Gedanken and Shining Black Holes

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“Black Holes: The Harmonic Oscillators of the 21st Century”

Thursday 2.4.10 ROOM 10-250 4:15PM

Refreshments ROOM 4-349 3:45PM

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X-ray Properties of Black-Hole Binaries

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ABSTRACT: We review the properties and behavior of 20 X-ray binaries that contain a dynamically-confirmed black hole, 17 of which are transient systems. During the past decade, many of these transient sources were observed daily throughout the course of their typically year-long outburst cycles using the large-area timing detector aboard the Rossi X-ray Timing Explorer. The evolution of these transient sources is complex. Nevertheless, there are behavior patterns common to all of them as we show in a comprehensive comparison of six selected systems. Central to this comparison are three X-ray states of accretion, which are reviewed and defined quantitatively. We discuss phenomena that arise in strong gravitational fields, including relativistically-broadened Fe lines, high-frequency quasi-periodic oscillations (100-450 Hz), and relativistic radio and X-ray jets. Such phenomena show us how a black hole interacts with its environment, thereby complementing the picture of black holes that gravitational wave detectors will provide. We sketch a scenario for the potential impact of timing/spectral studies of accreting black holes on physics and discuss a current frontier topic, namely, the measurement of black hole spin.

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The body of experimental information[1] on the characteristics (e.g. associated with relevant spectral, time and space resolutions) of the radiation emission from objects identified as black holes is a compelling reason for the formulation of a realistic theory of the plasmas that can be associated with these objects. In particular, the use of concepts (such as that of viscosity for the transport of angular momentum) and conditions that are suitable for ordinary gases has to be abandoned and the evolution of the appropriate plasma regimes in the entire phase space has to be considered.
Particle Distributions in Phase Space

e.g.

\[ f_e = f_e(x, y, z, p_x, p_y, p_z, t) \]

7 dimensions

“….not cursing the darkness but trying to light a candle….“
Composite Disk Structures

Theoretical developments and experimental observations point to the existence of relatively strong highly coherent and dynamically important magnetic field and plasma configurations in the central regions of disk structures surrounding compact objects.

Simplest cases:

i) Axisymmetric

ii) Current Carrying

iii) Non-accreting

Magnetic field represented by

\[ \mathbf{B} = \frac{1}{R} \left[ \nabla \psi \times \mathbf{e} + I(\psi) \mathbf{e}_\phi \right] \]

\[ \mathbf{B} \cdot \nabla \psi = 0 \]
Direct detection of a magnetic field in the innermost regions of an accretion disk

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Models predict that magnetic fields play a crucial role in the physics of astrophysical accretion disks and their associated winds and jets. For example, the rotation of the disk twists around the rotation axis the initially vertical magnetic field, which responds by slowing down the plasma in the disk and by causing it to fall towards the central star. The magnetic energy flux produced in this process points away from the disk, pushing the surface plasma outwards, leading to a wind from the disk and sometimes a collimated jet. But these predictions have hitherto not been supported by observations. Here we report the direct detection of the magnetic field in the core of the protostellar accretion disk FU Orionis. The surface field reaches strengths of about 1 kG close to the centre of the disk, and it includes a significant azimuthal component, in good agreement with recent models. But we find that the field is very filamentary and slows down the disk plasma much more than models predict, which may explain why FU Ori fails to collimate its wind into a jet.

Although magnetic fields have been reported in the external regions of a few protostellar disks, no field estimate yet exists for the innermost and densest parts of accretion disks from which most of the ejected plasma presumably originates. Obtaining constraints on the field strength and topology in accretion disk cores (such as those derived for cool star surfaces from time-resolved spectropolarimetry) would thus be extremely helpful for validating existing models of magnetized accretion/ejection structures around protostars and black holes. Thanks to its high mass-accretion rate of about $10^{-4}$ solar masses per year ($M_{\odot}$ yr$^{-1}$), FU Ori appears as a bright disk completely outshining the central protostar and is thus an obvious candidate for detecting magnetic fields in disk cores. Because models predict typical disk fields of equipartition strength (with roughly equal thermal and magnetic pressures), FU Ori should host fields of several hundred gauss in its innermost regions, within reach of modern spectropolarimetric techniques.

