Evolution of Isolated Rotating White Dwarfs

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Uniformly Rotating General Relativistic White Dwarfs
- WD EoS: relativistic Feynman-Metropolis-Teller
- General Relativistic FMT Theory of WDs
- Hartle-Thorne Formalism
- Stability criteria for rotating WDs

Rotating White Dwarfs As Progenitors (Precursors) of Type Ia Supernovae and Millisecond Pulsars
- Type Ia Supernovae From Very Long Delayed Explosion of Core-WD Merger
- Delayed gravitational collapse of Super Chandrasekhar WDs
- Spin-up and spin-down stages

Preliminary Results
- Evolution of the physical parameters
- Lifespan of isolated white dwarfs

Conclusion
How will isolated white dwarfs evolve?

**Super-Chandrasekhar WDs**
- slow down -> ?, no
- explode as a type Ia supernova - > ?, yes?
- collapse into a neutron star - > ?, yes?

**Sub-Chandrasekhar WDs**
- slow down - > ?, yes
- will live forever - >?, yes
# Equation of state

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical effects, parameters and approaches</td>
<td>Quantum statistics (pressure of degenerate electrons)</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td></td>
<td>Electromagnetic interaction (electron-electron and electron-nuclei Coulomb interactions)</td>
<td>no</td>
<td>yes, however point like nuclei, uniform distribution of electrons</td>
</tr>
<tr>
<td></td>
<td>Weak interaction (inverse beta decay, beta equilibrium)</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td></td>
<td>Nuclear interaction – determination of the correct mass of a nucleus</td>
<td>yes</td>
<td>yes</td>
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</table>

General Relativistic FMT Theory of WDs

(Rotondo, Rueda, Ruffini, Xue, Phys. Rev. D 84, 084007, 2011)

Critical masses and densities of He, C, O, Fe WDs

<table>
<thead>
<tr>
<th>Composition</th>
<th>( \rho_{\text{crit}} ) (g/cm(^3))</th>
<th>Instability</th>
<th>( M_{\text{max}}^{J=0} / M_\odot )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^4\text{He})</td>
<td>(1.56 \times 10^{10})</td>
<td>GR</td>
<td>1.40906</td>
</tr>
<tr>
<td>(^{12}\text{C})</td>
<td>(2.12 \times 10^{10})</td>
<td>GR</td>
<td>1.38603</td>
</tr>
<tr>
<td>(^{16}\text{O})</td>
<td>(1.94 \times 10^{10})</td>
<td>inverse (\beta)-decay</td>
<td>1.38024</td>
</tr>
<tr>
<td>(^{56}\text{Fe})</td>
<td>(1.18 \times 10^{9})</td>
<td>inverse (\beta)-decay</td>
<td>1.10618</td>
</tr>
</tbody>
</table>
Hartle’s Formalism


\[
ds^2 = e^{\nu(r)} [1 + 2h(r, \theta)] dt^2 - e^{\lambda(r)} \left(1 + \frac{2m(r, \theta)}{r - M^J=0(r)}\right) dr^2 - r^2 [1 + 2k(r, \theta)] \{d\theta^2 + \sin^2 \theta (d\phi - \omega dt)^2\}
\]

where 
- \(h(r, \theta) = h_0(r) + h_2(r)P_2(\cos \theta) + \ldots\)
- \(m(r, \theta) = m_0(r) + m_2(r)P_2(\cos \theta) + \ldots\)
- \(k(r, \theta) = k_2(r)P_2(\cos \theta) + \ldots\)
- \(e^{\lambda(r)} = [1 - 2M^J=0(r)/r]^{-1}\)

\(\omega(r)\), proportional to \(\Omega\)

\(h_0, h_2, m_0, m_2, k_2\), proportional to \(\Omega^2\)
Stability criteria for rotating WDs

