Gravitational energy as “dark energy”
Towards concordance cosmology without $\Lambda$

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[gr-qc/0702082];
[arxiv:0709.0732];
new results, to appear
B.M. Leith, S.C.C. Ng and DLW:
What is “dark energy”?

- Usual explanation: a homogeneous isotropic form of “stuff” which violates the strong energy condition. (Locally pressure $P = w \rho c^2$, $w < -\frac{1}{3}$; e.g., for cosmological constant, $\Lambda$, $w = -1$.)

- New explanation: in ordinary general relativity, a manifestation of global variations of those aspects of gravitational energy which by virtue of the equivalence principle cannot be localised – the cosmological quasilocal gravitational energy associated with dynamical gradients in spatial curvature generated by a universe as inhomogeneous. [Call this dark energy if you like. It involves energy; and “nothing” is dark.]
6df: voids & bubble walls (A. Fairall, UCT)
From smooth to lumpy

• Universe was very smooth at time of last scattering; fluctuations in the fluid were tiny \( \delta \rho / \rho \sim 10^{-5} \) in photons and baryons; \( \sim 10^{-3} \) in non–baryonic dark matter.

• FLRW approximation very good early on.

• Universe is very lumpy or inhomogeneous today.

• Recent surveys estimate that 40–50\% of the volume of the universe is contained in voids of diameter \( 30h^{-1} \) Mpc. [Hubble constant \( H_0 = 100h \) km sec\(^{-1}\) Mpc\(^{-1}\)] (Hoyle & Vogeley, ApJ 566 (2002) 641; 607 (2004) 751)

• Add some larger voids, and many smaller minivoids, and the universe is *void–dominated* at present epoch.

• Clusters of galaxies are strung in filaments and bubbles around these voids.
The Sandage-de Vaucouleurs paradox...

- Matter homogeneity only observed at $\gtrsim 200$ Mpc scales
- If “the coins on the balloon” are galaxies, their peculiar velocities should show great statistical scatter on scale much smaller than $\sim 200$ Mpc
- However, a nearly linear Hubble law flow begins at scales above 1.5–2 Mpc from barycentre of local group.
- Moreover, the local flow is statistically “quiet”.
- Can we explain this as an effect of dark energy? Maybe. Peculiar velocities are isotropized in FLRW universes which expand forever (regardless of dark energy).
Inhomogeneous cosmology

- Need an averaging scheme to extract the average homogeneous geometry

- Only exact approaches dealing with *averages* of full non-linear Einstein equations considered here (NOT perturbation theory: Kolb et al. . . ; NOT LTB models etc)

- Still many approaches, with different assumptions

- Do we average tensors on curves of observers (Zalaletdinov 1992, 1993) . . . recent work Coley, Pelavas, and Zalaletdinov, PRL 95 (2005) 151102; Coley and Pelavas, PR D75 (2007) 043506

For irrotational dust cosmologies, characterised by an energy density, \( \rho(t, x) \), expansion, \( \theta(t, x) \), and shear, \( \sigma(t, x) \), on a compact domain, \( \mathcal{D} \), of a suitably defined spatial hypersurface of constant average time, \( t \), and spatial 3–metric, average cosmic evolution in Buchert’s scheme is described by the exact equations

\[
\begin{align*}
3 \frac{\ddot{a}}{a^2} &= 8\pi G \langle \rho \rangle - \frac{1}{2} \langle \mathcal{R} \rangle - \frac{1}{2} Q \\
3 \frac{\dot{a}}{a} &= -4\pi G \langle \rho \rangle + Q \\
\partial_t \langle \rho \rangle + 3 \frac{\dot{a}}{a} \langle \rho \rangle &= 0 \\
Q &= \frac{2}{3} \left( \langle \theta^2 \rangle - \langle \theta \rangle^2 \right) - 2 \langle \sigma \rangle^2
\end{align*}
\]
Back-reaction

