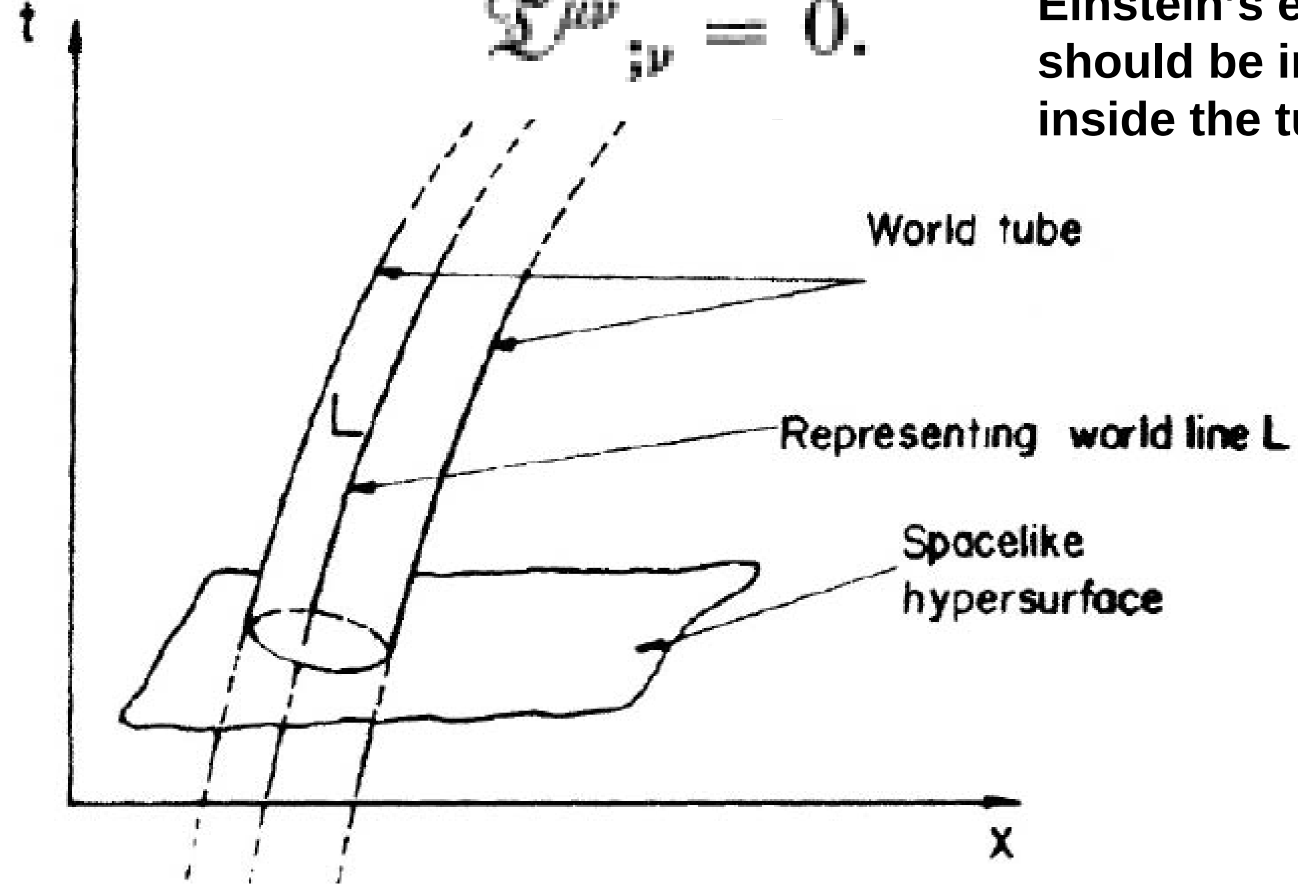
GR model for a test body

A test body in the spacetime is represented by a world line (roughly, the center of mass orbit) and a pair (or more) of tensors defined (and evolving) all along the CM world line (roughly, the way in which the body is allowed to move along the orbit due to its internal structure)

Extended body in the spacetime

$$\mathfrak{T}^{\mu\nu}{}_{;\nu} = 0.$$

Einstein's equations should be imposed inside the tube.



