Rapidly Rotating Black Holes and Neutron Stars in Einstein-Gauss-Bonnet-Dilaton Theory

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1st Scientific ICRANet Meeting in Armenia
Black Holes: the Largest Energy Sources in the Universe

30 June - 4 July 2014 - Yerevan (Armenia)
Outline

1. Motivation
2. Black Holes
3. Neutron Stars
4. Wormholes
5. Conclusions
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Motivation

- Incompatibility with Quantum Mechanics
- Singularities
- Dark Matter, Dark Energy
- Tests in Strong Fields
- ...

General Relativity
Motivation

GR or Alternative Theories of Gravity

- Scalar-tensor theories
- Tensor-vector-scalar theories
- $f(R)$ theories
- Higher curvature theories
- ...

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Motivation

For instance: String Theory

unification of all fundamental interactions

dimensional reduction to 4 spacetime dimensions:

compactification or dimensional reduction

low energy effective theories

- additional fields
  - dilaton
  - axion
  - Maxwell fields
  - Yang-Mills fields
  - ...

- higher order curvature corrections
  - Gauss-Bonnet term
  - ...

- ...

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Rapidly Rotating Black Holes and Neutron Stars ...
Black Holes: Yerevan 2014
For Instance: Einstein-Gauss-Bonnet-Dilaton Theory

- One of the simplest consistent modifications of GR
  - Compatible with all solar system tests
  - Observational consequences in the strong gravity regime
    - Black holes
    - Neutron stars
    - Wormholes
Einstein-Gauss-Bonnet-dilaton gravity

Action

\[ S = \frac{1}{16\pi} \int d^4 x \sqrt{-g} \left[ R - \frac{1}{2} (\partial_\mu \phi)^2 + \alpha' e^{-\gamma \phi} R_{GB}^2 \right] \]

Gauss-Bonnet term: quadratic in the curvature

\[ R_{GB}^2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \]

- \( \alpha' \) Gauss-Bonnet coupling constant
- \( \gamma \) dilaton coupling constant (\( \gamma = 1 \))

In 4 spacetime dimensions the coupling to the dilaton is needed. The resulting set of equations of motion are of second order.
Einstein-Gauss-Bonnet-dilaton gravity

EGBd equations

\[ \nabla^2 \phi = \alpha' \gamma e^{-\gamma \phi} R_{GB}^2 \]

\[ G_{\mu\nu} = \frac{1}{2} \left[ \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla_\lambda \phi \nabla^\lambda \phi \right] \]

\[ -\alpha' e^{-\gamma \phi} \left[ H_{\mu\nu} + 4 \left( \gamma^2 \nabla^\rho \phi \nabla^\sigma \phi - \gamma \nabla^\rho \nabla^\sigma \phi \right) P_{\mu\nu\rho\sigma} \right] \]

abbreviations

\[ H_{\mu\nu} = 2 \left[ R R_{\mu\nu} - 2 R_{\mu\rho} R_{\nu}^{\rho} - 2 R_{\mu\rho\nu\sigma} R^{\rho\sigma} + R_{\mu\rho\sigma\lambda} R_{\nu}^{\rho\sigma\lambda} \right] - \frac{1}{2} g_{\mu\nu} R_{GB}^2 \]

\[ P_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} + 2 g_{\mu[\sigma} R_{\rho]}^{\nu} + 2 g_{\nu[\rho} R_{\sigma]}^{\mu} + R g_{\mu[\rho g_{\sigma]}^{\nu} \]

- non-positive “energy density” \( \rightarrow \) repulsion
- dilaton “hair”
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Black Holes in GR

Schwarzschild 1916
- black hole with mass $M$
  - static spherically symmetric
  - event horizon

$$r_H = 2M$$

Kerr 1963
- black hole with mass $M$
  - and angular momentum $J$
  - stationary rotating
  - event horizon

$$r_H = M + \sqrt{M^2 - a^2}$$
Black Holes in GR

Israel, Penrose, Wheeler, ....

