The Fireshape model for GRBs

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(together with a very large number of ICRANet Faculty, researchers, post-docs, and Ph.D. students over the last 15 years)

1st Scientific ICRANet Meeting in Armenia
Yerevan – Armenia – June 30th 2014
(some) Key Features of the Model

- No need for beaming to reduce energy budget ($10^{55}$ ergs allowed).

- Prompt emission made of two different components: the Proper GRB (P-GRB) and the extended afterglow peak.

- (Quasi)-Thermal spectrum in the fireshell co-moving frame for the prompt emission.

- Short and Long GRBs differ only by their different fireshell baryon loading and surrounding medium.

- Need to distinguish “genuine” short and “disguised” short GRBs.
The Dyadosphere

\[ \mu = \frac{M}{M_\odot} \quad \zeta = \frac{Q}{M} \]

\[ r_+ \approx 1.47 \times 10^5 \mu \left( 1 + \sqrt{1 - \zeta^2} \right) \text{ cm} \]

\[ r_{ds} \approx 1.12 \times 10^8 \sqrt{\mu \zeta} \text{ cm} \]


The Dyadotorus

Thermalization of $e^+e^-\gamma$ plasma

Starting with a pure $e^+e^-$ configuration

Dotted lines: *Neglecting* inverse 3-body interactions (Cavallo & Rees, 1978).

Solid lines: *Considering* inverse 3-body interactions (Aksenov, et al., 2007).

Starting with a pure $\gamma$ configuration

The optically thick *fireshell*: 1) The PEM Pulse

**Sphere:** PEM pulse has an uniform density profile which decreases in time like a portion of Friedmann Universe.

**Slab 1:** PEM pulse thickness is constant if measured in the laboratory frame.

**Slab 2:** PEM pulse thickness is constant if measured in the co-moving frame.

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Engulfment of baryonic remnant

\[ B \overset{\text{def.}}{=} \frac{N_B m_p c^2}{E_{\text{dy}} a} \equiv \frac{M_B c^2}{E_{\text{dy}} a} \]

Assumptions:
- PEM pulse geometry unchanged;
- Fully inelastic collision;
- Thermal equilibrium between baryons and PEM pulse.

Valid if:
- \( B < 10^{-2} \);
- \( n_{e^+e^-}/n_B \) very high (e.g. > 10^6);
- PEM pulse \( \gamma \) very high: \( (\gamma \geq 100) \).

Results:
- Strong decrease in PEM pulse \( \gamma \) Lorentz factor:
  \[ \gamma_{\text{coll}} = \gamma_0 \frac{1 + B}{\sqrt{\gamma_0^2 (2B + B^2) + 1}} \]
- Rise of the PEM pulse internal energy measured in the co-moving frame:
  \[ \frac{E_{\text{coll}}}{E_{\text{dy}}} = \frac{1}{\gamma_0} \sqrt{\gamma_0^2 (2B + B^2) + 1} - \left( \frac{1}{\gamma_0} + B \right) \]
- A corresponding re-heating of the plasma in the co-moving frame, but not in the laboratory frame, a rise of \( e^+e^- \) pair number and a greater PEM pulse opacity.

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- Rise of the PEM pulse internal energy measured in the co-moving frame:

$$\frac{E_{\text{coll}}}{E_{\text{dya}}} = \frac{1}{\gamma_0} \sqrt{\gamma_0^2 (2B + B^2) + 1} - \left( \frac{1}{\gamma_0} + B \right);$$

- A corresponding re-heating of the plasma in the co-moving frame, but not in the laboratory frame, a rise of $e^+e^-$ pair number and a greater PEM pulse opacity.

The optically thick fireshell: 2) The PEMB Pulse

\[ \gamma_{\text{asym}} \overset{\text{def.}}{=} \frac{E_{\text{dy}}}{M_B c^2} \equiv \frac{1}{B} \]

At transparency point:

\[ \gamma \rightarrow 10^{-2} \rightarrow \gamma_{\text{asym}} \]

The percentage of PEMB Pulse internal energy converted into baryonic kinetic energy rises with rising \( B \).
Fireshell dynamics

Baryon conservation:
\[ \frac{n^0_B}{n_B} = \frac{V}{V_0} = \frac{\mathcal{V}}{V_0} \frac{\gamma}{\gamma_0} \quad V = \gamma \mathcal{V} \]

Adiabatic expansion:
\[ \frac{\tilde{\varepsilon}_0}{\varepsilon} = \left( \frac{V}{V_0} \right) \frac{\tilde{\gamma}}{\gamma_0} = \left( \frac{\mathcal{V}}{V_0} \right) \frac{\tilde{\gamma}}{\gamma_0} \]

Energy conservation:
\[ T^{0\nu} = 0 \]

Rate equation:
\[ \frac{\partial}{\partial t} N_{e^\pm} = -N_{e^\pm} \frac{1}{V} \frac{\partial V}{\partial t} + \frac{\sigma v}{\gamma^2} \left( N^2_{e^\pm} (T) - N^2_{e^\pm} \right) \]

\( V \) – comoving volume,
\( \mathcal{V} \) – lab volume,
\( n_B \) – baryon number density,
\( \tilde{\gamma} \) – thermal index,
\( \tilde{\varepsilon} \) – internal energy density,
\( \rho_B \) – rest mass density,
\( \sigma \) – cross section,
\( v \) – velocity,
\( N_{e^\pm} \) – number of pairs,
\( T \) – plasma temperature.