Angle brackets denote the spatial volume average, e.g.,

$$\langle \mathcal{R} \rangle \equiv \left( \int_D \text{d}^3 x \sqrt{\text{det} \, 3g} \mathcal{R}(t, x) \right) / \mathcal{V}(t)$$

$$\langle \theta \rangle = 3 \frac{\dot{a}}{a}$$

Generally for any scalar $\Psi$,

$$\frac{d}{dt} \langle \Psi \rangle - \langle \frac{d\Psi}{dt} \rangle = \langle \Psi \dot{\theta} \rangle - \langle \theta \rangle \langle \Psi \rangle$$

The extent to which the back–reaction, $Q$, can lead to apparent cosmic acceleration or not has been the subject of much debate.
Within a statistically average cell

Need to consider relative position of observers over scales of tens of Mpc over which $\frac{\delta \rho}{\rho} \sim 1$.

GR is a local theory: gradients in spatial curvature and gravitational energy can lead to calibration differences between our rods and clocks and volume average ones.
The Copernican principle

- Retain Copernican Principle - we are at an average position for observers in a galaxy
- Observers in bound systems are not at a volume average position in freely expanding space
- By Copernican principle other average observers should see an isotropic CMB
  
  BUT nothing in theory, principle nor observation demands that such observers measure the same mean CMB temperature nor the same angular scales in the CMB anisotropies

- Average mass environment (galaxy) can differ significantly from volume–average environment (void)
Dilemma of gravitational energy...

- In GR spacetime carries energy & angular momentum

\[ G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \]

- On account of the strong equivalence principle, \( T_{\mu\nu} \) contains localizable energy–momentum only

- Kinetic energy and energy associated with spatial curvature are in \( G_{\mu\nu} \): variations are “quasilocal”!

- Newtonian version, \( T - U = -V \), of Friedmann equation

\[ \frac{\dot{a}^2}{a^2} + \frac{k c^2}{a^2} = \frac{8\pi G \rho}{3} \]

where \( T = \frac{1}{2} m \dot{a}^2 x^2 \), \( U = -\frac{1}{2} km c^2 x^2 \), \( V = -\frac{4}{3} \pi G \rho a^2 x^2 m \);

\( r = a(t) x \).
Ricci curvature and gravitational energy

- For Lemaître–Tolman–Bondi models constant spatial curvature replaced by energy function with $E(r) > 0$ in regions of negative spatial curvature.

- In quasilocal Hamiltonian approach of Chen, Nester and Liu (MPL A22 (2007) 2039) relative to a fiducial static Cartesian reference frame a comoving observer in $k = -1$ FLRW universe sees negative quasilocal energy; or relative to the static frame the comoving observer has positive quasilocal energy.

- For perturbation theory I advocate “Machian gauge” of Bičak, Katz and Lynden–Bell (PR D76 (2007) 063501): uniform Hubble flow plus minimal shift distortion condition.
Thought experiment equivalent situations:

- **SR**: observers isotropically decelerate at different rates
- **GR**: regions of different density have different volume deceleration (for same initial conditions)
Cosmological Strong Equivalence Principle

- Even within pressureless dust there exist suitable small frames such that conformal volume expanding motions are locally indistinguishable from the equivalent uniform motion of particles in a static Minkowski space.

- Identify cosmic “rest frame” as the union of frames differing by an integrated “relative acceleration” of such pseudo-Minkowski space observers from same initial conditions, with same “local expansion” once cumulative “relative acceleration” is accounted for.

- Preserves isotropy of mean CMB temperature

- Implicitly solves the Sandage–de Vaucouleurs paradox.

- Voids appear to expand faster; but their local clocks tick faster, locally measured expansion can still be uniform.
Average isotropic observer rest frames

- Define them by average expansion over different regions being homogeneous, i.e.,

\[
\left\langle \frac{1}{\ell_r(\tau)} \frac{d\ell_r(\tau)}{d\tau} \right\rangle = \frac{1}{3} \langle \theta \rangle_1 = \frac{1}{3} \langle \theta \rangle_2 = \cdots = \bar{H}(\tau)
\]

- Average over regions in which (i) spatial curvature, shear and vorticity fluctuations average out; (ii) space is expanding at the boundaries, at least marginally.

