Nonlinear electrodynamics: The missing trigger for the formation of astrophysical charged black holes in gravitational core collapse supernovae

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General relativity and charged black holes

What is the problem? How to form an astrophysical charged BH

Maxwell vs. Nonlinear electrodynamics

Nonlinear electrodynamics
- Early XXth Century Approaches
- NLED Back reaction effect: The Solution

Theoretical framework: The Basics
- NLED B-R repulsive dynamics

Pulsar induced vacuum back reaction

Induced magnetization in classical electrodynamics

Making it longer neutralization timescale

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- Differential Rotation
- Chiral Plasma Instability
Abstract

Theorists of the general theory of relativity contend that in nature there exists electrically charged (Reissner-Nordstrom) black holes, celestial objects which a distant observer would characterize by their mass and charge. Notwithstanding, none astrophysical mechanism has been proved to self-consistently break up the universal global charge neutrality of most cosmic systems. Foundational arguments from nonlinear electrodynamics (NLED) provide a mechanism able to drive the formation of an astrophysical charged black hole upon the gravitational collapse of a massive star. Due to its repulsive action (nonlinear growing of the initial field in a rotating proto-neutron star (P-NS)) NLED allows, as compared to the gravitational timescale ($\Delta T_{\text{grav}} \simeq 1/\sqrt{G \rho_{\text{NS}}} \gtrsim 10^{-4}$ s), to make it longer the timescale for Coulombian (electrostatic) neutralization ($\Delta T \simeq \lambda_{\text{Debye}}/c \lesssim 10^{-20}$ s), which would otherwise take place at the phase transition created inner crust-upper mantle charge separation interface (separatrix), much earlier than the gravitational core collapse would take over. In such stalled charge separation state the aftermath of gravitational collapse of the P-NS inner core can be an astrophysical charged black hole.
Theorists of GR contend:

- **Einstein equations must somehow be realized in nature**
- **Argument: Exact mathematical solutions**
  - Space-time of charged black hole (Reissner-Nordstrom, BH)

**Metric** ($t, r, \theta, \phi$  Schwarzschild coordinates, signature (+, -, -, -), units $G = c = 1$. $M =$ mass, $Q =$ charge)

\[
ds^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 - \frac{dr^2}{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2, \quad (1)
\]

Most astrophysicists still pose the question:

- **Nature and mechanism able to break up global charge neutrality in astronomical objects?**
  - A few attempts to cope with this puzzle
  - None has conclusively shut off the debate[1].

**The issue remains a very open problem in relativistic astrophysics**
What is the problem? How to form an astrophysical charged BH

With regard to this tantalizing issue, it worth to quote that in an earlier paper [Mosquera Cuesta etal., Phys. Rev. D67 (2003) 087702] a mechanism inspired in brane-world physics was presented which allows for mass disappearance (electrons, rather than protons, leaking) from the brane to the bulk producing an asymmetry in an otherwise endlessly global neutral (+, -) charge distribution lying on the brane, e.g. a star. As a result, an astrophysical charged black hole may come out by end of a supernova gravitational core collapse. This mass leaking mechanism might have also been at work during the very early universe driving a matter-antimatter primordial asymmetry.
Maxwell theory: Electrodynamics Linear in Lorentz invariants $F$, $G$

Fundamental Problems

- Ionized gas: Naive (quantum mechanical) calculation of ground-state energy density yields infinity
- Divergence of the electric field of point charges
- Catastrophic instability of Bohr’s atomic model
- Infinite self-energy of point particles

NLED: Set up of relativistic invariance to prevent divergences in EM

Several approaches:

  (Study case here !!)
- Born-Infeld: Inspired on Special Relativity [3]
- Plebanski: Structurally robust framework (+ Plasma Physics)[4]
- Pagels-Tomboulis: QCD Inspired [?]
Lagrangians \((G = 0 :: \mu, b \text{ const.}):\)

\(a) L^H = -\frac{1}{4} F + \frac{\mu}{4} F^2 + ... \)

\(b) L^B = -\frac{b^2}{2} \left[ \left( 1 + \frac{F}{b^2} \right)^{1/2} - 1 \right] \leftarrow \left( 1 + \frac{v^2}{c^2} \right)^{1/2} \)

\(c) L = \alpha^2 F^2 - \frac{1}{4} F - \frac{\mu^2}{F} + \frac{\beta^2}{F^2} \)

\(d) L = -\gamma^2 \sqrt{1 + \beta F - \alpha^2 F^2} \)
Way outs:
- Setting an upper limit on the electric field strength upon promoting Electron: charged particle of finite radius (see Eq.(3) part b, above).

