Exact solutions of Einstein and Einstein-Maxwell equations

0.1 Topics

- Exact solutions in General Relativity and Supergravity
- Cosmology and Astrophysics
- Quantum Fields
- Fundamental Relativity

0.2 Participants

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0.4 Research activity

0.4.1 Exact solutions in General Relativity and Supergravity

• In 2014 ICRANet started the new program "Exact solutions in the supersymmetric General Relativity" in collaboration with the group of Prof. H. Nicolai at Albert Einstein Institute at Potsdam (Germany). This new direction is now is in the list of the thematic of the ICRANet sector "Exact Solutions of the Einstein and Einstein-Maxwell equations". During 2014-2015 the work have been dedicated to the extension of the generating technique known as

the Inverse Scattering Method (ISM) to the super-gravity (V. Belinski). Here we have two main problems: first to formulate the super-symmetric version of ISM for the two-dimensional integrable models in super-gravity and then to find a way to construct exact super-solitonic solutions. During this year both of these problems was solved for the 2-dimensional extended N=2 super-gravity and corresponding paper have been published in Physical Review D, reference [1].

- Some classes of the so called 'travelling wave' solutions of Einstein and Einstein–Maxwell equations in general relativity and of dynamical equations for massless bosonic fields in string gravity in four and higher dimensions was found (G.A. Alekseev). This work appeared already in the previous ICRANet report 2014 but it was publisehed only in 2015, reference [2].
- The exact solution of Einstein - Maxwell equations for a Schwarzschild black hole immersed in the static spatially homogeneous (in the absence of a black hole) space-time of Bertotti-Robinson magnetic universe have been constructed (G.A. Alekseev). In this solution, the black hole possesses a finite initial boost in the direction of the magnetic field and performs a "geodesic" oscillating motion interacting with the background gravitational and electromagnetic fields, reference [3].
- A completely analytical model of the process of collision and nonlinear interaction of gravitational and electromagnetic soliton wave pulses and strong electromagnetic travelling waves of arbitrary profiles propagating in the expanding universe (symmetric Kasner space-time) was constructed (G.A. Alekseev). In contrast to intuitive expectations that rather strong travelling waves can destroy the soliton, it occurs that the soliton survives during its interaction with electromagnetic wave of arbitrary amplitude and profile, but its parameters begin to evolve under the influence of this interaction. If a travelling electromagnetic wave possesses a finite duration, the soliton parameters after interaction take constant values again, but these values in general are different from those before the interaction. Based on exact solutions of Einstein - Maxwell equations, the model demonstrates a series of nonlinear phenomena, such as (a) creation of gravitational waves in the collision of two electromagnetic waves, (b) creation of electromagnetic soliton wave in the collision of gravitational soliton with travelling electromagnetic wave, (c) scattering of a part of soliton wave in the direction of propagation of travelling electromagnetic wave, (d) quasiperiodic oscillating character of fields in the wave interaction region and multiple mutual transformations of gravitational and electromagnetic waves in this region. The figures illustrate these

features of nonlinear wave interactions in General Relativity, reference [4].

• It was constructed a method for generating exact interior solutions of Einstein's equations in the case of static and axially symmetric perfect-fluid spacetimes (H. Quevedo et al.). The method is based upon a transformation that involves the metric functions as well as the density and pressure of the seed solution. In the limiting vacuum case, it reduces to the Zipoy-Voorhees transformation that can be used to generate metrics with multipole moments. All the metric functions of the new solution can be calculated explicitly from the seed solution in a simple manner. The physical properties of the resulting new solutions are shown to be completely different from those of the seed solution, reference [7].

0.4.2 Cosmology and Astrophysics

Cosmology

• The work on the book "Cosmological Singularity" (V.Belinski and M. Henneaux) has been continued. The project is in progress under agreement with Cambridge University Press. This year was dedicated to the chapter "Oscillatory Singularity in String Models" and to few sections dedicated to the influence of different kinds of matter on the character of the singularity.

• The influence of the shear viscosity (in the framework of Israel-Stewart non-equilibrium thermodynamics) on the character of the cosmological singularity has been investigated (V. Belinski) and results confirm the previous author's statement on the existence of some types of viscous matter which are able to generate the stable Friedman-like initial cosmological singularity without any fine tuning. The base of this work already appeared in the previous ICRANet 2014 report but it was published (with some new details) in 2015, reference [5].