The optically thick *fireshell*: The PEMB Pulse

\[ E_{e^+}^{\text{tot}} = 1.44 \times 10^{49} \text{ erg} \]

Graph showing the variation of the Lorentz \( \gamma \) factor with the fireshell radius (cm) for different values of magnetic field strength, \( B \).
The optically thick fireshell: The PEMB Pulse
Changing the baryon loading location

Fireshell pair number density: the role of the rate equation

$E_{\text{e}^\pm}^{\text{tot}} = 1.44 \times 10^{49} \text{ erg}$
Fireshell pair number density: the role of the rate equation

\[ E_{\text{tot}}^\text{1.22 \times 10^{55}} \text{ erg} \]

\[ E_{\text{tot}}^\text{1.22 \times 10^{55}} \text{ erg} \]

From Fermi integrals
Actual - B=10^{-2}
Actual - B=10^{-3}
Actual - B=10^{-4}
Actual - B=10^{-5}
Actual - B=10^{-6}
Actual - B=10^{-7}
Actual - B=10^{-8}
Fireshell Temperature

$E_{e^+}^{\text{tot}} = 1.44 \times 10^{49}$ erg

Doppler blue-shifted toward the observer

In the co-moving frame

Fireshell temperature (keV)

Fireshell radius (cm)
Fireshell Temperature

$E_e^{\text{tot}} = 1.22 \times 10^{55}$ erg

- Doppler blue-shifted toward the observer
- In the co-moving frame
**Fireshell transparency:**

the P-GRB emission and the optically thin *fireshell*

- All photons produced by the electron-positron pair recombination are emitted in a short but strong energy flash: *the P-GRB*

- Out of the PEMB pulse it remains only an optically thin *fireshell* of baryonic matter accelerated at ultra-relativistic velocities: the ABM Pulse, which gives origin to *the extended afterglow*.

- The ratio between the fraction of PEMB Pulse internal energy converted into ABM Pulse kinetic energy and that emitted in the P-GRB depends on the $B$ parameter.

$$B \overset{\text{def.}}{=} \frac{N_B m_p c^2}{E_{\text{dya}}} = \frac{M_B c^2}{E_{\text{dya}}}$$

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The Fireshell transparency

Comparison of the fireshell model with

**Shemi and Piran (1990)**

- **Piran, Shemi, Narayan (1993)**
  - Adiabatic expansion:
    \[
    \frac{\mathcal{E}}{\mathcal{E}_0} = \frac{T}{T_0} = \frac{R_0}{R}
    \]
  - Energy conservation:
    \[
    \gamma = \frac{\mathcal{E}_0 + \mathcal{M}c^2}{\mathcal{E} + \mathcal{M}c^2}
    \]


- Adiabatic expansion:
  \[
  \frac{\tilde{\mathcal{E}}}{\tilde{\mathcal{E}}_0} = \left(\frac{V}{V_0}\right)^\tilde{\eta} = \left(\frac{\mathcal{V}}{\mathcal{V}_0}\right)^\tilde{\eta} \left(\frac{\gamma}{\gamma_0}\right)^\tilde{\eta},
  \]
- Energy conservation:
  \[
  \frac{\gamma}{\gamma_0} = \sqrt{(\tilde{\mathcal{E}}_0 + \rho_B^0)\mathcal{V}_0}{(\tilde{\mathcal{E}} + \rho_B)\mathcal{V}}
  \]
- Baryon conservation:
  \[
  \frac{n_B^0}{n_B} = \frac{V}{V_0} = \frac{\mathcal{V}}{\mathcal{V}_0} \frac{\gamma}{\gamma_0}
  \]
- Rate equation:
  \[
  \frac{\partial}{\partial t} N_{e^\pm} = -N_{e^\pm} \frac{1}{\mathcal{V}} \frac{\partial \mathcal{V}}{\partial t} + \frac{\sigma v}{\gamma^2} (N_{e^\pm}^2(T) - N_{e^\pm}^2)
  \]

**Fireshell**

• Transparency is not an instantaneous process

• Even for $B$ close to $10^{-2}$ some fraction of the energy is carried by photons emitted at transparency point

