

Gravitational energy as “dark energy”

Towards concordance cosmology without Λ

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DLW: **New J. Phys. 9 (2007) 377**

[gr-qc/0702082];

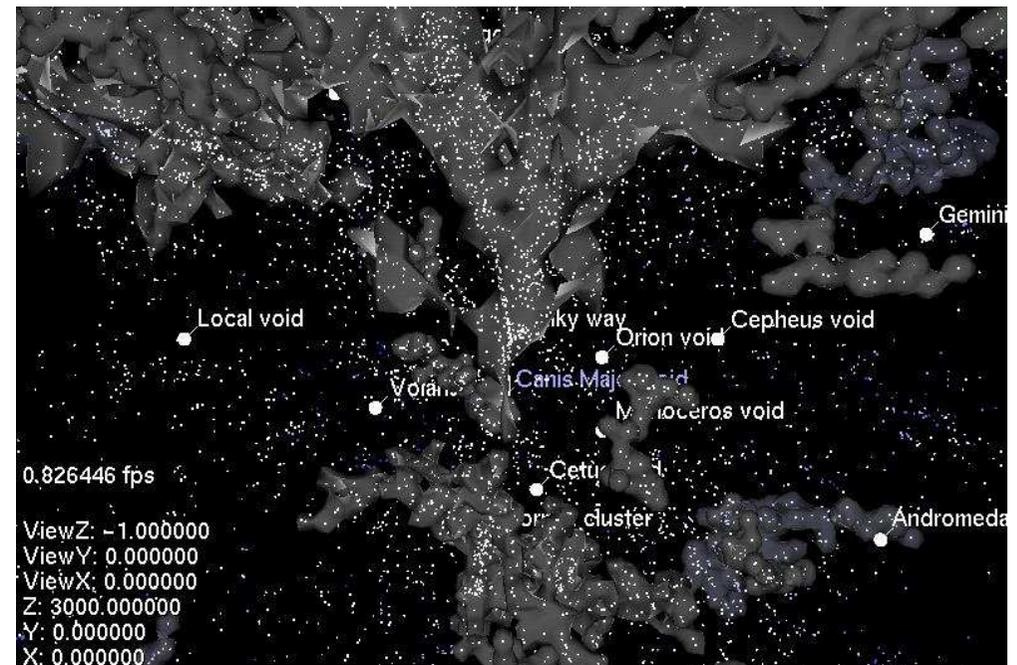
Phys. Rev. Lett. 99 (2007) 251101

[arxiv:0709.0732];

new results, to appear

B.M. Leith, S.C.C. Ng and DLW:

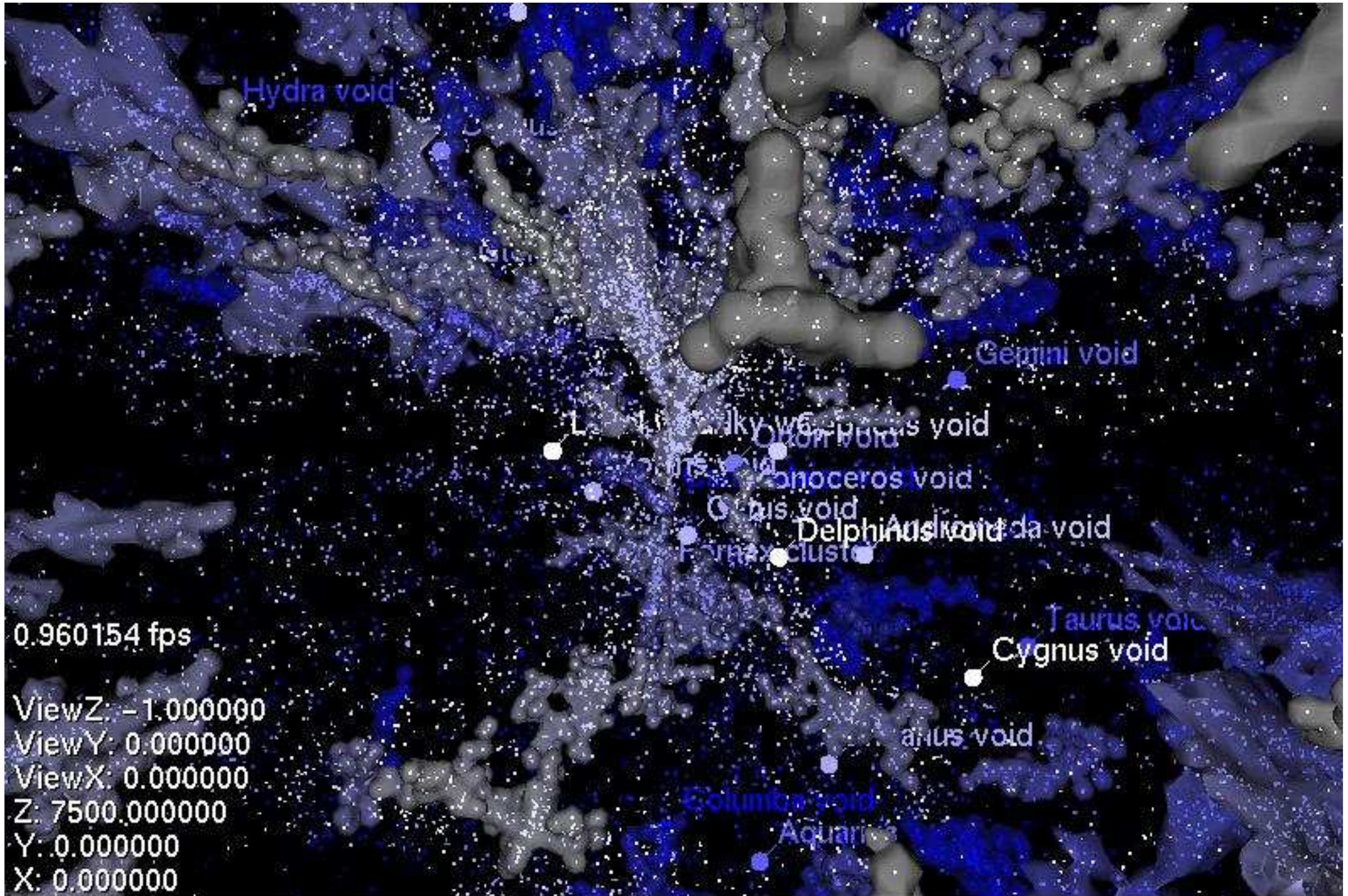
ApJ 672 (2008) L91 [arXiv:0709.2535]



What is “dark energy”?

- Usual explanation: a homogeneous isotropic form of “stuff” which violates the strong energy condition.
(Locally pressure $P = w\rho c^2$, $w < -\frac{1}{3}$;
e.g., for cosmological constant, Λ , $w = -1$.)
- New explanation: in ordinary general relativity, a manifestation of global *variations* of those aspects of gravitational energy which by virtue of the equivalence principle cannot be localised – the *cosmological quasilocal gravitational energy* associated with *dynamical gradients* in spatial curvature generated by a universe as inhomogeneous.
[Call this *dark energy* if you like. It involves *energy*, and “nothing” is dark.]

6df: voids & bubble walls (A. Fairall, UCT)



From smooth to lumpy

- Universe was very smooth at time of last scattering; fluctuations in the fluid were tiny ($\delta\rho/\rho \sim 10^{-5}$ in photons and baryons; $\sim 10^{-3}$ in non-baryonic dark matter).
- FLRW approximation very good early on.
- Universe is very lumpy or inhomogeneous today.
- Recent surveys estimate that 40–50% of the volume of the universe is contained in voids of diameter $30h^{-1}$ Mpc. [Hubble constant $H_0 = 100h \text{ km sec}^{-1} \text{ Mpc}^{-1}$] (Hoyle & Vogeley, ApJ 566 (2002) 641; 607 (2004) 751)
- Add some larger voids, and many smaller minivoids, and the universe is *void-dominated* at present epoch.
- Clusters of galaxies are strung in filaments and bubbles around these voids.

The Sandage-de Vaucouleurs paradox...

- Matter homogeneity only observed at $\gtrsim 200$ Mpc scales
- If “the coins on the balloon” are galaxies, their peculiar velocities should show great statistical scatter on scale much smaller than ~ 200 Mpc
- However, a nearly linear Hubble law flow begins at scales above 1.5–2 Mpc from barycentre of local group.
- Moreover, the local flow is statistically “quiet”.
- Can we explain this as an effect of dark energy? Maybe. Peculiar velocities are isotropized in FLRW universes which expand forever (regardless of dark energy).
- Empirical results do not appear to match best-fit Λ CDM parameters (Axenides & Perivolaropoulos, PRD 65 (2002) 127301).

Inhomogeneous cosmology

- Need an averaging scheme to extract the average homogeneous geometry
- Only exact approaches dealing with *averages* of full non-linear Einstein equations considered here (NOT perturbation theory: Kolb et al...; NOT LTB models etc)
- Still many approaches, with different assumptions
- Do we average tensors on curves of observers (Zalaletdinov 1992, 1993) ... recent work Coley, Pelavas, and Zalaletdinov, PRL 95 (2005) 151102; Coley and Pelavas, PR D75 (2007) 043506
- Can we get away with averaging scalars (density, pressure, shear ...)? (Buchert 2000, 2001) ... recent work Buchert CQG 23 (2006) 817; Astron. Astrophys. 454 (2006) 415; Gen. Rel. Grav. 40 (2008) 467 etc

Buchert's dust equations (2000)

For irrotational dust cosmologies, characterised by an energy density, $\rho(t, \mathbf{x})$, expansion, $\theta(t, \mathbf{x})$, and shear, $\sigma(t, \mathbf{x})$, on a compact domain, \mathcal{D} , of a suitably defined spatial hypersurface of constant average time, t , and spatial 3-metric, average cosmic evolution in Buchert's scheme is described by the exact equations

$$3 \frac{\dot{\bar{a}}^2}{\bar{a}^2} = 8\pi G \langle \rho \rangle - \frac{1}{2} \langle \mathcal{R} \rangle - \frac{1}{2} Q$$

$$3 \frac{\ddot{\bar{a}}}{\bar{a}} = -4\pi G \langle \rho \rangle + Q$$

$$\partial_t \langle \rho \rangle + 3 \frac{\dot{\bar{a}}}{\bar{a}} \langle \rho \rangle = 0$$

$$Q \equiv \frac{2}{3} (\langle \theta^2 \rangle - \langle \theta \rangle^2) - 2 \langle \sigma \rangle^2$$

Back-reaction

Angle brackets denote the spatial volume average, e.g.,

$$\langle \mathcal{R} \rangle \equiv \left(\int_{\mathcal{D}} d^3x \sqrt{\det {}^3g} \mathcal{R}(t, \mathbf{x}) \right) / \mathcal{V}(t)$$