- IMPORTANT POINT: $\bar{H}$ is the “locally” measured Hubble parameter, NOT the global average $H_{av}$ with respect to any one set of clocks, such as $\tau_w$.

- $\bar{H}$ is uniform whereas proper lengths $\ell_r(\tau_i)$ and proper time $\tau_i$ can be region dependent
Bound and unbound systems...

- Isotropic observers “at rest” within expanding space in voids may have clocks ticking at a rate $d\tau_v = \gamma(\tau_w, x)d\tau_w$ with respect to static observers in bound systems.

Volume average: $dt = \bar{\gamma}_w d\tau_w$, $\bar{\gamma}_w(\tau_w) = \langle -\xi^\mu n_\mu \rangle_H$

- We are not restricted to $\gamma = 1 + \epsilon$, $\epsilon \ll 1$, as expected for typical variations of binding energy.

- Observable universe is assumed unbound.

- With no dark energy I find $\gamma < \frac{3}{2} = \frac{H_{\text{Milne}}}{H_{\text{Einstein-de Sitter}}}$.

- Where is infinity? In 1984 George Ellis suggested a notion of *finite infinity*: a region within which isolated systems, such as stars or galaxies, or galaxy clusters can be considered as as approximately independent dynamical systems.
Where is infinity?

- Inflation provides us with boundary conditions.
- Initial smoothness at last–scattering ensures a uniform initial expansion rate. For gravity to overcome this a universal critical density exists. BUT if we assume a smooth average evolution we can overestimate the critical density today.

\[ \rho_{cr} \neq \frac{3H^2_{av}}{8\pi G} \]

- Identify finite infinity relative to demarcation between bound and unbound systems, depending on the time evolution of the true critical density since last-scattering.

- Normalise \textit{wall time}, \( \tau_w \), as the time at finite infinity, (close to galaxy clocks) by \( \langle -\xi^\mu n_\mu \rangle_{\mathcal{F}_I} = \langle \gamma(\tau_w, x) \rangle_{\mathcal{F}_I} = 1 \).
Define *finite infinity*, “*fi*” as boundary to minimal *connected* region within which *average expansion* vanishes $\langle \theta \rangle = 0$ or average curvature vanishes $\langle R \rangle = 0$.

Shape of *fi* boundary irrelevant (minimal surface generally): could typically contain a galaxy cluster.
Two/three scale model

\[ \bar{a}^3 = f_{w_i} a_w^3 + f_{v_i} a_v^3 \]

- Splits into void fraction with scale factor \( a_v \) and "wall" fraction with scalar factor \( a_w \). Assume \( 3\delta^2 H_w = \langle \sigma^2 \rangle_w \), \( 3\delta^2 H_v = \langle \sigma^2 \rangle_v \).

- Buchert equations for volume averaged observer, with \( f_v(t) = f_{v_i} a_v^3 / \bar{a}^3 \) (void volume fraction) and \( k_v < 0 \)

\[
\frac{\dot{\bar{a}}^2}{\bar{a}^2} + \frac{\dot{f}_v^2}{9 f_v (1 - f_v)} - \frac{\alpha^2 f_v^{1/3}}{\bar{a}^2} = \frac{8\pi G}{3} \bar{\rho}_0 \frac{\bar{a}_0^3}{\bar{a}^3},
\]

\[
\ddot{f}_v + \frac{\dot{f}_v^2 (2f_v - 1)}{2f_v (1 - f_v)} + 3\frac{\dot{a}}{\bar{a}} \dot{f}_v - \frac{3\alpha^2 f_v^{1/3} (1 - f_v)}{2\bar{a}^2} = 0,
\]

if \( f_v(t) \neq \text{const} \); where \( \alpha^2 = -k_v f_{v_i}^{2/3} \).
Two/three scale model

- Universe starts as Einstein–de Sitter, from boundary conditions at last scattering consistent with CMB; almost no difference in clock rates initially.