• The difference is essential for $\gamma(B), R_{tr}(B), E_{\gamma}(B), E_k(B)$

Shemi, Piran & Narayan
Fireshell model

Assumptions:
- sequence of fully inelastic collisions of the ABM pulse with a great number of ISM concentric spherical shells, thin, cold, at rest in the laboratory frame;
- ABM pulse thickness is constant if measured in the laboratory frame;
- the energy released in the collision is emitted instantaneously (fully radiative condition).

\[
\gamma_2 = \sqrt{1 + 2\gamma_1 \frac{\Delta M_{\text{ism}} c^2}{\rho B_1 V_1} + \left( \frac{\Delta M_{\text{ism}} c^2}{\rho B_1 V_1} \right)^2}
\]

\[
\Delta E_{\text{int}} = \rho B_1 V_1 \sqrt{1 + 2\gamma_1 \frac{\Delta M_{\text{ism}} c^2}{\rho B_1 V_1} + \left( \frac{\Delta M_{\text{ism}} c^2}{\rho B_1 V_1} \right)^2} - \rho B_1 V_1 \left( 1 + \frac{\Delta M_{\text{ism}} c^2}{\rho B_1 V_1} \right)
\]

Interaction ABM Pulse – ISM

Assumptions:
- sequence of fully inelastic collisions of the ABM pulse with a great number of ISM concentric spherical shells, thin, cold, at rest in the laboratory frame;
- ABM pulse thickness is constant if measured in the laboratory frame;
- the energy released in the collision is emitted instantaneously (fully radiative condition).

\[ \gamma_2 = \sqrt{1 + 2 \gamma_1 \frac{\Delta M_{\text{ism}} c^2}{\rho B_1 V_1} + \left( \frac{\Delta M_{\text{ism}} c^2}{\rho B_1 V_1} \right)^2} \]

\[ \Delta E_{\text{int}} = \rho B_1 V_1 \sqrt{1 + 2 \gamma_1 \frac{\Delta M_{\text{ism}} c^2}{\rho B_1 V_1} + \left( \frac{\Delta M_{\text{ism}} c^2}{\rho B_1 V_1} \right)^2} - \rho B_1 V_1 \left( 1 + \frac{\Delta M_{\text{ism}} c^2}{\rho B_1 V_1} \right) \]

**ABM Pulse – ISM: differential form**

\[
\begin{align*}
\frac{dE_{\text{int}}}{dM_{\text{ism}}} &= (\gamma - 1) c^2 dM_{\text{ism}} \\
\frac{d\gamma}{dM} &= -\frac{\gamma^2 - 1}{M} dM_{\text{ism}} \\
dM &= \frac{1 - \varepsilon}{c^2} dE_{\text{int}} + dM_{\text{ism}} \\
dM_{\text{ism}} &= 4\pi m_p n_{\text{ism}} r^2 dr
\end{align*}
\]

- **Fully radiative condition:** \( \varepsilon = 1 \)
  - \( \gamma_0 \gg \gamma \gg 1 \)
  - **Blandford – McKee**
  - Analytic integration

- **Fully adiabatic condition:** \( \varepsilon = 0 \)
  - \( \gamma_0^2 \gg \gamma^2 \gg 1 \)
  - **Blandford – McKee**
  - Analytic integration

- **Analytic integration**

\[
\begin{align*}
\gamma &= r^{-3} \\
t &= \frac{r}{c} \left( 1 + \frac{1}{14\gamma^2} \right)
\end{align*}
\]

\[
\begin{align*}
\gamma &= r^{-3/2} \\
t &= \frac{r}{c} \left[ 1 + \frac{1}{8\gamma^2 (r)} \right]
\end{align*}
\]

Comparison between exact and approximate afterglow equations of motion (radiative and adiabatic).

Exact solution

Effective power-law index: $\gamma \propto r^{-a}$

Comparison between exact and approximate afterglow equations of motion (radiative and adiabatic)

- Exact solution
  - Effective power-law index: $\gamma \propto r^{-a}$
  - $a = 3.0$ ($\gamma_0 \gg \gamma \gg 1$)
  - $a = 1.5$ ($\gamma_0^2 \gg \gamma^2 \gg 1$)

Comparison between exact and approximate afterglow equations of motion (radiative and adiabatic)

Exact solution

Effective power-law index: $\gamma \propto r^{-a}$

$a = 3.0$ ($\gamma_0 \gg \gamma \gg 1$)

$a = 1.5$ ($\gamma_0^2 \gg \gamma^2 \gg 1$)

$\gamma_0 = 10^2$

Comparison between exact and approximate afterglow equations of motion (radiative and adiabatic)

Exact solution

Effective power-law index: $\gamma \propto r^{-a}$

$a = 3.0 \ (\gamma_0 \gg \gamma \gg 1)$

$a = 1.5 \ (\gamma_0^2 \gg \gamma^2 \gg 1)$

$\gamma_0 = 10^3$

Comparison between exact and approximate afterglow equations of motion (radiative and adiabatic).

