Dark matter in dwarf spheroidal galaxies and their phase-space density distribution


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DM particle mass?

We don’t know, but ...

- Tremaine&Gunn set lower particle mass at sub-keV/c² range [1]
- Lyman-α forest raise the lower bound to approximately 1.2keV/c² [2]
- RAR model predicts a bound at keV/c² scales implying a quantum core [3]

Phase-space density

How much can the late-time coarse-grained phase-space density diminish compared to the primordial fine-grained?

- Tremaine&Gunn approach makes no predictions about the decrease
- numerical simulations conclude a decrease within one order of magnitude (see e.g. [4])

Outline

- Tremaine & Gunn approach
- Fermionic DM + RAR model
- Particle mass bound inferred from dSphs
- Phase-space density distribution of dSphs
- Conclusion
Tremaine & Gunn (1979)

- primordial fine-grained DF is fixed according to Liouville’s theorem

\[ f_{\text{FD}}^{\text{max}} = 4 \frac{gm^4}{2h^3} \]

- coarse-grained phase-space density decreases due to phase mixing effects

\[ F^{\text{max}} = ? \]
Tremaine & Gunn (1979)

- Maxwellian velocity distribution with maximum \((2\pi\sigma^2)^{3/2}\)
- Isothermal sphere behaviour in central region with max. density \(\rho_0\)

\[
R_K^2 = \frac{9\sigma^2}{4\pi G \rho_0}
\]

\[
f_{FD}^{\text{max}} \geq F^{\text{max}} \approx \frac{\rho_0}{(2\pi\sigma^2)^{3/2}}
\]

\[
m^4 \geq \frac{9}{4} \frac{h^3}{(2\pi)^{5/2} g G \sigma R_K^2}
\]

- Lower bound \(\sim 100 eV/c^2\) for typical dwarf spheroidals
Numerical simulations

Dark matter *ingredients*

- collision-less gas of fermions in hydrostatic equilibrium
- gravitational interaction + Pauli principle
Dark matter ingredients

- collision-less gas of fermions in hydrostatic equilibrium
- gravitational interaction + Pauli principle

\[
\frac{1}{r^2} \frac{d}{dr} \left[ r^2 \frac{d\Phi}{dr} \right] = 4\pi G \rho(r)
\]

\[
f(r, p) = \frac{1}{e^{p^2/(2m k_B T)} - \frac{\mu(r)}{k_B T} + 1}
\]
Fermionic DM

\[ \frac{\rho(r)}{\rho_B} = \frac{4}{\sqrt{\pi}} \int_0^\infty \frac{y^2}{ey^2-\theta(r)+1} \, dy \]

\[ \frac{R^2}{r^2} \frac{d}{d[r/R]} \left[ \frac{r^2}{R^2} \frac{d\theta}{d[r/R]} \right] = -\frac{8e^{-\alpha_0}}{\sqrt{\pi}} \int_0^\infty \frac{y^2}{ey^2-\theta(r)+1} \, dy \]

\[ \theta(0) = \theta_0, \quad \theta'(0) = 0 \]

- curve (dimensionless) depends on \( \theta_0 \) only

Substitutions

\[ \theta(r) = \frac{\mu(r)}{k_B T} \quad \quad y^2 = \frac{p^2}{2mk_B T} \]
\[
\frac{R^2}{r^2} \frac{d}{d[r/R]} \left[ \frac{r^2}{R^2} \frac{d\theta}{d[r/R]} \right] = -\frac{8e^{-\alpha_0}}{\sqrt{\pi}} \int_0^\infty \frac{y^2}{ey^2-\theta(r) + 1} dy
\]

- in sum: three parameter, e.g. \((m, \theta_0, \sigma)\)

\[
R^2 = \frac{\sigma^2}{2\pi G \rho_0}
\]

\[
e^{\alpha_0} = \frac{\rho_0}{\rho_B} = \frac{h^3}{gm^4} \frac{\rho_0}{(2\pi\sigma^2)^{3/2}}
\]

Substitutions

\[
\sigma^2 = \frac{k_B T}{m} \quad \rho_B = \frac{gm}{\lambda_B^3} \quad \lambda_B = \frac{h}{\sqrt{2\pi m k_B T}}
\]
\[
\frac{R^2}{r^2} \frac{d}{d[r/R]} \left[ \frac{r^2}{R^2} \frac{d\theta}{d[r/R]} \right] = -\frac{8e^{-\alpha_0}}{\sqrt{\pi}} \int_0^\infty \frac{y^2}{ey^2 - \theta(r) + 1} dy
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R^2 = \frac{\sigma^2}{2\pi G\rho_0}
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\[
e^{\alpha_0} = \frac{\rho_0}{\rho_B} = \frac{h^3}{gm^4} \frac{\rho_0}{(2\pi\sigma^2)^{3/2}}
\]

\[
\rho_0 = \rho_0(m, \theta_0, \sigma)
\]