No-hair theorem:
A stationary vacuum black hole is uniquely characterized by its mass $M$ and angular momentum $J$.


Multipole moments $M_{l}$ and $S_{l}$
All multiple moments can be expressed in terms of only two quantities:

$$M_{0} = M \quad \quad \quad S_{1} = J$$

$$M_{l} + iS_{l} = M \left(i \frac{J}{M}\right)^{l}$$

Quadrupole moment

$$M_{2} = Q = - \frac{J^{2}}{M}$$
Kerr black holes

- astrophysical black holes
- angular momentum bound

\[ \frac{J}{M^2} \leq 1 \]

- < 1 non-extremal black hole
- = 1 extremal black hole
- > 1 naked singularity (cosmic censorship)
Dilatonic black holes in higher curvature string gravity

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(Received 10 November 1995)

We give analytical arguments and demonstrate numerically the existence of black hole solutions of the 4D effective superstring action in the presence of Gauss-Bonnet quadratic curvature terms. The solutions possess nontrivial dilaton hair. The hair, however, is of “secondary type,” in the sense that the dilaton charge is expressed in terms of the black hole mass. Our solutions are not covered by the assumptions of existing proofs of the “no-hair” theorem. We also find some alternative solutions with singular metric behavior, but finite energy. The absence of naked singularities in this system is pointed out. [S0556-2821(96)01920-0]
Static EGBD Black Holes


Line element

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -e^{\Gamma(r)} dt^2 + e^{\Lambda(r)} dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \]

Expansion at the event horizon \( r_h \)

\[ e^{-\Lambda(r)} = \lambda_1 (r - r_h) + \lambda_2 (r - r_h)^2 + ... \]
\[ e^{\Gamma(r)} = \gamma_1 (r - r_h) + \gamma_2 (r - r_h)^2 + ... \]
\[ \phi(r) = \phi_h + \phi'_h (r - r_h) + \phi''_h (r - r_h)^2 + ... \]

Insertion into the eoms: relevant relation

\[ \phi'_h = \frac{r_h}{\alpha'} e^{-\phi_h} \left( -1 \pm \sqrt{1 - 6 \frac{\alpha'^2}{r_h^4} e^{2\phi_h}} \right) \]

Square root: \( \phi'_h \) real
Static EGBD Black Holes


critical black holes:

horizon expansion

$$\sqrt{1 - 6 \frac{\alpha' r^2}{r_h^4} e^{2\phi_h}}$$

lower bound on the horizon size for fixed $\alpha'$

lower bound on the mass
Static EGBD Black Holes


critical black holes:
horizon expansion

\[
\sqrt{1 - 6 \frac{\alpha'^2}{r_h^4} e^{2\phi_h}}
\]

lower bound
on the horizon size
for fixed \( \alpha' \)

lower bound on the mass

\[ GM / (\alpha')^{1/2} \]
Static EGBD Black Holes


asymptotically flat solutions

asymptotic expansion

\[ e^\Lambda(r) = 1 + \frac{2M}{r} + \frac{4M^2 - D^2}{r^2} + \ldots \]

\[ e^\Gamma(r) = 1 - \frac{2M}{r} + \ldots \]

\[ \phi(r) = \phi_\infty + \frac{D}{r} - \frac{2MD}{r^2} + \ldots \]

global charges: mass \( M \), dilaton charge \( D \)

linear stability radial perturbations ...
Black Holes

Static EGBD Black Holes


- region with negative energy density
- strange further solutions
Are black holes in alternative theories serious astrophysical candidates?
The case for Einstein-dilaton-Gauss-Bonnet black holes

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(Received 12 February 2009; published 21 April 2009)

**Slowly Rotating EGBD Black Holes**

**Lowest Order Perturbation Theory**

- **Solutions**
  - angular velocity: larger
  - ergoregion: larger

- **Orbits**

\[
\mathcal{L} = \frac{1}{2} e^{-2\beta \phi} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu
\]