• It was analyzed in the context of geometrothermodynamics a Legendre invariant metric structure in the equilibrium space of an ideal gas (H. Quevedo et al.). The concept of thermodynamic geodesic as a succession of points, each corresponding to a state of equilibrium, have been introduced in such a way that the resulting curve represents a quasi-static process. A rigorous geometric structure was derived in which the thermodynamic geodesics at a given point split the equilibrium space into two disconnected regions separated by adiabatic geodesics. This resembles the causal structure of special relativity, which one can use to introduce the concept of adiabatic cone for

thermodynamic systems. This result might be interpreted as an alternative indication of the inter-relationship between relativistic physics and classical thermodynamics, reference [6].

• The existence of current-time universe's acceleration is usually modeled by means of two main strategies. The first makes use of a dark energy barotropic fluid entering the energy-momentum tensor of Einstein's theory. The second lies on extending the Hilbert-Einstein action giving rise to the class of extended theories of gravity. In our group it was proposed a third approach, derived as an intrinsic geometrical effect of space-time, which provides repulsive regions under certain circumstances (H. Quevedo et al.). We demonstrate that the effects of repulsive gravity naturally emerge in the field of a homogeneous and isotropic universe. To this end, we use an invariant definition of repulsive gravity based upon the behavior of the curvature eigenvalues. Moreover, we show that repulsive gravity counterbalances the standard gravitational attraction influencing both late and early times of the universe evolution. This phenomenon leads to the present speed up and to the fast expansion due to the inflationary epoch. In so doing, we are able to unify both dark energy and inflation in a single scheme. Further, we argue that the spatial scalar curvature can be taken as vanishing because it does not affect at all the emergence of repulsive gravity. We check the goodness of our approach through two cosmological fits involving the most recent union 2.1 supernova compilation, reference [11].

Astrophysics

• It was investigated the circular motion of test particles in the equatorial plane of the ergoregion in the Kerr spacetime (D. Pugliese and H. Quevedo). This work appeared already in the ICRANet report 2014 but it was published only in this year, reference [8].

• A class of spherically symmetric Born-Infeld black holes which contains the mass, electric charge, Born-Infeld parameter and the cosmological constant as physical parameters have been investigated (H. Quevedo et al). I was found that for the mass to be an extensive thermodynamic variable it is necessary to consider the cosmological constant and the Born-Infeld parameter as thermodynamic variables as well. It was analyzed the properties of such a thermodynamic system, explore the range of values where the system is thermodynamically well-defined, and the phase transition structure. In addition, it was shown that the equilibrium manifold in the context of ge-

ometrothermodynamics reproduces correctly the thermodynamic properties of this black hole class, reference [9].

• The motion of test particles in the gravitational field of a static naked singularity generated by a mass distribution with quadrupole moment have been investigated (H. Quevedo et al.). We use the quadrupole-metric (q- metric) which is the simplest generalization of the Schwarzschild metric with a quadrupole parameter. We study the influence of the quadrupole on the motion of massive test particles and photons and show that the behavior of the geodesics can drastically depend on the values of the quadrupole parameter. In particular, we prove explicitly that the perihelion distance depends on the value of the quadrupole. Moreover, we show that an accretion disk on the equatorial plane of the quadrupole source can be either continuous or discrete, depending on the value of the quadrupole. The inner radius of the disk can be used in certain cases to determine the value of the quadrupole parameter. The case of a discrete accretion is interpreted as due to the presence of repulsive gravity generated by the naked singularity. Radial geodesics are also investigated and compared with the Schwarzschild counterparts., reference [12].

• It was investigated equatorial geodesics in the gravitational field of a rotating and deformed source described by the approximate Hartle -Thorne metric (H. Quevedo et al.). In the case of massive particles, have been derived (within the same approximation) analytic expressions for the orbital angular velocity, the specific angular momentum and energy, and the radii of marginally stable and marginally bound circular orbits. Moreover, it was possible to calculate the orbital angular velocity and the radius of lightlike circular geodesics. It was studied numerically the frame dragging effect and the influence of the quadrupolar deformation of the source on the motion of test particles. Also it was showed that the effects originating from the rotation can be balanced by the effects due to the oblateness of the source, reference [13].

• It was studied the stationary axially symmetric solutions of the Einstein vacuum field equations that can be used to describe the gravitational field of astrophysical compact objects in the limiting case of slow rotation and slight deformation (H. Quevedo et al.). It was derived explicitly the exterior Sedrakyan-Chubaryan approximate solution, and express it in analytical form, which makes it practical in the context of astrophysical applications. In the limiting case of vanishing angular momentum, the solution reduces to the well-known Schwarzschild solution in vacuum. The new solution is equiv-

alent to the exterior Hartle-Thorne solution. It was established the mathematical equivalence between the Sedrakyan-Chubaryan, Fock-Abdildin and Hartle-Thorne formalisms, reference [14].