$$\int \mathfrak{T}^{\mu\nu}\, dv, \quad \int \delta x^\rho \mathfrak{T}^{\mu\nu}\, dv, \quad \int \delta x^\rho\, \delta x^\sigma\, \mathfrak{T}^{\mu\nu}\, dv, \quad \ldots,$$

Multipolar approximation

Competing models

Mathisson-Papapetrou model

Dixon model

Variations of both

Purely spinning particles (Mathisson-Papapetrou model)

$$\frac{DP^\mu}{d\tau_U} = -\frac{1}{2} R^\mu{}_{\nu\alpha\beta} U^\nu S^{\alpha\beta}$$

$$\frac{DS^{\mu\nu}}{d\tau_U} = 2P^{[\mu}U^{\nu]}$$

4 + 6 equations

U=timelike unit tangent vector to the CM world line

P=generalized momentum of the particle (evolving along U)

S=spin tensor (evolving along U)

but

4(P)+6(S)+3(U)=13 unknowns!

Something is missing here: model incomplete or incorrect?

Supplementary conditions

1. Corinaldesi–Papapetrou conditions (CP): $S^{t\nu} = 0$,
2. Pirani conditions (P): $S^{\mu\nu}U_\nu = 0$,
3. Tulczyjew conditions (T): $S^{\mu\nu}P_\nu = 0$.

Mathisson M 1937 *Acta Phys. Pol.* **6** 167
Papapetrou A 1951 *Proc. R. Soc.* **209** 248
Corinaldesi E and Papapetrou A 1951 *Proc. R. Soc.* **209** 259
Pirani F 1956 *Acta Phys. Pol.* **15** 389
Tulczyjew W 1959 *Acta Phys. Pol.* **18** 393

Physically correct supplementary conditions

Tulczyjew conditions (T): $S^{\mu\nu} P_\nu = 0.$

Quadrupolar particles: Dixon's model

Fundamental equations:

$$\frac{DP^{\mu}}{d\tau_U} = -\frac{1}{2}R^{\mu}{}_{\nu\alpha\beta}U^{\nu}S^{\alpha\beta} - \frac{1}{6}J^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}{}^{;\mu} \equiv F^{(\mathrm{spin})\mu} + F^{(\mathrm{quad})\mu}$$

$$\frac{DS^{\mu\nu}}{d\tau_U} = 2P^{[\mu}U^{\nu]} - \frac{4}{3}J^{\alpha\beta\gamma[\mu}R^{\nu]}{}_{\alpha\beta\gamma}\ ,$$

$$P^{\mu} = mU_p^{\mu}\ (\text{with } U_p \cdot U_p = -1)$$

U Unit timelike tangent vector to the center of mass line of the body

$J^{\alpha\beta\gamma\delta}$ Quadrupole moment tensor

The quadrupole moment tensor

The tensor J is the quadrupole moment of the stress-energy tensor of the body, and has the same algebraic symmetries as the Riemann tensor. A “1+3” splitting gives

$$J^{\alpha\beta\gamma\delta} = \Pi^{\alpha\beta\gamma\delta} - \bar{u}^{[\alpha}\pi^{\beta]\gamma\delta} - \bar{u}^{[\gamma}\pi^{\delta]\alpha\beta} - 3\bar{u}^{[\alpha}Q^{\beta][\gamma}\bar{u}^{\delta]}$$

$$\Pi^{\alpha\beta\gamma\delta}$$

$$\Pi^{\alpha\beta\gamma\delta} = \Pi^{[\alpha\beta][\gamma\delta]}$$

$$\pi^{\alpha\beta\gamma} = \pi^{\alpha[\beta\gamma]} \qquad \pi^{[\alpha\beta\gamma]} = 0$$

$$Q^{\alpha\beta} = Q^{(\alpha\beta)}$$

Algebraic properties of J

The number of independent components of J is 20:

6 in $Q^{\alpha\beta}$

6 in $\Pi^{\alpha\beta\gamma\delta}$

and 8 in $\pi^{\alpha\beta\gamma}$

Physical meaning of the splitting fields

$\Pi^{\alpha\beta\gamma\delta}$	represents the stress of quadrupole moment
$\pi^{\alpha\beta\gamma}$	represents the linear momentum of quadrupole momentum
$Q^{\alpha\beta}$	represents the mass of quadrupole moment

Quadrupolar particles in Papapetrou model

$$\frac{DP^\mu}{\mathrm{d}\tau_U} = -\frac{1}{2}R^\mu{}_{\nu\alpha\beta}U^\nu S^{\alpha\beta}$$ **+ quardrupolar terms**

$$\frac{DS^{\mu\nu}}{\mathrm{d}\tau_U} = 2P^{[\mu}U^{\nu]}$$ **+ quardrupolar terms**

$$\frac{D \text{ quardrupolar terms}}{\mathrm{d}\tau_U} =$$ **+ spin + quadrupolar terms**

Never written explicitly up to now!