During the engineering tests of the new high-efficiency high-resolution spectropolarimeter ESPaDOnS, we secured six observations of FU Ori at three different epochs, each consisting of one unpolarized and one circularly polarized spectrum. Using least-squares deconvolution (LSD), we extracted the unpolarized (Stokes I) and circularly polarized (Stokes V) information from 4,700 spectral lines simultaneously, and produced mean LSD Stokes I and V profiles of FU Ori at each epoch. Whereas the unpolarized LSD profiles give information about the velocity field of the central regions of FU Ori, the circularly polarized LSD profiles give access to the putative disk magnetic field through the Zeeman signatures it generates in spectral lines. Averaging the four LSD Stokes V profiles recorded at the same epoch (30 November 2004), we obtain a clear Zeeman detection in the absorption lines of FU Ori (see Fig. 1); the corresponding line-of-sight magnetic field estimated from the first-order moment of the Zeeman signature is equal to $32 \pm 8$ G.

This is evidence that the field we detect comes from the disk and not from either the central star or the two recently detected cool companions, each $\sim 100$ fainter in the V band than the accretion disk, it would otherwise require the star to exhibit a 3 kG surface-averaged line-of-sight field and thus to host a large-scale magnetic dipole of polar strength $>9$ kG (ref. 17), far larger than the global dipolar field component observed on both protostars and young low-mass stars. The LSD Stokes V profiles collected at the two other epochs (24 September 2004 and 28 November 2004) are compatible within noise level with the Zeeman signature detected on 30 November (see Fig. 1). As our observing epochs are randomly phased with respect to each other, it suggests that, at first order, the temporal fluctuations from either rotational modulation or intrinsic

Figure 1 | Magnetic field detection in the protostellar accretion disk FU Ori. a, The circularly polarized (Stokes V) LSD profile of FU Ori (top curve, expanded by 100, normalized to the unpolarized continuum intensity I, and shifted by +1.05) shows a clear Zeeman signature in conjunction with the unpolarized (Stokes I) LSD profile (bottom curve). b, The LSD Stokes V profiles obtained at the other two epochs (dashed and dotted lines) are compatible at noise level with the detected Zeeman signature (solid line), suggesting that the parent magnetic structure is grossly axisymmetric.

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Master Equation for an Axisymmetric Rotating Plasma Structure
(in the gravitational field of a central object)

\[
\frac{\partial}{\partial z} \left[ R \rho \left( \Omega^2 - \Omega_k^2 \right) \right] - \Omega_k^2 \frac{\partial}{\partial R} \rho + \frac{1}{4\pi} \left[ B_z \nabla^2 B_R - B_R \nabla^2 B_z \right] = 0
\]

where

\[
\Omega_k (R) = \left( \frac{GM_*}{R^3} \right)^{1/2}
\]

is the Keplerian frequency

\[
\Omega = \Omega(\psi)
\]

is the rotation frequency

\[
\mathbf{B} = \nabla \psi \times \nabla \varphi + I(R,z) \nabla \varphi
\]

is the expression of the magnetic field in terms of the magnetic surface function \( \psi = \psi(R,z) \)

\[
\mathbf{B} \cdot \nabla \psi = 0 \quad \nabla \varphi = \frac{1}{R} \mathbf{e}_\phi
\]
The Master Equation relates the magnetic surface function to the density profile that characterizes an axisymmetric disk structure. This is derived by applying the $\mathbf{e}_\phi \cdot \nabla \times$ operator to the total momentum conservation equation. We note that

$$\nabla \times \left( -\nabla p + \frac{1}{c} \mathbf{J} \times \mathbf{B} \right) = \frac{1}{4\pi} \nabla \times (\mathbf{B} \cdot \nabla \mathbf{B})$$

and we have

$$\nabla \times \left[ \rho (\mathbf{V} \cdot \nabla \mathbf{V}) - \frac{1}{4\pi} (\mathbf{B} \cdot \nabla \mathbf{B}) \right] + \nabla \rho \times \nabla \Phi_G = 0,$$

where

$$\Phi_G = -\frac{GM_*}{\sqrt{R^2 + z^2}}, \quad -\nabla \Phi_G = -\frac{V^2_k}{R} \left( \mathbf{e}_R + \frac{z}{R} \mathbf{e}_z \right), \quad V^2_k \equiv \frac{GM_*}{R}.$$

Then,

$$0 = \frac{\partial}{\partial z} \left[ -\rho \left( \frac{V^2_\phi}{R} - \frac{V^2_k}{R} \right) - \frac{1}{4\pi} \left( B_R \frac{\partial}{\partial R} + B_z \frac{\partial}{\partial z} \right) B_R \right]$$

$$- \frac{\partial}{\partial R} \left[ -\frac{1}{4\pi} \left( B_R \frac{\partial}{\partial R} + B_z \frac{\partial}{\partial z} \right) B_z \right] - \left( \frac{\partial}{\partial R} \rho \right) \left( V^2_k \frac{z}{R} \right)$$
According to the Master Equation if we specify $\Omega$ as a function of $\Psi$, the particle density distribution $\rho$ becomes the source of the relevant magnetic field configuration. Consequently, the relevant pressure profile can be obtained from the vertical equilibrium equation

$$\frac{\partial p}{\partial z} \simeq -z \rho \Omega^2_k + \frac{1}{4\pi} B_R \left( \frac{\partial}{\partial R} B_z - \frac{\partial}{\partial z} B_R \right)$$