Mass-Shedding Limit

\[
\Omega_{\text{orb}}(r) = \Omega_0(r) \left[ 1 - j F_1(r) + j^2 F_2(r) + q F_3(r) \right],
\]

\[ j = cJ/(GM^2) \text{ and } q = c^4Q/(G^2M^3) \]

\[ \Omega_0 = \frac{M^{1/2}}{r^{3/2}}, \quad F_1 = \frac{M^{3/2}}{r^{3/2}}, \]

\[ F_2 = (48M^7 - 80M^6r + 4M^5r^2 - 18M^4r^3 + 40M^3r^4 + 10M^2r^5 + 15Mr^6 - 15r^7)/[16M^2r^4(r - 2M)] + F, \]

\[ F_3 = \frac{6M^4 - 8M^3r - 2M^2r^2 - 3Mr^3 + 3r^4}{16M^2r(r - 2M)/5} - F, \]

\[ F = \frac{15(r^3 - 2M^3)}{32M^3} \ln \frac{r}{r - 2M}. \]

Turning Points

\[
\left( \frac{\partial M(\rho_c, J)}{\partial \rho_c} \right)_{J} = 0
\]


Inverse beta decay

<table>
<thead>
<tr>
<th>Decay</th>
<th>$\beta$</th>
<th>$\rho_{\text{crit}}^{\beta, \text{relFMT}}$</th>
<th>$\rho_{\text{crit}}^{\beta, \text{unif}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4\text{He} \rightarrow^3\text{H} + n \rightarrow 4\text{n}$</td>
<td>20.596</td>
<td>$1.39 \times 10^{11}$</td>
<td>$1.37 \times 10^{11}$</td>
</tr>
<tr>
<td>$^{12}\text{C} \rightarrow^{12}\text{B} \rightarrow^{12}\text{Be}$</td>
<td>13.370</td>
<td>$3.97 \times 10^{10}$</td>
<td>$3.88 \times 10^{10}$</td>
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<tr>
<td>$^{16}\text{O} \rightarrow^{16}\text{N} \rightarrow^{16}\text{C}$</td>
<td>10.419</td>
<td>$1.94 \times 10^{10}$</td>
<td>$1.89 \times 10^{10}$</td>
</tr>
<tr>
<td>$^{56}\text{Fe} \rightarrow^{56}\text{Mn} \rightarrow^{56}\text{Cr}$</td>
<td>3.695</td>
<td>$1.18 \times 10^{9}$</td>
<td>$1.14 \times 10^{9}$</td>
</tr>
</tbody>
</table>
Equilibrium WD configurations


Carbon WD for RFMT EoS.

Oxygen WD for RFMT EoS.
Equilibrium WD configurations


Carbon WD for RFMT EoS.

Oxygen WD for RFMT EoS.
Rotating WD properties

<table>
<thead>
<tr>
<th>Composition</th>
<th>$\rho_{M^{J\neq 0}_{max}}$</th>
<th>$k$</th>
<th>$M^{J=0}<em>{max}/M</em>\odot$</th>
<th>$R_{M^{J=0}_{max}}$</th>
<th>$\sigma$</th>
<th>$P_{min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4\text{He}$</td>
<td>$5.46\times 10^9$</td>
<td>1.0646</td>
<td>1.40906</td>
<td>1163</td>
<td>0.26952</td>
<td>0.284</td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>$6.95\times 10^9$</td>
<td>1.0632</td>
<td>1.38603</td>
<td>1051</td>
<td>0.54692</td>
<td>0.501</td>
</tr>
<tr>
<td>$^{16}\text{O}$</td>
<td>$7.68\times 10^9$</td>
<td>1.0626</td>
<td>1.38024</td>
<td>1076</td>
<td>0.72343</td>
<td>0.687</td>
</tr>
<tr>
<td>$^{56}\text{Fe}$</td>
<td>$1.18\times 10^9$</td>
<td>1.0864</td>
<td>1.10618</td>
<td>2181</td>
<td>0.71685</td>
<td>2.195</td>
</tr>
</tbody>
</table>

**Maximum Mass and Minimum Period**

$$M^{J\neq 0}_{max} = k M^{J=0}_{max} \sim 1.500, 1.474, 1.467, 1.202 \ M_\odot$$

$$P_{min} = \sigma \left( \frac{M_\odot}{M^{J=0}_{max}} \right)^{\frac{1}{2}} \left( \frac{R_{M^{J=0}_{max}}}{10^3 \text{km}} \right)^{\frac{3}{2}} \text{sec} \sim 0.3, 0.5, 0.7, 2.2 \text{ sec}$$