For $a = 3.0$ ($\gamma_0 \gg \gamma \gg 1$):

$$a = 3.0$$

For $a = 1.5$ ($\gamma_0^2 \gg \gamma^2 \gg 1$):

$$a = 1.5$$

$\gamma_0 = 10^4$

Comparison between exact and approximate afterglow equations of motion (radiative and adiabatic)

Exact solution

Effective power-law index: $\gamma \propto r^{-a}$

- $a = 3.0$ ($\gamma_0 >> \gamma >> 1$)
- $a = 1.5$ ($\gamma_0^2 >> \gamma^2 >> 1$)

$\gamma_0 = 10^5$

Comparison between exact and approximate afterglow equations of motion (radiative and adiabatic).

Exact solution

Effective power-law index: $\gamma \propto r^{-a}$

- For $a = 3.0$ ($\gamma_0 > > \gamma > 1$)
  - $\gamma_0 = 10^6$

- For $a = 1.5$ ($\gamma_0^2 > > \gamma^2 > 1$)
Comparison between exact and approximate afterglow equations of motion: radiative and adiabatic.

**Exact solution**

Effective power-law index: $\gamma \propto r^{-a}$

- $a = 3.0 \ (\gamma_0 \gg \gamma \gg 1)$
- $a = 1.5 \ (\gamma_0^2 \gg \gamma^2 \gg 1)$

\[ \gamma_0 = 10^7 \]

Comparison between exact and approximate afterglow equations of motion (radiative and adiabatic).

**Exact solution**

Effective power-law index: \( \gamma \propto r^{-a} \)

- \( a = 3.0 \) (\( \gamma_0 \gg \gamma \gg 1 \))
- \( a = 1.5 \) (\( \gamma_0^2 \gg \gamma^2 \gg 1 \))

\( \gamma_0 = 10^8 \)

Comparison between exact and approximate afterglow equations of motion

radiative and adiabatic

Exact solution

Effective power-law index: $\gamma \propto r^{-a}$

$\gamma_0 = 10^9$

Comparison between exact and approximate afterglow equations of motion (radiative and adiabatic)

The power-law expansion *never* applies in GRB prompt emission

- $a = 3.0 \; (\gamma_0 \gg \gamma \gg 1)$
- $a = 1.5 \; (\gamma_0^2 \gg \gamma^2 \gg 1)$

$\gamma_0 = 10^9$

Comparison between exact and approximate afterglow equations of motion (radiative and adiabatic)

**Exact solution**

- Effective power-law index: $\gamma \propto r^{-a}$
  - $a = 3.0$ ($\gamma_0 >\gamma > 1$)
  - $a = 1.5$ ($\gamma_0^2 >\gamma^2 > 1$)
  - $\gamma_0 = 10^2$

**The relevant case for GRBs!**

Comparison between exact and approximate afterglow equations of motion (radiative and adiabatic).

The power-law expansion *never* applies to actual GRBs at all.

- $a = 3.0 \ (\gamma_0 \gg \gamma \gg 1)$
- $a = 1.5 \ (\gamma_0^2 \gg \gamma^2 \gg 1)$

The relevant case for GRBs!

$\gamma_0 = 10^2$

Moving sources:
Arrival time and emission time
Moving sources:
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Moving sources:
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Moving sources:
Arrival time and emission time

\[ t \approx \gamma R \]
Moving sources:
Arrival time and emission time

$t_0 + \Delta t$  

$\gamma \approx 4$  

$t$  

$R$
Moving sources: Arrival time and emission time

\[ t \to t_0 + \Delta t \]

\[ \gamma \approx 4 \]
Moving sources:
Arrival time and emission time

\[ t = \frac{r (\Delta t)}{c} = \frac{v}{c} \Delta t \]

\[ \gamma \approx 4 \]
Moving sources:
Arrival time and emission time

\[
\Delta t_a = \Delta t - \frac{r(\Delta t)}{c} = \Delta t \left(1 - \frac{v}{c}\right)
\]

\[
\gamma \approx 4
\]

\[
\Delta t \equiv t_0 + \Delta t
\]

\[
t \equiv t_0 + \Delta t
\]

\[
R
\]

Arrival time and emission time differ by an amount that depends on the velocity of the source.
Moving sources:

Arrival time and emission time

The arrival time on the Earth of a signal depends on the motion of the source!

\[ t_R = t_0 + \Delta t \]

\[ \gamma \approx 4 \]

\[ \Delta t_a = \Delta t - \frac{r(\Delta t)}{c} = \Delta t \left(1 - \frac{v}{c}\right) \]
The arrival time on the Earth of a signal depends on the motion of the source!

