Critical field phenomena in ultrastrong laser fields

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Outline

- Ultrastrong laser fields. Critical fields in c.m. frame
- Nondipole and magnetic field effects in pair production in laser fields
- Streaking with pairs: characterization of zeptosecond gamma-rays
- Interference and enhancement of vacuum polarization effects
- Summary
World’s most powerful laser will pull apart vacuum of space

1. High lasers combine to
2. Ghost particles normally annihilate each other too quickly to be detected
3. Laser creates intense electrical field that

I \sim 10^{22} \text{ W/cm}^2

ELI-Ultra High Field Facility
I \rightarrow 10^{25} \text{ W/cm}^2

Nuclear Physics Facility
Magurele, Romania

Beamlines Facility
Prague, Czech Rep.

Attosecond Facility
Szeged, Hungary
Ultrastrong laser fields: Nonperturbative multiphoton processes

The first laboratory evidence of multiphoton pair production.
Vacuum instability: $e^+e^-$ pair production

Oscillating electric field: $E = E_0 \cos \omega t$

$\xi = eE/mc\omega = \text{laser period/formation time}$

Multiphoton: $\xi << 1$

Multiphoton:

$$\frac{dP}{dV \, dt} = \frac{1}{32\pi} \left( \frac{E_L}{E_{cr}} \right)^2 \frac{c}{\lambda_c^4} \left( \frac{\xi}{2} \right)^{4mc^2/\hbar\omega}$$

Tunneling: $\xi >> 1$

Tunneling:

$$\frac{dP}{dV \, dt} = \frac{1}{8\pi^3} \left( \frac{E}{E_{cr}} \right)^2 \frac{c}{\lambda_c^4} e^{-\pi \frac{E_{cr}}{E}}$$

Brezin, Itzykson, PRD 2, 1191 (1970)
Vacuum instability: $e^+e^-$ pair production

In crossed laser beams

When nondipole/magnetic field effects are important?

$$l_{coh} \sim \frac{m}{eE} \geq \lambda \quad \Rightarrow \quad \xi \leq 1 \quad \omega \leq m$$

$$\xi = \frac{eE}{mc\omega}$$

Ruf, Mocken, Müller, KZH, Keitel
PRL 102, 080402 (2009)
Vacuum instability: $e^+e^-$ pair production in crossed laser beams

Disturbed Rabi oscillations: Pair production probability versus pulse length at $\xi = 1$. (a) The red dashed line and the black solid line show the OEF case for $\omega = 0.49072m$ and the CLP case for $\omega = 0.472m$, respectively, each corresponding to a 5-photon resonance.

Ruf, Mocken, Müller, KZH, Keitel
PRL 102, 080402 (2009)
Nondipole effects

The resonance peaks are shifted and split due to non-zero photon momentum:
For example, \( n = 5 = 3 \) (from left) + 2 (from right) = 4 + 1

\[ \omega = m_e (n_+ + n_-) / 2 n_+ n_- \]
Overview: Pair creation in an oscillating electric field

- Pure two level system due to momentum conservation

\[ q_0(p) = \frac{1}{T} \int_0^T dt \sqrt{(p - eA(t))^2 + m^2} \]

\[ m^* \equiv q_0(p = 0) \approx 1.21m \quad \text{for} \quad \xi = 1 \]

- Rabi-oscillations

- Resonances enforced by energy conservation

\[ 2q_0(0) = n\omega \]

see, e.g. V. S. Popov 1973
Nondipole effects

The resonance peaks are shifted and split due to non-zero photon momentum:
For example, \( n = 5 = 3 \) (from left) + 2 (from right) = 4 + 1

\[
\omega = m_\sigma (n_+ + n_-)/2n_+ n_-
\]
The resonance peaks are shifted and split due to non-zero photon momentum: 
For example, \( n = 5 = 3 \) (from left) + 2 (from right) = 4 + 1
Nondipole effects

Pair production process in a standing laser field as a three-level system. Similarity to Autler-Towne effect in strong field ionization.

Ruf, Mocken, Müller, KZH, Keitel
PRL 102, 080402 (2009)
Strong field pair production in use:
streaking method for characterization of ultra-short pulses of GeV $\gamma$-rays
from femtoseconds up to zeptoseconds

**SHEEP** - streaking at high energies with electrons and positrons

Time-resolved intra-nuclear dynamics

MeV-GeV excitation energy
Zeptosecond time-scale

Quark system
Yoctosecond double pulses of γ-rays

heavy-ion collisions

Characterization of ultra-short pulses
Streak-camera

E Goulielmakis et al.
Streaking mechanism
Conversion photons to electrons

For XUV 100 eV - 1 KeV:
Photoionization

For hard x-rays 10 KeV - 1 MeV:
Compton ionization

For γ-rays >MeV:
e+e- pair production

\[ \frac{N_C}{N_{e^+e^-}} \sim 2 \times 10^2 \alpha \rho_e \chi_C^3 \sim 10^{-8} \text{ at } \rho_e = 10^{23} \text{ cm}^{-3}, \]
Streak-camera for $\gamma$-rays via strong field $e^+e^-$ pair production