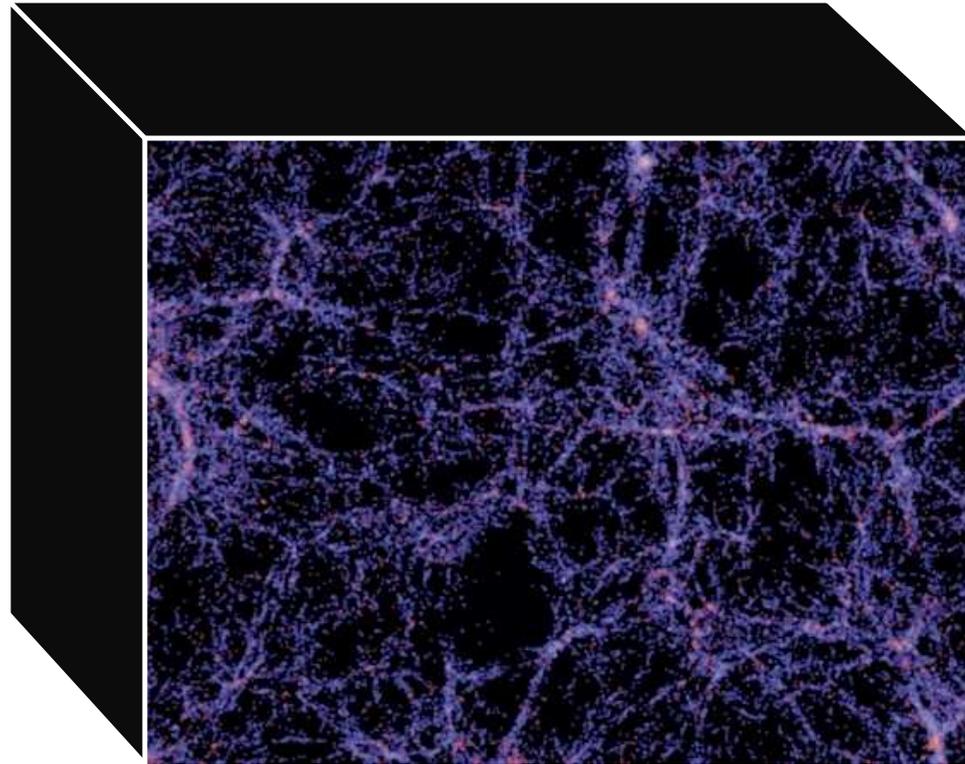
$$\langle \theta \rangle = 3 \frac{\dot{a}}{a}$$

Generally for any scalar Ψ ,

$$\frac{d}{dt} \langle \Psi \rangle - \left\langle \frac{d\Psi}{dt} \right\rangle = \langle \Psi \theta \rangle - \langle \theta \rangle \langle \Psi \rangle$$

- The extent to which the back-reaction, \mathcal{Q} , can lead to apparent cosmic acceleration or not has been the subject of much debate.

Within a statistically average cell



- Need to consider relative position of observers over scales of tens of Mpc over which $\delta\rho/\rho \sim 1$.
- GR is a local theory: gradients in spatial curvature and gravitational energy can lead to calibration differences between our rods and clocks and volume average ones

The Copernican principle

- Retain Copernican Principle - we are at an average position *for observers in a galaxy*
- Observers in bound systems are not at a volume average position in freely expanding space
- By Copernican principle other average observers should see an isotropic CMB
- BUT *nothing in theory, principle nor observation demands that such observers measure the same mean CMB temperature nor the same angular scales in the CMB anisotropies*
- Average mass environment (galaxy) can differ significantly from volume-average environment (void)

Dilemma of gravitational energy...

- In GR spacetime carries *energy & angular momentum*

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- On account of the strong equivalence principle, $T_{\mu\nu}$ contains localizable energy–momentum only
- Kinetic energy and energy associated with spatial curvature are in $G_{\mu\nu}$: variations are “quasilocal”!
- Newtonian version, $T - U = -V$, of Friedmann equation

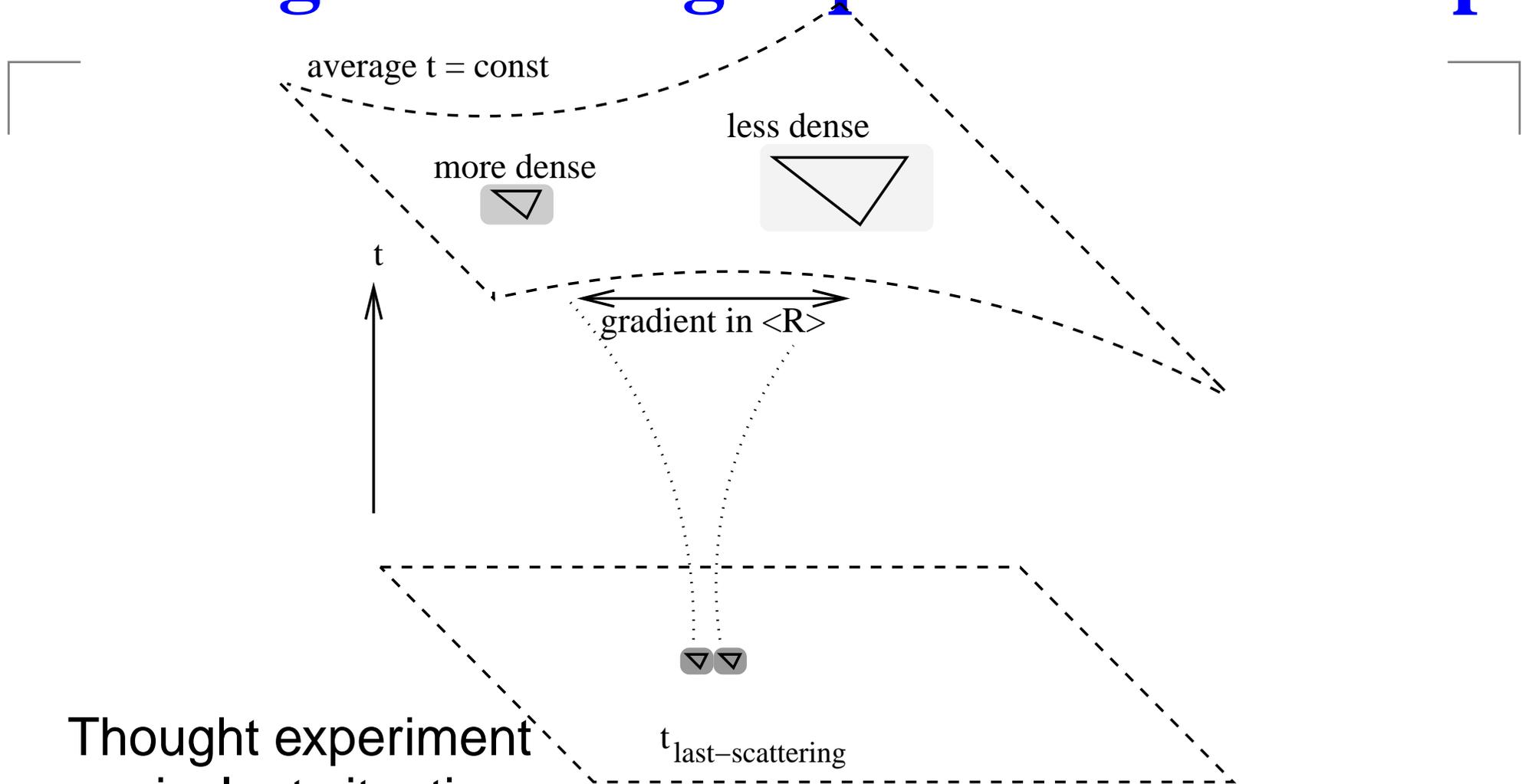
$$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G\rho}{3}$$

where $T = \frac{1}{2}m\dot{a}^2x^2$, $U = -\frac{1}{2}kmc^2x^2$, $V = -\frac{4}{3}\pi G\rho a^2x^2m$;
 $\mathbf{r} = a(t)\mathbf{x}$.

Ricci curvature and gravitational energy

- For Lemaître–Tolman–Bondi models constant spatial curvature replaced by energy function with $E(r) > 0$ in regions of negative spatial curvature.
- In quasilocal Hamiltonian approach of Chen, Nester and Liu (MPL A22 (2007) 2039) relative to a fiducial static Cartesian reference frame a comoving observer in $k = -1$ FLRW universe sees negative quasilocal energy; or relative to the static frame the comoving observer has positive quasilocal energy.
- For perturbation theory I advocate “Machian gauge” of Bičak, Katz and Lynden–Bell (PR D76 (2007) 063501): uniform Hubble flow plus minimal shift distortion condition.

Cosmological Strong Equivalence Principle



Thought experiment
equivalent situations:

- SR: observers isotropically decelerate at different rates
- GR: regions of different density have different volume deceleration (for same initial conditions)

Cosmological Strong Equivalence Principle

- Even within pressureless dust there exist suitable small frames such that conformal volume expanding motions are locally indistinguishable from the equivalent uniform motion of particles in a static Minkowski space.
- Identify cosmic “rest frame” as the union of frames differing by an integrated “relative acceleration” of such pseudo-Minkowski space observers from same initial conditions, with same “local expansion” once cumulative “relative acceleration” is accounted for.
- Preserves isotropy of mean CMB temperature
- Implicitly solves the Sandage–de Vaucouleurs paradox.
- Voids appear to expand faster; but their local clocks tick faster, locally measured expansion can still be uniform.

Average isotropic observer rest frames

- Define them by average expansion over different regions being homogeneous, i.e.,

$$\left\langle \frac{1}{\ell_r(\tau)} \frac{d\ell_r(\tau)}{d\tau} \right\rangle = \frac{1}{3} \langle \theta \rangle_1 = \frac{1}{3} \langle \theta \rangle_2 = \dots = \bar{H}(\tau)$$

- Average over regions in which (i) spatial curvature, shear and vorticity fluctuations average out; (ii) space is expanding at the boundaries, at least marginally.
- IMPORTANT POINT: \bar{H} is the “locally” measured Hubble parameter, NOT the global average H_{av} with respect to any one set of clocks, such as τ_w .
- \bar{H} is uniform whereas proper lengths $\ell_r(\tau_i)$ and proper time τ_i can be region dependent

Bound and unbound systems...

- Isotropic observers “at rest” within expanding space in voids may have clocks ticking at a rate $d\tau_v = \gamma(\tau_w, \mathbf{x})d\tau_w$ with respect to static observers in bound systems.
Volume average: $dt = \bar{\gamma}_w d\tau_w$, $\bar{\gamma}_w(\tau_w) = \langle -\xi^\mu n_\mu \rangle_{\mathcal{H}}$
- We are not restricted to $\gamma = 1 + \epsilon$, $\epsilon \ll 1$, as expected for typical variations of binding energy.
- Observable universe is assumed unbound.
- With no dark energy I find $\gamma < \frac{3}{2} = H_{\text{Milne}}/H_{\text{Einstein-de Sitter}}$.
- Where is infinity? In 1984 George Ellis suggested a notion of *finite infinity*: a region within which isolated systems, such as stars or galaxies, or galaxy clusters can be considered as approximately independent dynamical systems.

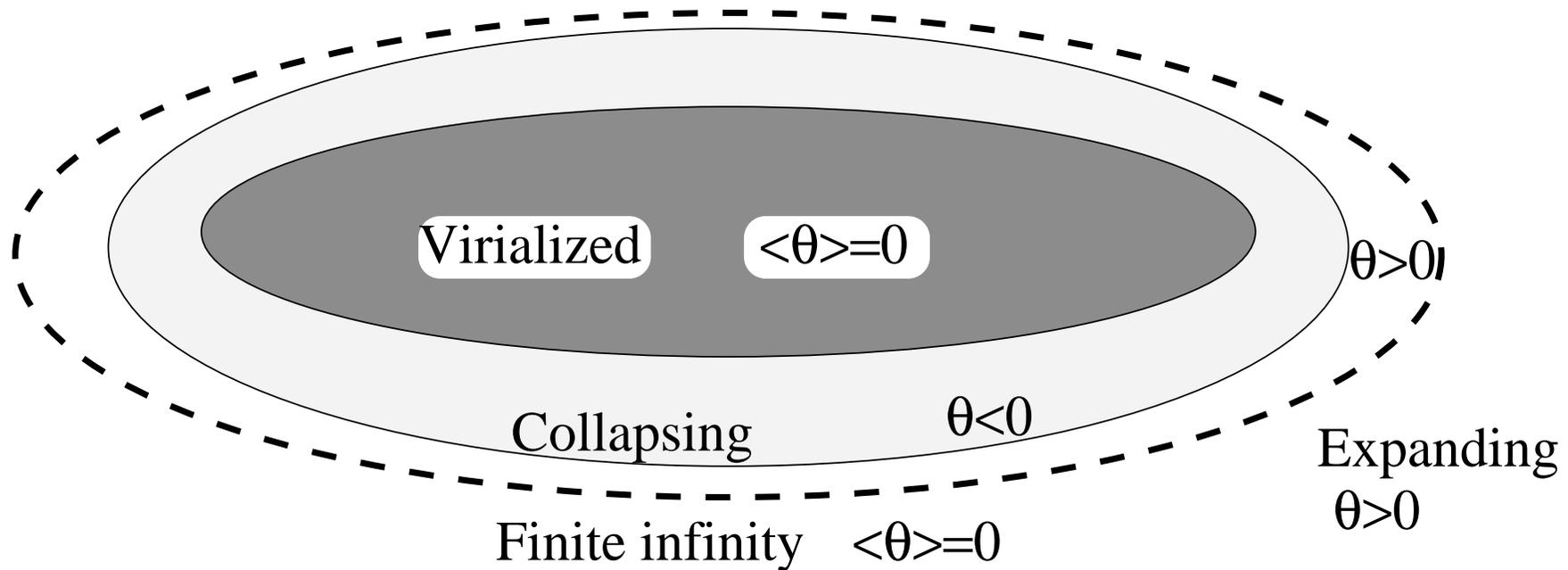
Where is infinity?

