Symmetries and Singularities

2nd Galileo-Xu Guangqi Meeting, 11-16 July 2010

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Based on joint work with
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(in various combinations)
A few words about AEI ...

An institution based in Potsdam and Hannover, devoted to research on all aspects of Einstein’s General Relativity, from pure theory to experiment:

- Mathematical Relativity and Geometric Analysis
- Quantum Gravity and Unified Theories
- Astrophysical and Numerical Relativity
- Gravitational Waves (GEO600)

Two Graduate Schools (IMPRS):

- Geometric Analysis, Gravitation and String Theory
- Gravitational Wave Astronomy
Main theme: Symmetry

... arguably the most successful principle of physics!

- **Space-time symmetries**
  - Rotations and translations in Newtonian physics
  - Special relativity and the Poincaré group
  - General relativity and general covariance

- **Internal symmetries**
  - Isospin $SU(2)$ symmetry: $m_{\text{neutron}} = 1.00135m_{\text{proton}}$ \cite{Heisenberg}
  - Flavor symmetry $SU(3)$ and the strong interactions
  - Standard model and $SU(3)_c \times SU(2)_w \times U(1)_Y$

- **The two fundamental theories of modern physics, General Relativity and the Standard Model of Particle Physics**, are based on and largely determined by symmetry principles!
Symmetry and Unification

Like a ferromagnet: symmetry is broken more and more with decreasing temperature as universe expands.
But where do we go from here?

Idea: symmetry enhancement as a guiding principle!

- Grand Unification → quark lepton unification,...
  \[ SU(3)_c \times SU(2)_w \times U(1)_Y \subset SU(5) \subset SO(10) \subset E_6 \subset \ldots? \]

- Higher dimensions: gauge group from isometries?
- Supersymmetry: relates Bosons ↔ Fermions, or:
  Forces (vector bosons) ↔ Matter (quarks & leptons)?
- Duality symmetries [Dirac,1931]
  \[ E + iB \rightarrow e^{i\alpha}(E + iB), \quad q + ig \rightarrow e^{i\alpha}(q + ig) \]
- ‘Fusion’ of space-time and internal symmetries?
- Quantum symmetry and quantum space-time?
- Expectation: new symmetry concepts needed at \( \ell_{Pl} \)!
Quantum gravity and hidden symmetries

- Basic question: what is the configuration space (or ‘moduli space’) \( \mathcal{M} \) of quantum gravity, and what are the symmetries acting on it?

- Hint from supergravity: *symmetry increases* with decreasing number of dimensions. In particular:
  - Maximal supergravity: \( E_{n(n)} \) in \( 11 - n \) dimensions. [Cremmer, Julia, 1979]

- An unexpected link with the exceptional Lie groups of *Killing and Cartan*: \( G_2, F_4, E_6, E_7, E_8 \)!

- \( E_{9(9)} \equiv E_8^{(1)} \): a solution generating symmetry acting on \( \mathcal{M} = E_{9(9)}/K(E_9) \) = moduli space of colliding plane wave solutions of maximal supergravity.

- ... suggests \( E_{10(10)} \) for \( D = 1 \): no space, only time?!
**Hypothesis:** for $T \to 0$ spatial points decouple and the system is effectively described by a continuous superposition of one-dimensional systems $\rightarrow$ effective dimensional reduction to $D = 1!$ [Belinski,Khalatnikov,Lifshitz (1972)]
BKL and Spacelike Singularities (II)

Near cosmological singularity parametrize metric as

\[ ds^2 = -N^2 dt^2 + g_{mn} dx^m dx^n , \quad g_{mn} = e_m^a e_n^a \]

Iwasawa decomposition of spatial zehnbein \( e_m^a \equiv e_m^a(t, x) \)

\[ e_m^a = e^{-\beta^a} \theta_m^a , \quad \det \theta_m^a = 1 \]

From classical BKL analysis we know that:

[Belinski,Khalatnikov,Lifshitz (1972); Misner (1969); Chitre (1972); DHN (2003)]

- Dynamics near singularity is dominated by logarithmic scale factors \( \beta^a \to \infty \) and effective potential which results from ‘integrating out’ non-diagonal metric and matter degrees of freedom.

- \( \Rightarrow \) off-diagonal metric components \( \theta_m^a \) and matter degrees of freedom ‘freeze’ as \( T \to 0 \).
Walls and Roots

• ‘Integrating out’ non-diagonal degrees of freedom leads to an effective description in terms of *cosmological billiards* in $\beta$-space of scale factors:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} n^{-1} G_{ab} \dot{\beta}^a \dot{\beta}^b + V_{\text{eff}}(\beta)$$

with *Lorentzian DeWitt metric* $G_{ab}$.