Ipp, Evers, Keitel, KZH, PLB 702, 383 (2011)
**Streak-camera for γ-rays via strong field e+e- pair production**

**intense pulse (IP)**

**test pulse (TP)**

**streaking pulse (SP)**

\[
N_{e^+e^-} \propto \frac{\alpha m^2}{\omega_t} \chi^{3/2} e^{-8/3 \chi} N_i \tau_i, \quad \chi \ll 1
\]

\[
\chi = \frac{2 \omega_i \omega_l \xi_i}{m^2} > \frac{8}{3}, \quad \xi = \frac{eE}{mc \omega}
\]

**IP+SP should not create e+e-:**

**Photoemission is negligible:**

\[
E_i', E_s' \ll E_{cr}
\]

\[
\alpha \xi_i N_i \ll 1
\]

\[
\alpha \xi_i N_i / \chi^{1/3} \ll 1
\]

**Cascade of pair production is suppressed:**

\[
\tau_{int} \ll \tau_{e^+e^-}
\]

\[
\phi_L, n - n_{th}, \theta, \phi
\]

\[
\uparrow
\]

\[
\bar{N}_x, p_y, \varepsilon_+, \varepsilon_-
\]

\[
\Delta \varepsilon \gg 1/\tau_t, \Delta \omega_t
\]

\[
\pi N / S \ll \omega_s \tau_t < \pi
\]

1st Scientific ICRANet Meeting in Armenia, July 1, 2014
Streak-camera for γ-rays: Parameters

### (a)

\[
\omega_i = 1 \text{ eV} \\
\omega_i = 10 \text{ eV} \\
\omega_i = 100 \text{ eV}
\]

### (b)

\[
\omega_s \tau_t \\
\xi_i = 1 \\
\xi_i = 10
\]

Photoemission is negligible: 
\[
\alpha \xi_i \mathcal{N}_i \ll 1 \quad \Rightarrow \quad \xi_i = 1 - 10, \mathcal{N}_i = 3 - 30
\]

\[
\chi = \frac{2 \omega_i \omega_t \xi_i}{m^2} > \frac{8}{3} \quad \Rightarrow \quad \omega_t
\]

\[
\omega_s \tau_t \xi_s / \xi_i >> \Delta \omega_t / \omega_t \sim 0.1
\]

\[
\pi N / S \ll \omega_s \tau_t < \pi \quad \Rightarrow \quad \xi_s
\]
Streak-camera for $\gamma$-rays: 0.3-30 GeV from fs up to zs resolution

<table>
<thead>
<tr>
<th>Test pulse: ~ 30 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>IR Laser pulse: $10^{22}$ W/cm$^2$</td>
</tr>
<tr>
<td>Streaking pulse: 100 eV-1 keV</td>
</tr>
<tr>
<td>SP intensity: $10^{22} - 10^{24}$ W/cm$^2$</td>
</tr>
<tr>
<td>TP resolution: 1-10 as – 0.1 - 1 zs</td>
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<td>Laser pulse, keV photon: $10^{24}$ W/cm$^2$</td>
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<tr>
<td>Streaking pulse: 100 eV-1 keV</td>
</tr>
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<tr>
<td>TP resolution: 1-10 as – 0.1 - 1 zs</td>
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</tbody>
</table>
Enhancement of photon-photon scattering with interference effects in nonuniform vacuum with modulated super-strong fields: Bragg scattering
Vacuum polarization

The Feynman box diagram for the lowest-order photon-photon scattering at the tree level. Cross-section in the optical regime is $10^{-64}$ cm$^2$ - 39 orders of magnitude smaller than the Thomson scattering cross-section.

$$E \ll E_{cr}, \quad I_{cr} \sim 10^{29} \text{W/cm}^2$$

**Vacuum birefringence**

Uniform medium

$$F = \text{Const}$$
Photon scattering

Nonuniform fields

$$\nabla F \neq 0$$

Nonuniform medium

$$E << E_{cr} \quad I_{cr} \sim 10^{29} \text{W/cm}^2$$

Di Piazza, KZH, Keitel, PRL 2005
Modulated vacuum: Bragg scattering

Bragg-scattering condition:

$$\vec{q} = \vec{k}_2 - \vec{k}_1 - n\vec{q} = 0$$

$$2d \sin \theta = n\lambda$$
Bragg scattering vs. photon-photon scattering

\[ \frac{d\dot{W}_{\text{Bragg}}}{d\Omega} \propto \alpha n^4 \frac{E^6}{E_{cr}^6} \left| \int e^{i \mathbf{q} \cdot \mathbf{r}} d^3 r \right|^2 \]

\[ \frac{d\dot{W}_{\text{Bragg}}}{d\Omega} \propto N^2 \]

phase-matching factor

\[ \mathcal{E} \equiv \frac{\dot{W}_{\text{Bragg}}}{\dot{W}_{\text{stim}}} \sim \frac{L_{z'}}{\lambda} \]

