Final Stages of a Neutron Star Binary System

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IK meeting 20th July, 2015
1. **Gravitational waves (GWs)**
   - Classical dynamics
   - Effective-one-body formalism

2. **On the fate of neutron star binary mergers**
   - Critical NS mass
   - On the stability of the post-merger core

3. **Short Gamma-ray Bursts**
   - Sub-classification
   - GRB 090227B (Family-2): Baryon load

4. **GWs emission from short GRBs**
   - Signal-to-Noise ratio
   - Detection rate in Ad.LIGO
In 1978, Taylor and Hulse reported that there was a systematic shift in the observed time of periastron relative to that expected if the orbital separation remained constant. The decay of the orbit of this system is exactly the rate predicted by GR and has provided direct evidence that gravitational radiation exists. 1993 Nobel Prize in Physics was awarded for this work.

\[
- \frac{dE_{\text{orb}}}{dt} = \frac{dE_{\text{GW}}}{dt}
\]
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The orbital angular velocity of the binary with components \((M_1, R_1)\) and \((M_2, R_2)\) orbiting each other in a circular orbit of radius \(r\), is given by \(\omega_k = \sqrt{G(M_1 + M_2)/r^3}\), and its total binding energy is,

\[
E_b = -\frac{1}{2} \frac{G M_1 M_2}{r}.
\] (1)

The leading term driving the loss of binding energy via gravitational wave emission is given by

\[
-\frac{dE_b}{dt} = \frac{32}{5} \frac{G^4}{c^5} \frac{(M_1 + M_2)(M_1 M_2)^2}{r^5},
\] (2)

which leads to a decreasing of the separation \(r\) with time and consequently a shortening of the orbital period \(P = 2\pi/\Omega\).

The loss of orbital binding energy by emission of GWs as a function of the frequency \(f\) as from the neutron star system in spiral phase can be written,

\[
\frac{dE_b}{df} = -\frac{1}{3} (\pi G)^{2/3} M^{5/3} f^{-1/3}.
\] (3)

where \(\omega_k = 2\pi f_k = \pi f\) and \(M = (M_1 M_2)^{3/5} / (M_1 + M_2)^{1/5}\) is the called chirp mass.
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The EOB formalism\(^1\) maps the conservative dynamics of a two-body system onto the geodesic dynamics of one body of mass \(\mu = \frac{M_1 M_2}{M^2}\).

\[
ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)
\]

where \(r = \frac{c^2 r_{12}}{GM}\) and the symmetric mass ratio \(\nu = \frac{M_1 M_2}{M^2}\).

\[
\begin{align*}
A[1PN] &= 1 - 2u \\
A[3PN] &= A[2PN] + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\nu u^4 \\
A[4PN] &= A[3PN] + \nu[a^c_5(\nu) + a^\ln_5 \ln u]u^5
\end{align*}
\]

4PN coefficients

\[
\begin{align*}
a^c_5(\nu) &= \frac{64}{5} \\
a^c_5 &= a^c_{5,0} + a^c_{5,1} \\
a^c_{5,0} &= \frac{-4237}{60} + \frac{2275\pi^2}{512} + \frac{256\log(2)}{5} + \frac{128}{5}\gamma \\
a^c_{5,1} &= \frac{-221}{6} + \frac{41}{32}\pi^2
\end{align*}
\]

Gravitational waves (GWs)  

Effective-one-body formalism

The real EOB Hamiltonian,

\[ H = M c^2 \sqrt{1 + 2\nu(\hat{H}_{\text{eff}} - 1)}. \]

The Effective Hamiltonian,

\[ \hat{H}_{\text{eff}}^2 = A(u) + p_\phi^2 B(u), \]

where \( B(u) = u^2 A(u) \).

The Angular momentum,

\[ p_\phi^2 = -\frac{A'(u)}{[u^2 A(u)]'}. \]

We need to write \( \hat{H}_{\text{eff}} \) as a function of \( \Omega \), or orbital frequency \( f \).

\[ GM\Omega(u) = \frac{1}{u} \frac{\partial H_{\text{EOB}}}{\partial j} = \frac{M A(u) p_\phi(u) u^2}{H_{\text{EOB}} \hat{H}_{\text{eff}}}. \]

Binary evolution up to the contact orbital frequency

\[ A'(u_{\text{LSO}}) B''(u_{\text{LSO}}) - A''(u_{\text{LSO}}) B'(u_{\text{LSO}}) = 0 \]

where \( u = 1/r \) and \( r_{\text{LSO}} = 6GM/c^2 \).