Applications:
- Cosmological and astrophysical contexts [5]
- Nonlinear optics, high power laser technology and plasma physics [6]
- Chiral plasma instability of electrons: Weak parity-violating electron-capture process in core collapse supernovae [7] ¹

¹ These authors concede not having identified what mechanism helps to enlarge the NS magnetic helicity, though they stress that the original B-field gives a positive feedback to itself, to grow exponentially, being this last the actual chiral instability. In our picture, this nonlinear enlargement of the field is a prove that NLED is doubtless at action inside just-born pulsars.
NLED Back reaction effect: The Solution

NLED features highlighted:

- **EM dynamics in a vacuum** $\leftrightarrow$ sort of repulsive action or **back reaction effect** (i.e. EM field feedback to itself), see Eq.(6)$^2$

- NLED Back reaction (B-R) effect: manifestation of induced magnetization of (quantum) vacuum medium by some acting magnetic field (the one of rotating magnetized neutron star (NS)) — NLED Vacuum friction [8]: EM interactions in ordinary media with M-E Opt properties: Retardation effects create time offset between vacuum induced magnetization and spinning magnetic dipole moment (simplest model of PSRs) [9].

  — Such dynamical state: Medium B-R to Inductor field $\rightarrow$ classical dissipative force

\(^{2}\)Further insights in Ref.[7] in connection to chirality imbalance (asymmetry) of electrons which appears due to self-interaction of the electron, proton and EM field amidst of, in the simplest atom semiclassical model
Theoretical framework

There exists several formulations of NLED:

a) electric permittivity: \( \varepsilon_0 (E, B) \) :: magnetic susceptibility \( \mu_0 (E, B) \)

b) \( L = L(F, G) \), e.g., the power series \( L = \sum_{j,k=0}^{\infty} c_{j,k} F^j G^k \), or

c) 4-dim effective theory from strings, M-theory, or AdS/CFT

Simplest NLED theory \(^3\) : \( S = \int \sqrt{-g} \, L(F, G) \, d^4 x \),

Field equation: By extremalizing \( L(F(\mathbf{A}_\mu)) \) w.r.t. \( \mathbf{A}_\mu \rightarrow (G = 0 :: \)

\[ L_F = \frac{dL}{dF} :: \quad L_{FF} = \frac{d^2L}{dF^2} \) \(^4\)

\[ \nabla_\nu (L_FF)^{\mu\nu} = 0 : \quad \nabla_\mu F^{\mu\nu} = J^\nu \equiv -\frac{L_{FF}}{L_F} F^{\mu\nu} F_{|\mu}. \] \(^6\)

Plus cyclic (Faraday) identity

\[ \nabla_\nu F^{*\mu\nu} = 0 \Leftrightarrow F_{\mu\nu|\alpha} + F_{\alpha\mu|\nu} + F_{\nu\alpha|\mu} = 0. \] \(^7\)
By taking the discontinuities of $F^{\mu\nu}$ Eq. (6) one gets (definitions in Hadamard’s book), and [12, 13] a)

$$L_F f^\mu k^\lambda + 2L_F F^{\alpha\beta} f_{\alpha\beta} F^{\mu\lambda} k^\lambda = 0 ,$$

(8)

+ discontinuity of the Bianchi identity, yields b)

$$f_{\alpha\beta} k_\gamma + f_{\gamma\alpha} k_\beta + f_{\beta\gamma} k_\alpha = 0 .$$

(9)

A scalar relation can be obtained by contracting this equation with $k^\gamma F^{\alpha\beta}$, which yields c)

$$(F^{\alpha\beta} f_{\alpha\beta} g^{\mu\nu} + 2F^{\mu\lambda} f_\lambda^\nu) k^\mu k^\nu = 0 .$$

(10)

Two distinct solutions appear:

1) when $F^{\alpha\beta} f_{\alpha\beta} = 0$, case in which such mode propagates along standard null geodesics, and

2) when $F^{\alpha\beta} f_{\alpha\beta} = \chi$, case in which equations (a8) and (c10) render propagation equation for field discontinuities.
Photons propagate in geodesics $\neq$ background S-T $g_{\mu\nu}$!

$$\left(g^{\mu\nu} - 4 \frac{L_{FF}}{L_F} F^{\mu\alpha} F_\alpha{}^\nu\right) k_\mu k_\nu = 0 . \quad (11)$$