Kryuchkyan, KZH, PRL 107, 053604 (2011)
Bragg scattering of light in vacuum with multiple laser beams

\[ E^{(0)} = E_0 \sum_{n=1}^{N} f(x, y, z - nd) \cos[\omega_L t - k_L x + \varphi(x, y, z - nd)] \]

\[ N = \frac{\alpha m^4 (2\pi)^{3/2}}{(360)^2 e^4} \left( \frac{I}{I_{cr}} \right)^3 (w_L x N d \tau) \left( \frac{N d}{\lambda} \right) W(\theta) \]

\[ I_{stim} = \frac{I_{tot}}{3} \quad I_{Bragg} = \frac{I_{tot}}{N + 1} ; \quad \frac{N_{Bragg}}{N_{stim}} = \frac{3^3 N^2}{N + 1^3} \approx 2 \quad N = 10 \]

\[ 3 \times 10^{22} \text{ W/cm}^2 \quad 100 \text{ fs} \quad N_{ph} \approx 5 \text{ / shot} \]

Kryuchkyan, KZH, PRL 107, 053604 (2011)
Summary

Bragg scattering can enhance photon-photon scattering

Nondipole effects: Autler-Towne splitting of pair production resonances

A detection scheme has been proposed for the characterization of short γ-ray pulses in the GeV energy range based on pair creation. Sub-attosecond time resolution could be achieved with high-order harmonic generation in the upcoming ELI facility.
Thank you for your attention!
Conclusion

• Nondipole effects: Autler-Townce splitting of pair production resonances

• A detection scheme has been proposed for the characterization of short γ-ray pulses in the GeV energy range based on pair creation. Sub-attosecond time resolution could be achieved with high-order harmonic generation in the upcoming ELI facility.

• Bragg scattering can enhance photon-photon scattering
<table>
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<tr>
<th>Parameter</th>
<th>High energy TP</th>
<th>Low energy TP</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Femto-</td>
<td>Atto-</td>
</tr>
<tr>
<td>$\omega_i$ [eV]</td>
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<td>1</td>
</tr>
<tr>
<td>$I_i$ [W/cm²]</td>
<td>$10^{20}$</td>
<td>$10^{20}$</td>
</tr>
<tr>
<td>$\xi_i$</td>
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<td>10</td>
</tr>
<tr>
<td>$N_i$</td>
<td>$\sim 3$</td>
<td>$\sim 3$</td>
</tr>
<tr>
<td>$\omega_s$ [eV]</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>$I_s$ [W/cm²]</td>
<td>$10^{18}$</td>
<td>$10^{22}$</td>
</tr>
<tr>
<td>$\xi_s$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\tau_t$ [as]</td>
<td>$10^2 - 10^3$</td>
<td>1 - 10</td>
</tr>
</tbody>
</table>

TABLE I: SHEEP parameters for different combinations of intense laser sources. $\Delta \omega_t / \omega_t \lesssim 0.1$, and $N/S = 10^{-2}$ are assumed. $(N_{e^+e^-}/N_t)|_{\omega_t=\omega_{t_{\min}}} \sim 10^{-2}$ in all cases. The XUV laser parameters can be realized in the ELI project [28].
Photon-photon scattering

\[ E \ll E_{cr} \quad I_{cr} \sim 10^{29} W / cm^2 \]

Box diagram = 0 in B = Const

Moulin et al. ZPC 1996
Bernard et al. EPJD 2000
Lundström et al. PRL 2006
Bragg scattering vs. Photon-photon scattering

\[ \sigma \sim \alpha^4 \left( \frac{\omega}{m} \right)^6 \lambda_C^2 \]

\[ j = \frac{E^2}{8\pi\omega} \quad N = \frac{E^2V}{8\pi\omega} \quad \rho_k = \frac{E^2}{8\pi\omega|\Delta^3k|} \]

\[ \Delta\dot{W}_{sp} \sim \sigma \cdot j_2 \cdot N_1 \sim \alpha^2 m^4 \left( \frac{E}{E_{cr}} \right)^4 \left( \frac{\omega}{m} \right)^6 V \Delta O \]

\[ \Delta\dot{W}_{stim} \sim \Delta\dot{W}_{sp} \cdot \rho_3 \sim \left[ \alpha m^4 \left( \frac{E}{E_{cr}} \right)^6 \right] \cdot \frac{\omega^3}{\Delta k} \cdot V \cdot \Delta O \quad \Delta k \sim \frac{1}{w_0} \]

\[ \Delta W_{Bragg} \sim R_0 \cdot \int e^{i\left( \vec{k}_1 - \vec{q} \right) \cdot \vec{r}} \cdot \omega^3 \cdot \Delta O \sim R_0 \cdot V^2 \cdot \omega^3 \cdot \Delta O \quad \Delta\Omega \sim 1/k^2 L_x L_y \]