- Inflation provides us with boundary conditions.
- Initial smoothness at last-scattering ensures a uniform initial expansion rate. For gravity to overcome this a universal critical density exists.
BUT if we assume a smooth average evolution we can overestimate the critical density today.

$$\rho_{\text{cr}} \neq \frac{3H_{\text{av}}^2}{8\pi G}$$

- Identify finite infinity relative to demarcation between bound and unbound systems, depending on the time evolution of the true critical density since last-scattering.
- Normalise *wall time*, τ_w , as the time at finite infinity, (close to galaxy clocks) by $\langle -\xi^\mu n_\mu \rangle_{\mathcal{F}_I} = \langle \gamma(\tau_w, \mathbf{x}) \rangle_{\mathcal{F}_I} = 1$.

Finite infinity



- Define *finite infinity*, “*fi*” as boundary to minimal *connected* region within which *average expansion* vanishes $\langle \theta \rangle = 0$ or *average curvature* vanishes $\langle R \rangle = 0$.
- Shape of *fi* boundary irrelevant (minimal surface generally): could typically contain a galaxy cluster.

Two/three scale model

$$\bar{a}^3 = f_{wi} a_w^3 + f_{vi} a_v^3$$

- Splits into void fraction with scale factor a_v and “wall” fraction with scalar factor a_w . Assume $3\delta^2 H_w = \langle \sigma^2 \rangle_w$, $3\delta^2 H_v = \langle \sigma^2 \rangle_v$.
- Buchert equations for volume averaged observer, with $f_v(t) = f_{vi} a_v^3 / \bar{a}^3$ (void volume fraction) and $k_v < 0$

$$\frac{\dot{\bar{a}}^2}{\bar{a}^2} + \frac{\dot{f}_v^2}{9f_v(1-f_v)} - \frac{\alpha^2 f_v^{1/3}}{\bar{a}^2} = \frac{8\pi G}{3} \bar{\rho}_0 \frac{\bar{a}_0^3}{\bar{a}^3},$$

$$\ddot{f}_v + \frac{\dot{f}_v^2(2f_v - 1)}{2f_v(1-f_v)} + 3\frac{\dot{\bar{a}}}{\bar{a}}\dot{f}_v - \frac{3\alpha^2 f_v^{1/3}(1-f_v)}{2\bar{a}^2} = 0,$$

if $f_v(t) \neq \text{const}$; where $\alpha^2 = -k_v f_{vi}^{2/3}$.

Two/three scale model

- Universe starts as Einstein–de Sitter, from boundary conditions at last scattering consistent with CMB; almost no difference in clock rates initially.
- We must be careful to account for clock rate variations. Buchert's clocks are set at the *volume average* position, with a rate between wall clocks and void clock extreme.

$$\bar{H}(t) = \bar{\gamma}_w H_w = \bar{\gamma}_v H_v; \quad H_w \equiv \frac{1}{a_w} \frac{da_w}{dt}, \quad H_v \equiv \frac{1}{a_v} \frac{da_v}{dt}$$

where $\bar{\gamma}_v = \frac{dt}{d\tau_v}$, $\bar{\gamma}_w = \frac{dt}{d\tau_w} = 1 + (1 - h_r) f_v / h_r$,
 $h_r = H_w / H_v < 1$.

- Need to be careful to obtain global H_{av} in terms of one set of isotropic observer wall clocks, τ_w .

Bare cosmological parameters

- Different sets of cosmological parameters are possible
- Bare cosmological parameters are defined as fractions of the true critical density related to the bare Hubble rate

$$\bar{\Omega}_M = \frac{8\pi G \bar{\rho}_{M0} \bar{a}_0^3}{3\bar{H}^2 \bar{a}^3},$$
$$\bar{\Omega}_k = \frac{\alpha^2 f_v^{1/3}}{\bar{a}^2 \bar{H}^2},$$
$$\bar{\Omega}_Q = \frac{-\dot{f}_v^2}{9f_v(1-f_v)\bar{H}^2}.$$

- These are the volume–average parameters, with first Buchert equation: $\bar{\Omega}_M + \bar{\Omega}_k + \bar{\Omega}_Q = 1$.

Dressed cosmological parameters

- Conventional parameters for “wall observers” in galaxies: defined by assumption (no longer true) that others in entire observable universe have synchronous clocks and same local spatial curvature

$$\begin{aligned} ds_{\mathcal{F}_I}^2 &= -d\tau_w^2 + a_w^2(\tau_w) [d\eta_w^2 + \eta_w^2 d\Omega^2] \\ &= -d\tau_w^2 + \frac{\bar{a}^2}{\bar{\gamma}_w^2} [d\bar{\eta}^2 + r_w^2(\bar{\eta}, \tau_w) d\Omega^2] \end{aligned}$$

where $r_w \equiv \bar{\gamma}_w (1 - f_v)^{1/3} f_{wi}^{-1/3} \eta_w(\bar{\eta}, \tau_w)$, and volume-average conformal time $d\bar{\eta} = dt/\bar{a} = \bar{\gamma}_w d\tau_w/\bar{a}$.

- This leads to conventional dressed parameters *which do not sum to 1*, e.g.,

$$\Omega_M = \bar{\gamma}_w^3 \bar{\Omega}_M .$$

Tracker solution PRL 99, 251101

- General exact solution possesses a “tracker limit”

$$\bar{a} = \frac{\bar{a}_0 (3\bar{H}_0 t)^{2/3}}{2 + f_{v0}} \left[3f_{v0}\bar{H}_0 t + (1 - f_{v0})(2 + f_{v0}) \right]^{1/3}$$

$$f_v = \frac{3f_{v0}\bar{H}_0 t}{3f_{v0}\bar{H}_0 t + (1 - f_{v0})(2 + f_{v0})},$$

- Void fraction $f_v(t)$ determines many parameters:

$$\bar{\gamma}_w = 1 + \frac{1}{2}f_v = \frac{3}{2}\bar{H}t$$

$$\tau_w = \frac{2}{3}t + \frac{2(1 - f_{v0})(2 + f_{v0})}{27f_{v0}\bar{H}_0} \ln \left(1 + \frac{9f_{v0}\bar{H}_0 t}{2(1 - f_{v0})(2 + f_{v0})} \right)$$

$$\bar{\Omega}_M = \frac{4(1 - f_v)}{(2 + f_v)^2}$$

Apparent cosmic acceleration

- Volume average observer sees no apparent cosmic acceleration

$$\bar{q} = \frac{2(1 - f_v)^2}{(2 + f_v)^2}.$$

As $t \rightarrow \infty$, $f_v \rightarrow 1$ and $\bar{q} \rightarrow 0^+$.

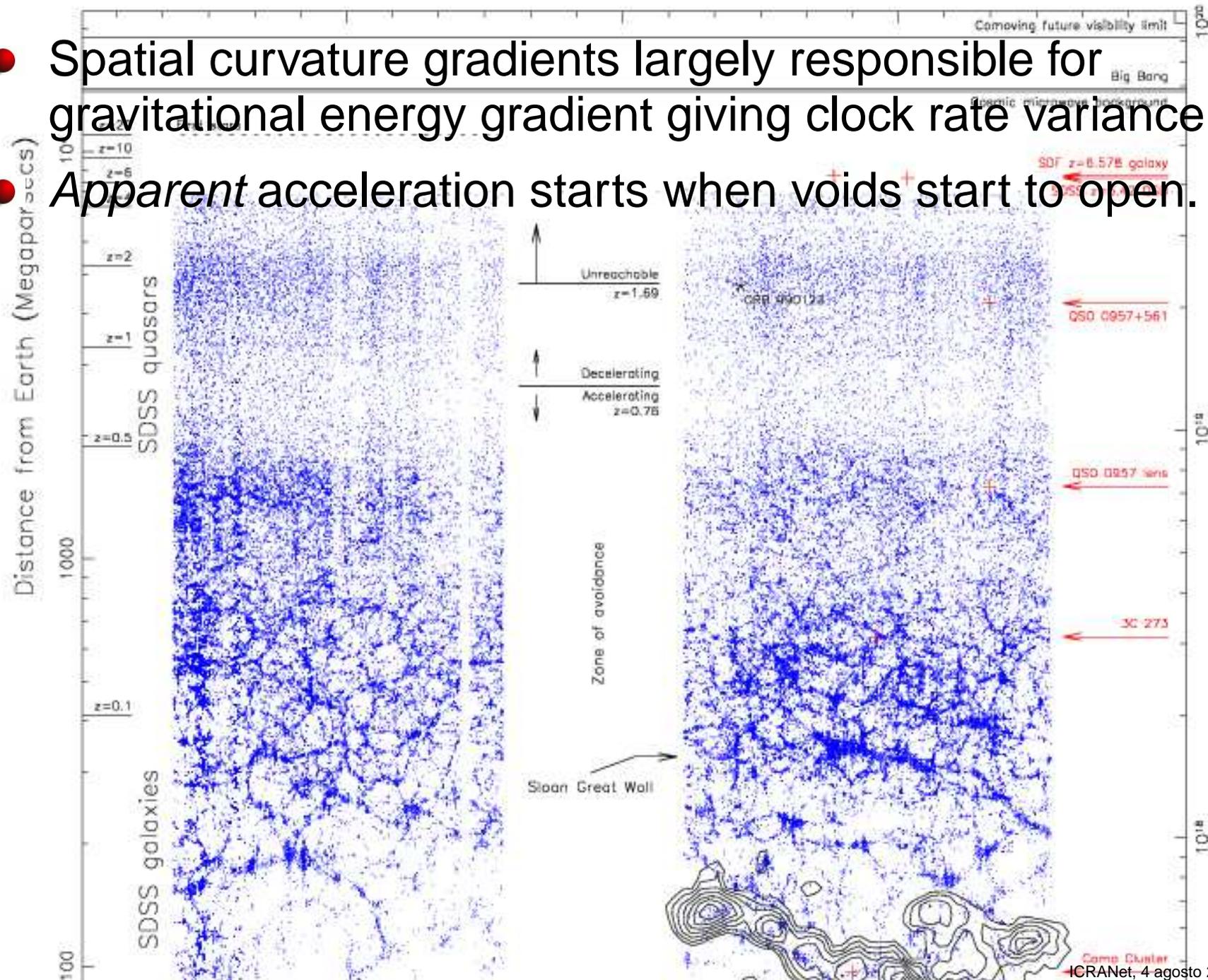
- A wall observer registers apparent cosmic acceleration