Astrophysical Implications of WDs
Type Ia Supernovae From Very Long Delayed Explosion of Core-WD Merger

It is assumed that $J$ is proportional to $\Omega$, i.e. the moment of inertia $I$ is constant.
Type Ia Supernovae From Very Long Delayed Explosion of Core-WD Merger


\[ \dot{E}_{rot} = \frac{dJ}{d\Omega} \frac{d\Omega}{dt} = -4\pi^2 I \frac{P}{P^3} \]

\[ \dot{E}_{EM} = -\frac{2 B^2 R^6}{3 c^3} \Omega^4 = -\frac{32\pi^4 B^2 R^6}{3 c^3 P^4} \]

\[ \tau_B \simeq \frac{I e^3}{B^2 R^6 \Omega_c^2} \left[ 1 - \left( \frac{\bar{\Omega}_0}{\bar{\Omega}_c} \right)^{-2} \right] (\sin \delta)^{-2} \approx 10^8 \left( \frac{B}{10^8 \text{ G}} \right)^{-2} \left( \frac{\bar{\Omega}_c}{0.7 \Omega_{\text{Kep}}} \right)^{-2} \]

\[ \times \left( \frac{R}{4000 \text{ km}} \right)^{-1} (\sin \delta)^{-2} \left( \frac{\beta_I}{0.1} \right) \left( \frac{\beta_I}{0.3} \right) \left[ 1 - \left( \frac{\bar{\Omega}_0}{\bar{\Omega}_c} \right)^{-2} \right] \text{ yr}, \]

It is assumed that \( J \) is proportional to \( \Omega \), i.e. the moment of inertia \( I \) is constant.

The result...

\[ 10^6 \text{ G} \lesssim B \sin \delta \lesssim 10^8 \text{ G} \]

\[ 10^7 \lesssim t \lesssim 10^{10} \text{ yr} \]
Moment of Inertia of NR and RWDs

\[ I^* = 10^{50} \text{ g } \times \text{cm}^2 \]

Carbon WD for RFMT EoS.

\[ \frac{I}{I^*} \]

\[ \rho_c \left[ \text{g/cm}^3 \right] \]

- Keplerian sequence
- Static configuration

Spin-up and spin-down stages

\[ \dot{\Omega} = \frac{\dot{E}}{\Omega} \left( \frac{\partial \Omega}{\partial J} \right)_{M_0,S,Z,A} \]

Delayed gravitational collapse of Super Chandrasekhar WDs

\[ t = -\frac{3}{2} \frac{c^3}{B^2} \int_{J_{\text{min}}}^{J_{\text{max}}} \frac{1}{R^6 \Omega^3} \, dJ, \quad R = R(\rho, J), \quad \Omega = \Omega(\rho, J) \]

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Preliminary results
Time versus magnetic field

Assuming constant $B$ over the course of time
Central density versus time

Assuming constant $B$ over the course of time

$B = 10^6 \, G$
Mean radius versus time

Assuming constant $B$ over the course of time
Central density versus time

Assuming magnetic flux conservation over the course of time
Mean radius versus time

Assuming magnetic flux conservation over the course of time
Moment of inertia versus time

Assuming magnetic flux conservation over the course of time
Concluding remarks

- Stability of rotating WDs is a delicate matter: all mass-shedding, inverse beta decay, and secular instability play role
- Both spin-down and spin-up stages by loosing angular momentum are possible in WDs
- Super Chandrasekhar WDs can only spin up by angular momentum loss
- The delayed time for gravitational collapse of a WD via magnetic braking is comprised in a variety of ranges upon the magnetic field value
Concluding remarks

• We showed that WDs composed of light elements (Helium, Carbon) are unstable against axisymmetric secular instability, whereas WDs with heavy elements (Oxygen,.., Iron) are stable.

• Evolution the physical parameters of WDs over time due to the angular momentum loss.

• The magnetic flux conservation shortens the lifespan of WDs .

• It will be interesting to consider the effects of temperature.

• Work in progress
Thank you for your attention!