phase-matching factor

Bragg-scattering condition:

\[ 2k \sin \frac{\vartheta}{2} = nq \]
Multiphoton Compton scattering

\[ F \sim \frac{\nu}{c} \times B \]

\[ \omega' = n\omega'_L \]

\[ a_0 = \frac{eE_0}{mc\omega_0} \]

\[ n \sim a_0^3 \]

\[ |\omega t - kr| < \pi \]

\[ \ell_{coh} = \frac{2\pi\nu}{\omega - kv} \sim \frac{1}{\omega} \gamma^2 \]

\[ \ell_f \sim \ell_{coh} \]

\[ \lambda_L \sim \frac{2\pi\nu}{\gamma} \]

\[ \omega \sim \gamma^3 \omega_0 \sim a_0^3 \omega_0 \]

\[ a_0 \sim 100 \quad l \sim 10^{22} W/cm^2 \]

\[ n \sim 10^6 \quad 50 \text{ MeV } \gamma - \text{rays} \]
Quantum effects in multiphoton Compton scattering

\[ \chi = \frac{e}{m^3} \sqrt{(F_{\mu\nu}p^\nu)^2} = \frac{E'}{E_S} \]

\[ E_S = \frac{m^2 c^3}{e\hbar} \]

\[ I_S = 2.3 \cdot 10^{29} \frac{W}{cm^2} \]

\[ \chi \sim \frac{\omega}{m} \]

\[ \chi \geq 1 \quad \text{Quantum regime: emitted photon recoil is significant} \]

\[ \chi \ll 1 \quad \text{Classical regime} \]

\[ E' \approx 2\gamma E \quad \chi \sim 2 \frac{\omega_0}{m} a_0 \gamma \sim 4 \cdot 10^{-6} a_0 \gamma \]

\[ a_0 = \frac{eE_0}{mc\omega_0} \]

\[ \chi \sim 1 : \ a_0 \sim 100, \gamma \sim 10^3 \]
Radiation Dominated Regime

The characteristic emitted photon energy:

$$\omega_c \sim m\gamma \chi$$

The probability of a photon emission on a coherence length:

$$\alpha \frac{1}{a_0}$$

Phase interval for a coherence length:

$$a_0$$

Number of coherence lengths on a laser period:

$$N_{ph} \sim \alpha a_0$$

Number of emitted photons during a laser period:

The electron radiative energy loss during a laser period:

$$\Delta \varepsilon_{rad} \sim N_{ph} \omega_c \sim \alpha a_0 \chi m\gamma$$

Radiation Dominated Regime (RDR):

$$\Delta \varepsilon_{rad} \geq m\gamma$$

$$R \equiv \alpha a_0 \chi \geq 1$$

Classical RDR:

$$\chi \ll 1 \Rightarrow R \geq 1 \rightarrow a_0 \geq 10^3$$
Radiation dominated dynamics in Thomson scattering

\[ R \geq \frac{4\gamma_0^2 - \xi^2}{2\xi^2} \]

\[ \gamma_d \sim \xi \sim \gamma_0 >> 1 \]

Electron trajectory

\[ \pi \gamma^2 c / \omega_c \approx \rho / \gamma \]


\[ \gamma = 80 \quad a_0 = 150 \quad I \sim 5 \times 10^{22} \text{ W/cm}^2 \]

10^9 electrons in beam
10^4 photons per pulse below 90°
1% of electrons contribute

Angle resolved radiation spectra
Quantum radiation dominated regime in Compton scattering

\[ \varepsilon = 1 \text{ GeV} \]
\[ \omega = 1.5 \text{ eV} \]
\[ I = 5 \times 10^{22} \text{ W/cm}^2 \]
\[ \xi = 154; \chi = 1.8; R_Q = 1 \]

Emission of 16 photons
Contribution of more photons 2%

Typical modification of spectrum in the quantum regime

Number of photons 10 keV-1MeV: \((N_{RR} - N_0)/N_0 \sim 40\%\)

Calculation method

The emitted radiation is calculated quantum mechanically, and the differential probability per unit phase interval is [1]

\[
\frac{dW_{ri}}{d\eta d\tilde{\omega}} = \frac{\alpha \chi m^2 [\int_{-\infty}^{\infty} K_{5/3}(x) dx + \tilde{\omega} \tau_{r} \chi^2 K_{2/3}(\chi)]}{\sqrt{3\pi} (k_0, p_i)},
\]

\[\tilde{\omega}_r = \tilde{\omega}/\rho_0,\] with the recoil parameter \(\rho_0 = 1 - \chi \tilde{\omega}\) and \(\tilde{\omega} = \omega'/(\gamma \chi)\). If \(\tilde{\omega}_r \geq 1\), \(\frac{dW_{ri}}{d\eta d\tilde{\omega}}\) is very small. Thus, \(\tilde{\omega}_r = \tilde{\omega}/\rho_0 = 1\), the cut-off frequency