On the other hand, the derivation of the Master Equation is compatible with pressure tensors of the form

$$\underline{p} = p_{th} \underline{I} + p_F \underline{e}_\phi \underline{e}_\phi$$

where $p_F$ indicates the anisotropic pressure of a fast particle population that may be present and $p_{th}$ the total thermal pressure.
Results

\[ \psi = \psi_0 + \psi_1, \quad \psi_0 = B_0 R_0 R, \quad \varepsilon_0^0 \equiv \frac{\psi_N}{B_0 R_0^2} < 1 \]

\[ B_z = B_0 + \left( \frac{\psi_N}{R_0 \delta_R} \right) \left\{ D_{*2} \left( \frac{\bar{z}}{R} \right) + \left( \cos R_* + \varepsilon_* \cos 2R_* \right) \exp \left( -\frac{\bar{z}^2}{2} \right) \right\} \]

\[ B_R \approx \frac{\psi_N}{R_0} \left\{ -\frac{1}{\Delta} \frac{dD_{*2}}{d\bar{z}} + \frac{1}{\Delta_{\bar{z}}} \bar{z} \left( \sin R_* + \frac{\varepsilon_*}{2} \sin 2R_* \right) \exp \left( -\frac{\bar{z}^2}{2} \right) \right\} \]

\[ \rho \approx \rho_N \left[ D_{*2} \left( \frac{\bar{z}}{R} \right) + D_0^0 \frac{3 \sin^2 R_*}{2(1 + \varepsilon_* \cos R_*)} \exp \left( -\frac{\bar{z}^2}{2} \right) \right] \]
\[ \varepsilon_* < \frac{1}{4} \]  

\[ \varepsilon_* = D_0^0 \]

\[ R_* \equiv \frac{R - R_0}{\delta_R} \]

\[ \frac{z}{\Delta} \equiv \frac{z}{\Delta z} \]

\[ \bar{z} \equiv \frac{z}{\Delta} \]

\[ \bar{z} \equiv \frac{z}{\Delta z} \]

\[ \text{e.g.} \quad D_{*2} = D_{*2}^0 \exp(-\bar{z}) \]
Scale Distances

\[ \equiv \Delta = R_0 \left( \frac{\delta_R R_0 B_0}{2 \psi_N} \right)^{1/2} \frac{\Omega_D}{\Omega_k} \sim \frac{R_0^{1/3}}{k_0^{1/3} \bar{\epsilon}_0^{1/3}} \]

\( \Omega_k \) = Keplerian frequency, \( \bar{\epsilon}_0 \equiv \frac{\psi_N}{B_0 R_0^2} \)

\[ \Omega_D^2 = -2 \frac{d\Omega_k}{dR} R \Omega_k \]

\[ \Delta_z = \epsilon_N \Delta \]

\( \epsilon_N < 1 \)

\[ \delta_R = \left( \frac{R_0}{k_N k_0} \right)^{1/3} = \left( \frac{\psi_N B_0}{R_0 \left( 4 \pi \rho_N \right) \Omega_D^2} \right)^{1/3} = \frac{\bar{\epsilon}_0^{1/3}}{k_0^{1/3}} R_0^{1/3} \]

\[ \frac{1}{k_0} \equiv \frac{B_0}{\left( 4 \pi \rho_N \right)^{1/2} \Omega_D} \]

\[ \frac{1}{k_N} \equiv \frac{\psi_N}{R_0^2 \left( 4 \pi \rho_N \right)^{1/2} \Omega_D} \]
Fig. 4. Density level lines as a function of $R_*(2\pi)$ and $z_*$. The heavy line corresponds to the surface represented in Fig. 5, corresponding to $D_0(R_*, z_*) = 0.5$. The function $D_0$ is defined by eqs. (103) and (109), and the variables $R_*$ and $z_*$ are introduced in § 9.
PLASMA DISKS AND RINGS WITH "HIGH" MAGNETIC ENERGY DENSITIES

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ABSTRACT

The nonlinear theory of rotating axisymmetric thin structures in which the magnetic field energy density is comparable with the thermal plasma energy density is formulated. The only flow velocity included in the theory is the velocity of rotation around a central object whose gravity is dominant. The periodic sequence, in the radial direction, of pairs of opposite current channels that can form is shown to lead to relatively large plasma density and pressure modulations, while the relevant magnetic surfaces can acquire a "crystal structure." A new class of equilibria consisting of a series of plasma rings is identified, in the regimes where the plasma pressure is comparable to the magnetic pressure associated with the fields produced by the internal currents. The possible relevance of this result to the formation of dusty plasma rings is pointed out.