\[ \begin{align*} 
  u_{\text{LSO}} &= 1/r = GM/\left(c^2 r_{\text{LSO}}\right) \\
  u_{\text{max}} &= 1/r_{\text{min}} = GM/\left(c^2 r_{\text{AB,min}}\right) \\
  A(u,0.25) &= 3\text{PN} \\
  P_3^1 &= A(u,0.25) = 3\text{PN} \\
  P_5^1 &= A(u,0.25) = 4\text{PN} 
\end{align*} \]
The binding energy as a function of the orbital frequency is

\[ E(\Omega) = H_{EOB} - M = M \{ \sqrt{1 + 2\nu(\hat{H}_{\text{eff}} - 1)} - 1 \} \]  

(4)

The gravitational energy spectrum is obtained through the derivative \( dE_b/d\Omega \).
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4J., Antoniadis et al., Science (2013)
A classification of short GRB emitted by NS mergers has been introduced by\textsuperscript{a} R. Ruffini et al., ApJ (2015)

- NS binary pulsars in our Galaxy\textsuperscript{4}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{neutron_star_binary_fate.png}
\end{figure}

\textsuperscript{4}J., Antoniadis et al., Science (2013)
A classification of short GRB emitted by NS mergers has been introduced by\textsuperscript{a}  

- NS binary pulsars in our Galaxy\textsuperscript{4}

- **Family-1 bursts**: are emitted by the formation of a massive NS

\textsuperscript{4}J., Antoniadis et al., Science (2013)
A classification of short GRB emitted by NS mergers has been introduced by\(^a\)


**NS binary pulsars in our Galaxy\(^4\)**

- Family-1 bursts: are emitted by the formation of a massive NS
- Family-2 bursts: are originated from a BH formation

\(^4\)J., Antoniadis et al., Science (2013)
Limits of stability for rotating NSs

- Secular instability and Keplerian or mass shedding limit

Figure: NS mass as a function of the central energy density $\epsilon_c$ for a sequence of constant angular momentum, $j$, with $j = cJ/(GM_\odot^2)$.

\[ \log(\epsilon_c/c^2 \text{ [g cm}^{-3}\text{]}) \]

\[ M/M_\odot \]

$\bullet$ Static sequence
$\bullet$ Keplerian sequence
$\bullet$ Secular Instability

$\Delta$ $f=50$ Hz
$\diamond$ $f=200$ Hz
$\Diamond$ $f=300$ Hz
$\square$ $f=500$ Hz
$\triangle$ $f=716$ Hz

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Limits of stability for rotating NSs

- Secular instability and Keplerian or mass shedding limit

![Graph showing neutron star mass as a function of central energy density](image)

**Figure**: NS mass as a function of the central energy density $\epsilon_c$ for a sequence of constant angular momentum, $j$, with $j = cJ/(GM_\odot^2)$.

- Figures taken from $^5$

On the fate of neutron star binary mergers

Critical NS mass

\[ M_{J,0}^{\text{crit}} = M_J = 0 \text{crit} (1 + kj_{\text{NS}}) \]

with

\[ j_{\text{NS}} = c J_{\text{NS}} / (G M^2_{\odot}) M_{J,0}^{\text{crit}} (M_{\odot})_{\text{LN}} k_p \]

\[ \text{Local charge neutrality (LN): based on relativistic meanfield (RMF) theory for the core and Baym-Pethick-Sutherland (BPS) EOS for the crust} \]

\[ \text{Global charge neutrality (GN): based on Einstein-Maxwell-Thomas-Fermi equations} \]

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7R. Belvedere et al., Nuclear Physics A, 2012
Static case

\[ M_{\text{crit}}^{J=0}(M_{\odot}) \]
\[
\begin{array}{l|cc}
\text{LN} & \text{GN} \\
\hline
\text{NL3} & 2.81 & 2.67 \\
\text{GM1} & 2.39 & 2.30 \\
\text{TM1} & 2.20 & 2.06 \\
\end{array}
\]

\[^6\text{F. Cipolletta et al., Phys. Rev. D (2015)}\]
\[^7\text{R. Belvedere et al., Nuclear Physics A, 2012}\]
On the fate of neutron star binary mergers

Critical NS mass

### Static case

\[ M_{\text{crit}}^{J=0}(M_\odot) \]

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### Rotating case

\[ M_{\text{crit}}^{j \neq 0} = M_{\text{crit}}^{J=0}(1 + k j_{\text{NS}}^p) \]

with \( j_{\text{NS}} = c J_{\text{NS}} / (G M_\odot^2) \)

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Rotating case $M^{J\neq 0}_{\text{crit}} = M^{J=0}_{\text{crit}}(1 + kj_{NS}^p)$

with $j_{NS} = cJ_{NS}/(GM_\odot^2)$

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- **Local charge neutrality (LN):** based on relativistic meanfield (RMF) theory for the core and Baym-Pethick-Sutherland (BPS) EOS for the crust\(^6\).