While electron dynamics can be classically calculated (because the electron de-Broglie wavelength is much smaller than the laser wavelength) [2]:

\[
\frac{d\rho^\alpha}{d\tau} = \frac{e}{m} F^{\alpha \beta} \rho_\beta - \frac{\tau}{m} \rho^\alpha + \tau e \frac{\tau}{T_c} F^{\alpha \beta} F_{\beta \gamma} \rho^\gamma,
\]

The rate of the electron radiation loss is: \(\mathcal{I} = \int d\tilde{\omega} (k_0 \cdot k) dW_{ri}/(d\eta d\tilde{\omega})\), and \(\mathcal{I}_c = 2\alpha \omega^2 \xi^2\) is the classical radiation loss rate.

Laser and electron parameters

Electron bunch:
\( \varepsilon = 500 \text{ MeV (} \gamma = 1000) \), Cyclinder shape: \( w_b = 3 \mu \text{m, } l_b = 6 \mu \text{m, and} \) \( n_e = 5.9 \times 10^{16} \text{cm}^{-3} \).

Laser pulses:
\( I \approx 7 \times 10^{22} \text{ W/cm}^2 \) (\( \xi = 230, \chi \approx 0.9 \) and \( R \approx 7 \)), \( w_0 = 10\lambda \), and \( \tau_0 = 0.5 \sim 40 T_0 \).

For a focused ultrashort laser pulse, the approximate solution of Maxwell equations should use two parameters on equal footing.

- Diffraction parameter \( (k_0 w_0)^{-1} [1] \)
- Temporal parameter \( (\omega_0 \tau_0)^{-1} [2] \)

\[
A = (E_0/k_0)[\hat{x} \psi(r, \eta) + i\hat{y} \psi(r, \eta)e^{i\pi/2}]e^{i\eta}
\]

with \( \psi = f(1 + i\eta/s^2)exp[i\phi_0 - f\rho^2 - \eta^2/(2s^2)], f = i/(i + \nu/z_r), \nu = z + \eta/(2k_0), \eta = \omega_0 t - k_0 z, \rho = r/w_0, r = \sqrt{x^2 + y^2}, \)
\( s = \omega_0 \tau_0/2 \sqrt{2 \log 2}, z_r = k_0 w_0^2/2 \) is the Rayleigh length, \( \phi_0 \) is the constant phase, \( E_0 \) is the laser amplitude, \( w_0, \omega_0, k_0, \) and \( \tau_0 \) is the waist radius, frequency, wave vector, and pulse duration of the laser.
Quantum RDR: 
\[ R = \alpha a_0 \chi \geq 1 \]
\[ \chi \approx 2 \gamma a_0 \omega / m \sim 1 \]

Spectra of electron radiation in laser pulses of various durations: the left column displays \( d\epsilon / d\Omega \) [GeV/sr] and the right \( d\epsilon / d\Omega d\omega \) [1/sr] for 1, 1.5 and 5 cycle pulses \( \lambda = 1 \mu m, w_0 = 10\lambda, a_0 = 230 \) and \( \gamma = 1000 \)
Robust signatures of quantum radiation reaction for nonlinear Compton scattering in focused ultrashort laser pulses

Spectra of electron radiation in laser pulses of various durations: the left column displays $d\varepsilon/d\Omega$ [GeV/sr] and the right $d\varepsilon/d\Omega d\omega$ [1/sr] for 1, 1.5 and 5 cycle pulses $\lambda = 1 \mu m$, $w_0 = 10 \lambda, a_0 = 230$ and $\gamma = 1000$

Quantum regime

$$R = a a_0 \chi \geq 1$$
$$\chi \approx 2\gamma a_0 \omega / m \sim 1$$

$$\Delta \theta = 180^\circ - \theta_b$$

$$\Delta \omega = \omega_b - 0$$

The quantum RR signatures in the quantum RDR. The boundary angle $\theta_b$ and the boundary frequency $\omega_b$ (b) vs the laser pulse duration.
Robust signatures of quantum radiation reaction for nonlinear Compton scattering in focused ultrashort laser pulses

Spectra of electron radiation in laser pulses of various durations: the left column displays $d\varepsilon/d\Omega$ [GeV/sr] and the right $d\varepsilon/d\Omega d\omega$ [1/sr] for 1, 1.5 and 5 cycle pulses $\lambda = 1 \mu m$, $w_0 = 10\lambda, a_0 = 230$ and $\gamma = 1000$

The RR signatures in the classical RR regime:

$R \ll 1$
$\chi \ll 1$

Quantum regime
$R = \alpha a_0 \chi \geq 1$
$\chi \approx 2\gamma a_0 \omega / m \sim 1$

The quantum RR signatures in the quantum RDR. The boundary angle $\theta_b$ and the boundary frequency $\omega_b$ (b) vs the laser pulse duration.