Subject headings: accretion, accretion disks — MHD — planets: rings — plasmas — stars: neutron — stars: rotation

1. INTRODUCTION

In the course of the search for the plasma collective modes that can provide the needed rate of angular momentum transport (Shakura & Sunyaev 1973) in accretion disks, we have realized that an elementary theory of the equilibrium configurations of differentially rotating axisymmetric plasma thin structures remained to be formulated. Thus, we have considered (Coppi 2005) the case where the rotation velocity is the only flow velocity present, the gravity of the central object is prevalent, and the magnetic field within the structure (ring sequence or disk) has two components: an "external" one in which the structure is embedded and an "internal" one due to currents flowing within the structure. No torque is associated with the resulting field configuration.

We note that the existing literature on rotating, perfectly conducting objects is extensive but addresses different issues. In particular, Ferraro (1937) dealt with the nonuniform rotation of the Sun and its magnetic field, Mestel (1961) with a rotating star that has a pure poloidal field initially, and Weber & Davis (1967) with the deceleration of the Sun through the action of magnetic stresses. In this context, more recently Lovelace (1976) and Blandford (1976) proposed accretion disk models in which the energy is extracted by electromagnetic torques, and Blandford & Payne (1982) treated the MHD flows from accretion disks and the associated production of radio jets.

Our elementary theory includes the role of marginally stable ballooning modes associated with both the differential rotation and the presence of an external field (Coppi & Coppi 2001; Coppi & Keyes 2003). We have started with the linear and nonlinear analysis (Coppi 2005) of disks in which the vertical confinement is due primarily to the vertical component of the gravitational force produced by the central object, as in the case of "gaseous" disks (Lynden-Bell 1969; Pringle & Rees 1972; Shakura & Sunyaev 1973), where the energy density of both the magnetic field in which they are immersed and that created by the currents within the disk is smaller than the thermal energy density (β < 1). A main result of this analysis is that the relevant poloidal magnetic field configuration can be characterized by a periodic sequence of pairs of toroidal current channels in which the currents have opposite directions and by the formation of a "crystal structure" of the magnetic surfaces. In fact, the vertical cross section of this structure can be visualized as a string of "field reverse configurations" (FRCs), the periodic reversal of the vertical field relative to the external field component being due to the currents within the structure.

An interesting question that arose from the same analysis (Coppi 2005) is whether a disk configuration would continue to exist in the case in which the magnetic field produced by the currents inside the plasma structure exceeds considerably the external magnetic field component, and the plasma pressure is contained (vertically) by the magnetic field pressure rather than by gravity. In fact, the results described in this paper show that the plasma density becomes strongly modulated by the current channels, and a sequence of plasma rings rather than a disk is formed. The theory is also given for the intermediate case, where the external field pressure is comparable with the plasma pressure and to the internally produced field. In this case a disk with a strongly corrugated (in the radial direction), vertical density profile is found.

Clearly this calls for a parallel analysis of self-gravitating plasma disks, which has been undertaken already. Moreover, other interesting questions arise. One is whether the tridimensional plasma modes that can arise in these configurations can produce sufficient rates of outward angular momentum transport and consequent rates of mass accretion that are consistent with the values of the effective diffusion coefficients (Shakura & Sunyaev 1973) employed for relevant models of accreting objects. A further question is whether an axisymmetric structure of the type identified in our analysis will evolve into a toroidally modulated coherent structure or a turbulent structure.

The present paper is organized as follows. In § 2 we discuss the basic equations for the vertical and radial momentum density balance in a differentially rotating plasma thin structure. The only velocity present is in the toroidal direction, and the magnetic field configuration is connected to it by the "isorotation condition" (Ferraro 1937) that is appropriate for relatively large electrical...
IV. Accretion Scenarios in the Presence of Magnetic Fields

\[ E_\phi = 0 \]

i) Simplest Disk Structure

\[ v_R \approx D_m \frac{1}{B_z} \left( \frac{\partial}{\partial R} B_z - \frac{\partial}{\partial Z} B_R \right) \]  \hspace{1cm} (1)

* The effects of a significant anomalous resistivity \( \eta = 4\pi D_m / c^2 \) would have to be important over the entire disk and a process to produce this resistivity would have to be identified.