$$q = \frac{-(1 - f_v)(8f_v^3 + 39f_v^2 - 12f_v - 8)}{(4 + f_v + 4f_v^2)^2},$$

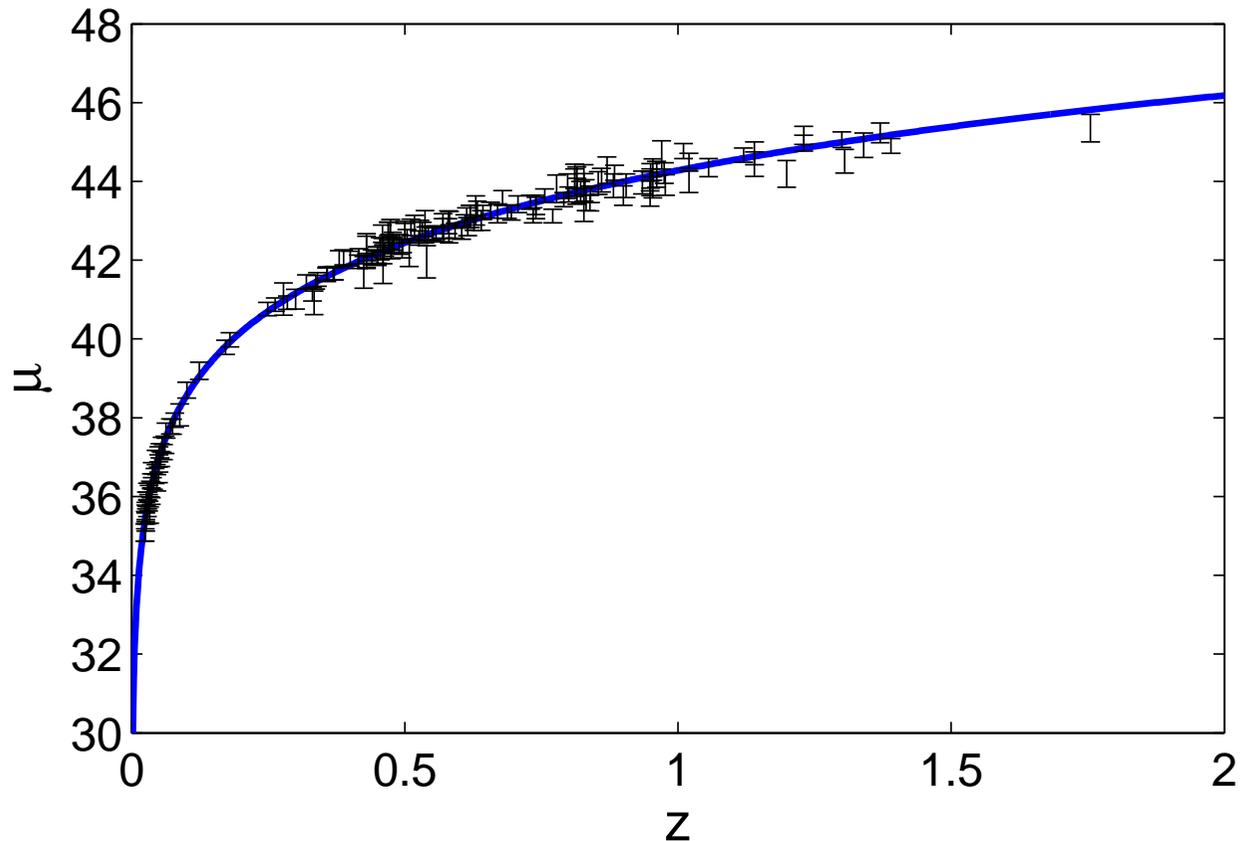
Effective deceleration parameter starts at $q \sim \frac{1}{2}$, for small f_v ; changes sign when $f_v = 0.58670773\dots$, and approaches $q \rightarrow 0^-$ at late times.

Cosmic coincidence problem solved

- Spatial curvature gradients largely responsible for gravitational energy gradient giving clock rate variance.
- *Apparent* acceleration starts when voids start to open.

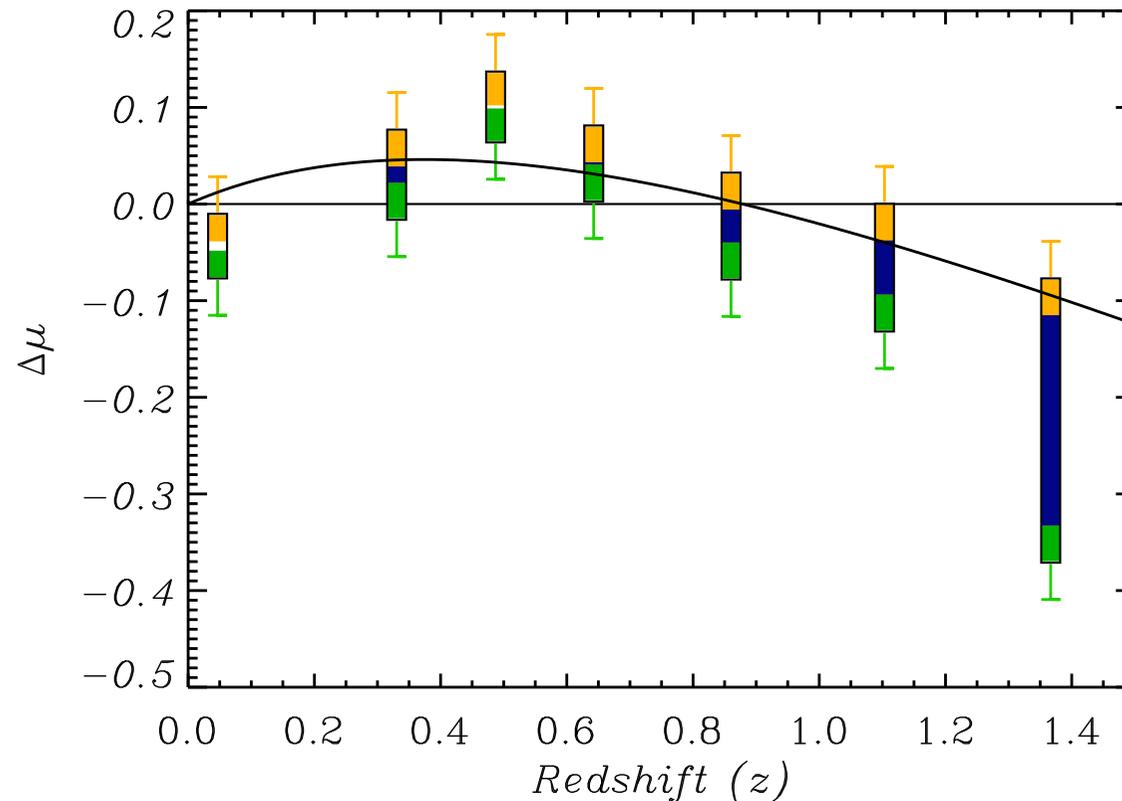


Test 1: Snela luminosity distances



- Type Ia supernovae of Riess07 Gold data set fit with χ^2 per degree of freedom = 0.9
- With $55 \leq H_0 \leq 75 \text{ km sec}^{-1} \text{ Mpc}^{-1}$, $0.01 \leq \Omega_{M0} \leq 0.5$, find Bayes factor $\ln B = 0.27$ in favour of FB model (marginally): statistically indistinguishable from Λ CDM.

Test 1: Snela luminosity distances



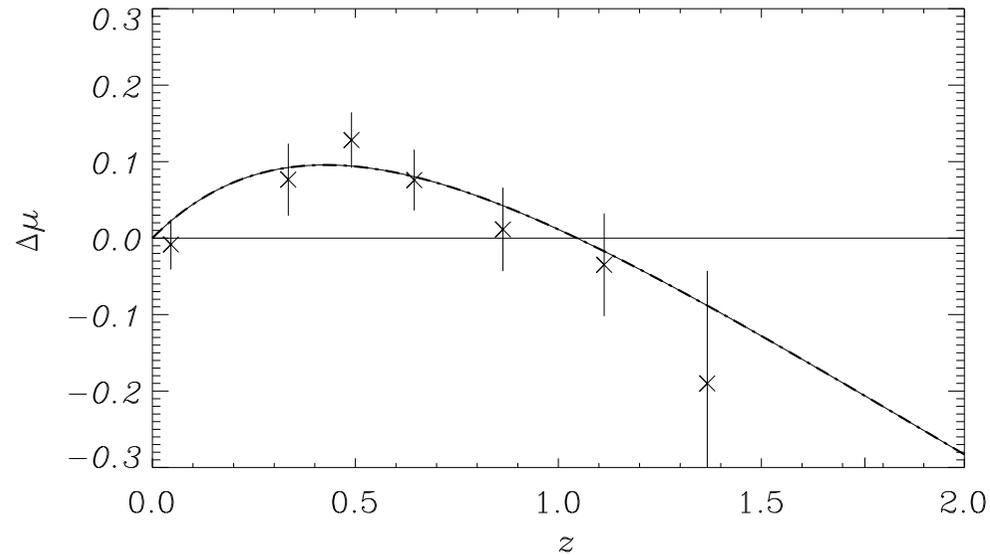
- Plot shows difference of model apparent magnitude and that of an empty Milne universe of same Hubble constant $H_0 = 61.73 \text{ km sec}^{-1} \text{ Mpc}^{-1}$. Note: residual depends on the expansion rate of the Milne universe subtracted (2σ limits on H_0 indicated by whiskers)

Comparison Λ CDM models

Best-fit spatially flat Λ CDM

$$H_0 = 62.7 \text{ km sec}^{-1} \text{ Mpc}^{-1},$$

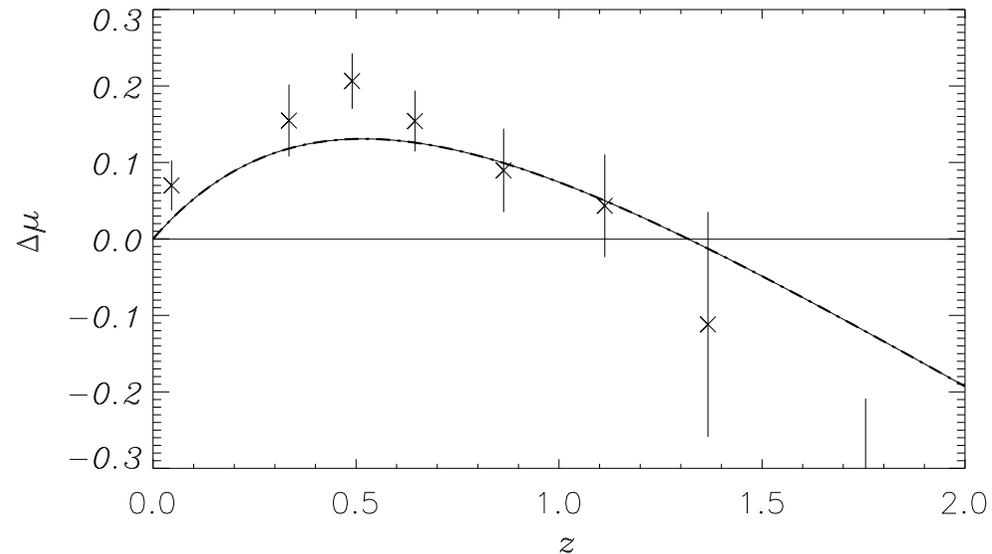
$$\Omega_{M0} = 0.34, \Omega_{\Lambda0} = 0.66$$