$\xi = 100, \gamma = 100$ ($\chi \approx 10^{-2}$, and $R \approx 10^{-2}$)
Estimations

\[ \frac{d\varepsilon}{dt} \sim \frac{\Delta\varepsilon}{\Delta t_{\text{coh}}} \]

\[ \Delta\varepsilon \sim \alpha \omega_c \quad \omega_c \sim \frac{m\chi\gamma}{1 + \chi} \]

\[ \Delta t_{\text{coh}} \sim \frac{l_{\text{coh}}}{c} \quad l_{\text{coh}} \sim \frac{2\pi\gamma^2 c}{\omega_c} \]

\[ \frac{d\varepsilon}{dt} \sim \frac{\alpha\omega_c^2}{2\pi\gamma^2} \]
The single electron's dynamics in a counterpropagating laser pulse. The dash, solid, and dash-dotted lines correspond to the results calculated with 1, 1.5 and 5 laser cycles pulse duration, respectively. The circles point out the places where the corresponding quantities are maximum. The other parameters employed here are $\lambda = 1 \text{m}$, $\gamma = 230$, $\theta = 1000$, and $w_0 = 10$. 

The back emission spectra 

$$d\varepsilon/d\eta \sim \Delta \varepsilon / \Delta \eta_{coh} \sim \alpha \omega_c \xi / (2\pi) \propto \chi^2$$

$$\chi \propto \xi \gamma$$

The back emission spectra 

$$p_m \sim m \xi (\eta_m) \text{ and } \theta_p \sim p_m / p_m$$
A single electron's dynamics in a counterpropagating laser pulse. But the radiation reaction effects are ignored. The dash, solid and dash-dot curves correspond to the results calculated with 0.5, 2, and 5 laser cycles pulse duration, respectively.
Necessary statistics

For instance, the photon emission total probability in the case of \( \tau = 5 T_0 \) is \( W_{ph}^{tot} \approx 39.42 \), while the probability for the photon emission in \( d\varepsilon/d\Omega|_{\max} \) and \( d\varepsilon/d\Omega|_{\varphi-\varphi_m, \theta-\theta_b} \) is \( W_{ph}^{m} \approx 0.0027 \) and \( W_{ph}^{b} \approx 0.007 \), respectively. The relative signal

\[
|W_{ph}^{b} - W_{ph}^{m}| / W_{ph}^{tot} \approx 10^{-4}
\]

The statistical error

\[
\delta_s = (W_{ph}^{tot} N_e N_{shot})^{-1/2} \sim 10^{-5} \quad (N_{shot} = 10)
\]

Besides, the absorption energy from the driving laser pulse by electron

\[
\varepsilon_{absorption} / \varepsilon_{laser} \approx 10^{-6}.
\]

Thus, an electron bunch of \( n_e \approx 10^{19} \text{cm}^{-3} \) also can be employed, and, for such case \( N_{shot} = 1 \).

Valid parameters of laser pulse

\[
\xi \leq \chi \leq 20 \xi \quad (R \geq 1 \text{ and } \chi \leq 1), \quad w_0 \geq 4 \lambda_0
\]

Valid parameters of electron bunch

\[
w_b \leq w_0 / 2, \quad 10^{15} \leq n_e \leq 10^{19} \text{ cm}^{-3} \quad (N_{shot} \leq 100)
\]
Conclusion

• We have identified signatures of quantum RDR in dependence of both the angular spread and the spectral bandwidth of the Compton radiation spectra on the laser pulse duration, which are distinct from those in the classical RR regime.

• Due to an interplay between laser beam focusing and quantum RR effects the angular spread of the main photon emission region has a prominent maximum at an intermediate pulse duration and decreases along the further increase of the pulse duration.

• The spectral bandwidths of the radiation in the quantum and classical regimes both monotonously decrease when the laser pulse duration is increased, but the former is by orders of magnitude larger due to much stronger RR effects.

• These signatures are robust and observable in a broad range of electron and laser beam parameters.
Competition between the Breit-Wheeler and Trident processes for the electron-positron pair production in laser fields
Multiphoton Compton scattering

\[ \chi_e \leq 1: \quad \frac{dW_\gamma}{dt} \approx \frac{\alpha m \chi_e}{6\pi \gamma} \]

Ritus, 1966

\[ a_0 \gg 1 \quad \text{Multiphoton processes} \]

\[ \chi_e = \frac{E'}{E_{cr}} \approx 2\gamma \frac{\omega}{m} a_0 \]

\[ \Omega \sim \chi_e \varepsilon_e \]

\[ \chi_e \geq 1 \quad \text{Quantum regime} \]

RDR: \[ R \equiv \alpha a_0 \chi_e \geq 1 \]

\[ N_\gamma \approx 0.6 \alpha a_0 N_L N_e \]

\[ I \to 10^{25} \text{ W/cm}^2 \]
Breit-Wheeler process

\[ \chi_\gamma < 1: \quad N_{BW} \approx \frac{9}{32} \frac{\alpha m^2}{\Omega} \left( \frac{\chi_\gamma}{2\pi} \right)^{3/2} e^{-\frac{8}{3\chi_\gamma N_\gamma \tau}} \]