Rather they follow $\rightarrow$ effective metric $\text{Eq.}(11)$ ($F^{\mu\alpha}$)

Now, by taking the derivative $\rightarrow$

$$k^\nu \nabla_\nu k_\alpha = 4 \left( \frac{L_{FF}}{L_F} F^{\mu\beta} F_\beta{}^\nu \right) k_\mu k_\nu \big|_\alpha . \quad (12)$$

$\rightarrow$ NLED brings in a term acting as a (field retarded self-energy) backreaction force which accelerates ($\rightarrow$ Higher energy $\rightarrow$ Higher pressure) or decelerates photon along its path$^4$

$^4$For astrophysical and cosmological consequences see [5].
— NLED \( L(F) \) produces perfect fluid energy-momentum tensor (E-M) in \( G_{\mu\nu} = T_{\mu\nu} \) \[13\]

\[
T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta L(F)}{\delta g^{\mu\nu}} \equiv T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu} .
\] (13)

The left-hand-side of Eq.(13) yields

\[
T_{\mu\nu} = -4L_F F_\mu^\alpha F_\alpha^\nu - Lg_{\mu\nu} .
\] (14)

By equating terms in Eqs.(13, 14), one gets (Maxwell \( L \rightarrow \rho = 3p = \frac{1}{2}(E^2 + B^2) \))

\[
\rho = -L - 4E^2 L_F , \quad p = L + \frac{4}{3}(E^2 - 2B^2)L_F .
\] (15)

\( L + \text{E-M T structure} \rightarrow \text{magnetic fluid: A collection of noninteracting fluids indexed by } k = -, 0, + \rightarrow \text{EoS: } p_k = \left( \frac{4k}{3} - 1 \right) \rho_k \rightarrow \) There is room for EoS to have negative pressure.
Other Lagrangians exhibiting repulsive force (EM field positive feedback to itself):

- **a) a truncated Laurent series** ($\alpha, \beta, \mu$ coupling constants) [13]

  \[
  L = \alpha^2 F^2 - \frac{1}{4} F - \frac{\mu^2}{F} + \frac{\beta^2}{F^2}. \tag{16}
  \]

  Thus, one obtains EoS describing ordinary radiation

  \[
  \rho_1 = -\alpha^2 F^2 = -4\alpha^2 B_s^4 \frac{1}{R^8} \quad \therefore p_1 = \frac{5}{3} \rho_1 \quad \therefore \rho_2 = \frac{1}{4} F = \frac{B_s}{2} \frac{1}{R^4} \quad \therefore p_2 = \tag{17}
  \]

  plus **fluids exerting repulsive action**

  \[
  \rho_3 = \frac{\mu^2}{F} = \frac{\mu^2}{2B_s^2} R^4 \quad \therefore p_3 = -\frac{7}{3} \rho_3 \tag{18}
  \]

  \[
  \rho_4 = -\frac{\beta^2}{F^2} = -\frac{\beta^2}{4B_s^4} R^8 \quad \therefore p_4 = -\frac{11}{3} \rho_4. \tag{19}
  \]
b) Lagrangian (16): purely phenomenological! Possible to regain repulsive dynamics by extending standard Born-Infeld $L(3)$ to [13]

$$L = -\gamma^2 \sqrt{1 + \beta F - \alpha^2 F^2} \quad : \quad b)p + \rho = \frac{\gamma^2 F(1 - 4\alpha^2 \gamma^2 F)}{3\rho}. \quad (20)$$

Check for such a property: Eq.(20-b) $\rightarrow$ field transition value:

$F \equiv F_{\text{trans}} \quad : \quad \rho + p > 0 \text{ for } F < F_{\text{trans}}, \text{ while } \rho + p < 0 \rightarrow$ (violation of SEC!) for $F > F_{\text{trans}}$!! (see details in [13])

$L(20)$ produces repulsive dynamics: Property looked for to keep in a stalled state the P-NS charge separatrix $\rightarrow$ gravitational core collapse can take over!!

