Quantum gravitational aspects inside a black hole

Brahma and DY, 1804.02821
Chen, Sasaki and DY, 1806.03766
Bouhmadi-Lopez, Brahma, Chen, Chen and DY, 1902.07874
Brahma and DY, 1906.06022
Bouhmadi-Lopez, Brahma, Chen, Chen and DY, in preparation

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\[
    ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2
\]

Inside a Schwarzschild black hole, there exists a **singularity**. One cannot extend geometry beyond this region.
At the singularity, all differential geometrical tools never work. How can we overcome this?
Perhaps, one way is to resolve the singularity by some method and to extend the geometry beyond the singularity. This is so-called a regular black hole.
However, after careful evaluations, one can see that a regular black hole constructed by usual matter may fail to resolve all singularities.
This strongly indicates that we need to investigate quantum gravitational effects inside a black hole.
Recently, Haggard and Rovelli proposed that quantum gravity effects will modify both inside and outside a black hole at the same time.
Effective black-to-white hole bounces: The cost of surgery

Suddhasattwa Brahma\textsuperscript{1*} and Dong-han Yeom\textsuperscript{1,2†}

However, we have shown that such a surgery modifies not only near the black hole but also up to infinity.
Recently, Ashtekar, Olmedo and Singh reported that LQG can explain a big bounce inside a black hole.
Comment on “Quantum Transfiguration of Kruskal Black Holes”

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However, there have been criticisms such that their solution looks unphysical in several aspects.
However, there have been criticisms such that their solution looks unphysical in several aspects.
Let us obtain a wisdom from a traditional approach.

溫故知新
Quantum Theory of Gravity. I. The Canonical Theory

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(Received 25 July 1966; revised manuscript received 9 January 1967)

\[
\left[ G_{ijkl} \frac{\delta}{\delta \gamma_{ij}} \frac{\delta}{\delta \gamma_{kl}} + \gamma^{1/2} (3) R \right] \Psi^{(3)G} = 0
\]
The Wheeler–DeWitt equation

\[ \hat{\mathcal{H}} \Psi = 0 \]

quantum Hamiltonian constraint

wave function of field space

Let us study the quantum gravitational wave function inside a black hole.
\[ ds^2 = -N^2(t)dt^2 + e^{2X(t)}dR^2 + r_s^2e^{2(Y(t)-X(t))}d\Omega_2^2 \]

One way to present a metric inside a black hole

\[ X(t) = \log \tan \frac{t}{2} \]

\[ Y(t) = \log \frac{1}{2} \sin t \]

A classical solution inside a Schwarzschild black hole.
\[ ds^2 = -N^2(t) dt^2 + e^{2X(t)} dR^2 + r_s^2 e^{2(Y(t)-X(t))} d\Omega_2^2 \]

One way to present a metric inside a black hole

\[ Y = -\log(e^X + e^{-X}) \]

One can remove time and show the on-shell solution as a relation between \( X(t) \) and \( Y(t) \).
\[ ds^2 = -N^2(t)dt^2 + e^{2X(t)}dR^2 + r_s^2 e^{2(Y(t) - X(t))}d\Omega_2^2 \]

One way to present a metric inside a black hole

\[
\left( \frac{\partial^2}{\partial X^2} - \frac{\partial^2}{\partial Y^2} + 4r_s^2 e^{2Y} \right) \Psi[X, Y] = 0
\]

The Wheeler–DeWitt equation presented by \(X\) and \(Y\).
This was also known previously, e.g., gr-qc/9411070, hep-th/0107250, etc.
\[ ds^2 = -N^2(t)dt^2 + e^{2X(t)}dR^2 + r_s^2 e^{2(Y(t)-X(t))}d\Omega_2^2 \]

One way to present a metric inside a black hole

\[
\left( \frac{d^2}{dX^2} + k^2 \right) \phi[X] = 0
\]

\[
\left( \frac{d^2}{dY^2} - 4r_s^2 e^{2Y} + k^2 \right) \psi[Y] = 0
\]