Ritus, 1966

\[ \chi_\gamma = 2 \frac{\Omega \omega_L}{m \omega_e} a_0; \quad \chi_\gamma \approx \chi_e, \text{ if } \chi_e \sim 1 \]

\[ \chi_\gamma \approx 2 \frac{\Omega \omega_L}{m \omega_e} a_0 \approx \chi_e^2, \text{ if } \chi_e \ll 1 \]

\[ \Omega \approx 2\gamma_d^2 \omega_L \bar{n} \approx 2\gamma^2 \omega_L a_0 (\omega / m)^2 \]
\[ \bar{n} \approx a_0^3, \gamma_d \approx \gamma / a_0 \]

\[ N_{BW} \approx \frac{9}{32} \alpha a_0 N_L N_\gamma \sqrt{\frac{\chi_\gamma}{2\pi}} e^{-\frac{8}{3\chi_\gamma}} \]
Compton +Breit-Wheeler process

\[ N_{C+BW} \approx \frac{9}{16} (\alpha a_0 N_L)^2 N_e \sqrt{\frac{\chi_\gamma}{2\pi}} e^{-\frac{8}{3 \chi_\gamma}} \]

\[ \chi_\gamma = 2 \frac{\Omega \omega_L}{m m} a_0; \]

\[ N_L = 1, R = \alpha a_0 \chi_e \geq 1 \]

\[ a_0=100; \gamma = 2000 \quad N_{C-BW}=10^7 \]

\[ N_{C-BW}=170/20000: \quad a_0=100; \gamma = 525; \quad I = 10^{22} W/cm^2 \]

\[ a_0=27; \gamma = 2000; \quad I = 7 \cdot 10^{20} W/cm^2 \]

\[ a_0=3; \gamma = 2 \cdot 10^4; \quad I = 10^{19} W/cm^2 \]
Breit-Wheeler vs Trident process

\[ \frac{r_e^2}{\gamma} J_L \tau \sim \alpha a_0 N_L \]

\[ \alpha \frac{r_e^2}{\gamma} J_L \tau \sim \alpha^2 a_0 N_L \]

\[ (\alpha a_0 N_L)^2 \]

\[ \frac{BW}{T} \sim a_0 N_L \]
Breit-Wheeler vs Trident process
Momentum distribution

Threshold: c.m. frame

\[
\begin{align*}
n\omega' &= \Omega' = m_* \\
2\gamma n\omega &= \frac{\Omega}{2\gamma} = m_* \\
q_+ &= q_- = 0 \\
q_+ = q_- &= -n\Omega/2 \approx -n\varepsilon_e/2
\end{align*}
\]

\[
\begin{align*}
n\omega' &= q'_e \\
n\omega' + \varepsilon'_e &= 3m_* \\
q'_e &= \frac{4m_*}{3}, \varepsilon'_0 = \frac{5m_*}{3} \\
q_+ &= q_- = -n\varepsilon_e/3
\end{align*}
\]
Finite beam sizes

\[ W_{e^+e^-} = 1 \quad a_0 = 100 \; ; \; \gamma = 2000 \]

\[ W_{e^+e^-} = 0.3, \; a'_0 = 83 \]

\[ l' = l_0 e^{-x'^2} \; ; \; x' = 0.61 \]

\[ \frac{2}{\sqrt{\pi}} \int_0^{x'} e^{-x'^2} \; dx' \approx 0.61 \]

\[ \frac{V'T'}{VT} \sim 0.61^4 \approx 0.13 \]

<table>
<thead>
<tr>
<th>$a_0$</th>
<th>$l$ [$W/cm^2$]</th>
<th>$\gamma$</th>
<th>$N_{C-BW}$</th>
</tr>
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<tbody>
<tr>
<td>100</td>
<td>$10^{22}$</td>
<td>2000</td>
<td>$10^6$</td>
</tr>
<tr>
<td>100</td>
<td>$10^{22}$</td>
<td>550</td>
<td>170/20000</td>
</tr>
<tr>
<td>3.2</td>
<td>$10^{19}$</td>
<td>20000</td>
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</tr>
<tr>
<td>29</td>
<td>$10^{21}$</td>
<td>2000</td>
<td>170/20000</td>
</tr>
</tbody>
</table>
Deviation from counterpropagating geometry

