Toward a General Relativistic Thomas-Fermi Theory of Neutron Stars

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Recent News from the GeV and TeV Gamma-Ray Domains:
Results and Interpretations
Pescara 21-26 March 2011
Neutron Stars: hostile places...

Liquid Core: Density > nuclear density = 2.7e14 g/cc

Solid Crust: Density < nuclear density

Atmosphere: Density < 1e3-1e4 g/cc

High Density and High Temperature Environment !!!
Neutron Stars: special places to do physics...

Liquid Core Physics:
- Strong Interaction
- Electromagnetic Interaction
- Neutrons
- Protons
- Electrons
- Weak interaction
- Gravitational Interaction

Solid Crust Physics:
- Solid State Physics
- Nuclei Lattice
- Electrons
- Neutrons (possibly)
- Nuclear Physics

Total Number of Particles $1 \times 10^{57}$
Neutron Stars: special places to do physics...

How to solve this puzzle self-consistently???
Traditional Approach

Electromagnetic Interaction: NO

Local Charge Neutrality $n_e = n_p$ is assumed

- Weak Int.
- Strong Int.
- Fermi-Dirac Stat.

Electrom. Int.

General Relativity
### Traditional Approach

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<thead>
<tr>
<th>Electromagnetic Interaction:</th>
<th>Local Charge Neutrality ne=np is assumed</th>
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<th>Relativistic Fermi-Dirac Statistics</th>
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<td>EOS</td>
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<td>GENERAL RELATIVITY</td>
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<td>Strong Interaction</td>
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**Microphysics and Macrophysics are decoupled by the assumption ne=np !!**

\[
P = P(\rho) \\
\frac{dm}{dr} = 4 \pi r^2 \rho \\
\frac{dP}{dr} = -(P + \rho)(4 \pi r^3 P + m)/(r (r-2m))
\]

**Microphysics embedded in General Relativity !!**
Traditional Approach: current status...

EOS + General Relativity

Nuclear Physics People decouple from General Relativity People
Traditional Approach: current status...
And if we don't assume ZERO electromagnetic interaction???

We cannot decouple microphysics from macrophysics !!!
Learning to deal with the EM interaction: lessons from the Thomas-Fermi atom...

Ne = Z non-relativistic electrons

Non-relativistic Fermi-Dirac statistics
Electromagnetic equilibrium
Electromagnetism equation

Point-like nucleus +eZ
Wigner-Seitz cell
What did we learn??

How to put together Fermi-Dirac statistics with electromagnetism:

**CRUCIAL POINT**

To request the equilibrium of particles:

$$\mu_e - eV = \text{constant}$$
Next Step: going to relativistic regimes...
(see Rotondo's talk...)

- Ne = Z non-relativistic electrons
- Ne = Z relativistic electrons in Beta equilibrium with nucleons
- Wigner-Seitz cells
- Point-like nucleus +eZ
- Extended nucleus +eZ and A-Z neutrons

What did we learn??

Relativistic Fermi-Dirac statistics
Electromagnetic interaction between protons and electrons
Weak equilibrium between neutrons, protons, electrons

**KEY ISSUE:**
Global equilibrium of particles (specificially the electrons)

\[ \mu_e - eV = \text{constant} \]
Next Step: adding General Relativity...

<table>
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<th>Relativistic Thomas-Fermi model</th>
<th>General Relativistic Thomas-Fermi model</th>
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<td>EM interaction $p$ and $e$</td>
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<td>Weak equilibrium $n, p, e$</td>
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<td>EM equilibrium</td>
<td>Grav. and EM equilibrium</td>
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<td>Maxwell equations</td>
<td>Einstein-Maxwell equations</td>
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The simplest possible solution: a self-gravitating system of n,p,e in beta equilibrium

Relat. Fermi-Dirac stat.
Weak equilibrium n,p,e
Einstein-Maxwell equations
Grav. and EM equilibrium

\[ \sqrt{g_{00}} \mu_i - q_i V = \text{constant} \]

Boundary Conditions:
Global Charge Neutrality
\[ ne \neq np \quad Ne = Np \]