It’s possible to observe superluminal velocities!

\[
\Delta t_a = \Delta t - \frac{r(\Delta t)}{c} = \Delta t \left(1 - \frac{v}{c}\right)
\]

\[
\gamma \approx 4
\]
Moving sources: Constant speed vs. variable speed

\[
\Delta t_a = \Delta t - \frac{r(\Delta t)}{c} = \Delta t \left(1 - \frac{v}{c}\right) \approx \frac{\Delta t}{2\gamma^2} = \frac{\Delta t}{32}
\]

Moving sources:
Constant speed vs. variable speed

\[ \Delta t_a = \Delta t - \frac{r(\Delta t)}{c} = \Delta t \left(1 - \frac{v}{c}\right) \approx \frac{\Delta t}{2\gamma^2} = \frac{\Delta t}{32} \]

\[ \Delta t_a = \Delta t - \frac{r(\Delta t)}{c} = \Delta t - \frac{1}{c} \int_{t_0}^{t_0 + \Delta t} v(t) \, dt \]

\( \gamma \approx 4 \)

1 ≤ \( \gamma \) ≤ 300 (in GRBs)

\[ \Delta t_a = \Delta t - \frac{1}{c} \left[ \int_{t_0}^{t_0+\Delta t} v(t) \, dt + r(t_0) \right] \cos \vartheta + \frac{r(t_0)}{c} \]

The EQuiTemporal Surfaces (EQTSSs)

\[ t_{\alpha}^d = (1 + z) \left[ t(r) - \frac{r}{c} \cos \theta + \frac{r^*}{c} \right] \]

The EQuiTemporal Surfaces (EQTSs)

\[ t_a^d = (1 + z) \left[ t(r) - \frac{r}{c} \cos \theta + \frac{r^*}{c} \right] \]

\[ t(r) = \frac{r}{v} \]

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\[ t^d_a = (1 + z) \left[ t(r) - \frac{r}{c} \cos \vartheta + \frac{r^*}{c} \right] \]

\[ t(r) = \frac{r}{v} \quad r^* = 0 \]

\[ r(\vartheta) = \frac{\frac{t^d_a}{v}}{1 + \frac{z}{v}} \frac{1}{1 - \frac{\cos \vartheta}{c}} \]

The EQuiTemporal Surfaces (EQTSs)

Ellipsoid with eccentricity $v/c$

$$t^d_a = (1 + z) \left[ t(r) - \frac{r}{c} \cos \vartheta + \frac{r^*}{c} \right]$$

$$t(r) = \frac{r}{v}$$

$$r^* = 0$$

$$r(\vartheta) = \frac{v \frac{t^d_a}{1 + z}}{1 - \frac{v}{c} \cos \vartheta}$$

LES AURÉOLES LUMINEUSES DES NOVAE

par Paul Couderc

Sommaire. — Trois sortes d'auréoles ont été découvertes successivement autour de Nova Persei 1901. L'auréole la plus célèbre présenta un accroissement très rapide : elle provenait d'une diffusion de la lumière par des nébuleuses immobiles voisines de la Nova.

Je montre que cette apparence se propage en général avec une vitesse de beaucoup supérieure à celle de la lumière. Le rayon de ces auréoles, lieu des points simultanément éclairés, croît comme $\sqrt{t}$, dans les conditions ordinaires d'observation.

D'après les clichés de 1901, les auréoles de Nova Persei vérifient parfaitement les prévisions théoriques ; la distribution dans l'espace des nébuleuses et leur structure en nappes minces se déduisent de cet examen.

Enfin, une étude théorique de la brillance dans l'auréole envisage diverses lois possibles de réflexion. Aucune ne paraît convenir aux clichés de Ritchey, ce qui suggère quelques remarques sur l'ordre de grandeur probable des particules réfléchissantes.

I

Introduction.

Le 21 août 1901, de Juvissy, Camille Flammarion et M. Antoniadi annonçaient par télexgramme à Max Wolf qu'une nébuleuse circulaire était visible sur des clichés de Nova Persei découvertes le 22 février 1901.

Max Wolf, avec les deux tubes de 41 cm d'ouverture de l'instrument de Heidelberg, reproduisit l'observation et démontra magistralement qu'il s'agissait d'une auréole parasite, due à ce que les objectifs employés n'étaient pas corrigés pour la lumière ultraviolette qu'envoyait la Nova [1]. Ce phénomène d'aberration chromatique provenait surtout des ondes $0 \mu$, 346.

Mais l'étude du phénomène parasite amena la découverte d'un phénomène céleste bien réel : sur d'excellents clichés, avec 4 h. de pose, Wolf reconnut la présence d'arcs nuageux finement structurés

ou, si $t = \tau$, $r = \rho$, rayon apparent de la sphère nébulaire,

$\omega^{2} + \tau^{2} - 2\tau = 0$.

La courbe représentative des variations de $\omega$ est un demi-cercle

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C.N.R.S. • Provided by the NASA Astrophysics Data System
In this article it is suggested that the central parts of some radio sources may expand with relativistic velocities. Relativistic effects could then have a decisive influence on the observable properties of these sources. In particular, the flux from a radio source expanding with relativistic velocity can change sufficiently rapidly to account for the variations observed in some sources, even if they are at cosmological distances. A related relativistic effect would imply that the ages of some types of radio sources may be significantly less than the usual estimates.