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On the fate of neutron star binary mergers

Critical NS mass

Observational constrains

- Massive NS observed $2M_\odot$ Antoniadis et al., Science (2013)

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\[ M \text{ } [M_\odot] \quad R_{\text{eq}} \text{ } [\text{km}] \]

X-Rays Data

\[ 8 \text{F. Cipolletta et al., Phys. Rev. D (2015)} \]
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Figure taken from  

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Which are the mass and angular momentum of the merger core?

Mass-ratio of the binary $M_1/M_2 \sim 1$ for galactic BNSs
Degree at which baryon and angular momentum are conserved
(mass and angular momentum loss, mass and angular momentum of a surrounding disk)

$$(M_1, M_2) \rightarrow (M_{b,1}, M_{b,2}) \rightarrow M_{b,f} = \alpha(M_{b,1} + M_{b,2}) \text{ where } \alpha \leq 1 \quad (5)$$

$$J_{mc} = \beta J_i \sim \beta J_{bin}(contact) \text{ where } \beta \leq 1 \quad (6)$$
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- Neutron Star Binding Energy\(^9\)

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- Neutron Star Binding Energy\(^9\)

### Static configurations

$$ \frac{M_b}{M_\odot} \approx \frac{M}{M_\odot} + \frac{13}{200} \left( \frac{M}{M_\odot} \right)^2 $$

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- Neutron Star Binding Energy\(^9\)

**Static configurations**

\[\frac{M_b}{M_\odot} \approx \frac{M}{M_\odot} + \frac{13}{200} \left( \frac{M}{M_\odot} \right)^2 \]

**Rotating configurations**

\[\frac{M_b}{M_\odot} = \frac{M}{M_\odot} + \frac{13}{200} \left( \frac{M}{M_\odot} \right)^2 \left( 1 - \frac{1}{130} j_{NS}^{1.7} \right) \]

Fate of the post-merger core?

- Stable range of masses: $M_1 = M_2 = 1.30 - 1.47 \, M_{\odot}$
- Secular Instability
- Mass-Shedding
Fate of the post-merger core?

- **EOS GM1**

![Diagram](image-url)

- Stable range of masses: $M_1=M_2 = 1.30-1.47$ M$_\odot$
Fate of the post-merger core?

- EOS GM1
- 90% of the angular momentum at the merger assumed to be kept by the new compact core
Fate of the post-merger core?

- EOS GM1
  - 90% of the angular momentum at the merger assumed to be kept by the new compact core
  - $1.0M_\odot < M_1 = M_2 < 2.0M_\odot$
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Short GRBs:

$E_p$, $i - E_{\text{iso}}$ relation


see the talk of Marco Muccino!
Family-1 bursts: $E_{iso} < 10^{52}$ erg
$M_{tot} \lesssim M_{crit}$
Originate: formation of a massive NS
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Short Gamma-ray Bursts

**Sub-classification**

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see the talk of Marco Muccino!

(ICRANet)
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Originate: formation of a massive NS

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Originate: formation of a BH
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Estimation of the theoretical baryon load for Short GRB 090227B (Family-2)

\[ B_{\text{th}} = \eta M_{\text{tot}} \times \frac{c^2}{E_{\text{GRB}}}, \]

where \( \eta \) is the fraction of the crustal mass ejected.