- We must be careful to account for clock rate variations. Buchert’s clocks are set at the volume average position, with a rate between wall clocks and void clock extreme.

\[
\bar{H}(t) = \bar{\gamma}_w H_w = \bar{\gamma}_v H_v; \quad H_w \equiv \frac{1}{a_w} \frac{da_w}{dt}, \quad H_v \equiv \frac{1}{a_v} \frac{da_v}{dt}
\]

where \(\bar{\gamma}_v = \frac{dt}{d\tau_v}, \bar{\gamma}_w = \frac{dt}{d\tau_w} = 1 + (1 - h_r)f_v/h_r,\)

\(h_r = H_w/H_v < 1.\)

- Need to be careful to obtain global \(H_{av}\) in terms of one set of isoptropic observer wall clocks, \(\tau_w\).
Bare cosmological parameters

- Different sets of cosmological parameters are possible
- Bare cosmological parameters are defined as fractions of the true critical density related to the bare Hubble rate

\[ \bar{\Omega}_M = \frac{8\pi G \bar{\rho}_0 \bar{a}^3}{3 \bar{H}^2 \bar{a}^3}, \]

\[ \bar{\Omega}_k = \frac{\alpha^2 f_v^{1/3}}{\bar{a}^2 \bar{H}^2}, \]

\[ \bar{\Omega}_Q = \frac{-f_v^2}{9 f_v (1 - f_v) \bar{H}^2}. \]

- These are the volume–average parameters, with first Buchert equation: \( \bar{\Omega}_M + \bar{\Omega}_k + \bar{\Omega}_Q = 1. \)
Dressed cosmological parameters

Conventional parameters for “wall observers” in galaxies: defined by assumption (no longer true) that others in entire observable universe have synchronous clocks and same local spatial curvature

\[
\begin{align*}
 ds^2_{\mathcal{F}_I} &= -d\tau_w^2 + a_w^2(\tau_w) \left[ d\eta_w^2 + \eta_w^2 d\Omega^2 \right] \\
&= -d\tau_w^2 + \frac{\bar{a}^2}{\bar{\gamma}_w^2} \left[ d\bar{\eta}^2 + r_w^2(\bar{\eta}, \tau_w) \, d\Omega^2 \right]
\end{align*}
\]

where \( r_w \equiv \bar{\gamma}_w \left( 1 - f_v \right)^{1/3} f_{wi}^{-1/3} \eta_w(\bar{\eta}, \tau_w) \), and volume–average conformal time \( d\bar{\eta} = dt/\bar{a} = \bar{\gamma}_w \, d\tau_w/\bar{a} \).

This leads to conventional dressed parameters which do not sum to 1, e.g.,

\[
\Omega_M = \bar{\gamma}_w^3 \bar{\Omega}_M.
\]
Tracker solution PRL 99, 251101

- General exact solution possesses a “tracker limit”

\[ \ddot{a} = \frac{\ddot{a}_0(3\bar{H}_0 t)^{2/3}}{2 + f_{v0}} \left[ 3f_{v0} \bar{H}_0 t + (1 - f_{v0})(2 + f_{v0}) \right]^{1/3} \]

\[ f_v = \frac{3f_{v0} \bar{H}_0 t}{3f_{v0} \bar{H}_0 t + (1 - f_{v0})(2 + f_{v0})}, \]

- Void fraction \( f_v(t) \) determines many parameters:

\[ \bar{\gamma}_w = 1 + \frac{1}{2} f_v = \frac{3}{2} \bar{H} t \]

\[ \tau_w = \frac{2}{3} t + \frac{2(1 - f_{v0})(2 + f_{v0})}{27 f_{v0} \bar{H}_0} \ln \left( 1 + \frac{9f_{v0} \bar{H}_0 t}{2(1 - f_{v0})(2 + f_{v0})} \right) \]

\[ \bar{\Omega}_M = \frac{4(1 - f_v)}{(2 + f_v)^2} \]
Apparent cosmic acceleration

- Volume average observer sees no apparent cosmic acceleration
  \[
  \bar{q} = \frac{2(1 - f_v)^2}{(2 + f_v)^2}.
  \]
  As \( t \to \infty, f_v \to 1 \) and \( \bar{q} \to 0^+ \).