In the parameter $\chi$:

$$2\gamma \rightarrow \gamma (1 + \beta \cos \theta) \approx 2\gamma(1 - \theta^2 / 4)$$

$$\gamma_{eff} = \gamma(1 - \theta^2 / 4)$$

At non-collinear geometry the PP rate is the same as in the case of the counterpropagating case when $\gamma \rightarrow \gamma_{eff}$
The scalar potential $\phi$ can be calculated from the Lorentz gauge condition
\[
\partial \phi / \partial t + \nabla \cdot \mathbf{A} = 0.
\]
The electromagnetic fields are derived from
\[
\mathbf{E} = -\partial \mathbf{A} / \partial t - \nabla \phi, \quad \mathbf{B} = \nabla \times \mathbf{A}.
\]

herewith, $\mathbf{E} = \mathbf{E}^{(\hat{x})} + \mathbf{E}^{(\hat{y})}, \mathbf{B} = \mathbf{B}^{(\hat{x})} + \mathbf{B}^{(\hat{y})}$:

\[
E_{x}^{(\hat{x})} = \frac{-E_{1}}{C_{1}^{2}} \left[ C_{1}^{3} - f^{2} C_{1} \frac{x^{2}}{z_{r}^{2}} + \frac{fC_{1}}{k_{0} z_{r}} - \frac{2i f k_{0} x^{2}}{z_{r} (2k_{0} C_{2} + \eta)^{2}} \right],
\]
\[
E_{y}^{(\hat{x})} = \frac{E_{1} f x y}{C_{1}^{2} z_{r}^{2}} \left[ fC_{1} + \frac{2i k_{0} z_{r}}{(2k_{0} j_{z} + \eta)^{2}} \right],
\]
\[
E_{z}^{(\hat{x})} = \frac{-E_{1} f x}{C_{1}^{2} z_{r}^{2}} \left[ i f C_{1} \frac{s^{2}}{s^{2} + i \eta} - C_{1} z_{r} \left( i + \frac{if^{2} r^{2}}{4z_{r}^{2}} - \frac{\eta}{s^{2}} \right)
- \frac{i C_{1} z_{r}}{s^{2} + i \eta} - z_{r} C_{2} \right],
\]
\[
B_{x}^{(\hat{x})} = 0,
\]
\[
B_{y}^{(\hat{x})} = E_{1} \left( \frac{if}{2k_{0} z_{r}} - i - \frac{if^{2} r^{2}}{4z_{r}^{2}} + \frac{\eta}{s^{2}} + \frac{1}{is^{2} - \eta} \right),
\]
\[
B_{z}^{(\hat{x})} = \frac{E_{1} f y}{z_{r}},
\]
\[ E_y^{(y)} = \frac{-E_1}{C_1^2} \left[ C_1^3 - f^2 C_1 \frac{y^2}{z_r^2} + \frac{f C_1}{k_0 z_r} - \frac{2 i k_0 y^2}{z_r (2 k_0 C_2 + \eta)^2} \right], \]

\[ E_x^{(y)} = \frac{E_1 f x y}{C_1^2 z_r^2} \left[ f C_1 + \frac{2 i k_0 z_r}{(2 k_0 j_z + \eta)^2} \right], \]

\[ E_z^{(y)} = \frac{-E_1 f y}{C_1^2 z_r^2} \left[ \frac{i f C_1}{k_0} - C_1 z_r \left( i + \frac{i f^2 r^2}{4 z_r^2} - \frac{\eta}{s^2} \right) - \frac{i C_1 z_r}{s^2 + i \eta} - z_r C_2 \right], \]

\[ B_y^{(y)} = 0, \]

\[ B_x^{(y)} = -E_1 \left( \frac{i f}{2 k_0 z_r} - i - \frac{i f^2 r^2}{4 z_r^2} + \frac{\eta}{s^2} + \frac{1}{i s^2 - \eta} \right), \]

\[ B_z^{(y)} = \frac{-E_1 f x}{z_r}, \]

where \( E_1 = E_0 \psi e^{i \eta} / S_0 \), the normalization parameter \( S_0 = [-s_1^2 - 1/(k_0 z_r)] / s_1 \), \( s_1 = i[1 + 1/(2 k_0 z_r) + 1/s^2] \), \( j_z = z + i z_r \) and \( s \), \( \eta \), \( C_1 \), \( C_2 \) are determined by:

\[ C_1 = i + \frac{ik_0 r^2 - 2k_0 j_z - \eta}{(2 k_0 j_z + \eta)^2} + \frac{s^2 + i \eta s^2 - \eta^2}{s^2 (\eta - i s^2)}, \]

\[ C_2 = \frac{-1}{(2 k_0 j_z + \eta)^2} + \frac{4 k_0 j_z + 2 \eta - 2ik_0 r^2}{(2 k_0 j_z + \eta)^3} \]

\[ + \frac{\eta^2 - s^2 - i \eta s^2}{(i \eta s + s^3)^2} + \frac{2i \eta + s^2}{i \eta s^2 + s^4}. \]
QED with intense laser pulses

Historical Remark: SLAC Experiment

The first laboratory evidence of multiphoton pair production

\[ e + N\omega \rightarrow e' + e^+e^- \]

\[ e + \omega \rightarrow e' + \gamma \]

\[ \gamma + N\omega \rightarrow e^+e^- \]