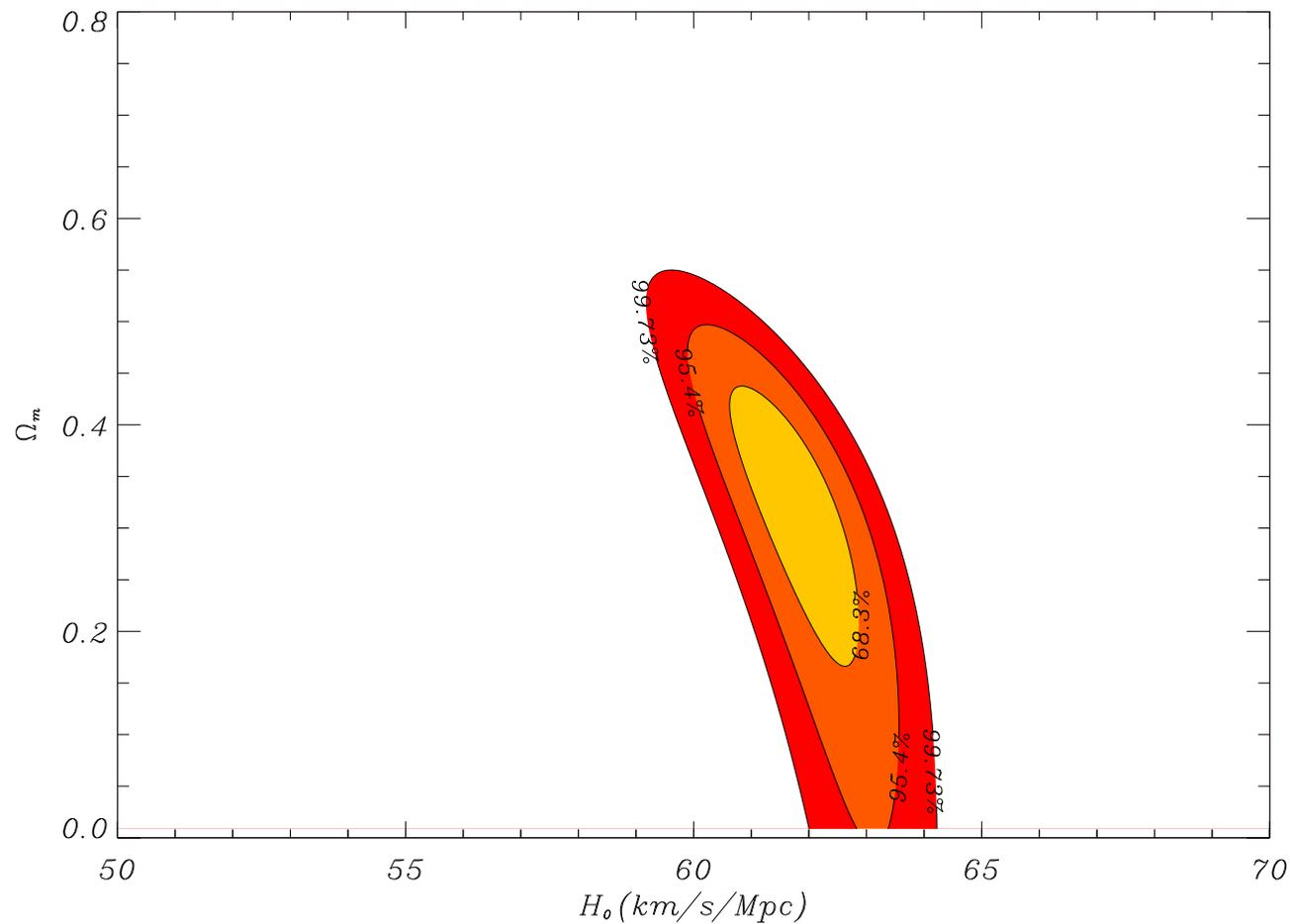
Riess astro-ph/0611572, p. 63

$$H_0 = 65 \text{ km sec}^{-1} \text{ Mpc}^{-1},$$

$$\Omega_{M0} = 0.29, \Omega_{\Lambda0} = 0.71$$

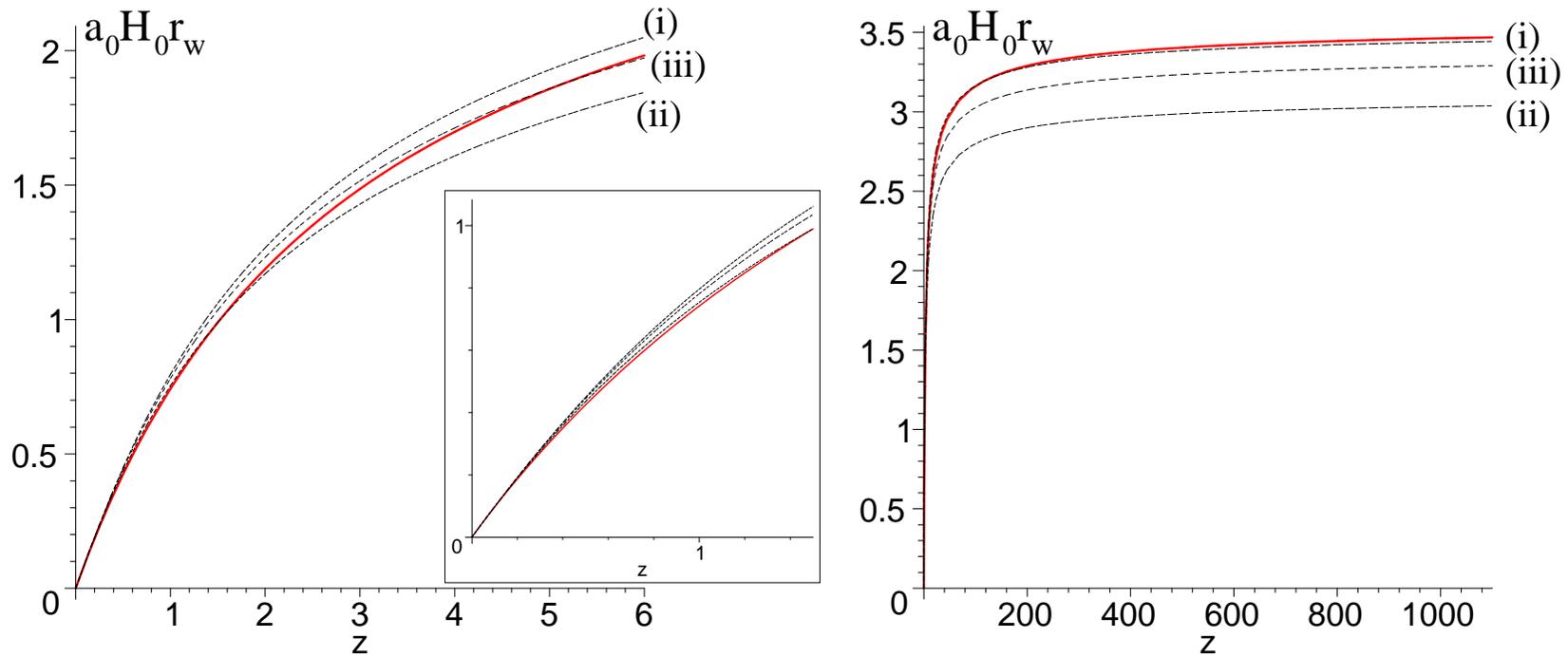


Test 1: Snela luminosity distances



Best-fit H_0 agrees with HST key team, Sandage et al.,
 $H_0 = 62.3 \pm 1.3$ (stat) ± 5.0 (syst) km sec⁻¹ Mpc⁻¹ [ApJ 653
(2006) 843].

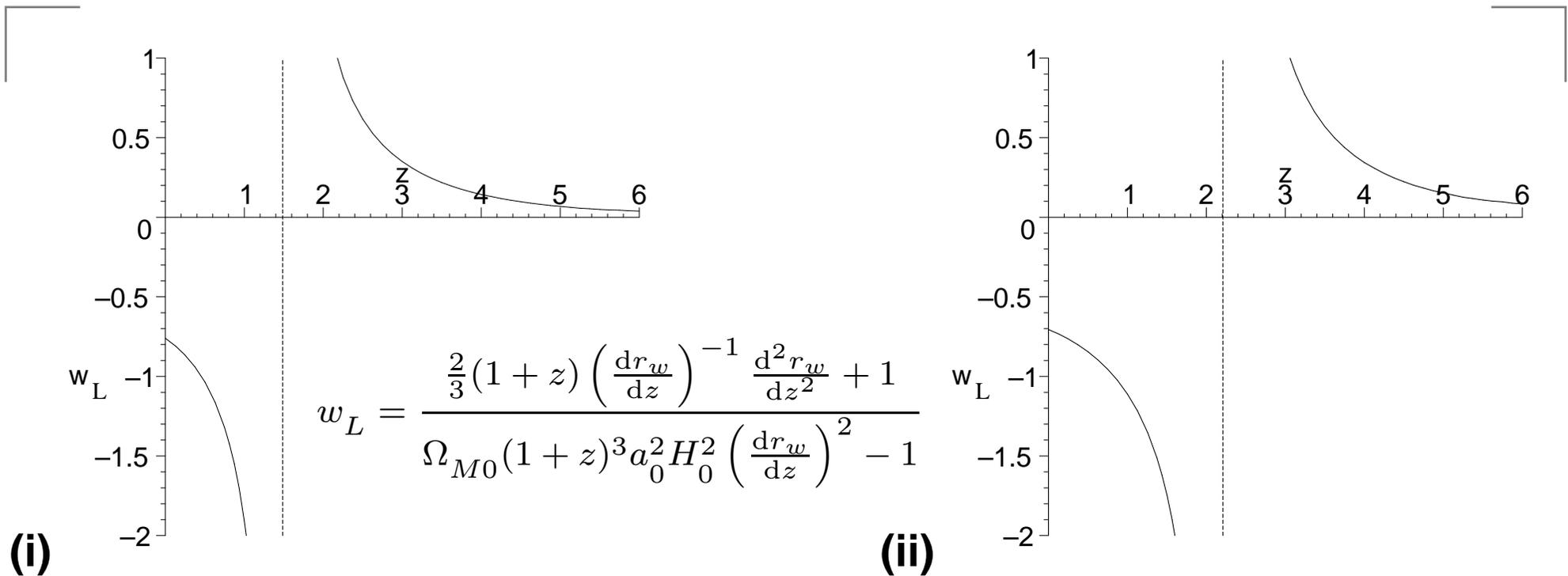
Dressed “comoving distance” $r_w(z)$



Best-fit FB model (**red line**) compared to 3 spatially flat Λ CDM models: **(i)** best-fit to WMAP5 only ($\Omega_\Lambda = 0.751$); **(ii)** best-fit to (Riess07) Snela only ($\Omega_\Lambda = 0.66$); **(iii)** joint WMAP5 + BAO + Snela fit ($\Omega_\Lambda = 0.721$)

- FB model closest to best-fit Λ CDM to *Snela only* result ($\Omega_{M0} = 0.34$) at *low redshift*, and to *WMAP5 only* result ($\Omega_{M0} = 0.249$) at *high redshift*

