Filtration Combustion of Solid Fuels: Models and Stability Problem

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Filtration Combustion of Solid Fuels, FCSF, (Combustion of Porous Media, Smoldering) is the old and simultaneously young type of combustion.

It is old not only due bad human habit of smoking, but...
... also by old technological usage, e.g. patenting agglomeration process in 1887.

F. Heberlein and T. Huntington, 1887.
Up-draft and **down draft** sintering in a blast furnace.
Instability is one of the main problems for the development of FCSF technologies.

Air flow rate: 680 \([\text{m}^3/\text{h} \cdot \text{m}^2]\)

Development of instability in the co-current inverse wave of FCSF. 80% of activated charcoal BAU-A and 20% of chamotte (1-3 mm); air flow rate 680 \([\text{m}^3/\text{h} \cdot \text{m}^2]\).
Instability of plane filtration combustion front.
Downward co-current inverse wave.
80% of charcoal and 20% of chamotte (5-7 mm).
Time interval between frames is 3 min.
Ignition of the mixture near the reactor wall and different shapes of combustion front depending on charge particle size.

Upward co-current wave.

<table>
<thead>
<tr>
<th>Ignition</th>
<th>Plane front</th>
<th>Developed inclined combustion front structures</th>
</tr>
</thead>
<tbody>
<tr>
<td>1α</td>
<td>2α</td>
<td>$d_p = 8.5 \text{ mm}$</td>
</tr>
<tr>
<td>3α</td>
<td>4α</td>
<td>$d_p = 6.0 \text{ mm}$</td>
</tr>
<tr>
<td>5α</td>
<td>6α</td>
<td>$d_p = 4.0 \text{ mm}$</td>
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<tr>
<td>7α</td>
<td></td>
<td>$d_p = 2.5 \text{ mm}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d_p = 1.5 \text{ mm}$</td>
</tr>
</tbody>
</table>
Ya. B. Zel'dovich, the father of investigation on flame stability


But the main role in the problem of stability of FCSF is belonged to the interaction between hydrodynamic in a porous medium and thermal fields, i. e. to the thermal-hydrodynamic instability.

This work is aimed at clarification of this interaction features
Problems of theoretical description of FCSF

• Anisotropy (dispersion) of heat and mass transfer coefficients
• Shrinkage of fuel particles
• Contraction of burned layer and displacement of particle bed as a whole
• Correct consideration of all kinds heat transfer in porous media
Two basic causes for instabilities in FCSF

- **Burnout of solid fuel and decrease of filtration resistance in burned-out regions**

- **Strong dependence of filtration resistance on the temperature (approx. as T^{-1.5}-T^{-2.0}).**

- **Thermal-hydrodynamic instability mechanism works in all cases**

In filtration combustion of gases:

Goals of the work

- To develop the model of FCSF instabilities with the only thermal-hydrodynamic mechanism
- To study the conditions for instability development in 2D FCSF
- To investigate numerically the dynamics of instability of FCSF leading to incomplete burning of solid fuel
Restrictions of the approach

Let me:

- neglect all structural changes in a burning porous medium
- to treat thermophysical properties of the system as constants
- to consider the integral mass flow rate as constant
- to use the Darcy’s law for gas filtration with the Sutherland’s viscosity dependence on temperature
- and, finally, to treat chemical interactions as global Arrhenius type reaction
We consider the co-current combustion waves

Instability for co-current inverse wave

Counter-current combustion waves are stable when only thermal-hydrodynamic mechanism is taken into account
System of Governing Equations

\[ c_{p,s}^V \frac{\partial T}{\partial t} + c_{p,g}^m \nabla T = \lambda_{ef} \nabla^2 T + QW \]

\[ \nabla \left( \hat{G}_\xi \right) = -\nu W \]

\[ \nabla \left( \hat{G}_\rho \right) = 0 \]

\[ \hat{G} = -\chi \nabla \rho \nabla p; \]

\[ \chi \equiv \frac{\varepsilon^{5.5} d_{eq}^2 M_g (T + C_A)}{5.6 R \mu_0 T^{2.5}} \]

\[ \frac{\partial \rho_{s,1}^m}{\partial t} = -W \]

\[ W = \begin{cases} k_0 \exp \left( \frac{E}{RT} \right) & \text{при } \xi > 0, \ \rho_{s,1}^m > 0 \\ 0 & \text{при } \xi = 0 \text{ или } \rho_{sm,1}^m = 0. \end{cases} \]
Dimensionless variables

$$\Theta = \frac{T - T_*}{\Delta T_*} \tau = t/t_* \quad \tilde{x} = x/x_* \quad \tilde{u} = u/u_* \quad (u_* = x_* / t_*) \quad \tilde{G} = G/G_*$$

$$\eta = \left[ \rho_s^{m-1} - \rho_{s,1}^{m-1} / \rho_{s,1}^{m-1} \right] \tilde{\xi} = \xi / \xi_0$$