* A large scale violation of the hyperconductivity condition \( E + v \times B / c = 0 \),

\[ v = \alpha v B + \Omega(\psi) Re_\phi \]  \hspace{1cm} (2)

such as that represented by Eq. (1), would prevent the consideration of important class of plasma modes (e.g. driven by the differential rotation and the vertical plasma pressure) relying on the validity of Eq. (2) and needed to produce outward angular momentum transport, “winds”, etc.
3D Standing Spirals (tightly wound + trailing)

* Do not shear apart

* Radial width of localization

\[ \Delta_R \approx \left( \frac{\gamma_0}{m_\phi \Omega_k} \right)^{1/2} \left( \frac{R_0}{k_0} \right) \sim \left( \frac{\gamma_0 v_A R_0}{\Omega_k} \right)^{1/2} \]

\( \gamma_0 = \) growth rate

* Height (vertical)

\[ \Delta_z \sim \left( \frac{c_s v_A}{\Omega_k} \right)^{1/2} \]

\[ \frac{\Delta_R}{\Delta_z} \sim \left( \frac{\gamma_0 R_0}{c_s} \right)^{1/2} \sim \left( \frac{\gamma_0 R_0}{\Omega_k H_0} \right)^{1/2} \]

Analytical Expression

\[ \hat{\xi}_z \approx \tilde{\xi}_z (R - R_0, z) \exp\left[ \gamma_0 t - i m_\phi \left( \Omega_0 t - \phi \right) + i k_R (R - R_0) \right] \]

\[ \Omega_0 = \Omega_k (R_0) \quad k_R \approx k_0 = \sqrt{3} \frac{\Omega_k}{v_A} \]
\[ \tilde{\xi}_z(R - R_0, z) \approx \tilde{\xi}_z^0 \exp \left\{ - \left[ \frac{(R - R_0)^2}{\Delta_R^2} + \frac{z^2}{2\Delta_z^2} \right] \right\} G_0^0(z) \]

\[ m_\phi k_R \frac{d\Omega_k}{dR} < 0 \quad \text{(trailing spirals)} \]

\[ \Delta_R^2 \approx - \frac{\gamma_0}{m_\phi k_R \frac{d\Omega_k}{dR}} > 0 \]

\[ G_0^0(z) = \text{eigenfunction’s vertical modulation} \]
General Relativity Effects

Schwartzchild Metric (Non-rotating)

\[ ds^2 = -\left(1 - \frac{2R_G}{r}\right)(cdt)^2 + \left(1 - \frac{2R_G}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2\right) \]

where \( R_G \equiv GM_*/c^2 \). In this case the radius of the marginally stable orbit (also known as ISCO) is

\[ R_{Ms} = 6R_G. \]

The phenomena we consider to guide the presented theory such as High Frequency Quasi Periodic Oscillations (HFQPOs) are estimated [Coppi & Rebusco, 2008] to be related to processes taking place at distances \( R \geq 10R_G \). Therefore, when considering a non-rotating black hole we can take General Relativity effects into account by adopting an effective potential such as the Paczynsky-Wiita gravitational potential

\[ \phi_G = -\frac{GM_*}{R - 2R_G}. \]
Kerr Metric (Rotating BH)

The radius $R_{MS}$ depends in a significant way on the value of the angular momentum $J = J e_z$ that a black hole can have. This is characterized by the dimensionless parameter

$$a_* = \frac{J}{M_* c R_G}$$

with $0 < a_* < 1$ , $a_* \to 1$ being the so-called “extreme Kerr” limit. In particular, $R_{MS}$ is different for direct and retrograde orbits and is a function of $a_*$. Specifically, when $a_* \to 1$ (extreme Kerr), $R_{MS} = R_G$ (for a direct orbit), $R_{MS} = 9 R_G$ (for a retrograde orbit). Clearly, for a direct orbit, the decrease of $\bar{R}_{MS}$ from 6, when $a_* = 0$, to 1 when $a_* \to 1$ is significant. As is well known, the relevant (Kerr) metric, of which the Schwartzchild metric is a special case, is

$$ds^2 = -\left(1 - \frac{2R_G r}{r_a^2}\right)(cdt)^2 - (2F_K)(ad\phi)(cdt)$$

$$+ \left( r^2 + a^2 + a^2 F_k \right) \sin^2 \theta (d\phi)^2 + \frac{r_a^2}{\Delta_a} dr^2 + r_a^2 (d\phi)^2 .$$

Here Boyer-Lindquist coordinates are used, $r_a^2 \equiv r^2 + a^2 \cos \theta$, $a \equiv a_* R_G = J / (M_* c)$, $\Delta_a^2 = r^2 \left(1 - 2 R_G / r\right) + a^2$ and $F_K = \left(2 r R_G / r_a^2 \right) \sin^2 \theta$. 