Variable Radio Sources

Intensity variations have been observed in several radio sources associated with quasi-stellar objects, the redshifts of which indicate that they are probably at cosmological distances. Well-known arguments concerning self-absorption enable us to place a lower limit on the dimensions of a source which is at a known distance (if the radio flux is synchrotron radiation), and it has proved difficult to understand how a source at a cosmological distance can vary with the observed rapidity unless its size is less than this limit. This problem arises if the assumption is made that the 'time-scale' of the variations cannot be much less than the time light would take to cross the emitting region. If (a) the variations are periodic, or (b) they result from changes in the brightness of a region of fixed size, this assumption is certainly necessary, but I shall describe a type of model where special relativistic effects arise which permit it to be relaxed.

In the model proposed, the source is assumed to be spherical and the radio variations are principally due to changes in its apparent diameter rather than in its surface brightness (in contrast to (b)). We shall show that, if the boundary of the source expands with velocity \( \sim \), the apparent rate of increase in its angular size and luminosity can be extremely high. Before discussing the physics of the model, and arguing the reasonableness of relativistic velocities, we shall illustrate its essential geometrical properties.

Consider a sphere, centre \( S \), the surface of which expands with radial velocity \( \) from zero initial radius. An observer \( O \) at a large distance \( R \) from \( S \) (and at rest relative to \( S \)) will observe an increase in the apparent size of the sphere.

If \( \) the observed angular diameter will be \( \frac{2}{\sqrt{e}} \), where \( t \) is the time measured from the moment when the expansion is seen to begin. However \( \) this result needs modification. The locus of points from which radiation reaches the observer at time \( t \) is not a sphere, but a spheroid with \( S \) as its axis along \( S0 \), eccentricity \( e/ \) and semi-latus rectum \( \gamma \), as shown in Fig. 1. Owing to aberration, the parts with velocity perpendicular to the observer are invisible if the sphere is opaque, and the apparent limb, in fact, has a velocity making an angle \( \cos^{-1} e/ \) (sin\(^{-1} e/ \) with \( S0 \). Furthermore the apparent component of velocity of the limb at right angles to the line of sight is \( \gamma \) (which can be \( \) if \( \) is large) and the observed angular diameter of the sphere at time \( t \) is \( \frac{2}{\sqrt{e}} \). If \( \gamma = 5 \), the apparent diameter of the source will increase by almost 10 light years each year. Because the observed intensity of a source, for a given surface brightness, is proportional to the apparent size, it is already clear that an expanding source could exhibit a rate of increase of flux density high enough to explain the observations.

This geometrical fact is obviously relevant to a variety of physical models, of which only one will be described here. Suppose that the source consists of a massive object (probably identifiable with a quasi-stellar object) which accelerates particles to relativistic energies. Suppose also that the surrounding magnetic field is weak, in the sense that \( \gamma \) is small compared with the kinetic energy density of the particles. A burst of relativistic particles ejected from the massive object will not be confined by the field, but will expand outwards, 'dragging' the field with it (that is, there will not be an electric field in a reference frame sharing the mean outward motion of the particles, though there will generally be in other frames). This explosion will have a relativistic velocity provided that the thermal gas density is small. We assume for simplicity that the outward radial velocity is constant, we have an expanding shell resembling the sphere already described.

The electron density and the field strength will decrease as the shell expands, and the spectrum of the synchrotron radiation will alter. The total observed flux at time \( t \) is obtained by integrating the contributions from all parts of the shell, but this calculation is complicated by the fact that the emission from different parts is emitted at different stages in the expansion. Also the Doppler blue-shift varies from \( \gamma \) on the limb to \( \gamma (1 + e/\) at the...
The EQuiTemporal Surfaces (EQTSSs)

\[ t^d_a = (1 + z) \left[ t(r) - \frac{r}{c} \cos \vartheta + \frac{r^*}{c} \right] \]

With not constant speed

\[ t(r) = \frac{r}{v} \]
\[ r^* = 0 \]

Ellipsoid with eccentricity \( v/c \)

\[ r(\vartheta) = \frac{v}{1 + z} \frac{t^d_a}{v} \frac{1}{1 - \frac{v}{c} \cos \vartheta} \]

EQTS analytic expression: radiative condition

\[ t^d_a = (1 + z) \left[ t(r) - \frac{r}{c} \cos \theta + \frac{r^*}{c} \right] \]
EQTS analytic expression: radiative condition

\[ t_a^{d} = (1 + z) \left[ t(r) - \frac{r}{c} \cos \vartheta + \frac{r^*}{c} \right] \]