Dixon’s vs Papapetrou model

The fundamental difference between the two Models is that Dixon’s model does not include any evolution equation for the quadrupole tensor.

The quadrupole moment of the body should be considered as given and representative of the material structure of the body itself.

Correctness of Dixon’s model

(true also up to the quadrupolar approximation)

In order Dixon’s model to be mathematically correct the following additional conditions should be imposed to the spin tensor:

$$S^{\mu\nu} U_{p\nu} = 0$$

Such supplementary conditions (or Tulczyjew-Dixon conditions) ensure the correct definition of
the various multipolar terms.

N. Backreaction should always be negligible!

Ehlers approximations

In all our works we have always considered Dixon’s model under the further simplifying assumption that the only contribution to the complete quadrupole moment J stems from the mass quadrupole moment Q, so that

$$\pi^{\alpha\beta\gamma} = 0 = \Pi^{\alpha\beta\gamma\delta}$$

$$J^{\alpha\beta\gamma\delta} = -3 \underset{p}{U}^{[\alpha} Q^{\beta][\gamma} \underset{p}{U}^{\delta]} \ , \qquad Q^{\alpha\beta} U_{p\,\beta} = 0$$

Spin vector

Let us introduce the spin vector by spatial (with respect to U_p) duality:

$$S^{\beta} = \frac{1}{2}\eta_{\alpha}{}^{\beta\gamma\delta}U_p^{\alpha}S_{\gamma\delta}$$

$$\eta_{\alpha\beta\gamma\delta} = \sqrt{-g}\epsilon_{\alpha\beta\gamma\delta}$$

It is also convenient to introduce the scalar

$$s^2 = \frac{1}{2}S_{\mu\nu}S^{\mu\nu}$$

Small spin approximation

(taking into account the smallness of S since the beginning)

$$m\frac{DU^\mu}{\mathrm{d}\tau_U} = -[R^*]^{\mu\nu\rho\sigma}U_\nu U_\sigma S_\rho \ ,$$

$$\frac{DS^\mu}{\mathrm{d}\tau_U} = 0 \ ,$$

$$S^\sigma \doteq sN^\sigma$$

(**up to first order in spin**)

$$\frac{DU^\mu}{\mathrm{d}\tau_U} = -\left(\frac{s}{m}\right)[R^*]^{\mu\nu\rho\sigma}U_\nu U_\sigma N_\rho \ ,$$

$$s\frac{DN^\mu}{\mathrm{d}\tau_U} = 0 \ .$$

Motion of extended bodies in Schwarzschild spacetime: an example

Consider the case of a Schwarzschild spacetime

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Introduce an orthonormal frame:

$$e_{\hat{t}} = (1 - 2M/r)^{-1/2}\partial_t,\ e_{\hat{r}} = (1 - 2M/r)^{1/2}\partial_r,\ e_{\hat{\theta}} = \frac{1}{r}\partial_\theta,\ e_{\hat{\phi}} = \frac{1}{r\sin\theta}\partial_\phi$$

Simplifying assumptions

- Assume that U is tangent to a (timelike) spatially circular orbit.
- Limit considerations to equatorial plane motion.
- Assume that U_p also is tangent to a circular orbit.

More formally…but… without too many details

$$U = \Gamma[\partial_t + \zeta \partial_\phi] = \gamma[e_{\hat{t}} + \nu e_{\hat{\phi}}], \qquad \gamma = (1-\nu^2)^{-1/2}$$

$$\zeta = \sqrt{-\frac{g_{tt}}{g_{\phi\phi}}}\nu$$

$$U_p = \gamma_p \left[e_{\hat{t}} + \nu_p e_{\hat{\phi}}\right] , \quad \gamma_p = (1-\nu_p^2)^{-1/2}$$

$$\theta = \pi/2 \qquad\qquad P = m\, U_p$$

Notation

(for a later convenience)