Basic temperature:

$$T_* = T_0 + \frac{q}{\delta - 1}; \quad \delta > 1$$

for a co-current FCSF-wave with the inverse structure

Transition from the regime of complete burning of solid fuel to the regime with incomplete burning

$$T_* = T_0 + \frac{q}{1 - \delta}; \quad \delta < 1$$

for a counter-current FCSF-wave with the normal structure

Transition from the kinetic regime to the filtration regime
Scales

\[ \Delta T_\star = \frac{RT^2}{E} \quad t_\star = t_r^\star \quad t_r^\star = \frac{1}{W} \left( \frac{1}{E} \right) \exp \left( \frac{E}{RT_\star} \right) \]

\[ u^\star = \frac{x_\star}{t_\star} = \sqrt{\alpha_s} \frac{1}{t_\star} \quad G_0^\star = \frac{\nu \rho_{s,0} m \xi_0 u^\star}{\xi_0} \quad V_0^\star = \frac{\nu \rho_{s,0} m \xi_0 u^\star}{\rho_{g,m} \xi_0} \]

Estimations for the mixture of coke with alumina:

\[ T_\star \sim 1000–1100 \text{ K}, \quad \Delta T_\star \sim 100–150 \text{ K}, \quad t_\star \sim 100 \text{ c} \text{ (depends on the size of particles)} \]

\[ x_\star \sim 0.01 \text{ m}, \quad u^\star \sim 0.01 \text{ m/c}, \quad V_0^\star \sim 0.1–0.3 \text{ m/c}. \]

Dimensionless parameters:

\[ \gamma = \frac{\Delta T_\star c_{p,s}^V}{Q \left( \Omega_s \right)} = \frac{\Delta T_\star}{Q} ; \quad \beta = \frac{\Delta T_\star}{T_\star} ; \quad \delta = \frac{\nu c_{p,s}^m \xi_0}{\xi_0 c_{p,s}^m} \]

\[ \Theta_0 = -\frac{1}{\gamma \xi - \delta} \text{ for the co-current wave with normal structure,} \]

\[ \Theta_0 = -\frac{1}{\gamma \xi - 1} \text{ for the co-current wave with inverse structure.} \]

Critical flow rate for transition between different regimes has a simple form in Zeldovich’s approximation:

\[ \tilde{G}_0^{tr} = \tilde{u}^{tr} = \sqrt{2\gamma} \]
Dimensionless equations

\[
\frac{\partial \Theta}{\partial \tau} + \delta \left( \tilde{G} \nabla \Theta \right) = \tilde{\nabla}^2 \Theta + \frac{1}{\gamma} \tilde{W}
\]

\[
\tilde{\nabla} \left( \tilde{G} \tilde{\xi} \right) = -\tilde{W}
\]

\[
\frac{\partial \eta}{\partial \tau} = \tilde{W}
\]

\[
\tilde{W} = \left\{ \begin{array}{ll}
\exp \left( \frac{\Theta}{1 + \beta \Theta} \right) & \text{при } \eta > 0, \tilde{\xi} > 0; \\
0 & \text{при } \eta = 0 \text{ или } \tilde{\xi} = 0.
\end{array} \right.
\]

\[
\tilde{\nabla} \left( \tilde{G} \tilde{p} \right) = 0
\]

\[
\tilde{\chi} \Phi = \left( \frac{1 + \beta \Theta_0}{1 + \beta \Theta} \right)^{5/2} \frac{1 + \beta \Theta + \tilde{C}_A}{1 + \beta \Theta_0 + \tilde{C}_A}
\]
Results and discussion

Main dimensionless parameters: $\gamma = 0.2$, $\beta = 0.077$, $\delta = 1.5$ (for inverse wave)

Test: comparison with Zeldovich’s approximation for plane co-current inverse wave.

Zeldovich’s approximation – solid lines

$\gamma = 0.2$, $\beta = 0.077$, $\tilde{G}_0 = 2.0$, $\tilde{H} = 4.0$
Instability leads to incomplete burning of solid fuel
Conditions for the inclination instability

\[ \alpha_f = \arctg \left( \tilde{K}_{f,r} - \tilde{X}_{f,l} \right) \tilde{H} \]

- angle of front inclination

\[ \gamma = 0.2, \beta = 0.077, \delta = 1.5 \]
Critical behavior of combustion front shape with the porous layer width $H$

$\tilde{H} = 4.0$
$\tilde{H} = 8.0$
$\tilde{H} = 12.0$
\tilde{H} = 16.0
Initiation: “spike”

\[ \tilde{H} = 16.0 \]
Changes in structure and behavior of combustion front with increase in reactor width $H$

- "Half-wave" stable
- "Half-wave" unstable
- "1-wave" stable
- "1-wave" unstable

$H \sim H$
Highlights of modeling results

• Co-current waves with inverse structure are unstable for sufficiently large values of \( \tilde{H} \) and \( \tilde{G}_0 \).
• There was no minimal threshold of initial disturbance for generation of instability observed.
• Instability of plane combustion front leads to drastic fall of solid fuel conversion.