- A wall observer registers apparent cosmic acceleration
  \[
  q = -\frac{(1 - f_v)(8f_v^3 + 39f_v^2 - 12f_v - 8)}{(4 + f_v + 4f_v^2)^2},
  \]
  Effective deceleration parameter starts at \( q \sim \frac{1}{2} \), for small \( f_v \); changes sign when \( f_v = 0.58670773 \ldots \), and approaches \( q \to 0^- \) at late times.
Cosmic coincidence problem solved

- Spatial curvature gradients largely responsible for gravitational energy gradient giving clock rate variance.
- Apparent acceleration starts when voids start to open.
Test 1: SneIa luminosity distances

- Type Ia supernovae of Riess07 Gold data set fit with $\chi^2$ per degree of freedom $= 0.9$

- With $55 \leq H_0 \leq 75 \text{ km sec}^{-1} \text{ Mpc}^{-1}$, $0.01 \leq \Omega_{M0} \leq 0.5$, find Bayes factor $\ln B = 0.27$ in favour or FB model (marginally): statistically indistinguishable from $\Lambda$CDM.
Test 1: SneIa luminosity distances

Plot shows difference of model apparent magnitude and that of an empty Milne universe of same Hubble constant $H_0 = 61.73 \text{ km sec}^{-1} \text{ Mpc}^{-1}$. Note: residual depends on the expansion rate of the Milne universe subtracted (2$\sigma$ limits on $H_0$ indicated by whiskers)
Comparison $\Lambda$CDM models

Best-fit spatially flat $\Lambda$CDM
$H_0 = 62.7 \text{ km sec}^{-1} \text{ Mpc}^{-1}$, 
$\Omega_{M0} = 0.34$, $\Omega_{\Lambda0} = 0.66$

Riess astro-ph/0611572, p. 63
$H_0 = 65 \text{ km sec}^{-1} \text{ Mpc}^{-1}$, 
$\Omega_{M0} = 0.29$, $\Omega_{\Lambda0} = 0.71$
Test 1: SNeIa luminosity distances

Best–fit $H_0$ agrees with HST key team, Sandage et al.,
$H_0 = 62.3 \pm 1.3$ (stat) $\pm 5.0$ (syst) km sec$^{-1}$ Mpc$^{-1}$ [ApJ 653 (2006) 843].
Dressed “comoving distance” $r_w(z)$

Best-fit FB model (red line) compared to 3 spatially flat $\Lambda$CDM models: (i) best-fit to WMAP5 only ($\Omega_\Lambda = 0.751$); (ii) best-fit to (Riess07) SNeIa only ($\Omega_\Lambda = 0.66$); (iii) joint WMAP5 + BAO + SNeIa fit ($\Omega_\Lambda = 0.721$)

FB model closest to best-fit $\Lambda$CDM to SNeIa only result ($\Omega_{M0} = 0.34$) at low redshift; and to WMAP5 only result ($\Omega_{M0} = 0.249$) at high redshift
A formal “dark energy equation of state” \( w_L(z) \) for the best-fit FB model, \( f_{v0} = 0.76 \), calculated directly from \( r_w(z) \): (i) \( \Omega_{M0} = 0.33 \); (ii) \( \Omega_{M0} = 0.279 \).

Description by a “dark energy equation of state” makes no sense when there is no physics behind it; but average value \( w_L \simeq -1 \) for \( z < 0.7 \) makes empirical sense.
Test 2: Angular scale of CMB Doppler peaks

Power in CMB temperature anisotropies versus angular size of fluctuation on sky
Angular scale is related to spatial curvature of FLRW models.

Relies on the simplifying assumption that spatial curvature is same everywhere.

In new approach spatial curvature is not the same everywhere.

Volume-average observer measures lower mean CMB temperature ($\bar{T}_0 \sim 1.98$ K, c.f. $T_0 \sim 2.73$ K in walls) and a smaller angular anisotropy scale.

Relative focusing between voids and walls.

Integrated Sachs–Wolfe effect needs recomputation.