Introduce co/counter rotating geodesics:

$$U_{\pm} = \gamma_K [e_{\hat{t}} \pm \nu_K e_{\hat{\phi}}] \,, \qquad \nu_K = \left[\frac{M}{r-2M}\right]^{1/2}$$

$$\zeta_{\pm} \equiv \pm\zeta_K = \pm(M/r^3)^{1/2}$$

Introduce also the Lie relative curvature of a circular orbit

$$k_{(\mathrm{lie})} = -\partial_{\hat{r}} \ln \sqrt{g_{\phi\phi}} = -\frac{1}{r}\left(1 - \frac{2M}{r}\right)^{1/2} = -\frac{\zeta_K}{\nu_K}$$

Adapted frame

Introduce a frame adapted to U_p

$$e_0 = U_p \ , \qquad e_1 = e_{\hat{r}} \ , \qquad e_2 = \gamma_p \left[\nu_p e_{\hat{t}} + e_{\hat{\phi}} \right] \ , \qquad e_3 = e_{\hat{z}}$$

Within this frame we have automatically

$$S = S^1 e_1 + S^2 e_2 + S^3 e_3$$

$$Q_{00} = Q_{01} = Q_{02} = Q_{03} = 0$$

Additional assumptions

- Let us assume that the spin of the body is aligned with the z axis and constant along the path: $$S^1 = 0 = S^2$$

Quasi-rigidity of the body according to Ehlers-Rudolph definition: the surviving components of the mass quadrupole moment are all constant along the path.

Write the equations…

From the spin evolution equations we have

$$Q_{12} = Q_{13} = Q_{23} = 0$$

Let us introduce the notation:

$$Q_{11} = Q_{33} + f, \qquad Q_{22} = Q_{33} + f'$$

f,f' are the structure functions of the body

Finally, the spin equations reduce to

$$0 = (\nu\nu_p - \nu_K^2)s + m\frac{\nu_K}{\zeta_K}(\nu - \nu_p) + 3\nu_K\zeta_K\frac{\gamma_p\nu_p}{\gamma}\,f$$

Momentum equations

After manipulations the momentum equations reduce to:

$$0 = (\nu\nu_p - \nu_K^2)(-f' + 2\gamma_p^2 f) + \gamma_p^2[2\nu\nu_p\nu_K^2 + \nu_p^2(\nu\nu_p + \nu_K^2) + 2\nu_K^2\nu_p^2]f$$
$$- \frac{2}{3}\frac{m\gamma\gamma_p}{\zeta_K^2}(\nu^2\nu_p^2 + 2\nu_p^2\nu_K^2 - 3\nu\nu_p\nu_K^2 - \nu^2\nu_K^2 + \nu_K^4) \ ,$$

Note that as soon as the dipolar (s) and quadrupolar (f,f') structure of the body is known the previous two equations are enough to the determine the motion: ν ν_p

TF property of the quadrupole moment

As for the classical case we can assume that the quadrupole moment of the body is trace-free; in this case Q is completely represented by f,f':

$$Q_{11} = \frac{2}{3}f - \frac{1}{3}f', \quad Q_{22} = -\frac{1}{3}f + \frac{2}{3}f', \quad Q_{33} = -\frac{1}{3}(f + f').$$

Rescaled adimensional quantities

It is quite natural to introduce the following rescaled dimensionless angular and quadrupolar momentum quantities

$$\sigma = \frac{s}{m}\zeta_K \ , \quad F = \frac{f}{m}\zeta_K^2 \ , \quad F' = \frac{f'}{m}\zeta_K^2$$

due to the fact that along a circular orbit r =const.

Smallness

The quantities $\sigma,\ F \text{ and } F'$ are necessarily small.

Although the quadrupolar terms f and f’ are small only for a quasi-spherical body, the further rescaling by ζ_K makes indeed them small in any case.