Substitutions

\[
\sigma^2 = \frac{k_B T}{m} \quad \rho_B = \frac{gm}{\lambda_B^3} \quad \lambda_B = \frac{h}{\sqrt{2\pi mk_B T}}
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\[ \frac{R^2}{r^2} \frac{d}{d[r/R]} \left[ \frac{r^2}{R^2} \frac{d\theta}{d[r/R]} \right] = -\frac{8e^{-\alpha_0}}{\sqrt{\pi}} \int_0^\infty \frac{y^2}{ey^2 - \theta(r) + 1} dy \]

- in sum: three parameter, e.g. \((m, \theta_0, \sigma)\)

\[ R^2 = \frac{\sigma^2}{2\pi G \rho_0} \]

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Substitutions

\[ \sigma^2 = \frac{k_B T}{m} \]

\[ \rho_B = \frac{gm}{\lambda_B^3} \]

\[ \lambda_B = \frac{h}{\sqrt{2\pi m k_B T}} \]
Fermionic DM – density profile
Fermionic DM – rotation curve
Fermionic DM – rotation curve

The RAR model is described by:

- **quantum core** ($\theta_0 \geq 10$)
- **halo** is defined as second maxima of rotation curve
RAR model – density profile

The diagram shows the density profile of a ferminic DM model with different parameters: 
- Blue solid line: fermionic DM ($\theta_0 = 30$)
- Blue dashed line: degenerate core ($\theta_0 = 30$)
- Blue dotted line: corresponding IS ($\theta_0 = -22.9$)
- Black dashed line: pseudo IS

Key points:
- Core $R_c$
- King radius $R_K$
- Halo $R_h$
RAR model – rotation curve

rotation curve

- fermionic DM ($\theta_0 = 30$)
- degenerate core ($\theta_0 = 30$)
- corresponding IS ($\theta_0 = -22.9$)
- pseudo IS

$\frac{\sigma(r)}{\sigma}$

$10^{-2}$
$10^{-1}$
$10^{0}$
$10^{1}$
$10^{2}$

$r/R$

core $R_c$

King radius $R_K$

halo $R_h$
Central density

- we know $\rho(r)/\rho_0$ and $r/R$ (numerical solution for given $\theta_0$)
- half-light radius $R_K$ and $\sigma$ are given (from observations)

\[
\frac{R_K^2}{R^2} = q
\]
\[
R^2 = \frac{\sigma^2}{2\pi G \rho_0}
\]

\[
\rho_0 = \frac{q}{2\pi} \frac{\sigma^2}{GR_K^2}
\]

**Note:** $R_K \approx \frac{4}{3} r_c$ is half-light radius of corresponding IS where $\rho(r_c) = \rho_0/2$
Workflow

\[ \rho_0 = \rho_0(\theta_0, \sigma, R_K) \]

\[ m = m(\theta_0, \rho_0, \sigma) \]
Density profile

\[ \rho(r) \left[ \frac{M_\odot}{pc^3} \right] \]

quantum core

\[ \begin{align*}
\theta_0 = 20, & \quad m \approx 23.35 \text{ keV}/c^2 \\
\theta_0 = 15, & \quad m \approx 10.07 \text{ keV}/c^2 \\
\theta_0 = 10, & \quad m \approx 4.85 \text{ keV}/c^2 
\end{align*} \]

Sculptor

\[ R_K \approx 160 \pm 40 \text{ pc} \]
\[ \sigma \approx 10.1 \pm 0.3 \text{ km/s} \]
Particle mass bound

WDM particle mass bound inferred from RAR-model

mean ($m_{\text{min}} \approx 4.8 \pm 1.8 \text{ keV}/c^2$)
Lyman-α forest ($m \geq 1.2 \text{ keV}/c^2$)

$m \geq 4.8 \pm 1.8 \text{ keV}/c^2$
Phase-space density

- approximation of phase-space density by Dalcanton \[^{[4]}\]

\[
Q(r) = \frac{\rho(r)}{\langle v^2(r) \rangle^{3/2}}
\]

\[
f_{\text{FD}}^{\text{max}} = \frac{gm^4}{2h^3}
\]

Conclusion

Assuming dSphs harbor a quantum core...

- DM particle mass $m \geq 3 \text{keV}/c^2$
- at least three magnitudes decrease of late-time coarse-grained phase-space density
Thank you!