Lectures in Core Collapse Supernova Theory
“Lasciate ogne speranza, voi ch’intrate.”

Dante Alighieri, La Comedia di Dante, L’Inferno, Canto III
Core Collapse Supernova Paradigm

Pre-supernova Structure

representative of a 15 $M_\odot$ star

Core Collapse and Explosion

1. $\text{He}$
2. $\text{C, O}$
3. $\text{O, Ne, Mg}$
4. Earth
5. $2.2 \times 10^8 \text{ cm}$
6. $\text{Si}$
7. $\text{Fe}$
8. $\text{neutrinos}$
9. $\text{shock}$
**Important Neutrino Emissivities/Opacities**

**“Standard” Emissivities/Opacities**

\[ e^{- (+)} + p(n), A \leftrightarrow n(p), A' \]

\[ e^+ + e^- \leftrightarrow \nu_{e, \mu, \tau} + \bar{\nu}_{e, \mu, \tau} \]

\[ \nu + n, p, A \rightarrow \nu + n, p, A \]

\[ \nu + e^-, e^+ \rightarrow \nu + e^-, e^+ \]

\[ N + N \leftrightarrow N + N + \nu_{e, \mu, \tau} + \bar{\nu}_{e, \mu, \tau} \]

\[ \nu_e + \nu_e \leftrightarrow \nu_{\mu, \tau} + \bar{\nu}_{\mu, \tau} \]

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- Nucleons in nucleus independent.
- No energy exchange in nucleonic scattering.

**Langanke et al. PRL, 90, 241102 (2003)**
- Include correlations between nucleons in nuclei.

- (Small) Energy is exchanged due to nucleon recoil.
- Many such scatterings.

**Burrows and Sawyer, PRC, 59, 510 (1999)**

- New source of neutrino-antineutrino pairs.


**Janka et al. PRL, 76, 2621 (1996)**

Electron-neutrino mean free path decreases much more rapidly with density than core size, and the neutrinos become trapped in the core.

⇒ A degenerate electron-neutrino Fermi sea develops.
When shock passes electron neutrinosphere, they escape in a burst.
Large positron population. Neutrino production via:

\[ e^- + p \rightarrow \nu_e + n \]
\[ e^+ + n \rightarrow \bar{\nu}_e + p \]
\[ e^- + e^+ \rightarrow \nu_{e,\mu,\tau} + \bar{\nu}_{e,\mu,\tau} \]
\[ N + N \leftrightarrow N + N + \nu_{e,\mu,\tau} + \bar{\nu}_{e,\mu,\tau} \]
\[ \nu_e + \bar{\nu}_e \rightarrow \nu_{\mu,\tau} + \bar{\nu}_{\mu,\tau} \]
Anatomy of Neutrino Luminosities

Increase in neutrino rms energies as PNS initially contracts.

Electron-Neutrino Burst

Accretion Luminosity

How is the supernova shock wave revived?

The most fundamental question in supernova theory

- Gravity
- Neutrino Heating
- Convection
- Shock Instability
- Nuclear Burning
- Rotation
- Magnetic Fields

*New Ingredient*
Neutrino heating depends on neutrino luminosities, spectra, and angular distributions.

\[ \dot{\epsilon} = \frac{X_n}{\lambda_0^2} \frac{L_{\nu_e}}{4 \pi r^2} \left( \frac{1}{F} \right) + \frac{X_p}{\lambda_0^2} \frac{L_{\bar{\nu}_e}}{4 \pi r^2} \left( \frac{1}{F} \right) \]

Neutrino heating is sensitive to all three (most sensitive to neutrino spectra).

⇒ Must compute neutrino distributions.

\[ f(t, r, \theta, \phi, E, \theta_p, \phi_p) \]

Multifrequency

\[ E_R(t, r, \theta, \phi, E) = \int d\theta_p \, d\phi_p \, f \]

Multifrequency

\[ E_R(t, r, \theta, \phi) = \int dE \, d\theta_p \, d\phi_p \, f \]

(\text{Parameterize Isotropy})

Gray

(\text{Parameterize Isotropy and Spectra})
Mean opacities in gray approximation require neutrino spectra, which are not known in this approximation.
The Boltzmann equation contains the same information as an infinite hierarchy of equations for the angular “moments” of the neutrino distribution function:

\[ \int d\mu \left[ \frac{\partial f}{\partial t} = L[f] \right] \Rightarrow \frac{\partial \psi^0}{\partial t} = \ldots \]

\[ \psi^0 = \frac{1}{2} \int d\mu f \]

\[ \int d\mu \mu \left[ \frac{\partial f}{\partial t} = L[f] \right] \Rightarrow \frac{\partial \psi^1}{\partial t} = \ldots \]

\[ \psi^1 = \frac{1}{2} \int d\mu \mu f \]