In fact, the radius of the orbit is assumed to be large enough in comparison with certain natural length scales like |s|/m (also known as the Møller radius of the body):

$$(|f|/m)^{1/2},\ (|f'|/m)^{1/2}$$

Final set of equations

$$0 = (\nu\nu_p - \nu_K^2)\sigma + \nu_K(\nu - \nu_p) + 3\nu_K \frac{\gamma_p \nu_p}{\gamma} F$$

$$0 = (\nu\nu_p - \nu_K^2)(-F' + 2\gamma_p^2 F) + \gamma_p^2[2\nu\nu_p\nu_K^2 + \nu_p^2(\nu\nu_p + \nu_K^2) + 2\nu_K^2\nu_p^2]F$$
$$- \frac{2}{3}\gamma\gamma_p(\nu^2\nu_p^2 + 2\nu_p^2\nu_K^2 - 3\nu\nu_p\nu_K^2 - \nu^2\nu_K^2 + \nu_K^4) \; .$$

The above equations can be solved to obtain the velocities in terms of spin and quadrupole parameters. All the various special cases like

(i) $\underline{\sigma = 0,\ F \neq 0, F' \neq 0}.$

have been examined.

Limit of small spin and quadrupole

If the contribution of quadrupolar terms can be considered negligible with respect to the dipolar ones and comparable with second order terms in the spin itself:

$$\nu_{\pm} \simeq \pm\nu_K - \frac{3}{2}\nu_K\sigma \pm \frac{3}{8}(2F + \sigma^2)\nu_K$$

$$\nu_p^{(\pm)} \simeq \nu_{\pm} \pm 3(F - \sigma^2)\nu_K$$

where the signs $\pm$ correspond to co/counter rotating orbits.

In terms of angular velocities…

$$\frac{1}{\zeta_\pm} \simeq \pm \frac{1}{\zeta_K} + \frac{3}{2\zeta_K}\sigma \pm \frac{3}{8\zeta_K}(5\sigma^2 - 2F)$$

One can then evaluate the period of revolution around the central source which consists of three different terms

$$T = \frac{2\pi}{|\zeta_\pm|} = T_K\,|1 \pm +\lambda_d + \lambda_Q\,|$$

$$T_K = \frac{2\pi}{\zeta_K}\ , \qquad \lambda_d = \frac{3}{2}\sigma\ , \qquad \lambda_Q = \frac{3}{8}(5\sigma^2 - 2F)\ .$$

Consequences

A direct measurement of T will then allow to estimate the quantity F determining the quadrupolar structure of the body, if its spin is known.

Note that the fraction due to the spin is different depending on whether the body is spinning up or down, whereas the term Q due to the quadrupole has a definite sign once the shape of the body is known (F cannot change its sign).

Applications and “numbers” (case of Schwarzschild)

In the case of the Earth:

$\sigma \approx 2.3 \times 10^{-15}$ and $F = F' \approx -1.8 \times 10^{-20}$, since $s/m_{\oplus} \approx 3.4 \times 10^{2}$ cm

$f = f' = -J_2 m_{\oplus} r_{\oplus}^2$, with $J_2 \approx 10^{-3}$

$r \approx 1.5 \times 10^{13}$ cm (distance Earth-Sun)

$T_K \approx 9.425 \times 10^{17}$ cm

$\lambda_d \approx 3.4 \times 10^{-15}$

$\lambda_Q \approx 1.3 \times 10^{-20}$

Applications and “numbers” (case of Kerr)

An interesting opportunity to test the quadrupole effect of an extended body would arise, for instance, from the motion of ordinary or neutron stars around Sgr A*, the supermassive ($M \simeq 10^6 M_\odot$) rotating ($a \in [0.5, 1]M$) black hole located at the galactic center [10, 14, 19].

To illustrate the order of magnitude of the effect, we may consider a binary pulsar system similar to PSR J0737-3039 as orbiting Sgr A* at a distance of $r \simeq 10^9$ km. The PSR J0737-3039 system consists of two close neutron stars (their separation is only $d_{AB} \sim 8 \times 10^5$ km) of comparable masses $m_A \simeq 1.4 M_\odot$, $m_B \simeq 1.2 M_\odot$), but very different intrinsic spin period (23 ms of pulsar A versus 2.8 s of pulsar B) [20]. Its orbital period is about 2.4 h, the smallest yet known for such an object. Since the intrinsic rotations are negligible with respect to the orbital period, we can treat the binary system as a single object with reduced mass $\mu_{AB} \simeq 0.7 M_\odot$ and intrinsic rotation equal to the orbital period. The spin parameter thus turns out to be equal

Table 1. The estimates of geodesic period T_+^g as well as the corrections λ_d^+ and λ_q^+ due to both the dipolar and quadrupolar structures of the PSR J0737-3039 binary system are listed for different values of Sgr A* rotational parameter a/M. Note that in order to resolve the dipolar and quadrupolar effects the measured period should be known with very high accuracy.