• The *Lie algebra connection* : identify space of logarithmic scale factors $\{\beta^a\}$ with the *Cartan subalgebra* of some indefinite *Kac Moody algebra*.

• Effective potential $V_{\text{eff}}(\beta)$: sharp walls from *spacelike normal vectors = simple roots* of this algebra.

• For maximal supergravity this Kac Moody algebra is the *maximally extended hyperbolic algebra* $E_{10}$.

• Half-maximal theories: $DE_{10} \subset E_{10}$ and $BE_{10}$. 
Cosmobilliards in ‘β-spacetime’

The ‘Kasner billiard ball’ moves in the ‘billiard chamber’ on lightlike straight lines (‘free Kasner flights’), bouncing off the walls of the chamber (‘Kasner bounces’). Chaotic oscillations of metric if chamber is contained within forward lightcone, otherwise ‘AVD’ behavior.
**Conjecture:** for $0 < T < T_P$ space-time ‘de-emerges’, and space-time based (quantum) field theory is replaced by (quantized) $E_{10}/K(E_{10})$ $\sigma$-model [Cf. DN, 0705.2643]
What is $E_{10}$?

(No one knows, really....)

$E_{10}$ is the ‘group’ associated with the Kac-Moody Lie algebra $g \equiv \mathfrak{e}_{10}$ defined via the Dynkin diagram [e.g. Kac]

Defined by generators $\{e_i, f_i, h_i\}$ and relations via Cartan matrix $A_{ij}$ (‘Chevalley-Serre presentation’)

$$[h_i, h_j] = 0, \quad [e_i, f_j] = \delta_{ij} h_i,$$

$$[h_i, e_j] = A_{ij} e_j, \quad [h_i, f_j] = -A_{ij} f_j,$$

$$(\text{ad } e_i)^{1-A_{ij}} e_j = 0 \quad (\text{ad } f_i)^{1-A_{ij}} f_j = 0.$$

$\mathfrak{e}_{10}$ is the free Lie algebra generated by $\{e_i, f_i, h_i\}$ modulo these relations $\rightarrow$ infinite dimensional as $A_{ij}$ is indefinite $\rightarrow$ Lie algebra of exponential growth!
Infinite Complexity from simple recursion

A Mandelbrot set generated from $z_{n+1} = f_c(z_n)$. 
Why $E_{10}$ is very special

- $E_{10}$ occupies a **uniquely distinguished place among all infinite-dimensional Lie algebras** (much like $E_8$ among the finite-dimensional Lie algebras)

- In BKL approximation, classical dynamics of SUGRA$_{11}$ near the initial singularity is well approximated by cosmological billiards in Weyl chamber of $E_{10}$

- $E_{10}$ may provide **Lie-algebraic mechanism for the ‘de-emergence’ of space and (upon quantization) time** near the singularity (that is, for $0 < T < T_P$)

- $E_{10}$ ‘knows all’ about maximal supersymmetry:
  - Different ‘slicings’ of the $E_{10}$ algebra yield correct supermultiplets for maximal supergravities (SUGRA$_{11}$, mIIA, IIB, ...)
  - $E_{10}/K(E_{10})$ $\sigma$-model dynamics at low levels matches with respective equations of motion when truncated to first order spatial gradients
Outlook

• Symmetry by no means exhausted as a *guiding principle of physics* but many open questions remain.

• Analysis of cosmological singularities à la BKL for maximal supergravity reveals key role of $E_{10}$.

• $E_{10}$ is a *uniquely distinguished* Lie algebra, but to find a manageable representation for it remains an outstanding mathematical challenge (after 40 years).

• $E_{10}$ ‘knows all’ about maximal supersymmetry and unifies many known ($S, T, U, ...$) dualities.

• Exponentially increasing complexity of $E_{10}$ algebra $\to$ an element of *non-computability* for $T \to 0$?
$E_{10}$ Versatility

$\mathfrak{sl}(10) \subseteq e_{10}$

\[ D = 11 \text{ SUGRA} \]

$\mathfrak{so}(9, 9) \subseteq e_{10}$

\[ \text{mIIA } D = 10 \text{ SUGRA} \]

$\mathfrak{sl}(9) \oplus \mathfrak{sl}(2) \subseteq e_{10}$

\[ \text{IIB } D = 10 \text{ SUGRA} \]

$\mathfrak{sl}(3) \oplus e_7 \subseteq e_{10}$

\[ \mathcal{N} = 8, \ D = 4 \text{ SUGRA} \]