\ldots

Approximation:

- Truncate hierarchy at the level of the “zeroth” moment (neutrino energy density).
- **Closure**: Relate the first moment (momentum density/flux) to the energy density so as to satisfy known limits:

\[ \psi^1 = -\Lambda \left( \frac{\partial \psi^0}{\partial r} \right) \]

\[ \Lambda = \frac{1}{\frac{3}{\lambda} \frac{1}{\psi^0} \frac{\partial \psi^0}{\partial r}} \]

**Diffusion Limit: Fick’s Law**

\[ \psi^1 \rightarrow -\frac{\lambda}{3} \frac{\partial \psi^0}{\partial r} \]

**Free Streaming Limit**

\[ \psi^1 \rightarrow \psi^0 \]
The Boltzmann equation contains the same information as an infinite hierarchy of equations for the angular “moments” of the neutrino distribution function:

\[
\int d\mu \left[ \frac{\partial f}{\partial t} = L[f] \right] \Rightarrow \frac{\partial \psi^0}{\partial t} = ... \quad \psi^0 = \frac{1}{2} \int d\mu f
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\]

...

**Approximation:**

- Truncate hierarchy at level of “first” moment (neutrino momentum density).
- **Closure:** Relate the second (and third) moments to the zeroth moment using “Eddington factors,” which are the ratio of these higher moments to the zeroth moment.

Eddington factors can be computed at different levels of approximation:

- “Prescribed” closure (e.g., maximum entropy closure).
- Approximate Boltzmann solution.
- Exact Boltzmann solution.
The simulation of core collapse supernovae with fully general relativistic, multi-angle, multi-frequency, Boltzmann neutrino transport has been achieved for spherically symmetric cases.

What’s missing?
- Better weak interaction physics?
- Better EOS?
- Neutrino mixing?
- Multi-D effects.

Mezzacappa et al., PRL, 86, 1935 (2001)  
Liebendoerfer et al., PRD, 63, 103004 (2001)
General Relativistic Boltzmann Equation

\[ p^\mu \mathcal{L}^\mu \hat{\mu} \frac{\partial f}{\partial x^\mu} + (eF^j_{\nu\rho}p^\nu - \Gamma^j_{\nu\rho}p^\nu p^\rho) \frac{\partial u^i}{\partial p^j} \frac{\partial f}{\partial u^i} = C[f] \]

1. Geometric Effects
2. Special Relativistic Effects
3. General Relativistic Effects

E.G.: Describes increase in neutrino Fermi energy in trapped regions as density increases.

<table>
<thead>
<tr>
<th>Spatial Dimensions</th>
<th>Netwonian or GR</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Partial Weak Interactions (Thompson et al. (2003))</th>
<th>Complete Weak Interactions</th>
<th>Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liebendoerfer et al. (2004) Lentz et al. (2010)</td>
<td>1</td>
<td>GR</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>Full GR</td>
</tr>
</tbody>
</table>
"Standard" Emissivities/Opacities

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Janka et al. PRL, 76, 2621 (1996)
Comparison of 1D Simulations; 15 W-H Progenitor

Shock Radii vs Post Bounce Time

- No Obsrvr Correc--Newt
- Reduced--Newt
- Full--Newt
- Full--GR

Shock Radius [km]

Post Bounce Time [s]
Comparison of 1D Simulations; 15 W-H Progenitor

E-Neutrino Luminosity vs Radius at 100 ms Post Bounce

Comparison of 1D Simulations; 15 W-H Progenitor

E-Antineutrino Luminosity vs Radius at 100 ms Post Bounce

Comparison of 1D Simulations; 15 W-H Progenitor

X-Neutrino Luminosity vs Radius at 100 ms Post Bounce

Comparison of 1D Simulations; 15 W-H Progenitor

X-Antineutrino Luminosity vs Radius at 100 ms Post Bounce
Comparison of 1D Simulations; 15 W-H Progenitor