E-M T conservation preserves Gauss law: $B = \frac{B_s}{R_{\text{NS}}^2}$: High energy astrophysics: $\rightarrow$ B-field of nascent or glitching pulsars [14, 9], e.g. Eq.(11) in [7], or in any P-NS structural rearrangement, usually a catastrophic phase transition [15, 9].
Let us turn back to the study case: a P-NS core collapsing in a SN

- EoS feature: decidedly attractive \( p = -\rho! \) \( \rightarrow \) onset of P-NS phase transition (PT) \( \rightarrow \) keeping stalled the charge separation state

- Characteristic timescale for Coulombian neutralization can grow longer!! (B-field positive exponential self-interaction) [7], \( \rightarrow \) gravitational core collapse can take over ending up in a charged BH

- Such NLED repulsion also prevents overlaying crust to plunge onto the core.

- Bunch of astrophysical mechanisms for this to happen: [15, 9, 16, 17, 18]. For instance: “separatrix during gravitational core collapse” [19]

- This astrophysical stage: Prelude of formation of charged BH [19]. Huge amount of work (realistic characterization) on structural configuration of static, rotating and collapsing NSs [16, 17, 18, 20]

- Indeed, the P-T may transiently produce hybrid star or quark star [15], before ending up by forming a charged BH
Vacuum Induced Magnetization (VIM)

Classical electrodynamics [22] defines magnetization (magnetic dipole moment per volume) as \( F = 2(\varepsilon_0 E^2 - \frac{B^2}{\mu_0}) :: E = 0 \rightarrow F = -2\frac{B^2}{\mu_0} \)

\[
H = -\frac{\partial L}{\partial B} = \frac{B}{\mu_0} - m_{br} .
\] (21)

Vacuum induced magnetization (VIM :: the response \( m_{br} \) to PSR dipole B-field action):

a) Born-Infeld in Eq.(3),

\[
\frac{\partial L^B_I}{\partial B} = \left( \frac{1}{\sqrt{1 - \frac{2B^2}{b^2\mu_0}}} \right) \frac{B}{\mu_0} :: m_{br}\bigg|_I^B = \left( \frac{1}{2\sqrt{1 - \frac{2B^2}{b^2\mu_0}}} \right) \frac{B}{\mu_0} \] (22)

b) Heisenberg-Euler in Eq.(3)

\[
\frac{\partial L^H_E}{\partial B} = \frac{B}{\mu_0} - 4\mu\left( \frac{B^2}{\mu_0} \right) \frac{B}{\mu_0} :: m_{br}\bigg|_E^H = 4\mu\left( \frac{B^2}{\mu_0} \right) \frac{B}{\mu_0} \] (23)
Vacuum Induced Magnetization (VIM) — Continued

c) extended Born-Infeld :: $L_F = -\frac{\gamma^2}{2} \left( \frac{\beta - 2\alpha^2 F}{\sqrt{1 + \beta F - \alpha^2 F^2}} \right)$,

$$\frac{\partial L^{B-I}_{Ext}}{\partial B} = -\frac{\gamma^2}{2} \left( \frac{-4\beta - 16\alpha^2 \frac{B^2}{\mu_0}}{\sqrt{1 - 2\beta \frac{B^2}{\mu_0} - 4 \frac{\alpha^2}{\mu_0^2} B^4}} \right) \frac{B}{\mu_0} \cdots$$

$$m_{br}|_{B-I_{Ext}} = \left( \frac{8\alpha^2 \gamma^2 \frac{B^2}{\mu_0}}{\sqrt{1 - 2\beta \frac{B^2}{\mu_0} - 4 \frac{\alpha^2}{\mu_0^2} B^4}} \right) \frac{B}{\mu_0}. \tag{24}$$

Eq.(23) can be compared to Eq.(6) in Ref.[8] computation up to $\alpha$

From Eqs.(22, 23, 24), VIM as functional $\mathcal{F}$ of P-NS external field reads

$$m_{br} = \mathcal{F} \left( \frac{B}{\mu_0} \right) \bigg| \frac{B}{L \mu_0}. \tag{25}$$
Collapse theory: some pre-SN stellar cores can rotate near Keplerian equatorial break-up frequency: $\Omega_K \geq \left(\frac{2}{3} \frac{G_{N} M}{R^3}\right)^{1/2} \rightarrow P_K \sim 0.6 \text{ s}$, after core bounce

Moreover, submillisecond PSRs spinning at $\Omega \sim 1122$ Hz do exist [23]. Thus, $P \rightarrow \frac{\Omega R}{c} \ll 1$ indicates the (spin) region where VIM is at work!