Here just calculate angular–diameter distance of sound horizon.
Parameters within the \((\Omega_m, H_0)\) plane which fit the angular scale of the sound horizon \(\delta = 0.01\) rad deduced for WMAP, to within 2%, 4% and 6%.
Test 3: Baryon acoustic oscillation scale

- In 2005 Cole et al. (2dF), and Eisenstein et al. (SDSS) detected the signature of the comoving baryon acoustic oscillation in galaxy clustering statistics
- Powerful independent probe of “dark energy”
- Here the effective dressed geometry should give an equivalent scale
Test 3: Baryon acoustic oscillation scale

Parameters within the $(\Omega_m, H_0)$ plane which fit the effective comoving baryon acoustic oscillation scale of $10^4h^{-1}$ Mpc, as seen in 2dF and SDSS.
Agreement of independent tests

Best-fit parameters: $H_0 = 61.7^{+1.2}_{-1.1}$ km sec$^{-1}$ Mpc$^{-1}$, $\Omega_m = 0.33^{+0.11}_{-0.16}$ (1$\sigma$ errors for SNeIa only) [Leith, Ng & Wiltshire, ApJ 672 (2008) L91]
By equivalence principle the instantaneous relative deceleration of backgrounds gives an instantaneous 4-acceleration of magnitude \( \alpha = H_0 c \sqrt{\gamma} / (\sqrt{\gamma^2 - 1}) \) beyond which weak field cosmological general relativity will be changed from Newtonian expectations: (i) as absolute scale nearby; (ii) scaled for Hubble parameter to large \( z \).

\[ \alpha_0 = 1.2^{+0.3}_{-0.2} \times 10^{-10} \text{ m s}^{-2} h_{75}^2 = 8.1^{+2.5}_{-1.6} \times 10^{-11} \text{ m s}^{-2} \text{ for } h_0 = 61.7 \text{ km sec}^{-1} \text{ Mpc}^{-1} . \]
Li abundance anomaly

Big-bang nucleosynthesis, light element abundances and WMAP with ΛCDM cosmology.
Resolution of Li abundance anomaly?

- Tests 2 & 3 shown earlier use the baryon–to–photon ratio $\eta_{B\gamma} = 4.6-5.6 \times 10^{-10}$ admitting concordance with lithium abundances favoured prior to WMAP in 2003.

- Conventional dressed parameter $\Omega_{M0} = 0.33$ for wall observer means $\bar{\Omega}_{M0} = 0.127$ for the volume–average.

- Conventional theory predicts the volume–average baryon fraction. With old BBN favoured $\eta_{B\gamma}$:
  
  $\bar{\Omega}_{B0} \simeq 0.027-0.033$; but this translates to a conventional dressed baryon fraction parameter $\Omega_{B0} \simeq 0.072-0.088$.

- The mass ratio of baryonic matter to non–baryonic dark matter is increased to 1:3.

- Enough baryon drag to fit peak heights ratio.
Spatial curvature: ellipticity anomaly

- Negative spatial curvature should manifest itself in other ways than angular–diameter distance of sound horizon.

- Indeed it does: greater geodesic mixing from negative spatial curvature registers ellipticity in the CMB anisotropy spectrum.

- Ellipticity has been detected since COBE, and statistical significance increases with each data release (Gurzadyan et al., Phys. Lett. A 363 (2007) 121; Mod. Phys. Lett. A 20 (2005) 813, …).

- For FLRW models this is an anomaly; here it is expected; but still needs quantitative analysis.
Alleviation of age problem

- Old structures seen at large redshifts are a problem for $\Lambda$CDM.

- Problem alleviated here; expansion age is increased, by an increasingly larger relative fraction at larger redshifts, e.g., for best-fit values $\Lambda$CDM $\tau = 0.85$ Gyr at $z = 6.42$, $\tau = 0.365$ Gyr at $z = 11$
  FB $\tau = 1.14$ Gyr at $z = 6.42$, $\tau = 0.563$ Gyr at $z = 11$

- Present age of universe for best-fit is $\tau_0 \simeq 14.7$ Gyr for wall observer; $t_0 \simeq 18.6$ Gyr for volume–average observer.