E-Neutrino RMS Energy vs Radius at 100 ms Post Bounce

Comparison of 1D Simulations; 15 W-H Progenitor

E-Antineutrino RMS Energy vs Radius at 100 ms Post Bounce

Comparison of 1D Simulations; 15 W-H Progenitor

X-Neutrino RMS Energy vs Radius at 100 ms Post Bounce

Comparison of 1D Simulations; 15 W-H Progenitor

X-Antineutrino RMS Energy vs Radius at 100 ms Post Bounce

4/11/11
Impact of Energy Exchange in Neutrino Scattering on Nucleons

Shock Radii vs Time from Bounce
W-H 15 Solar Mass Progenitor; Effect of Dimensionality and Neutrino Rates
# SELECT NEUTRINO TRANSPORT MILESTONES

<table>
<thead>
<tr>
<th>Publication</th>
<th>Milestone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bruenn (1975) Ann. N.Y. Acad. Sci. 262 80</td>
<td>1D MGFLD Implementations</td>
</tr>
<tr>
<td>Kotake et al. (2006) AIP Conf. Proc. 847 421</td>
<td>2D MGFLD Implementations</td>
</tr>
<tr>
<td>Cardall et al. (2005), in <em>Open Issues in Core Collapse Supernova Theory</em>, eds. Mezzacappa and Fuller</td>
<td></td>
</tr>
</tbody>
</table>
The simulation of core collapse supernovae with fully general relativistic, multi-angle, multi-frequency, Boltzmann neutrino transport has been achieved for spherically symmetric cases.

<table>
<thead>
<tr>
<th>Transport Method</th>
<th>Dimensions: Spherical Symmetry</th>
<th>Total Number of Dimensions</th>
<th>Dimensions: Axisymmetry</th>
<th>Total Number of Dimensions</th>
<th>Dimensions: No Symmetries</th>
<th>Total Number of Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>MGFLD</td>
<td>r,E</td>
<td>2</td>
<td>r,θ,E</td>
<td>3</td>
<td>r,θ,φ,E</td>
<td>4</td>
</tr>
<tr>
<td>MGVET</td>
<td>r,E</td>
<td>2</td>
<td>r,θ,E</td>
<td>3</td>
<td>r,θ,φ,E</td>
<td>4</td>
</tr>
<tr>
<td>Boltzmann</td>
<td>r,μ,E</td>
<td>3</td>
<td>r,θ,μ,η,E</td>
<td>5</td>
<td>r,θ,φ,μ,η,E</td>
<td>6</td>
</tr>
</tbody>
</table>

Laying the theoretical and numerical foundations for 2D/3D Boltzmann studies:

General relativistic treatments:
There are more things in heaven and earth, Horatio, Than are dreamt of in your philosophy.

Stationary Accretion Shock Instability (SASI)


Shock wave unstable to non-radial perturbations.

SASI has axisymmetric and nonaxisymmetric modes that are both linearly unstable!


- Decreases advection velocity in gain region.
- Increases time in the gain region.
- Moves shock toward silicon/oxygen layers.
- Generates convection.
Simulation Building Blocks

- **“RbR-Plus” MGFLD Neutrino Transport**
  - $O(v/c)$, GR time dilation and redshift, GR aberration (in flux limiter)

- **2D PPM Hydrodynamics**
  - GR time dilation, effective gravitational potential, adaptive radial grid

- **Lattimer-Swesty EOS**
  - 180 MeV (nuclear compressibility), 29.3 MeV (symmetry energy)

- **Nuclear (Alpha) Network**
  - 14 alpha nuclei between helium and zinc

- **2D Effective Gravitational Potential**

- **Neutrino Emissivities/Opacities**
  - “Standard” + Elastic Scattering on Nucleons
    + Nucleon–Nucleon Bremsstrahlung

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**“Ray-by-Ray-Plus” Approximation**

- Solve set of 1D problems.
- Ignore differences in lateral fluxes across 1D problems.
  
Shock Radii vs Time from Bounce
Effect of Progenitor Mass

Explosion Energy versus Progenitor Mass
Wossley-Heger 12, 15, 20, 25 Solar Mass Nonrotating Progenitors; 256 x 256 Spatial Resolution

Advection and Heating Timescales vs Time from Bounce
25 W-H Progenitor

Explosion Energy as a Function of Post-Bounce Time
W-H 25 Solar Mass Progenitor

$t_{\text{explosion}} \sim 200$ ms

$E_{\text{explosion}} \sim "several" \times 10^{50}$ erg

And what about the progenitors?

Old and new progenitors have very different structure at critical junctures/times.

And progenitors are not 1D, not even close…

See Meakin and Arnett, astro-ph/1101.5646v1 and Dave’s lectures here.
What can we expect from 3D?