\(\beta = \text{const.} \ (= 1/2 \text{ for heterotic string theory})

- orbital frequency: smaller
- ISCO: larger

**Higher Order Perturbation Theory**
Rapidly Rotating EGBD Black Holes


line element (Lewis-Papapetrou Ansatz in isotropic coordinates)

\[ ds^2 = -f_0 dt^2 + f_1 (dr^2 + r^2 d\theta^2) + f_2 r^2 \sin^2 \theta (d\phi - \omega dt)^2 \]

global charges

mass \( M \), angular momentum \( J \), dilaton charge \( D \)

\[ f_0 \rightarrow 1 - \frac{2M}{r}, \quad \omega \rightarrow \frac{2J}{r^3}, \quad \phi \rightarrow -\frac{D}{r}. \]
horizon properties

horizon $r_H$: \[ f_0(r_H) = 0 \]

horizon angular velocity: \[ \Omega = \omega(r_H) \]

surface gravity $\kappa_{sg}$: \[ \kappa_{sg}^2 = -1/4(\nabla_\mu \chi_\nu)(\nabla^\mu \chi^\nu) \]

Hawking temperature: \[ T_H = \kappa_{sg}/2\pi \]

horizon area: \[ A_H = \int_{\Sigma_h} d^2x \sqrt{h} \]

entropy: \[ S = \frac{1}{4} \int_{\Sigma_h} d^2x \sqrt{h}(1 + 2\alpha'e^{-\gamma\phi}\tilde{R}) \]

Smarr formula

\[ M = 2T_H S + 2\Omega J - \frac{D}{2\gamma} \]

relative error of Smarr formula in the calculations $< 10^{-5}$
Rapidly Rotating EGBD Black Holes

ergo region

\( \rho < 0 \)

horizon
Rapidly Rotating EGBD Black Holes

Horizon area versus angular momentum

smaller area
angular momenta beyond the Kerr limit
Domain of existence of stationary rotating black hole solutions

- static black holes ($J = 0$)
- Kerr black holes ($\alpha = 0$)
- critical black holes
  - branches of solutions stop
- “extremal” black holes ($T_H = 0$)
  - dilaton diverges, but spacetime is regular
Rapidly Rotating EGBD Black Holes

Hawking temperature versus angular momentum

\[ T_H \rightarrow 0: \text{extremal limit} \]
Quadrupole moment in EGBd theory as compared to Kerr
full calculation and perturbative calculation

larger quadrupole moment
$R_{isco}$ circumferential radius innermost stable circular orbit in the equatorial plane

Testparticles have larger ISCO radius
Testparticles in EGBD Black Hole Spacetimes

\( \nu_{isco} \) orbital frequency of innermost stable circular orbit in the equatorial plane

Testparticles have smaller orbital frequency
Neutron Stars

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Equations of state

Demorest et al., Nature 476, 1081 (2010)
Equations of state

perfect fluid

\[ T_{\text{matt}}^{\mu\nu} = (\rho + P)u^\mu u^\nu + g^{\mu\nu} P \]

polytropic equation of state

\[ \rho = nm_b + K \frac{n_0 m_b}{\Gamma - 1} \left( \frac{n}{n_0} \right)^\Gamma \]
\[ P = Kn_0 m_b \left( \frac{n}{n_0} \right)^\Gamma \]

- Polytropic (Damour et al. PRL70, 2220 (1993)).
- FPS (Lorenz et al. PRL70, 379 (1993)).
- APR (Akmal et al. PRC58, 1804 (1998)).
- Causal Limit (Hebeler et al. PRL105, 161102 (2010)).
Equations of state


I-Love-Q: Unexpected Universal Relations for Neutron Stars and Quark Stars
Equations of state

Chakrabarti et al., PRL (2014)

Extension of universal relations to rapid rotation

\[ \hat{Q} \]

\[ \hat{I} \]

\[ a \]
Equations of state

Chakrabarti et al., PRL (2014)
Compact stars in alternative theories of gravity: Einstein-Dilaton-Gauss-Bonnet gravity