Using the approximate solution for \( t = t(r) \)
(Panaitescu & Mészáros, 1998)
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\[ \vartheta = 2 \arcsin \left[ \frac{1}{2\gamma_o} \sqrt{\frac{2\gamma_o^2 c t_a}{r}} - \frac{1}{7} \left( \frac{r}{r_o} \right)^6 \right] \]
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Using analytic solution for \( t = t(r) \)

\[ \cos \vartheta = \frac{M_B - m_i^o}{2r \sqrt{C}} (r - r_\odot) + \frac{m_i^o r_\odot}{8r \sqrt{C}} \left[ \left( \frac{r}{r_\odot} \right)^4 - 1 \right] \]

\[ + \frac{r_\odot \sqrt{C}}{12 r m_i^o A^2} \ln \left\{ \frac{[A + (r/r_\odot)]^3 (A^3 + 1)}{[A^3 + (r/r_\odot)^3] (A + 1)^3} \right\} + \frac{c t_\odot}{r} - \frac{c t_a^d}{r (1 + z)} \]

\[ + \frac{r^*}{r} + \frac{r_\odot \sqrt{3C}}{6 r m_i^o A^2} \left[ \arctan \frac{2 (r/r_\odot) - A}{A \sqrt{3}} - \arctan \frac{2 - A}{A \sqrt{3}} \right] \]
Using analytic solution for $t = t(r)$

Using the approximate solution for $t = t(r)$ (Panaitescu & Mészáros, 1998)
EQTS analytic expression: radiative condition

Using analytic solution for $t = t(r)$

Using the approximate solution for $t = t(r)$ (Panaitescu & Mészáros, 1998)

Distance (cm)
EQTS analytic expression: radiative condition

\[ \cos \theta \bigg|_t \geq \frac{v(t)}{c} \]

Distance (cm)

Invisibile

EQTS analytic expression: adiabatic condition

\[ t_a^d = (1 + z) \left[ t(r) - \frac{r}{c} \cos \vartheta + \frac{r^*}{c} \right] \]

Using the analytic solution for \( t = t(r) \)
(Panaitescu & Mészáros, 1998)

\[ \vartheta = 2 \arcsin \left[ \frac{1}{2 \gamma_0} \sqrt{\frac{2 \gamma_0^2 c t_a}{r} - \frac{1}{4} \left( \frac{r}{r_0} \right)^3} \right] \]

Using the approximate solution for \( t = t(r) \)

\[ \cos \vartheta = \frac{m_i^0}{4M_B \sqrt{\gamma_0^2 - 1}} \left[ \left( \frac{r}{r_0} \right)^3 - \frac{r_0}{r} \right] + \frac{c t_0}{r} \]

\[ - \frac{c t_a}{r} + \frac{r^*}{r} - \gamma_0 - \frac{m_i^0 / M_B}{\sqrt{\gamma_0^2 - 1}} \left[ \frac{r_0}{r} - 1 \right] \]
Using the analytic solution for $t = t_r$ (Panaitescu & Mészáros, 1998)
Using the analytic solution for $t = t(r)$

Using the approximate solution for $t = t(r)$ (Panaitescu & Mészáros, 1998)

EQTS analytic expression: adiabatic condition

adiabatic

radiative
EQTS analytic expression: adiabatic condition

Using the analytic solution for $t = t(r)$

Using the approximate solution for $t = t(r)$ (Panaitescu & Mészáros, 1998)

EQTS exact and approximate (radiative cd.)

$t = 35$ seconds:

$t = 4$ days:
EQTS exact and approximate (adiabatic cd.)

\[ t = 4 \text{ days: } \]

\[ t = 35 \text{ seconds: } \]

“Canonical GRB” Bolometric luminosity

Two different phases:

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Two different phases:

**P-GRB**: emitted at the PEMB pulse transparency point.

(still work in progress)

“Canonical GRB” Bolometric luminosity

Two different phases:

- **P-GRB**: emitted at the PEMB pulse transparency point. (still work in progress)

- **Extended Afterglow**: due to the interaction between ABM pulse and ISM. Includes E-APE

\[ \frac{dE_\gamma}{dt_a^d d\Omega} = \int_{E_QTS} \left( \frac{d^d}{4\pi} \right) \frac{\Delta E}{V \cos \theta} \Lambda^{-4} \frac{dt}{dt_a^d} d\Sigma \]

\[ t_a^d = (1 + z) \left( t - \frac{\int_0^t \nu(t') dt'}{c} + \frac{r_{ds}}{c} \cos \theta \right) \]


“Canonical GRB” Bolometric luminosity

Two different phases:

**P-GRB:** emitted at the PEMB pulse transparency point.

*(still work in progress)*

**Extended Afterglow:** due to the interaction between ABM pulse and ISM. Includes E-APE

\[
\frac{dE_\gamma}{dt_a d\Omega} = \int_{EQTS} \frac{\Delta \varepsilon}{4\pi} v \cos \vartheta \Lambda^{-4} \frac{dt}{dt_a} d\Sigma
\]

\[
t_a^d = (1 + z) \left( t - \frac{\int_0^t v(t') dt' + r_{ds}}{c} \cos \vartheta + \frac{r_{ds}}{c} \right)
\]


The *Fireshell* transparency

The observed spectrum (e.g. for GRB 031203): a double convolution of thermal spectra

The observed spectrum (e.g. for GRB 031203): a double convolution of thermal spectra

**Observed spectrum:** Convolution of instantaneous spectra over the observation time, i.e. convolution of convolutions of thermal spectra.