• It was considered the stability properties of test particles moving along circular orbits around a mass with quadrupole and showed that the quadrupole modifies drastically the properties of an accretion disk made of such test particles (h. Quevedo et al.), reference [15].

• It was found approximate exterior and interior solutions of Einstein's equations which describe the gravitational field of a static deformed mass distribution (H. Quevedo et al.). The deformation of the source is taken into account up to the first order in the quadrupole, reference [16].

• It was considered the problem of orbital stability of the motion of a test particle in the restricted three-body problem, by using the orbital moment and its time derivative (H. Quevedo et al.). It turns out that is possible to get some insight into the stability properties of the motion of test particles, without knowing the exact solutions of the motion equations, reference [17].

• It was analytically computed, through the six-and-a-half post-Newtonian order, the second-order-in-eccentricity piece of the Detweiler-Barack-Sago gauge-invariant redshift function for a small mass in eccentric orbit around a Schwarzschild black hole (T. Damour et al.). Using the first law of mechanics for eccentric orbits [A. Le Tiec, Phys. Rev. D **92**, 084021] it was transcribed the result into a correspondingly accurate knowledge of the second radial potential of the effective one-body formalism [A. Buonanno and T.Damour, Phys. Rev. D **59**, 084006]. Comparison have been made of the newly acquired analytical information to several different numerical self-force data and it was found good agreement, within estimated error bars. Also, for the first time, independent analytical checks of the recently derived approximation to the post-Newtonian dynamics [T. Damour, P. Jaranowski and G. Shaefer, Phys. Rev. D **89**, 064058] have been obtained, reference [19].

• It was analytically computed, through the eight-and-ahalf post-Newtonian order and the fourth-order in spin, the gravitational self-force correction to Detweiler's gauge invariant redshift function for a small mass in circular orbit around a Kerr black hole (T. Damour et al.). Using the first law of mechanics for black hole binaries with spin [L. Blanchet, A. Buonanno and A. Le Tiec, Phys. Rev. D **87**, 024030] one can transcribe the results into a knowledge of various spin-dependent couplings, as encoded within the spinning effective-one-body model of T. Damour and A. Nagar [Phys. Rev. D **90**, 044018]. Also it was compared the analytical results to the (corrected) numerical self-force

results of A.G. Shah, J.L. Friedman and T.S. Keidl [Phys. Rev. D **86**, 084059], from which it is possible to show how to extract physically relevant spin-dependent couplings, reference [20].

• It was presented a new effective one-body (EOB) Hamiltonian with next-to-leading order (NLO) spin-spin coupling for black hole binaries endowed with arbitrarily oriented spins (T. Damour et al.). The Hamiltonian is based on the model for parallel spins and equatorial orbits developed in [Physical Review D **90**, 044018], but differs from it in several ways. In particular, the NLO spin-spin coupling is not incorporated by a redefinition of the centrifugal radius, but by separately modifying certain sectors of the Hamiltonian, which are identified according to their dependence on the momentum vector. The used gauge-fixing procedure allows to reduce the 25 different terms of the NLO spin-spin Hamiltonian in Arnowitt-Deser-Misner coordinates to only 9 EOB terms. This is an improvement with respect to the EOB model recently proposed [Physical Review D **91**, 064011], where 12 EOB terms were involved. Another important advantage is the remarkably simple momentum structure of the spin-spin terms in the effective Hamiltonian, which is simply quadratic up to an overall square root. Moreover, a Damour-Jaranowski-Schäfer type gauge could be established, thus allowing one to concentrate, in the case of circular and equatorial orbits, the whole spin-spin interaction in a single radial potential, rererence [21].

• It was developed an improved numerical relativity (NR) calibration of the new effective one-body (EOB) model for coalescing non precessing spinning black hole binaries recently introduced by Damour and Nagar [Physical Review D **90**, 044018] (T. Damour et al.). It was done so by comparing the EOB predictions to both the phasing and the energetics provided by two independent sets of NR data covering mass ratios and dimensionless spin range. The EOB model presented here has only two calibration parameters, one entering the non spinning sector, as a 5PN effective correction to the interaction potential, and one in the spinning sector. These parameters are determined by comparing the EOB phasing with the SXS phasing, the consistency of the energetics being checked afterwards. The quality of the analytical model for gravitational wave data analysis purposes is assessed by computing the EOB/NR faithfulness, that is found to range, over the NR data sample, between 99% and 99.99% with a median value 99.865%, reference [22].