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\[
S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} (\partial_\mu \phi)^2 + \alpha' e^{-\gamma \phi} R_{\text{GB}}^2 + \mathcal{L}_{\text{matt}} \right]
\]

\( \alpha' \) Gauss-Bonnet coupling constant

\( \gamma = 1, \beta = \sqrt{2} \) dilaton coupling constant
Neutron Stars

Slowly Rotating Neutron Stars in EGBd Theory

Pani et al, PRD84, 104035 (2011)

static neutron stars with APR EoS: dependence on $\alpha$ and $\beta$

branches end

expansion around origin: square roots

reality condition: condition on $\alpha\beta$, maximum central density
Pani et al, PRD84, 104035 (2011)

perturbative result: moment of inertia $I = J/\Omega$ for slow rotation

$I = (2.38 \pm 0.24) \times 10^{45} \text{ g cm}^2$

$M = (1.97 \pm 0.04) M_\odot$
Kleihaus et al., in progress

polytropic equation of state: $\Gamma = 1 + \frac{1}{N}$, $N = 0.746$

static versus mass shedding case
Kleihaus et al., in progress

mass-shedding limit

\[ \sigma = 1.34, \sigma = 1.353, \sigma = 1.365. \]
Neutron Stars

Rapidly Rotating Neutron Stars in EGBD Theory

Kleihaus et al., in progress

Extension of universal relations to EGBd

\[ \frac{J}{M^2} = 0.4 \]

\[ \frac{N}{\alpha} = 0.7463, \alpha = 0 \]

\[ \text{FPS, } \alpha = 0 \]

\[ \frac{N}{\alpha} = 0.7463, \alpha = 1 \]

\[ \text{FPS, } \alpha = 1 \]

\[ \frac{N}{\alpha} = 0.7463, \alpha = 2 \]

\[ \text{FPS, } \alpha = 2 \]
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PHYSICAL REVIEW D 76, 024016 (2007)

Wormholes as black hole foils

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(Received 20 April 2007; published 27 July 2007)

We study to what extent wormholes can mimic the observational features of black holes. It is surprisingly found that many features that could be thought of as “characteristic” of a black hole (endowed with an event horizon) can be closely mimicked by a globally static wormhole, having no event horizon. This is the case for the apparently irreversible accretion of matter down a hole, no-hair properties, quasi-normal-mode ringing, and even the dissipative properties of black hole horizons, such as a finite surface resistivity equal to 377 Ohms. The only way to distinguish the two geometries on an observationally reasonable time scale would be through the detection of Hawking’s radiation, which is, however, too weak to be of practical relevance for astrophysical black holes. We point out the existence of an interesting spectrum of quantum microstates trapped in the throat of a wormhole which could be relevant for storing the information lost during a gravitational collapse.

DOI: 10.1103/PhysRevD.76.024016

PACS numbers: 04.70.Dy
Wormholes

GR Wormholes


embedding diagram

2 asymptotically flat regions

sphere of minimal surface/radius

no horizon

no singularity
Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity

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(Received 16 March 1987; accepted for publication 17 July 1987)

Rapid interstellar travel by means of spacetime wormholes is described in a way that is useful for teaching elementary general relativity. The description touches base with Carl Sagan’s novel Contact, which, unlike most science fiction novels, treats such travel in a manner that accords with the best 1986 knowledge of the laws of physics. Many objections are given against the use of black holes or Schwarzschild wormholes for rapid interstellar travel. A new class of solutions of the Einstein field equations is presented, which describe wormholes that, in principle, could be traversed by human beings. It is essential in these solutions that the wormhole possess a throat at which there is no horizon; and this property, together with the Einstein field equations, places an extreme constraint on the material that generates the wormhole's spacetime curvature: In the wormhole’s throat that material must possess a radial tension \( \tau_r \) with the enormous magnitude \( \tau_r \sim (\text{pressure at the center of the most massive of neutron stars}) \times (20 \text{ km})^2/(\text{circumference of throat})^2 \). Moreover, this tension must exceed the material’s density of mass-energy, \( \rho_c c^2 \). No known material has this \( \tau_r > \rho_c c^2 \) property, and such material would violate all the “energy conditions” that underlie some deeply cherished theorems in general relativity. However, it is not possible today to rule out firmly the existence of such material; and quantum field theory gives tantalizing hints that such material might, in fact, be possible.
Desired properties of traversible wormholes