a/M	T_+^g (cm)	λ_d^+	λ_q^+
0.52	$1.622\,36 \times 10^{16}$	8.83×10^{-8}	-7.05×10^{-10}
0.75	$1.622\,38 \times 10^{16}$	8.75×10^{-8}	-6.99×10^{-10}
0.983	$1.622\,40 \times 10^{16}$	8.66×10^{-8}	-6.92×10^{-10}
0.996	$1.622\,41 \times 10^{16}$	8.66×10^{-8}	-6.92×10^{-10}
1	$1.622\,41 \times 10^{16}$	8.66×10^{-8}	-6.92×10^{-10}

to $\sigma \approx 6 \times 10^{-8}$, whereas the quadrupolar parameters are $F = F' \approx 9.6 \times 10^{-10}$, since we have taken $f = f' = \mu_{AB} d_{AB}^2$ as a rough estimate.

Since the rotation parameter of Sgr A* is not small we must use the exact expression (2.30). In the literature one finds different estimates of the spin parameter of the galactic center black hole, ranging from $a \simeq 0.52M$ [21] all the way up to $a \simeq 0.983M$ [22, 23] or even $a \simeq 0.996M$ [24, 25]. We list in table 1 the corresponding values of the geodesic period T_+^g of the PSR J0737-3039 binary system as well as the corrections $\lambda_d^\pm$ and $\lambda_q^\pm$ due to its dipolar and quadrupolar structures respectively.

It turns out that no relevant differences arise for the selected values of the black hole spin parameter. In particular, the results are not sensitive to the black hole being either fast rotating [23] or near extreme [25] or even extreme ($a = M$). In the latter case equations (2.30) reduce to

$$
\begin{aligned}
T^g_{\pm} &= 2\pi\left(M \pm \frac{1}{\zeta_K}\right),\\
\lambda^{\pm}_d &= \pm\frac{3}{2}\sigma\Lambda^{\pm}, \qquad (3.1)\\
\lambda^{\pm}_q &= -\frac{3}{4}\Lambda^{\pm}\left[\pm F\left(1 - 6r^3\zeta_K^3\Lambda^{+}\right) + 2\sigma^2\left(\pm 1 - \frac{3}{4}\Lambda^{\pm}\left(2 \pm 1 - r^3\zeta_K^3\right)\right)\right],
\end{aligned}
$$

where

$$
\Lambda^{\pm} = \frac{1 - r\zeta_K}{1 + r^3\zeta_K^3}\frac{1 \pm r^3\zeta_K^3}{1 + r^3\zeta_K^3}. \qquad (3.2)
$$

The detection of pulsars in Sgr A* is difficult because of the intense scattering region located in front of Sgr A*. However, these pulsars may be detectable in the next decade by the SKA detector, which promises high frequency sensitivity and large collecting area [13].

Another possible application of our calculations could be the orbital motions of the so-called S-stars [21], i.e. the massive ($(30 \div 120)M_{\odot}$), young (<10 Myr) stars within the influence of the supermassive black hole. However, in this case the orbits are no longer circular and the problem of discriminating quadrupole effects would deserve further investigation.

Related works

Extensions of this work have been already considered for different spacetimes (Kerr, GW) and releasing some of the stringent assumptions adopted here.

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Quadrupole effects on the motion of extended bodies in Kerr spacetime

Donato Bini[1,2,3], Pierluigi Fortini[4], Andrea Geralico[2,5] and Antonello Ortolan[6]

Abstract. The motion of a body endowed with dipolar as well as quadrupolar structure is investigated in the Kerr background according to the Dixon's model, extending a previous analysis done in the Schwarzschild background. The full set of evolution equations is solved under the simplifying assumptions of constant frame components for both the spin and the quadrupole tensors and that the center of mass moves along an equatorial circular orbit, the total four-momentum of the body being aligned with it. We find that the motion deviates from the geodesic one due to the internal structure of the body, leading to measurable effects. Corrections to the geodesic value of the orbital period of a close binary system orbiting the Galactic Center are discussed assuming that the Galactic Center is a Kerr supermassive black hole.