- P-NS: $m$ magnetic dipole moment, $R$ radius and $B_s$ surface B-field ($B_s \simeq \frac{\mu_0 m}{4\pi R^3}$ :: $m = \|m\|$) $\rightarrow$ dipole B-field leading term reads [22]

$$B(r,t) \simeq \frac{\mu_0}{4\pi} \left[ \frac{3r(m(t - \frac{r}{c}) \cdot r)}{r^5} - \frac{m(t - \frac{r}{c})}{r^3} \right].$$ (26)

The term $t - \frac{r}{c}$ in $m$ accounts for retardation effects.
At point \( r \) the vacuum B-R induced magnetic moment reads (its origin can be traced back to Eq.(6):

\[
\nabla_\mu F^{\mu\nu} = J^\mu, \quad J^\mu = J_{\text{ind}}^\mu + J_{\text{ext}}^\mu = -\frac{L_{FF}}{L_F} F^{\mu\nu} F_{\nu}, \quad \text{i.e. even if} \quad J_{\text{ext}}^\mu = 0, \quad \text{induced current stems from field feedback on itself}
\]

\[
dm_{br}(r,t) = \left[ \mathcal{F} \left( \frac{B}{\mu_0} \right) \right]_{I}^{B}, \left[ \mathcal{F} \left( \frac{B}{\mu_0} \right) \right]_{E}^{H}, \left[ \mathcal{F} \left( \frac{B}{\mu_0} \right) \right]_{\text{Ext}}^{B-I} \mathbf{B}(r,t) \, dV(r, \theta, \phi)
\]

\text{(27)}

with \( dV = r^2 \sin \theta \, dr \, d\theta \, d\phi \), \( (r, \theta, \phi) \) :: \((x, y, z)\) coordinates

At time \( t + \frac{r}{c} \) : B-field \( dB_{br} \) produced by \( dm_{br}(r,t) \) at PSR center \( r \)

\[
dB_{br}(0,t + \frac{r}{c}) \simeq \frac{\mu_0}{4\pi} \left[ \frac{3r(dm_{br}(r,t) \cdot r)}{r^5} - \frac{dm_{br}(r,t)}{r^3} \right]. \quad \text{(28)}
\]

This VIM interacts with P-NS spinning magnetic dipole moment by dissipating energy
Quantum Vacuum can ever be thought of as an ordinary medium! Classical electrodynamics rate of energy lost: [22] (unit vector $\mathbf{u}_z \parallel \Omega_z$ :: $\Omega = \frac{2\pi}{P}$ rotation frequency)

$$\dot{d}E_{br} = - \left( \mathbf{m}(t + \frac{r}{c}) \times dB_{br}(0,t + \frac{r}{c}) \right) \Omega \cdot \mathbf{u}_z . \quad \text{(29)}$$

Integration: star radius to infinity (averaging over periods: $P$) Eq. (29) yields

$$\dot{E}_{br} = \int_{R}^{\infty} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \langle d\dot{E}_{br} \rangle_P . \quad \text{(30)}$$
For the moment, let us focus on the study case: à la Heisenberg-Euler: Eq.(3)

- For P-NS Ref.[21] showed that this Lagrangian leads to
  \[ p = \frac{1}{3} \rho - \rho_\gamma, \quad \text{with} \quad \rho_\gamma = \frac{16}{3} c_1 B^4. \]
  For supercritical fields \( \rho_\gamma \) dominates so that the EoS becomes negative \( \rightarrow \) the condition to provide repulsive dynamics is reached!!

- In connection to Eq.(30), for nearly overcritical B-fields Ref. [8] showed that
  \[ \dot{E}_{br} \simeq \alpha \left( \frac{18\pi^2}{45} \right) \frac{\sin^2 \theta}{\mu_0 c} \frac{R^4}{B_c^2 P^2} B_s^4, \]  \hspace{1cm} (31)

  while à la Maxwell the energy dissipation rate reads [9]
  \[ \dot{E}_{Maxw} = \left( \frac{128\pi^5}{3} \right) \frac{\sin^2 \theta}{\mu_0 c^3} \frac{R^6}{P^4} B_s^2. \]  \hspace{1cm} (32)
A confrontation of these energy losses hints at fundamental changes w.r.t. the method currently in use to estimate the B-field strength of pulsars [14, 9].

B-R energy lost depends on $B_s^4$, while standard one grows as $B_s^2$. Astronomers measure the luminosity, flux, spin rate and spin down of PSRs.

B-field strength on PSR surface is inferred by assuming that pulsar EM power release is explained by classical dipole model [14, 9].

To correctly infer B-field strength of extremely magnetized, slow pulsars: Take into account B-R or vacuum frictional effects otherwise such fields will be severely underestimated, e.g. the “magnetars” [8].