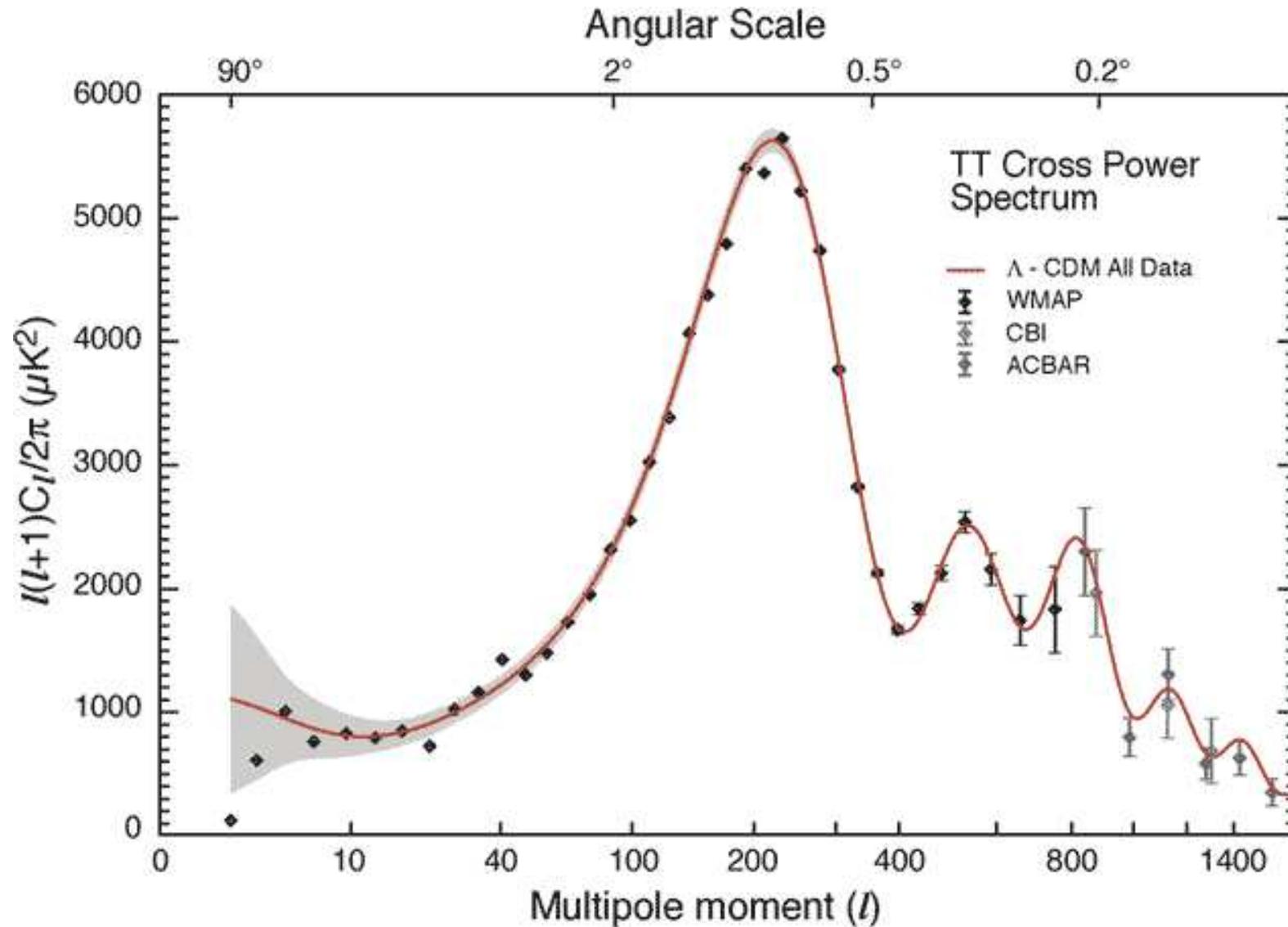
Equivalent “equation of state”?



A formal “dark energy equation of state” $w_L(z)$ for the best-fit FB model, $f_{v0} = 0.76$, calculated directly from $r_w(z)$: **(i)** $\Omega_{M0} = 0.33$; **(ii)** $\Omega_{M0} = 0.279$.

- Description by a “dark energy equation of state” makes no sense when there is no physics behind it; but average value $w_L \simeq -1$ for $z < 0.7$ makes empirical sense.

Test 2: Angular scale of CMB Doppler peaks

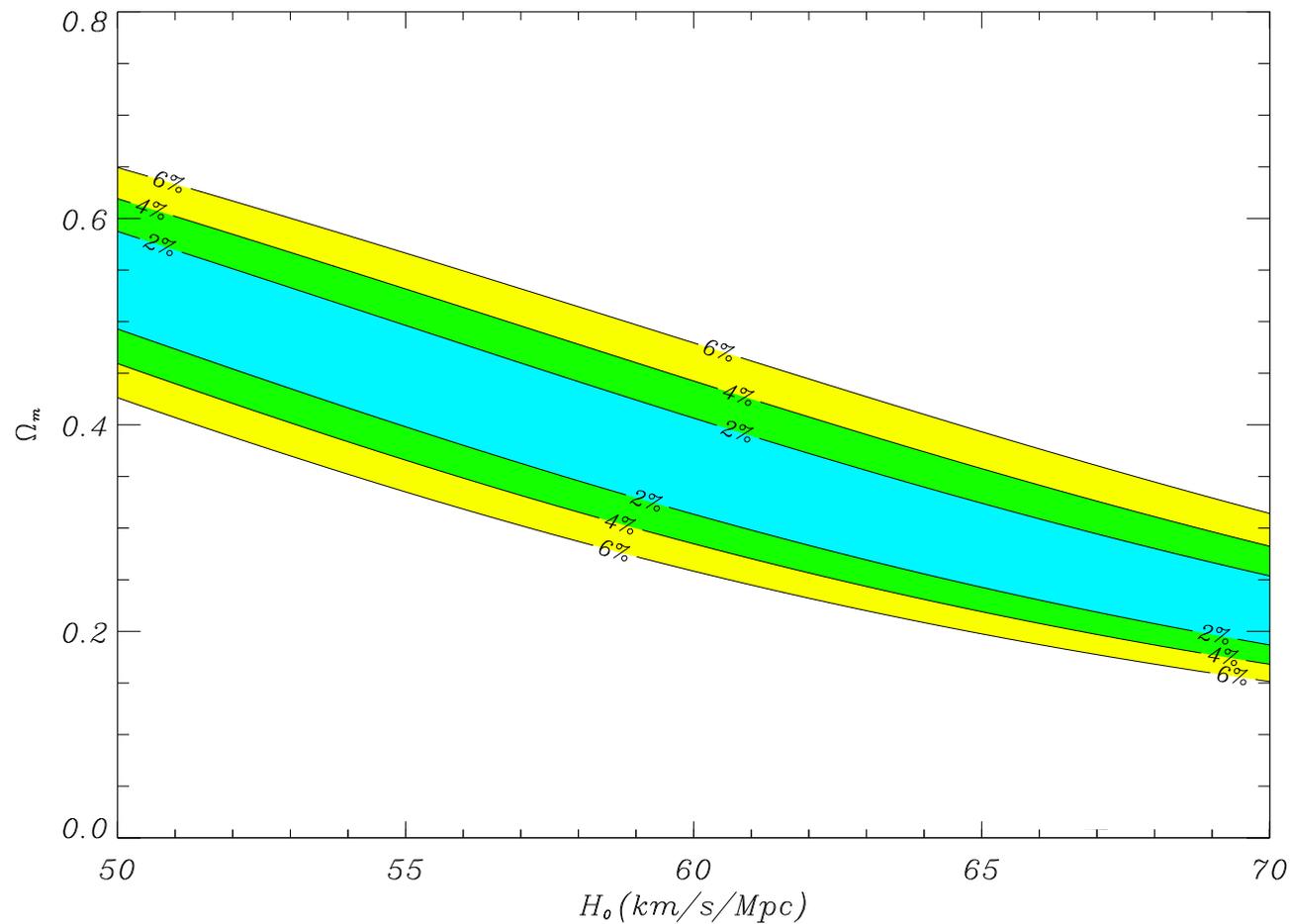


Power in CMB temperature anisotropies versus angular size of fluctuation on sky

Test 2: Angular scale of CMB Doppler peaks

- Angular scale is related to spatial curvature of FLRW models
- Relies on the simplifying assumption that spatial curvature is same everywhere
- In new approach spatial curvature is not the same everywhere
- Volume–average observer measures lower mean CMB temperature ($\bar{T}_0 \sim 1.98$ K, c.f. $T_0 \sim 2.73$ K in walls) and a smaller angular anisotropy scale
- Relative focussing between voids and walls
- Integrated Sachs–Wolfe effect needs recomputation
- Here just calculate angular–diameter distance of sound horizon

Test 2: Angular scale of CMB Doppler peaks

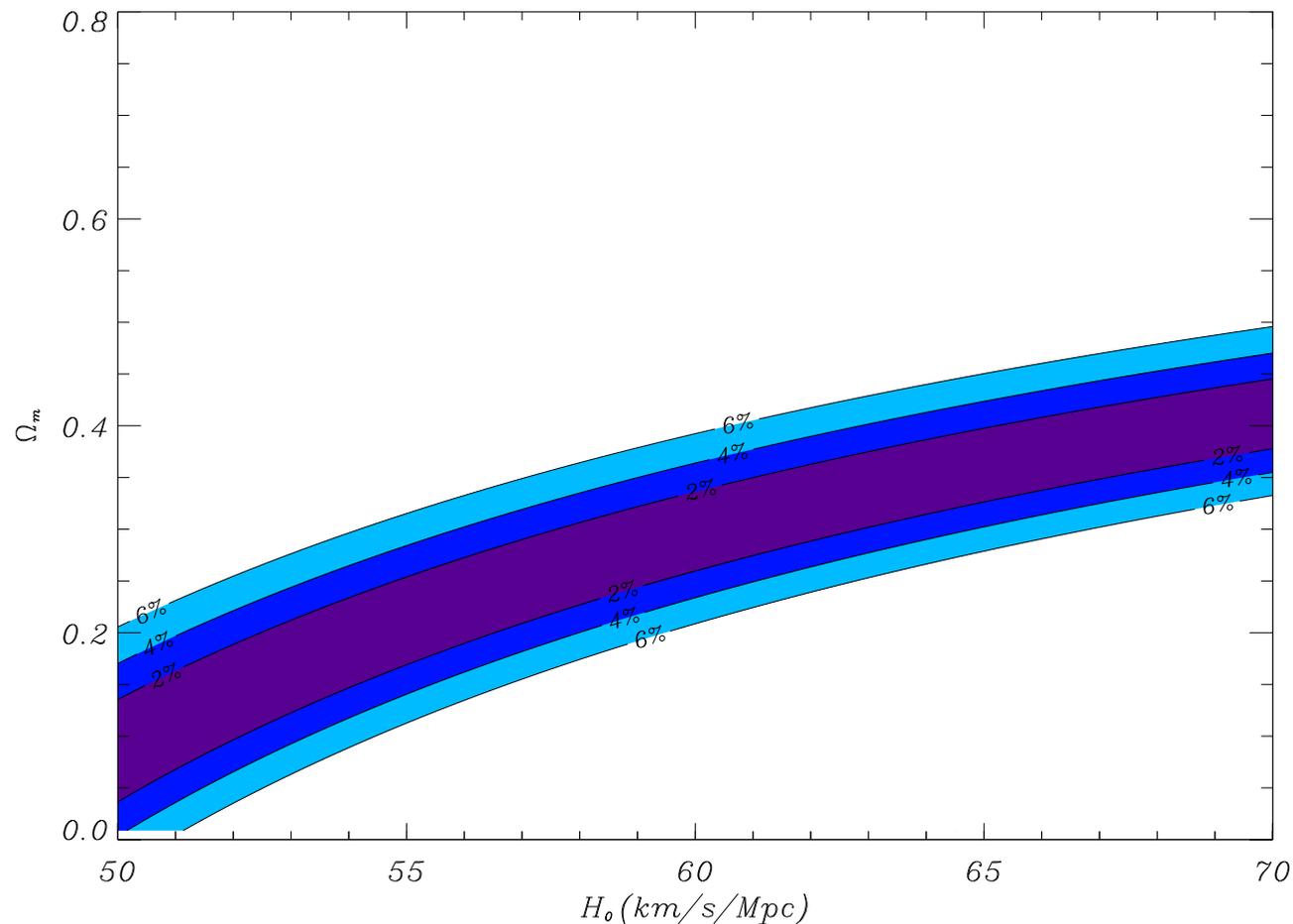


Parameters within the (Ω_m, H_0) plane which fit the angular scale of the sound horizon $\delta = 0.01$ rad deduced for WMAP, to within 2%, 4% and 6%.