Separation of variable
\[ ds^2 = -N^2(t) dt^2 + e^{2X(t)} dR^2 + r_s^2 e^{2(Y(t) - X(t))} d\Omega_2^2 \]

One way to present a metric inside a black hole

\[ \phi_k [X] = e^{\pm ikX} \]

\[ \psi_k [Y] = C_1 I_{ik}(2r_s e^Y) + C_2 K_{ik}(2r_s e^Y) \]

hyperbolic Bessel function

General analytic solution
\[ ds^2 = -N^2(t)dt^2 + e^{2X(t)}dR^2 + r_s^2 e^{2(Y(t)-X(t))}d\Omega_2^2 \]

One way to present a metric inside a black hole

\[ \phi_k[X] = e^{\pm ikX} \]

\[ \psi_k[Y] = C_1 I_{ik}(2r_s e^Y) + C_2 K_{ik}(2r_s e^Y) \]

Since the second term diverges as \( Y \) increases, we will ignore the term.
\[ \Psi[X,Y] = \int_{-\infty}^{\infty} f(k) e^{-ikX} I_{ik}(2r_s e^Y) dk \]

This is the most generic solution of the WDW equation for inside a Schwarzschild black hole.
\[ ds^2 = -N^2(t)dt^2 + e^{2X(t)}dR^2 + r_s^2 e^{2(Y(t)-X(t))}d\Omega_2^2 \]

One way to present a metric inside a black hole

\[ Y = -\log(e^X + e^{-X}) \]

In this solution, the \textbf{event horizon} is located at \( X, Y \to -\infty \), while the \textbf{singularity} is located at \( X \to \infty \) and \( Y \to -\infty \).
\[ \Psi[X, Y] = \int_{-\infty}^{\infty} f(k) e^{-ikx} I_{ik}(2r_s e^Y) \, dk \]

\[ f(k) = \frac{2A e^{-\sigma^2 k^2 / 2}}{\Gamma(-ik)r_s^{ik}} \]

We will impose the boundary condition such that the wave function as a (Gaussian) peak at the event horizon, because it is reasonable to assume that the solution is classical at the horizon.
This is the numerical plot of the wave function.
This is the numerical plot of the wave function. The red curve is the peak of the wave function, i.e., the steepest-descent.
This steepest-descent coincides well with the classical trajectory.

\[ Y = -\log(e^X + e^{-X}) \]
One way to present a metric inside a black hole

\[
ds^2 = -N^2(t)dt^2 + e^{2X(t)}dR^2 + r_s^2 e^{2(Y(t) - X(t))}d\Omega_2^2
\]

\[
\left( \frac{d^2}{dX^2} + k^2 \right) \phi[X] = 0
\]

\[
\left( \frac{d^2}{dY^2} - 4r_s^2 e^{2Y} + k^2 \right) \psi[Y] = 0
\]

This result is not so surprising because there is a potential barrier in the \( Y > 0 \) region.
$ds^2 = -N^2(t) dt^2 + e^{2X(t)} dR^2 + r_s^2 e^{2(Y(t) - X(t))} d\Omega_2^2$

One way to present a metric inside a black hole

\[
\left( \frac{d^2}{dX^2} + k^2 \right) \phi[X] = 0
\]

\[
\left( \frac{d^2}{dY^2} - 4r_s^2 e^{2Y} + k^2 \right) \psi[Y] = 0
\]

Incoming Gaussian pulse at the horizon ($X, Y \to -\infty$) will reach the potential barrier nearby $Y = 0$ and bounced to the singularity ($X \to \infty$ and $Y \to -\infty$).
This steepest–descent coincides well with the classical trajectory.

\[ Y = -\log(e^X + e^{-X}) \]
This steepest-descent coincides well with the classical trajectory.