In this case $R_{MS}$ can be evaluated by considering the effective potential for particles orbits in the plane $z = 0$, whose radial velocity is given by \( \dot{R}^2/(2c^2) + V_{\text{eff}}(R, E_N, L) = E_N/c^2 = \mathcal{E} \), where

\[
V_{\text{eff}} = -\frac{R_G}{R} + \frac{L^2/c^2 - 2a^2 \mathcal{E}}{2R^2} - \frac{R_G}{R^3} \left( \frac{L}{c} - a\sqrt{\mathcal{E} + 1} \right)^2
\]

(*)

and $L$ is the particle specific angular momentum. For circular orbits $V_{\text{eff}} = \mathcal{E}$ and $dV_{\text{eff}}/dR = 0$, that give $\mathcal{E}$ and $L$ as functions of $R$. The radius $R_{MS}$ is given by $d^2V_{\text{eff}}/dR^2 = 0$ and, for $a \rightarrow R_G$, $R_{MS}^+/R_G \rightarrow 5 \mp 4$. Thus we may adopt Eq. (*) to add General Relativity corrections to the relevant theory developed in the Newtonian limit.
Plasma Regions in the Vicinity of Black Holes

i) Buffer Region

\[ R_{Er} < R \]

\( R_{Er} \) = outer radius of the Ergosphere.

BH rotational energy converted into the plasma energy within this region.

ii) Three-Regime Region

\[ R - \Delta_{sp} < R < R_{QPO} + \Delta_{sp} \]

\( R_{QPO} \) = co-rotation radius of plasma spirals.

\( \Delta_{sp} \) = radial half-width of spiral pattern.

iii) Structured Peripheral Region

\[ R_{QPO} + \Delta_{sp} < R \]

Accreting Ring Structure. Thermal emission.
Three-Regime Region

1) Extreme Regime (Runaway-like)

High Frequency QPOs emission
Steep spiral structures power law spectrum (without a breakup to energies of 1 MeV or higher)

2) Non-thermal Regime (Slideaway-like)

Jet emitting structures
Hard power law spectrum (80% of the 2-20keV flux)

3) Dissipative Thermalized Regime (Classical Resistive)

Dissipating ring structures
Thermalized component of spectrum much stronger than power law component

This scenario is reminiscent of the three-regimes that have been identified in magnetically confined current carrying plasmas by varying the streaming parameter $\xi = J_{\parallel} / (n_e v_{the})$, where $J_{\parallel} = J \cdot B / B$. 
HFQPOs

- Highly Coherent Peaks in the X-ray power spectra
- 0.1-1200 Hz
  HF -> few hundred Hz
- Show up alone OR in pairs OR more
- In **Black Holes**:
  stable 3:2

The experimental observations show that HFQPOs have frequencies connected to those of orbits related to the MS orbit (e.g., Remillard & McClintock 2006). Moreover HFQPOs occur at the same fixed frequencies for the same black hole candidate (the maximum of the relevant frequency shifts is about 15%) and these scale as $f_0 \propto 1/M_*$ for different sources, $M_*$ being the black hole mass.
universal Fermi interaction predated the Tiomno–Wheeler paper by about a month, but neither it nor Puppi’s later exposition of his ideas on the subject contained such a diagram.15

Even in his first years as a researcher in the 1930s, Wheeler could see that cosmic rays offered a source of particles far more energetic than could be imagined coming from any accelerator then envisioned. Right after the war, he promoted and secured a cosmic-ray laboratory at Princeton. “I was in a hurry,” he wrote, “and I wanted to see particle research conducted on my doorstep.”1

There is one more story to tell from Wheeler’s “everything is particles” period. In the summer of 1949, armed with a Guggenheim fellowship, he and his family, with his graduate student John Toll in tow, traveled to Europe. Wheeler meant to divide his time between some very far-out ideas (a purely electromagnetic world and a world without spacetime) and the more mundane physics of nuclear structure. He chose to settle in Paris, from where he could travel now and then to Copenhagen. As he wrote later, “I did not want to get back fully into the conversational culture of [Bohr’s] institute.”41