Test 3: Baryon acoustic oscillation scale

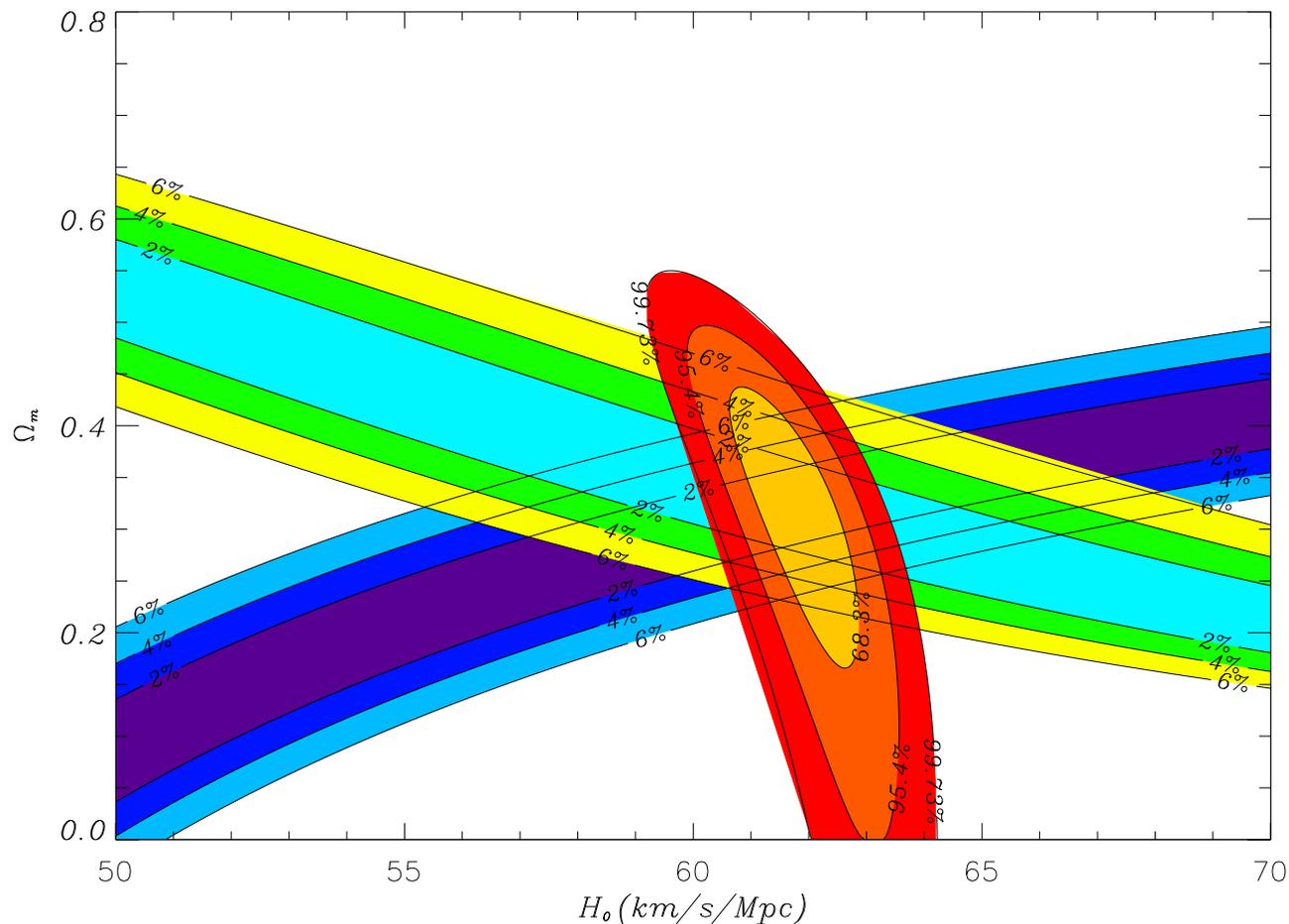
- In 2005 Cole et al. (2dF), and Eisenstein et al. (SDSS) detected the signature of the comoving baryon acoustic oscillation in galaxy clustering statistics
- Powerful independent probe of “dark energy”
- Here the effective dressed geometry should give an equivalent scale

Test 3: Baryon acoustic oscillation scale



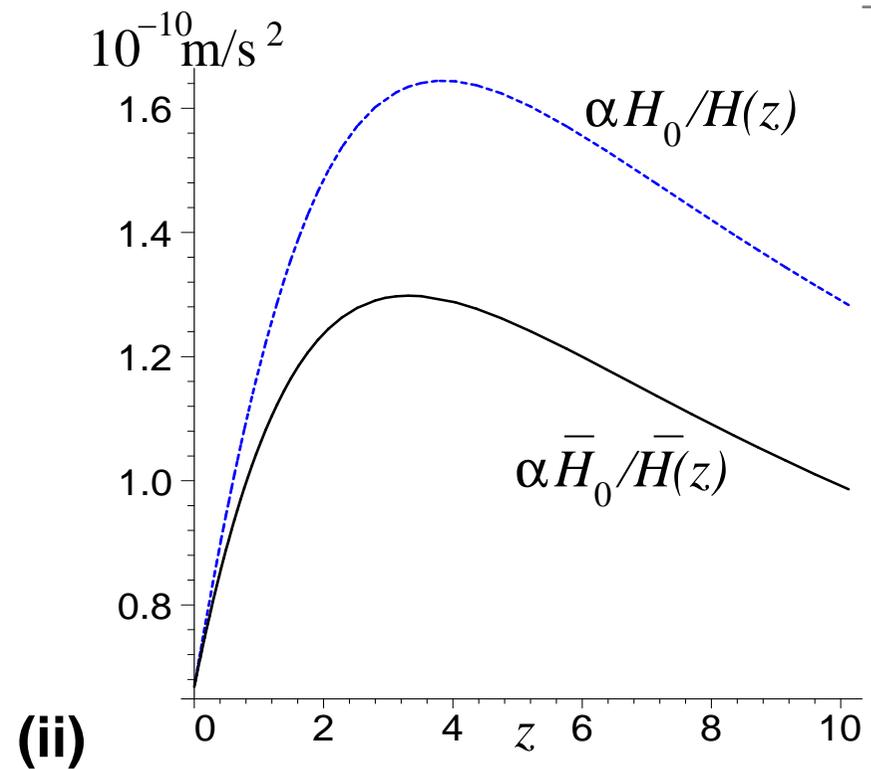
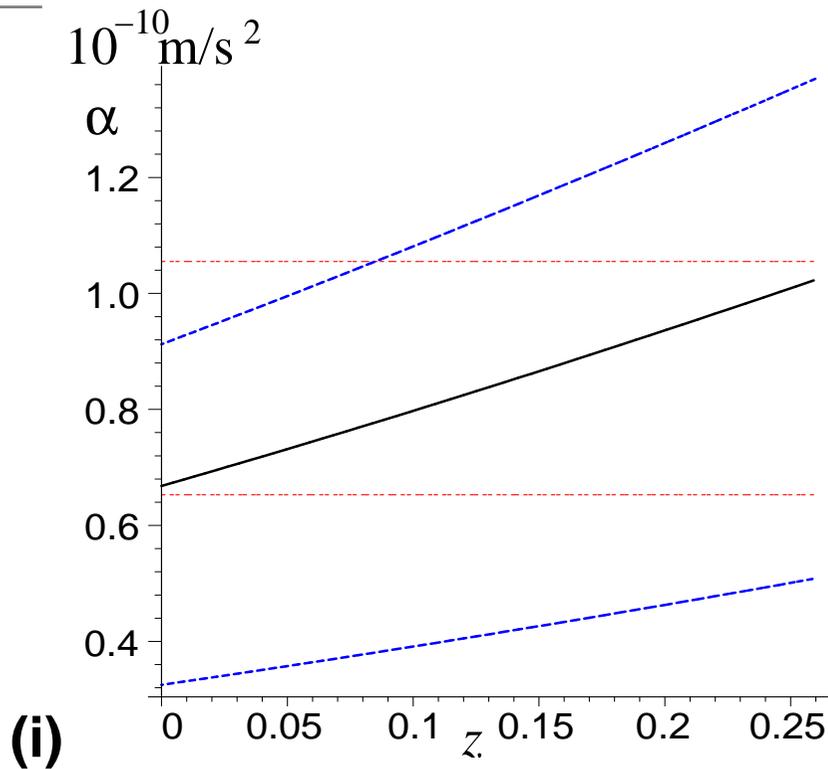
Parameters within the (Ω_m, H_0) plane which fit the effective comoving baryon acoustic oscillation scale of $104h^{-1}$ Mpc, as seen in 2dF and SDSS.

Agreement of independent tests



Best-fit parameters: $H_0 = 61.7^{+1.2}_{-1.1} \text{ km sec}^{-1} \text{ Mpc}^{-1}$,
 $\Omega_m = 0.33^{+0.11}_{-0.16}$ (1σ errors for Snela only) [Leith, Ng & Wiltshire, ApJ 672 (2008) L91]

SEP relative acceleration scale



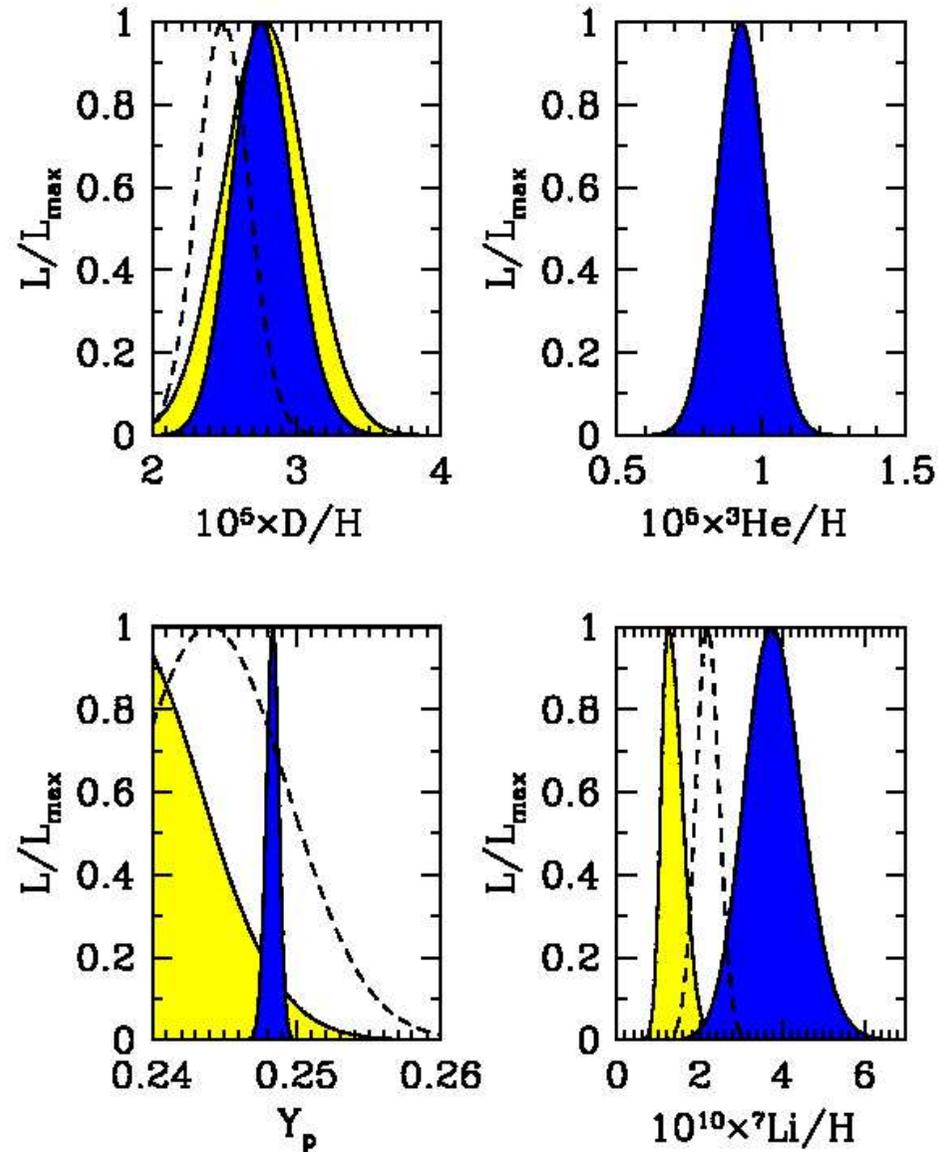
By equivalence principle the instantaneous relative deceleration of backgrounds gives an instantaneous 4-acceleration of magnitude $\alpha = H_0 c \bar{\gamma} \dot{\bar{\gamma}} / (\sqrt{\bar{\gamma}^2 - 1})$ beyond which *weak field cosmological general relativity* will be changed from Newtonian expectations: **(i)** as absolute scale nearby; **(ii)** scaled for Hubble parameter to large z .