- Suggests problems of under–emptiness of voids in Newtonian N-body simulations may be an issue of using volume–average time?? The simulations need to carefully reconsidered.
Variance of Hubble flow

- Relative to “wall clocks” the global average Hubble parameter $H_{av} > \bar{H}$
- $\bar{H}$ is nonetheless also the locally measurable Hubble parameter within walls

**TESTABLE PREDICTION:**

$$H_{av} = \bar{\gamma}_w \bar{H} - \bar{\gamma}_w^{-1} \bar{\gamma}'_w$$

With $H_0 = 62 \text{ km sec}^{-1} \text{ Mpc}^{-1}$, expect according to our measurements:

- $\bar{H}_0 = 48 \text{ km sec}^{-1} \text{ Mpc}^{-1}$ within ideal walls (e.g., around Virgo cluster?); and
- $\bar{H}_{v0} = 76 \text{ km sec}^{-1} \text{ Mpc}^{-1}$ across local voids (scale $\sim 45 \text{ Mpc}$)
Explanation for Hubble bubble

- As voids occupy largest volume of space expect to measure higher average Hubble constant locally until the global average relative volumes of walls and voids are sampled at scale of homogeneity; thus expect maximum $H_0$ value for isotropic average on scale of dominant void diameter, $30h^{-1}\text{Mpc}$, then decreasing til levelling out by $100h^{-1}\text{Mpc}$.

- Consistent with observed Hubble bubble feature (Jha, Riess, Kirshner ApJ 659, 122 (2007)), which is unexplained (and problem for) $\Lambda$CDM.

- Intrinsic variance in apparent Hubble flow exposes a local scale dependence which may partly explain difficulties astronomers have had in converging on a value for $H_0$. 
N. Li and D. Schwarz, arxiv:0710.5073v1–2
Best fit parameters

- Hubble constant \( H_0 = 61.7^{+1.2}_{-1.1} \text{ km sec}^{-1} \text{ Mpc}^{-1} \)
- Present void volume fraction \( f_{v0} = 0.76^{+0.12}_{-0.09} \)
- Bare density parameter \( \Omega_M = 0.125^{+0.060}_{-0.069} \)
- Dressed density parameter \( \Omega_M = 0.33^{+0.11}_{-0.16} \)
- Non-baryonic dark matter / baryonic matter mass ratio \( \frac{\Omega_M - \Omega_B}{\Omega_B} = 3.1^{+2.5}_{-2.4} \)
- Bare Hubble constant \( H_0 = 48.2^{+2.0}_{-2.4} \text{ km sec}^{-1} \text{ Mpc}^{-1} \)
- Mean lapse function \( \bar{\gamma}_0 = 1.381^{+0.061}_{-0.046} \)
- Deceleration parameter \( q_0 = -0.0428^{+0.0120}_{-0.0002} \)
- Wall age universe \( \tau_0 = 14.7^{+0.7}_{-0.5} \text{ Gyr} \)
## Model comparison

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<th>$\Lambda$CDM</th>
<th>FB scenario</th>
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<td>SneIa luminosity distances</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td>BAO scale (clustering)</td>
<td>Yes</td>
<td>Yes</td>
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<td>Sound horizon scale (CMB)</td>
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<td>Doppler peak fine structure</td>
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<td>Integrated Sachs–Wolfe effect</td>
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<td>Primordial $^7$Li abundances</td>
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<td>CMB ellipticity</td>
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<td>CMB low multipole anomalies</td>
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Conclusion

- Apparent cosmic acceleration can be understood purely within general relativity; by (i) treating geometry of universe more realistically; (ii) understanding fundamental aspects of general relativity which have not been fully explored – quasi–local gravitational energy, of gradients in spatial curvature etc.

- The “fractal bubble” model passes three major independent tests which support ΛCDM and may resolve significant puzzles and anomalies.

- Every cosmological parameter requires subtle recalibration, but no “new” physics beyond dark matter: no Λ, no exotic scalars, no modifications to gravity.

- Questions raised – otherwise unanswered – should be addressed irrespective of phenomenological success.