Let us now proceed to estimate B-field strength needed to delay the electrostatic neutralization process at the separatrix.
Making it longer the (+, -) neutralization timescale

- The NS total mechanical energy reads: \( E_{NS} = E_{grav} + E_{spin} \)
- Estimate how much longer the electrostatic timescale can go on by equating it to the timescale dictated by gravity:
  \[
  \Delta T^{NLED} = \frac{\dot{E}_{NS}}{\dot{E}_{Maxw} + \dot{E}_{br}} \iff \Delta T^{grav} = \frac{1}{\sqrt{G\rho}} \gtrsim 10^{-4} \text{ s}
  \]
- Such a relation can be cast in the form
  \[
  \frac{1}{\sqrt{G\rho}} = \alpha \left( \frac{18\pi^2}{45} \right) \frac{G M_{NS}^2}{R_{NS}^5} + \frac{2}{5} M_{NS} \Omega_{NS}^2 R_{NS}^2 + \left( \frac{128\pi^5}{3} \right) \frac{B_s^4}{\mu_0 c^2 P^4} B_s^2
  \]

By solving for \( B_s \) this fourth order quadratic equation (fiducial period \( P \sim 1 \text{ ms} [23] \) and \( \sin \theta = 1, \frac{1}{2} \)), one obtains the B-field strength at separatrix: \( B_s \sim 10^{18-19} \text{ G} \), which is several orders of magnitude higher than any electric field that may appear at the separatrix, e.g. as in Ref.[20].
The state-of-the-art in astrophysics is called for next, see [14, 9].

- **A newly-born NS may undergo vigorous convection** during the first 30-60 s. **If it spins differentially extremely fast** \( P \lesssim 1 \text{ ms} \) **conditions are created for the \( \alpha - \Omega \) dynamo to get into action**! (It may survive depletion due to turbulent diffusion).

- **Under collapse conditions**, B-fields \( B \sim 10^{17-18} \left( \frac{P}{1 \text{ ms}} \right) \) G may be generated as long as the differential rotation is dragged out by the growing magnetic stresses. For this process to efficiently operate the ratio: spin rate \( P \)/convection overturn timescale \( \tau_{\text{conv}} \), the Rossby number \( R_0 \), should be \( R_0 \leq 1 \).

- Then, an ordinary dipole \( B_{\text{dip}} \sim [10^{12} - 10^{13}] \) G can be built by incoherent superposition of small dipoles of characteristic size \( \lambda \sim [\frac{1}{3} - 1] \) km \( \rightarrow \) surface saturation strength \( B_{\text{sat}} = (4\pi \rho)^{1/2} \lambda / \tau_{\text{conv}} \sim 10^{16-17} \) G can be reached, as very recently proved by [24]. Indeed, the dipole B-field approximation \( \rightarrow \) induced magnetization \( B_{\text{mag}}^{\text{ind}} \sim 10^{20} \) G can be reached at the very km-scale deep inner core.
The here purported timescale could be made even more longer by large magnetic helicity \( \mathcal{H} = \int \mathrm{d}x \mathbf{A} \cdot \mathbf{B} \) \( \mathbf{A} \) vector potential) from large chiral imbalance of electrons (plasma instability) caused by exponential growing of P-NS initial B-field in the parity-violating weak process of deleptonization during the SN [7].

CONCLUSION:

At P-T interface, fields this high surely drive the star to collapse \( \rightarrow \) charged BH, and to à la Schwinger instability of the vacuum, triggering a sort of second SN explosion. The signature of this vacuum explosion can be similar to that from r-process due to P-NS crust abundance of neutrons, and would produce a late time bump or re-brightening in the SN light curve.

Picture realization: Many astrophysical contexts: Models of gamma-ray bursts (GRBs): The very central engine. Reissner-Nordstrom BH: can afford polarization and à la Schwinger pair creation [25], and full relativistic hydrodynamics and light curve evolution characterizing GRBs.
That’s all for the time being
!!! Thanks for your kind attention !!!
Final Discussion: B-field amplification :: Two mechanisms

**Chiral Plasma Instability**


Final Discussion: B-field amplification :: Two mechanisms

Chiral Plasma Instability


J. Hadamard, “Leçons sur la propagation des ondes et les equations de l’Hydrodynamique” (Hermann, Paris 1903)


Final Discussion: B-field amplification :: Two mechanisms

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Chiral Plasma Instability


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