● Coincides with empirical MOND scale

$$\alpha_0 = 1.2_{-0.2}^{+0.3} \times 10^{-10} \text{ ms}^{-2} h_{75}^2 = 8.1_{-1.6}^{+2.5} \times 10^{-11} \text{ ms}^{-2} \text{ for } h_0 = 61.7 \text{ km sec}^{-1} \text{ Mpc}^{-1}.$$

Li abundance anomaly

Big-bang nucleosynthesis, light element abundances and WMAP with Λ CDM cosmology.

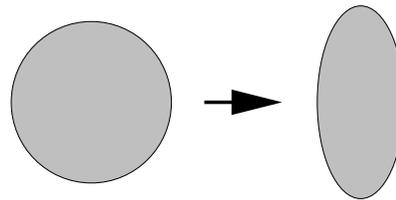


Resolution of Li abundance anomaly?

- Tests 2 & 3 shown earlier use the baryon-to-photon ratio $\eta_{B\gamma} = 4.6\text{--}5.6 \times 10^{-10}$ admitting concordance with lithium abundances favoured prior to WMAP in 2003
- Conventional dressed parameter $\Omega_{M0} = 0.33$ for wall observer means $\bar{\Omega}_{M0} = 0.127$ for the volume-average.
- Conventional theory predicts the *volume-average baryon fraction*. With old BBN favoured $\eta_{B\gamma}$:
 $\bar{\Omega}_{B0} \simeq 0.027\text{--}0.033$; but this translates to a conventional dressed baryon fraction parameter $\Omega_{B0} \simeq 0.072\text{--}0.088$
- The mass ratio of baryonic matter to non-baryonic dark matter is increased to 1:3
- Enough baryon drag to fit peak heights ratio

Spatial curvature: ellipticity anomaly

- Negative spatial curvature should manifest itself in other ways than angular–diameter distance of sound horizon
- Indeed it does: greater geodesic mixing from negative spatial curvature registers ellipticity in the CMB anisotropy spectrum



- Ellipticity has been detected since COBE, and statistical significance increases with each data release (Gurzadyan et al., Phys. Lett. A **363** (2007) 121; Mod. Phys. Lett. A **20** (2005) 813, . . .)
- For FLRW models this is an anomaly; here it is expected; but still needs quantitative analysis

Alleviation of age problem

- Old structures seen at large redshifts are a problem for Λ CDM.
- Problem alleviated here; expansion age is increased, by an increasingly larger relative fraction at larger redshifts, e.g., for best-fit values
 Λ CDM $\tau = 0.85$ Gyr at $z = 6.42$, $\tau = 0.365$ Gyr at $z = 11$
FB $\tau = 1.14$ Gyr at $z = 6.42$, $\tau = 0.563$ Gyr at $z = 11$
- Present age of universe for best-fit is $\tau_0 \simeq 14.7$ Gyr for wall observer; $t_0 \simeq 18.6$ Gyr for volume-average observer.
- Suggests problems of under-emptiness of voids in Newtonian N-body simulations may be an issue of using volume-average time?? The simulations need to be carefully reconsidered.

Variance of Hubble flow

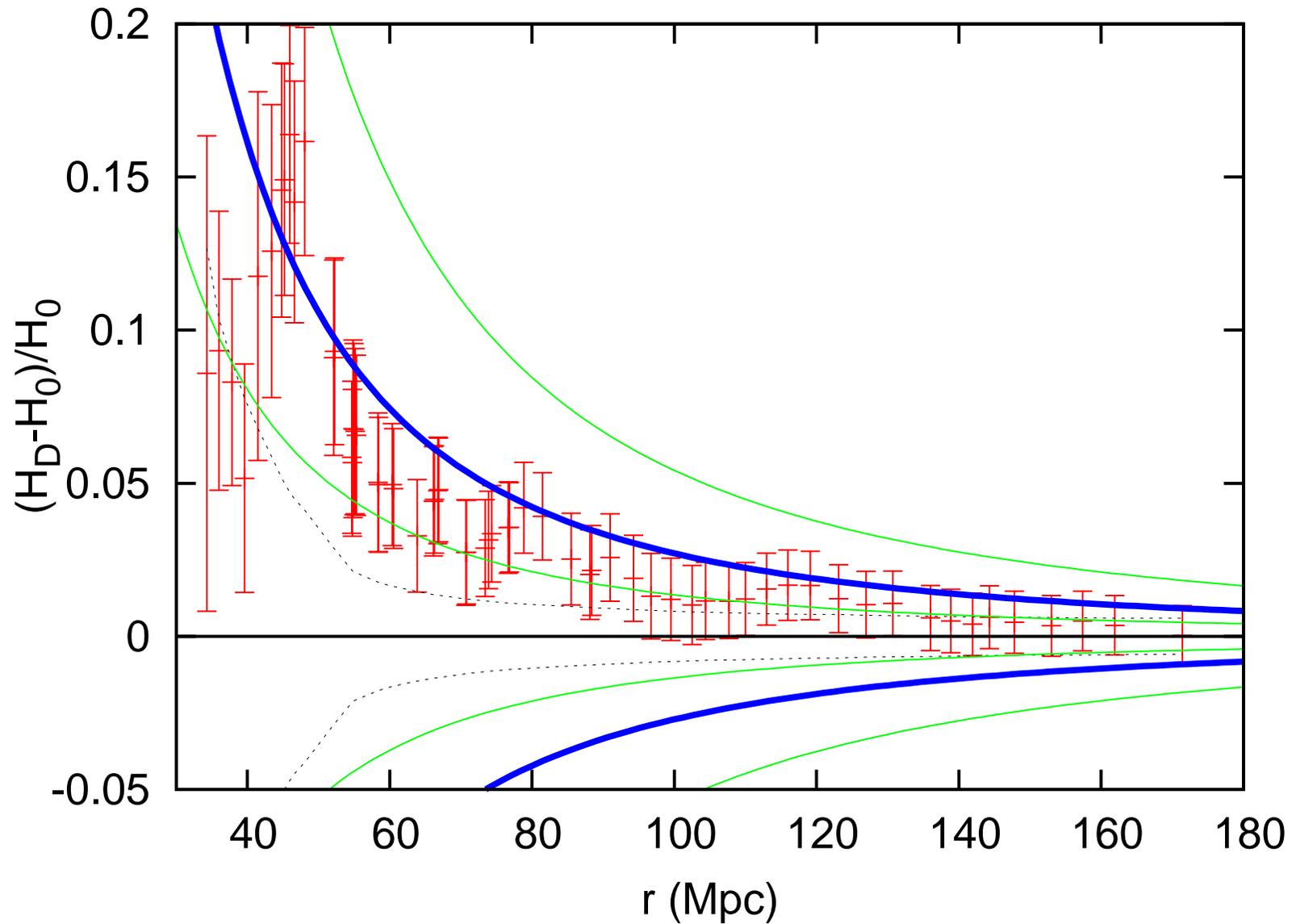
- Relative to “wall clocks” the global average Hubble parameter $H_{\text{av}} > \bar{H}$
- \bar{H} is nonetheless also the locally measurable Hubble parameter within walls
- TESTABLE PREDICTION:

$$H_{\text{av}} = \bar{\gamma}_w \bar{H} - \bar{\gamma}_w^{-1} \bar{\gamma}'_w$$

- With $H_0 = 62 \text{ km sec}^{-1} \text{ Mpc}^{-1}$, expect according to our measurements:
 $\bar{H}_0 = 48 \text{ km sec}^{-1} \text{ Mpc}^{-1}$ within ideal walls (e.g., around Virgo cluster?); and
 $\bar{H}_{v0} = 76 \text{ km sec}^{-1} \text{ Mpc}^{-1}$ across local voids (scale $\sim 45 \text{ Mpc}$)

Explanation for Hubble bubble

- As voids occupy largest volume of space expect to measure higher average Hubble constant locally until the global average relative volumes of walls and voids are sampled at scale of homogeneity; thus expect maximum H_0 value for isotropic average on scale of dominant void diameter, $30h^{-1}$ Mpc, then decreasing till levelling out by $100h^{-1}$ Mpc.
- Consistent with observed Hubble bubble feature (Jha, Riess, Kirshner ApJ 659, 122 (2007)), which is unexplained (and problem for) Λ CDM.
- Intrinsic variance in apparent Hubble flow exposes a local scale dependence which may partly explain difficulties astronomers have had in converging on a value for H_0 .



N. Li and D. Schwarz, arxiv:0710.5073v1-2

Best fit parameters

- Hubble constant $H_0 = 61.7_{-1.1}^{+1.2} \text{ km sec}^{-1} \text{ Mpc}^{-1}$
- present void volume fraction $f_{v0} = 0.76_{-0.09}^{+0.12}$
- bare density parameter $\bar{\Omega}_{M0} = 0.125_{-0.069}^{+0.060}$
- dressed density parameter $\Omega_{M0} = 0.33_{-0.16}^{+0.11}$
- non-baryonic dark matter / baryonic matter mass ratio
 $(\bar{\Omega}_{M0} - \bar{\Omega}_{B0}) / \bar{\Omega}_{B0} = 3.1_{-2.4}^{+2.5}$
- bare Hubble constant $\bar{H}_0 = 48.2_{-2.4}^{+2.0} \text{ km sec}^{-1} \text{ Mpc}^{-1}$
- mean lapse function $\bar{\gamma}_0 = 1.381_{-0.046}^{+0.061}$
- deceleration parameter $q_0 = -0.0428_{-0.0002}^{+0.0120}$
- wall age universe $\tau_0 = 14.7_{-0.5}^{+0.7} \text{ Gyr}$

Model comparison

	Λ CDM	FB scenario
Sn Ia luminosity distances	Yes	Yes
BAO scale (clustering)	Yes	Yes
Sound horizon scale (CMB)	Yes	Yes
Doppler peak fine structure	Yes	[still to calculate]
Integrated Sachs–Wolfe effect	Yes	[still to calculate]
Primordial ${}^7\text{Li}$ abundances	No	Yes?
CMB ellipticity	No	[Maybe]
CMB low multipole anomalies	No	[Foreground void: Rees–Sciama dipole]
Hubble bubble	No	Yes
Nucleochronology dates of old globular clusters	Tension	Yes
X-ray cluster abundances	Marginal	Yes
Emptiness of voids	No	[Maybe]
Sandage-de Vaucouleurs paradox	No	Yes
Coincidence problem	No	Yes

Conclusion

- Apparent cosmic acceleration can be understood purely within general relativity; by (i) treating geometry of universe more realistically; (ii) understanding fundamental aspects of general relativity which have not been fully explored – *quasi-local gravitational energy*, of *gradients* in spatial curvature etc.
- The “fractal bubble” model passes three major independent tests which support Λ CDM and may resolve significant puzzles and anomalies.
- Every cosmological parameter requires subtle recalibration, but no “new” physics beyond dark matter: no Λ , no exotic scalars, no modifications to gravity.
- Questions raised – otherwise unanswered – should be addressed irrespective of phenomenological success.