**Instantaneous spectrum:** Convolution of thermal spectra over the EQTS.

The observed spectrum of GRB 031203

Prompt emission spectrum as a convolution of quasi thermal spectra

\[ I_\nu = \frac{2h\nu^3}{c^2} \left( \frac{\nu}{k_B T} \right)^\alpha \frac{1}{\exp\left( \frac{\nu}{k_B T} \right) - 1} \]

\[ E_{\text{dy}} = 1.0 \times 10^{54} \text{ erg} \]

\[ B = 2.5 \times 10^{-3} \]

\[ n_{\text{ism}} \sim 6.06 \text{ #/cm}^3 \]

\[ \alpha = -1.8 \]

**GRB 080319B**

GRB 050315: BAT Light curve (15-350 keV)

$E_{dya} = 1.46 \times 10^{53}$ erg

$B = 4.55 \times 10^{-3}$

$E_{P-GRB} = 1.35\% \ E_{dya}$

$n_{cbm} \sim 1.0 \#/cm^3$
“Canonical GRB”

$B \rightarrow 0$

Detector arrival time (s)

Lorentz $\gamma$ factor

Radius (cm)

Observed flux (erg/cm$^2$-s)
"Canonical GRB"

$B \rightarrow 0$

![Graph showing Lorentz $\gamma$ factor vs. radius (cm).](image)

![Graph showing observed flux (erg/cm$^2$-s) vs. detector arrival time (s).](image)
"Canonical GRB"

$B \rightarrow 0$

"Genuine short" GRBs
"Canonical GRB"

\[ B \rightarrow 10^{-2} \]
"Canonical GRB"

\[ B \rightarrow 10^{-2} \]
“Canonical GRB”

$B \rightarrow 10^{-2}$
“Canonical GRB”

$B \rightarrow 10^{-2}$
"Canonical GRB"

$B \rightarrow 10^{-2}$

"Long" GRBs
A new class of GRBs

• GRB 060614 (Gehrels et al., 2006; Mangano et al., 2007):
  – A short pulse $\sim 4$ s + long-lasting multi-peaked structure.
  – “Hybrid” properties between short and long bursts.
    $\Rightarrow$ Hard to classify in terms of short/long categories.

• BATSE + HETE-II + Swift catalogue (Norris & Bonnell, 2006):
  – “Occasional softer, extended emission lasting tens of seconds after the initial spikelike emission”.
  – Softer extended emission “relatively strong, with peak intensities only 2-10 lower than the spike emission”.
  – “The strength of the extended emission converts an otherwise short burst ... making it appear to be a long burst”.
    $\Rightarrow$ The current nomenclature for the two classes (short – hard / long – soft) is at best misleading.
GRB 970228
prompt emission
(BeppoSAX)

GRB 970228 prompt emission (BeppoSAX)

\[ E_{\text{dy}} = 1.4 \times 10^{54} \text{ erg} \]
\[ B = 5.0 \times 10^{-3} \]
\[ E_{P-GRB} = 1.1\% \ E_{\text{dy}} \]
\[ n_{\text{ism}} \sim 10^{-3} \# / \text{cm}^3 \]

GRB 970228

prompt emission
(BeppoSAX)

\[ E_{\text{dya}} = 1.4 \times 10^{54} \text{ erg} \]

\[ B = 5.0 \times 10^{-3} \]

\[ E_{\text{P-GRB}} = 1.1\% \ E_{\text{dya}} \]

\[ n_{\text{ism}} \sim 10^{-3} \ \#/ \text{cm}^3 \]

GRB 970228 detected in the halo of its host galaxy.

The role of the low ISM density

GRBM observations in 40-700 keV band

Afterglow light curve in 40-700 keV band with $<n_{\text{ism}}>$ = $10^{-3}$ #/cm$^3$

Afterglow light curve in 40-700 keV band with $<n_{\text{ism}}>$ = 1 #/cm$^3$

P-GRB light curve in 40-700 keV band
The role of the low ISM density

The role of the low ISM density

"Deflates" ext. afterglow peak luminosity with respect to the P-GRB (increasing its duration, since total energy is fixed by $B$)

"Disguised" short GRBs

"Canonical GRB"

$B \rightarrow 10^{-2}$
“Canonical GRB”

$B \rightarrow 10^{-2}$

“Long” GRBs
and/or
“Disguised short” GRBs