• It was found the exact expressions of the tenth and tenth-and-a-half post-Newtonian terms (T. Damour et al.). It was also introduced a new approach to the analytic computation of self-force regularization parameters based on

a WKB analysis of the radial and angular equations satisfied by the metric perturbations, reference [23].

• The conservative dynamics of gravitationally interacting two-point-mass systems has been recently determined at the fourth post-Newtonian (4PN) approximation [T. Damour, P. Jaranowski, and G. Schäfer, Phys. Rev. D **89**, 064058], and found to be nonlocal in time. Now it was shown (T. Damour et al.) how to transcribe this dynamics within the effective one-body (EOB) formalism. It was compared this improved analytical knowledge to previous, numerical gravitational self-force computation of precession effects, reference [24].

• Continuing the analytic computation of the first-order self-force contribution to Detweiler's redshift variable it was found the exact expressions of the ninth and ninth-and-a-half post-Newtonian terms (T. Damour et al.), reference [25].

• The data analysis of the gravitational wave signals emitted by coalescing neutron star binaries requires the availability of an accurate analytical representation of the dynamics and waveforms of these systems. In our group it was proposed (T. Damour et al.) an effective one-body (EOB) model that describes the general relativistic dynamics of neutron star binaries from the early inspiral up to merger. Our EOB model incorporates an enhanced attractive tidal potential motivated by recent analytical advances in the post-Newtonian and gravitational self-force description of relativistic tidal interactions. No fitting parameters are introduced for the description of tidal interaction in the late, strong field dynamics. We compare the model energetics and the gravitational wave phasing with new high resolution multi-orbit numerical relativity simulations of equal mass configurations with different equations of state. We find agreement within the uncertainty of the numerical data for all configurations. Our model is the first semi-analytical model which captures the tidal amplification effects close to merger. It thereby provides the most accurate analytical representation of binary neutron star dynamics and waveforms currently available, reference [26].

• The 1974 discovery, by Russell A. Hulse and Joseph H. Taylor, of the first binary pulsar PSR B1913+16, opened up new possibilities for the study of relativistic gravity. PSR B1913+16, as well as several other binary pulsars, provided observational proofs that gravity propagates at the velocity of light and has a quadrupolar structure. Binary pulsars also provided accurate tests of the strong field regime of relativistic gravity. General Relativity has passed all the binary pulsar tests with flying colors. The discovery of binary pulsars

had also very important consequences for astrophysics: accurate measurement of neutron star masses, improved understanding of the possible evolution scenarios for the co-evolution of binary stars, proof of the existence of binary neutron stars emitting gravitational waves for hundreds of millions of years, before coalescing in catastrophic events radiating intense gravitational wave signals, and probably leading also to important emissions of electromagnetic radiation and neutrinos. The Damour's article reviews the history of the discovery of the first binary pulsar, and describes both its immediate impact, and its longer term effect on theoretical and experimental studies of relativistic gravity, reference [27].

0.4.3 Quantum Fields

• The work has been done which summarises recent progress obtained by the mini-superpspace quantization of N=1, d=4 supergravity, formulated in the framework of the Bianchi IX cosmological model (T. Damour and Ph. Spindel). The emphasis is put on three main results : the completeness of the solution space obtained, the elements suggesting a hidden Kac-Moody structure of the theory and those leading to conjecture an avoidance of the cosmological singularity by some branches of the wave function of the Universe, reference [18].

0.4.4 Fundamental Relativity

• It was proved that the maximally symmetric vacuum solutions of General Relativity emerge from the geometric structure of statistical mechanics and thermodynamic fluctuation theory (H. Quevedo et al). It was showed that the pseudo-Riemannian structure of the Thermodynamic Phase Space is a solution to the vacuum Einstein-Gauss-Bonnet theory of gravity with a cosmological constant. Then one can use the geometry of equilibrium thermodynamics to demonstrate that the maximally symmetric vacuum solutions of Einstein's Field Equations (for Minkowski, de-Sitter and Anti-de-Sitter spacetimes) correspond to thermodynamic fluctuations. Moreover, these might be the only possible solutions that can be derived in this manner. Thus, the results are the first concrete examples of spacetimes effectively emerging from the thermodynamic limit over an unspecified microscopic theory without any further

assumptions, reference [10].

0.5 References

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