It is indeed an eigenvalue problem!!
The simplest possible solution: a self-gravitating system of n,p,e in beta equilibrium

\[ n_e = n_p \]

\[ E_n^F + m_n c^2 = E_p^F + m_p c^2 + E_e^F + m_e c^2 \]

\[ \frac{1}{r} \frac{dv}{dr} + \frac{1 - e^\lambda}{r^2} = \frac{8 \pi G}{c^4} e^\lambda \left[ P \right], \]

\[ \frac{dP}{dr} + \frac{1}{2} \frac{dv}{dr} (E + P) = 0, \]

\[ \frac{dM}{dr} = 4 \pi r^2 \frac{E}{c^2} \]

\[ \frac{dM}{dr} = 4 \pi r^2 \frac{E}{c^2} - \frac{4 \pi r^3 e^{-v/2}}{dV/c^2} \frac{dV}{dr} (n_p - n_e), \]

\[ \frac{1}{r} \frac{dv}{dr} + \frac{1 - e^\lambda}{r^2} = \frac{8 \pi G}{c^4} e^\lambda \left[ P - \frac{e^{-(v+\lambda)}}{8 \pi \alpha \hbar c} \left( \frac{dV}{dr} \right)^2 \right], \]

\[ \frac{dP}{dr} + \frac{1}{2} \frac{dv}{dr} (E + P) = -\frac{dP_{\text{em}}}{dr} - \frac{4P_{\text{em}}}{r}, \]

\[ \frac{d^2 \dot{V}}{dr^2} + \frac{d \dot{V}}{dr} \left[ \frac{2}{r} - \frac{1}{2} \left( \frac{dv}{dr} + \frac{d\lambda}{dr} \right) \right] = -4 \pi \alpha \hbar c e^{v/2} e^\lambda \left\{ n_p \right\} \]

\[ - \frac{e^{-3v/2}}{3 \pi^2 \hbar^3 c^3} \left[ (\dot{V} + m_e c^2)^2 - m_e^2 c^4 e^v \right]^{3/2} \]

Jorge A. Rueda, M. Rotondo, R. Ruffini, and S.-S. Xue, submitted to PRL
Fermi energy of particles assuming local charge neutrality
Coulomb and Gravitational potential in the globally neutral solution
Particle density inside the star...

\[ \frac{n_i}{n_{\text{nuc}}} \]

- **-** electrons
- **--** protons
- ***-*** neutrons

\[ r \text{ (km)} \]

\[ \times 10^{-3}, \times 10^{-4}, \times 10^{-5}, \times 10^{-6}, \times 10^{-7}, \times 10^{-8}, \times 10^{-9}, \times 10^{-10} \]
Electron density approaching the boundary of the star: eigenvalue problem...
Next Step: adding the strong interaction...
What do we ask to the nuclear interaction theory?

Relativistic theory
“Good” thermodynamic limit for large # particles
Consistent with observed properties of nuclei

Walecka Model
Extending the Walecka model to General Relativity (see Xue's talk)

Relat. Fermi-Dirac stat.
Weak equilibrium n,p,e
Strong interaction n,p
Einstein-Maxwell equations
Grav-EM-strong equilibrium

Boundary Conditions

- Constancy of General Relativistic Fermi energy of all particle species
- Continuity of GR particle Fermi energy at the core-crust boundary
- Global Charge Neutrality
What is new? Boundary conditions based on continuity of GR particle Fermi energy (Xue and Belvedere's talk)
See Belvedere's talk

\[ e^{-\lambda(r)} \left( \frac{1}{r^2} - \frac{\lambda'}{r} \right) - \frac{1}{r^2} = -8\pi GT_0^0, \]

\[ e^{-\lambda(r)} \left( \frac{1}{r^2} + \frac{\nu'}{r} \right) - \frac{1}{r^2} = -8\pi GT_1^1. \]

\[ \Phi' + \frac{\nu'}{2}(E + P) = -g_{\sigma ns} \sigma' - \omega' g_i^0 \omega - \rho' g_{\rho}^0 - V' e_j^0. \]

\[ V'' + V' \left[ \frac{2}{r} - \frac{(v' + \lambda')}{2} \right] = -e^\lambda e_j^0. \]

\[ \sigma'' + \sigma' \left[ \frac{2}{r} + \frac{(v' - \lambda')}{2} \right] = e^\lambda \left[ \sigma U(\sigma) + g_{n s} n_s \right]. \]

\[ \omega'' + \omega' \left[ \frac{2}{r} - \frac{(v' + \lambda')}{2} \right] = -e^\lambda \left[ g_{\omega}^0 - m_\omega^2 \omega \right]. \]

\[ \rho'' + \rho' \left[ \frac{2}{r} - \frac{(v' + \lambda')}{2} \right] = -e^\lambda \left[ g_{\rho}^0 - m_\rho^2 \rho \right]. \]

\[ E_e^F = e^{v/2} \mu_e - eV = \text{constant}, \]

\[ E_n^F = E_p^F + E_e^F. \]