Anatomy of a Mach Reflection

- Symmetry of Mach reflection broken in three dimensions.
- Internal shock (orthogonal to supernova shock) leads to two counter-rotating flows.
Ramifications...

SASI-induced counter rotating flows.

Inner flow capable of spinning up remnant NS to 50 ms periods, even beginning with spherically symmetric initial conditions.

Implications for
- the growth of B fields?
- the supernova mechanism?
- supernova observables?

Ongoing 3D Multi-Physics Simulations


Simulation Building Blocks

✦ “RbR-Plus” MGFLD Neutrino Transport
  • O(v/c), GR time dilation and redshift, GR aberration (in flux limiter)

✦ 3D PPM Hydrodynamics
  • GR time dilation, effective gravitational potential, adaptive radial grid

✦ 3D Effective Gravitational Potential

✦ Nuclear (Alpha) Network
  • 14 alpha nuclei between helium and zinc

✦ Lattimer-Swesty EOS
  • 220 MeV (nuclear compressibility), 29.3 MeV (symmetry energy)

✦ Neutrino Emissivities/Opacities
  • “Standard” + Elastic Scattering on Nucleons
    + Nucleon–Nucleon Bremsstrahlung

Resolution

512 X 128 X 256
⇒ 32,768 processors

~ 2000 hours

~ 65 M processor-hours
15 M$_\odot$ Heger

- Shock Radius 1D
- Min/Mean/Max Shock Radius 2D
- Min/Mean/Max Shock Radius 3D

Shock Radius [km]

Time from Bounce [s]
Turbulence introduced by SASI-induced shear flow amplifies magnetic field strength.

- Field topology is complex, consisting of numerous intertwined tubules.
- Size of the tubules and field strength is limited by numerical resolution.
- Field strength is not amplified to dynamically significant levels.
- Field strength is amplified to levels observed in neutron stars.


See also Obergaulinger and Janka (2011), astro-ph/1101.1198v1
• Core Bounce: Requires realistic (3D) GR electron-neutrino transport.
  1D: Liebendoerfer et al. 2001. PRD 63, 103004.

• PNS Instabilities: Require realistic 3D GR multiflavor neutrino transport.
  1. Prompt convection.
  2. PNS convection.

• Neutrino-Driven Convection and SASI: Require realistic 3D explosion models.
Prediction from parameterized model.

Yakunin et al. 2010, *Class. Quant. Grav.* 27, 194005
ANATOMY OF A GRAVITATIONAL WAVE SIGNAL

- Bounce
- Prompt Convection
- Early Shock Deceleration

- Lower-Frequency Envelope: SASI-Induced Shock Excursions
- Higher-Frequency Variations: Impingement of Downflows on PNS from Neutrino-Driven Convection and SASI (+ PNS Convection).

- Rise: Prolate Explosion
ANATOMY OF A GRAVITATIONAL WAVE SIGNAL

Shock Deceleration

Prompt Convection

PNS Convection/SASI-Induced PNS Flows

Explosion
Neutrino-Driven Convection
Post-Shock SASI Flows

2D MODELS: GRAVITATIONAL WAVE ENERGY EMITTED

\[ E_{GW} \left(10^{-8} M_\odot c^2\right) \]

- 12M_\odot
- 15M_\odot
- 25M_\odot

\[ \text{time from bounce (ms)} \]

### SELECT MILESTONES IN CORE COLLAPSE SUPERNova THEORY

<table>
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<th>Publication</th>
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</table>
Wilson (1974) PRL **32**, 849 | Core collapse supernovae can be neutrino driven!  
First numerical models. |
Framed contemporary core collapse supernova theory. |
Revolutionized core collapse supernova theory. |
| Mezzacappa et al. (2001) PRL **86** 1935  
Liebendoerfer et al. (2001) PRD 63 3004  
Missing link. |

3D awaits!
Multi-institution, multi-investigator, multi-disciplinary effort.

Applied Math/CS Collaborators
- Closures, Solvers: Hauck, D’Azevedo
- Data Management: Klasky and collaborators
- Networking: Beck, Rao, and collaborators
- Visualization: Ahern, Ma, Meredith, Pugmire, Toedte
- Cray Center of Excellence: Levesque, Wichmann

Funded by
Office of Science
U.S. Department of Energy
“Per aspera ad astra.”

Lucius Annaeus Seneca
Roman Philosopher, Statesman, Dramatist
4 BC – 65 AD