In the spring of that year he had submitted a draft manuscript to Bohr, suggesting that it be coauthored by Bohr, Wheeler, and Hill. Bohr agreed, and in a letter said, “I should like to think a few days whether I might suggest some smaller alterations or additions.”39 Three years later Bohr, still not wholly satisfied with the paper, suggested that it be published by Wheeler and Hill alone, as it finally was.39 In the fall of 1949, early in that hiatus, Wheeler came up with an explanation for the large deformations observed in some nuclei. He conveyed the idea to Bohr with the hope that it would be incorporated into their joint paper. Bohr sat on the idea and a year later Wheeler received a preprint from James Rainwater setting forth the same idea.39 For that work, Rainwater shared the 1975 Nobel Prize in Physics. Was Wheeler at least a little irked with Bohr? Not one whit, as far as I have been able to determine. The most that Wheeler could bring himself to say was, “I learned a lesson. When one discovers something significant, it is best to publish it promptly and not wait to incorporate it into some grander scheme. Waiting to assemble all the pieces might be all right for a philosopher, but it is not wise for a physicist.”39

Weapons work

Most physicists who were beyond their teen years in 1941, even if only by a little, signed up for war work. In January 1942, a month after Pearl Harbor, Wheeler joined Arthur Compton’s “Metallurgical Laboratory” at the University of Chicago, at first leaving his wife and two children in Baltimore, where, later that year, his third child, Alison, was born. In characteristic fashion, Wheeler looked beyond the first reactor (or “pile”), on which Fermi and some of his colleagues were working. He began thinking about the large plutonium-production reactors that would follow.

After being assigned by Compton to be the lab’s principal liaison scientist to DuPont, the company charged with designing and building the new reactors, Wheeler went in 1943 to the company’s headquarters in Wilmington, Delaware. He had already recognized the possibility that some as-yet-unknown fission fragments might have anomalously large neutron-absorption cross sections and thus might poison the chain reaction. So he supported the overdesigning of the reactors—something that DuPont’s conservative engineers were inclined to do anyway.

Wheeler’s caution paid off. On 27 September 1944, the first Hanford reactor, within hours of being powered up to its initial plateau of 9 megawatts, began to fizzle and die. After being turned off and “rested” overnight, the reactor was powered up again, only to exhibit the same distressing pattern of gradual demise. Wheeler, analyzing the ups and downs of the reactor’s behavior over its first two days, concluded that the culprit was an isotope with a half-life somewhat less than 11 hours that was itself the daughter of another radioactive isotope. It took him only a few minutes standing before a chart of the nuclides on the wall outside his Hanford office to conclude that the offending isotope was xenon-135, with a half-life of 9.2 hours, the daughter of iodine-135. That...
Relativistic magnetohydrodynamical effects of plasma accreting into a black hole

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By an explicit analytic solution it is shown how, in the accretion of a poloidally magnetized plasma into a Kerr black hole, a torque is exerted on the infalling gas, implying the extraction of rotational energy from the black hole. The torque arises from the twisting of magnetic field lines by the frame-dragging effect. It is also shown how, under suitable conditions, a sizable charge separation can be found in the magnetosphere of accreting black holes and hence an electric charge is expected to be induced on the black hole.

A large number of gravitationally collapsed objects (neutron stars or black holes) are observed to be members of binary systems of which one of the components is a normal star.\(^1\) Due to the frequent occurrence of magnetic fields in stellar atmospheres, it is to be expected that magnetic and electric fields will play a prominent role in the accretion processes. The treatment presented here could therefore be of great relevance in realistic astrophysical processes and at the same time answer two major fundamental issues in black-hole physics: (a) the possibility of extracting rotational or Coulomb energy from a black hole,\(^2\) and (b) the possibility of building a magnetosphere around a black hole with a sizable charge separation.\(^3\) We consider accretion into a Kerr black hole\(^4\) of a plasma embedded in a magnetic field which at infinity approaches a poloidal configuration (that is, an axially symmetric field with zero components about the axis of symmetry). The following simplifying assumptions are made: (a) The plasma is assumed to have negligible pressure; (b) infinite conductivity or \(F_\theta^\mu U^\nu = 0\), \(U^\nu\) being the four-velocity of the plasma stream and \(F_\mu^\nu\) the electromagnetic tensor; (c) the mass of the accreting plasma is assumed to be small by comparison to the mass of the black hole; and (d) the magnetic fields are assumed to be too weak to appear from the need of obtaining simple analytic solutions for the accretion process. More realistic regimes can then be further analyzed by numerical work. The background geometry is assumed to be given by the Kerr metric, which in the Boyer-Lindquist coordinates assumes the form

\[
d s^2 = \Sigma^{-1} d\phi^2 + \Sigma d\theta^2 + \Sigma \sin^2 \theta [(r^2 + a^2) d\phi - a dt]^2 - \Sigma^{-1} \Delta (dt - a \sin \theta d\phi)^2, \tag{1}\]

with \(\Delta = r^2 - 2Mr + a^2\) and \(\Sigma = r^2 + a^2 \cos^2 \theta\), \(M\) being the mass, and \(a\) the angular momentum per unit mass of the black hole.