\[ Y = -\log(e^X + e^{-X}) \]
However, I was disappointed because the steepest-descent simply moves toward the singularity. Then there is no new effect from quantum gravity and the singularity is not resolved.
However, note that there is an ambiguity to define the arrow of time.
One can interpret that there is only one arrow of time. However, it is fair to say that there are two arrows of time.
If we interpret following the latter way, it is very interesting. The two parts of black hole spacetime is **annihilated** at a hypersurface.
Annihilation to nothing.
This may be an opposite process of the *creation from nothing*. 
Also, a possible realization of the DeWitt boundary condition.
Now let us see both of inside and outside. If we only focus outside, then it is semi-classical and the probability of each slice will not vary.
However, if we evaluate the probability of outside and inside together, it will approach to zero.
This process is definitely non-unitary and we will lose information.
\[ |f\rangle = \sum_j a_j |f^j\rangle \]

\[ \langle f | i \rangle = \int_{i \rightarrow f_j} DgD\phi \; e^{is} \]

path integral as a propagator

However, we need to see the entire wave function.
\[ |f\rangle = \sum_j a_j |f^j\rangle \]

\[ \langle f | i \rangle = \int_{i \rightarrow f} Dg D\phi e^{-S_E} \] Euclidean analytic continuation

However, we need to see the entire wave function.
\[ |f\rangle = \sum_j a_j |f^j\rangle \]

\[ \langle f | i \rangle = \int_{i \to f^j} Dg D\phi \ e^{-S_E} \]

\[ \cong \sum_{i \to f^j} e^{-S_{E}^{\text{on-shell}}} \quad \text{steepest-descent approximation} \]

\[ \quad \text{need to find/sum instantons} \]

However, we need to see the entire wave function.
\langle f | i \rangle = \int_{i \to f} DgD\phi \ e^{-S_E}
\[ \langle f | i \rangle = \int_{i \rightarrow f} Dg D\phi \ e^{-S_E} \]
\[ \langle f | i \rangle = \int_{i \to f} Dg D\phi \ e^{-S_E} \]

\[ |f\rangle = \sum_{i} a_i |f^i\rangle \]

Hartle and Hertog, 2015
\[ \langle f | i \rangle = \int_{i \to f} DgD\phi \ e^{-S_E} \]

\[ |f\rangle = \sum_j a_j |f_j\rangle \]
\[ \langle f | i \rangle \equiv \sum_{i \to f} e^{-S^\text{on-shell}_E} \]

\[ |f\rangle = \sum_j a_j |f^j\rangle \]
Let us see the entire wave function.
In the path integral, there exists a tunneling channel such that there is no formation of a black hole, even though the probability is very low (Chen, Sasaki and DY, 1806.03766).
\[ p_1[h_A] \sim 1 \]

\[ p_2[h_A] \sim e^{-M^2} \]

From the beginning, the first history is dominant in terms of probability.
\[ p_1[h_A] \sim 1 \]
\[ \rightarrow p_1[h_{\text{vac}} \cup h_B] = 0 \]
\[ p_2[h_A] \sim e^{-M^2} \]

However, as the time slice evolves, the probability of the first history will decay to zero.
\[ p_1 + p_2 + \ldots = 1 \]

\[ p_1[h_A] \sim 1 \quad \rightarrow \quad p_1[h_{\text{vac}} \cup h_B] = 0 \]

\[ p_2[h_A] \sim e^{-M^2} \quad \rightarrow \quad p_2[h_B] \sim 1 \]

If the sum of the probabilities should be preserved, then the probability of the second history should be dominated later.
So, in the late time, the wave function is dominated by the trivial geometry which has no loss of information.
So in the end everyone was right in a way. Information is lost in topologically non-trivial metrics like black holes. This corresponds to dissipation in which one loses sight of the exact state. On the other hand, information about the exact state is preserved in topologically trivial metrics. The confusion and paradox arose because people thought classically in terms of a single topology for spacetime. It was either $R^4$ or a black hole. But the Feynman sum over histories allows it to be both at once. One can not tell which topology contributed to the observation, any more than one can tell which slit the electron went through in the two slits experiment. All that observation at infinity can determine is that there is a unitary mapping from initial states to final and that information is not lost.

Hawking, 2005
Revisit the conceptual picture of Hartle and Hertog.
This is based on a very brave interpretation of the WDW wave function.
However, this will definitely shed some light to the resolution of the information loss paradox.
Thank you very much