- **stability**
  - non-linear stability analysis
    Shinkai, Hayward, PRD 2002
    - additional negative energy causes expansion
    - reduced negative energy causes collapse
  - linear stability analysis
    Gonzales et al., CQG (2009)
    - unstable radial mode
    - instability seems generic

- reasonable stress-energy tensor

- small tidal forces
  (allowing human beings to travel)

- short travel time
  (allowing human beings to travel)
GR Wormholes

Exotic (phantom) matter

<table>
<thead>
<tr>
<th>Component</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atoms</td>
<td>4.9%</td>
</tr>
<tr>
<td>Dark Matter</td>
<td>26.8%</td>
</tr>
<tr>
<td>Dark Energy</td>
<td>68.3%</td>
</tr>
</tbody>
</table>

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EGBD Wormholes


no exotic matter

domain of existence

• lower boundary: black hole

• left boundary: $f_0 \to \infty$

• right boundary: singularity

throat area
Multi-throat EGBd wormholes

At the right boundary a singularity is encountered.
Wormholes

EGBD Wormholes

Stability

![Graph showing stability of EGBD Wormholes]

- **Lilac**: stable
- **Red**: unstable
- **White**: undecided

**Equations and Parameters**

\[ \frac{A}{16\pi M^2} \]

\[ \frac{\alpha/r^2}{\sigma^2} = 0 \]

\[ D/M \]

\[ A/16\pi M^2 \]

\[ \frac{\alpha/r^2}{\sigma^2} = 0 \]

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**EGBD Wormholes**

- Geodesics from Lagrangian

\[ L = \frac{1}{2} e^{-2\beta \phi} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \]

\[ \beta = \text{const.} \quad (= 1/2 \text{ for heterotic string theory}) \]

- Effective potential: \[ V_{\text{eff}}^2 (l, L) = e^{2\nu} \left( e^{-2\beta \phi} + \frac{L^2}{r_0^2 + l^2} \right) \]

- \[ E^2 \geq V_{\text{eff}}^2 (l, L) \]

- Turning points \( l_i \):

\[ E^2 - V_{\text{eff}}^2 (l_i, L) = 0 \]

- No horizon

- Bound orbits:

  motion around the throat

  motion across the throat
EGBD Wormholes

$L=2, E=0.90$

$L=2, E=0.98$

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acceleration of a traveler at the throat?

- $g_\oplus$: acceleration of gravity at the surface of the earth

- acceleration on the order of $g_\oplus$: throat radius on the order of $(10 - 100)$ light-years
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Conclusions

Gravity with String Theory corrections
- dilaton field
- Gauss-Bonnet term

Comparison with Kerr black holes
- EGBd black holes with $J/M^2 > 1$ exceed the Kerr bound
- EGBd black holes have smaller horizon area than Kerr black holes (except for large $J/M^2$)
- EGBd black holes have larger quadrupole moment
- Testparticles have larger ISCO radius
- Testparticles have smaller orbital frequency
Conclusions

- **Einstein-Gauß-Bonnet neutron stars**
  - slowly rotating
  - rapidly rotating
    - smaller mass
    - smaller moment of inertia
    - larger quadrupole moment
    - universal relations?
    - ...

- **Einstein-Gauß-Bonnet wormholes**
  - no exotic matter
  - static: stable wormholes
  - rotating wormholes?
  - ...

UNDER CONSTRUCTION
check back soon
THANKS