The geodesics equations can be integrated.\(^5\) It follows that if \(U_\theta = -1\) and \(U_\varphi = 0\) at infinity then \(U_\varphi\) is a constant of the motion. We then have for the velocity field of the freely falling particles

\[
V^\mu = U^\mu / U^t = -\frac{\Delta \left[ -\Delta U_\theta^2 + 2M r (r^2 + a^2) \right]^{1/2}}{\Sigma (r^2 + a^2) + 2Mr a^2 \sin^2 \theta}, \tag{2a}\]

\[
V^\theta = U^\theta / U^t = \frac{\Delta U_\theta}{\Sigma (r^2 + a^2) + 2Mr a^2 \sin^2 \theta}, \tag{2b}\]

\[
V^\varphi = U^\varphi / U^t = \frac{R^\varphi / g^\varphi}{\Sigma}. \tag{2c}\]

Let us now consider the electromagnetic field associated with accreting plasma. Since we consider an axially symmetric configuration the electromagnetic field is totally described by the component \(A_\varphi\) of the vector potential and by the component \(H_\varphi = F_{\rho \varphi}\) of the magnetic field. We have

\[
F_{\rho \theta} = H_\varphi, \tag{3a}\]

\[
F_{\rho \varphi} = A_{\rho \varphi}, \tag{3b}\]

\[
F_{\theta \varphi} = A_{\rho \varphi}. \tag{3c}\]

From the \(\theta\) and \(\varphi\) component of the condition of infinite conductivity \(F_\mu U^\mu = 0\) we obtain two more components of the electromagnetic field tensor,

\[
F_{t \varphi} = V^\varphi H_\varphi + V^\theta \frac{\partial A_\theta}{\partial \varphi}, \tag{4a}\]

\[
F_{t \theta} = - V^\varphi H_\varphi + V^\theta \frac{\partial A_\theta}{\partial \theta}. \tag{4b}\]

The equation for \(H_\varphi\) can be obtained from the Maxwell equation \(* F_{\mu \nu} = 0\) and the stationarity condition \((\partial A_\varphi / \partial t) = 0\). We then have

\[
\frac{\partial}{\partial \varphi} (H_\varphi V^\varphi) + \frac{\partial}{\partial \theta} (H_\varphi V^\theta) + \frac{\partial V^\varphi A_\theta}{\partial \varphi} - \frac{\partial V^\theta A_\varphi}{\partial \theta} = 0. \tag{5}\]

From the \(\varphi\) component of the equation of infinite conductivity and the condition of stationarity \(\partial A_\varphi / \partial t = 0\) we obtain
will take many eons to exhaust the hole’s enormous store of spin energy.

Your crew and countless generations of their descendants can call this artificial world “home” and use it as a base for future explorations of the Universe. But not you. You long for the Earth and the friends whom you left behind, friends who must have been dead now for more than 4 billion years. Your longing is so great that you are willing to risk the last quarter of your normal, 200-year life span in a dangerous and perhaps foolhardy attempt to return to the idyllic era of your youth.

Time travel into the future is rather easy, as your voyage among the holes has shown. Not so travel into the past. In fact, such travel might be completely forbidden by the fundamental laws of physics. However, DAWN tells you of speculations, dating back to the twentieth century, that backward time travel might be achieved with the aid of a hypothetical space warp called a *wormhole*. This space warp consists of two entrance holes (the wormhole’s *mouths*), which look like holes but without horizons, and which can be far apart in space (Figure P.7). Anything that enters one mouth finds itself in a short tube (the wormhole’s *throat*) that leads to and out of another mouth. The tube cannot be seen from our Universe because it passes through *hyperspace* rather than through normal space, and is possible for time to hook up through the wormhole in a way through our Universe, DAWN explains. By traversing the tube in one direction, say from the left mouth to the right, go backward in our Universe’s time, while traversing in the other direction, from right to left, one would go forward. Such a tube would be a time warp, as well as a space warp.

The laws of quantum gravity demand that exceeding gravity holes of this type exist, DAWN tells you. These quantum holes must be so tiny, just $10^{-51}$ centimeter in size, that their existence—far too brief, $10^{-45}$ second, to be usable for time...