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RESEARCH ARTICLE

Dixon's extended bodies and weak gravitational waves

**Donato Bini · Christian Cherubini ·
Andrea Geralico · Antonello Ortolan**

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Abstract General relativity considers Dixon's theory as the standard theory to deal with the motion of extended bodies in a given gravitational background. We discuss here the features of the "reaction" of an extended body to the passage of a weak gravitational wave. We find that the body acquires a dipolar moment induced by its quadrupole structure. Furthermore, we derive the "world function" for the weak field limit of a gravitational wave background and use it to estimate the deviation between geodesics and the world lines of structured bodies. Measuring such deviations, due to the existence of cumulative effects, should be favorite with respect to measuring the amplitude of the gravitational wave itself.

3 Discussion and general results

In the expressions (30) we can distinguish a harmonic part and a secular one; the latter is given by

$$
\begin{aligned}
\omega\, x_{\mathrm{sec}} &= -\frac{A_\times}{2}(2\sigma_0^2\sigma_0^3 + q_0^{23})\,\omega\tau_U\,,\\
\omega\, y_{\mathrm{sec}} &= \frac{1}{4}\left[2A_\times(\sigma_0^1\sigma_0^3 + q_0^{13}) + A_+\sigma_0^3(2 + f_0)\right]\omega\tau_U\,,\\
\omega\, z_{\mathrm{sec}} &= \frac{1}{4}\left[2A_\times(\sigma_0^1\sigma_0^2 + q_0^{12}) + A_+\sigma_0^2(2 - f_0)\right]\omega\tau_U\,,
\end{aligned}
\tag{31}
$$

and it is important in view of "cumulative" effects which it originates.

Dixon's extended bodies and impulsive gravitational waves

D. Bini[a,b], P. Fortini[c], A. Geralico[b], A. Ortolan[e]

[a]*Istituto per le Applicazioni del Calcolo "M. Picone", CNR I-00161 Rome, Italy*

[b]*International Center for Relativistic Astrophysics - I.C.R.A.*
University of Rome "La Sapienza", I-00185 Rome, Italy

[c]*Department of Physics, University of Ferrara and INFN Sezione di Ferrara, I-44100 Ferrara, Italy*

[d]*Physics Department, University of Rome "La Sapienza", I-00185 Rome, Italy*

[e]*INFN - National Laboratories of Legnaro, I-35020 Legnaro (PD), Italy*

Abstract

The "reaction" of an extended body (spinning and endowed with a quadrupolar structure) to the passage of an exact plane gravitational wave is discussed following Dixon's model under the condition that backreaction effects can be neglected. The analysis performed evidentiates several general features, e.g. even if initially absent, the body acquires a spin induced by the quadrupole structure, the center of mass moves from its initial position. Furthermore, special situations may exist in which certain spin components undergo "spin-flip" or "spin-glitch" effects which can be eventually observed.

“Glitches” observed in pulsars

A number of interesting results have been evidentiated. For instance, in general a) even if initially absent, the body acquires a dipolar moment induced by the quadrupole tensor; b) the center of mass moves from its initial position and the projection of the orbit on the wave front is a straight line, whose inclination depends on the initial spin of the body; c) special situations may occur in which certain spin components change their magnitude leading to effects (e.g. spin-flip) which can be eventually observed.

This interesting feature recalls the phenomenon of glitches observed in pulsars: a sudden increase in the rotation frequency, often accompanied by an increase in slow-down rate (see e.g. [11–13] and references therein). Currently, only multiple glitches of the Crab and Vela pulsars have been observed and studied extensively. Larger glitches in younger pulsars are usually followed by an exponential recovery or relaxation back toward the pre-glitch frequency, while for older pulsars and small glitches the jump tends to be permanent. The physical mechanism triggering glitches is not well understood yet, even if these are commonly thought to be caused by internal processes.

If one models a pulsar by a Dixon's extended body, then the present analysis shows that a sort of glitch can be generated by the passage of a strong gravitational wave, due to the pulsar quadrupole structure. In fact, from Eq. (27) we see that the profile of a polarization function can be suitably selected in order to fit observed glitches and in particular to describe the post-glitch behavior.

Work in progress

Structured body of astrophysical interest (pulsars) to be modeled by a similar analysis are still under consideration.

A constellation of satellites to be considered as a single extended body with quadrupolar structure is under consideration too, in view of possible applications of all this formalism to GW detectors like LISA.

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