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Properties of Neutron Star\(^{11}\)

<table>
<thead>
<tr>
<th>(M(M_\odot))</th>
<th>(R(\text{km}))</th>
<th>(M_{\text{crust}}(M_\odot))</th>
<th>(R_{\text{crust}}(\text{km}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.335</td>
<td>12.24</td>
<td>(3.6 \times 10^{-5})</td>
<td>0.47</td>
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</tbody>
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Numerical simulation of the dynamical evolution of DNS:

\[ M_B \sim 10^{-3} - 10^{-2} M_\odot, \text{ where LN was employed.}\textsuperscript{13} \]

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Outline

1. Gravitational waves (GWs)
   - Classical dynamics
   - Effective-one-body formalism

2. On the fate of neutron star binary mergers
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3. Short Gamma-ray Bursts
   - Sub-classification
   - GRB 090227B (Family-2): Baryon load

4. GWs emission from short GRBs
   - Signal-to-Noise ratio
   - Detection rate in Ad.LIGO
The signal-to-noise ratio SNR

\[ \text{SNR}^2 = 4 \int_0^\infty \frac{|\tilde{h}(f)|}{S_h(f)} df \]  \hspace{1cm} (8)

where \( \tilde{h}(f) \) is the Fourier transform of \( h(t) \) and \( S_h(f) \) is the strain noise spectral density in the interferometer.

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- The average of the square SNR \( \langle \text{SNR}^2 \rangle \) over all orientations and directions to the source, depends on the energy spectrum \( dE/df \) of the emitted GWs.  

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\langle \text{SNR}^2 \rangle = \int_{f_{\text{min}}}^{f_{\text{max}}} df_d \frac{h_c^2(f_d)}{5f_d^2 S_n^2(f_d)},
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\]  

(9)

- The amplitude \( h_c \) is defined as the Fourier transform of the gravitational waveform \( h(t) \), \( h_c(f) = f|\tilde{h}(f)| \)

\[
h_c^2(f) = \frac{2(1 + z)^2}{\pi^2 d_L^2} \frac{dE}{df} [(1 + z)f_d],
\]

(10)

Gravitational waves emission from progenitors of short GRBs (Family-2 bursts)
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The coalescence rate for the family-2 was calculated by

\[ R_c = \left(0.2 - 6.2 \right) \times 10^{-4} \text{Gpc}^{-3} \text{yr}^{-1} \]

Using the coalescence rates \( R_c \), we made an estimation of the number of coalescing NS binaries that advanced LIGO could detect in the next years

\[ N_d \approx 4 \pi \frac{3}{2} R_c D_h^3 T \left( \frac{M_c}{M_1 + M_2} \right)^{15/6} \]

where \( R_c \) is the coalescence rate, \( T \) is the observed time, \( M_c = \left( \frac{M_1 + M_2}{3} \right)^{2/5} \left( \frac{M_1 M_2}{2} \right)^{-1/5} \) is the chirp mass of the binary and the Adv. LIGO horizon distances is \( D_h \).

\[^{15}\text{Ruffini et al., ApJ (2015)}\]
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(11)

where \(R_c\) is the coalescence rate, \(T\) is the observed time, \(\mathcal{M}_c = (M_1 + M_2)^{3/5}(M_1M_2)^{-1/5}\), is the chirp mass of the binary and the Adv. LIGO horizon distances is \(D_h\).

- Sensitivity and Horizon distance in adv.LIGO\textsuperscript{16}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
Epoch & Estimated Run Duration & $E_{GW} = 10^{-2}M_\odot c^2$ & BNS Range (Mpc) & Number of BNS Detections & \% BNS Localized within 5 deg$^2$ & \% BNS Localized within 20 deg$^2$
\hline
2015 & 3 months & LIGO 40 – 60 & Virgo – & 0.0004 – 3 & – & –
2016–17 & 6 months & LIGO 60 – 75 & Virgo 20 – 40 & 0.006 – 20 & 2 & 5 – 12
2017–18 & 9 months & LIGO 75 – 90 & Virgo 40 – 50 & 0.04 – 100 & 1 & 2 & 10 – 12
2019+ & (per year) & LIGO 105 & Virgo 40 – 80 & 0.2 – 200 & 3 & 8 & 8 – 28
2022+ (India) & (per year) & LIGO 105 & Virgo 80 & 0.4 – 400 & 17 & 48
\hline
\end{tabular}
\end{table}

\textsuperscript{16}J. Aasi et al. (LIGO Scientific Collaboration and Virgo15 Collaboration), arXiv 1304, 0670 (2013)
Family-1 bursts: \( \text{NS+NS} = \text{MNS} \)

**Figure**: Number of detections, \( N_d \), of the coalescing NS binary systems in Advanced LIGO per year, using the coalescence rate \( R_c \), for family-1 bursts (NS+NS=MNS).
Family-2 bursts: NS+NS=BH

Figure: Number of detections, $N_d$ of the coalescing NS binary systems in Advanced LIGO per year, using the coalescence rate $R_c$, for family-2 bursts (NS+NS=BH).
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