7. FORMATION OF IMAGES WITH RÖNTGEN RAYS

“Formazione di immagini coi raggi Röntgen,”

Röntgen rays do not undergo reflections nor refractions, at least in the usual sense of the word, since the reflection of diffraction occurs only under well defined incidence angles. As a consequence in X-ray optics the problem of obtaining images cannot be solved, as in ordinary optics, by means of spherical lenses or mirrors.

Gouy\(^1\) suggested theoretically a method for obtaining monochromatic images with X-rays, by means of a cylinder of mica. In a few words it is the following:

Let us consider a circular cylinder of mica and suppose that at a point of its axis there is a source S of monochromatic Röntgen rays. They will be reflected on the mica at those points where Bragg’s relation is fulfilled: these points obviously are on circular sections of the cylinder. And the rays reflected at one of these circles will obviously gather in a point I on the axis, symmetric to S with respect to the plane of the reflecting circle, where one will have a real monochromatic image of S.

If S were in the neighborhood of the axis, another image of it would be formed in the neighborhood of the axis\(^2\). Suppose now we have, in the neighborhood of the axis, a planar figure from whose points monochromatic X-rays come out, and we place a plate in the position where its image is formed.

Let \( r \) be the mirror-object distance, \( R \) the radius of the cylinder of mica, \( \theta \) the Bragg incidence angle, \( r' \) the image-mirror distance. If we project everything onto a plane perpendicular to the axis of the cylinder of mica, the projections of \( r \) and \( r' \) will be \( r \cos \theta \), \( r' \cos \theta \); and then according to the usual formulae for spherical mirrors we will have

\[
\frac{1}{r \cos \theta} + \frac{1}{r' \cos \theta} = \frac{2}{R},
\]

from which

\[
r' = \frac{Rr}{2r \cos \theta - R}.
\]

The linear coefficient of enlargement of the segments orthogonal to \( r \) and the

\(^1\)C. R. Gouy, *C. R.*, 161, 176 (1915).

\(^2\)Of course, provided that the cylinder is confined in a small enough region contained between two generatrices.
axis of the cylinder will be

\[ \mu_1 = \frac{r'}{r} = \frac{R}{2r \cos \theta - R} \]  

(1)

If the object is close to the axis we have approximately \( \mu_1 = 1 \).

To calculate the enlargement of the segments parallel to the plane of the axis and of \( r \), let us indicate by \( \varphi \) and \( \varphi' \) the angles that the lines perpendicular to the plane of the object and of the plate form with \( r \) and \( r' \) respectively.

Then one immediately sees that the desired enlargement is

\[ \mu_2 = \frac{\cos \varphi'}{\cos \varphi} \]  

(2)

Suppose now we photograph an aperture placed perpendicularly to the plane of \( r \) and the axis by a flat plate of mica of length \( l \). If \( h \) is the length of the aperture, the length of its image will be \( 2l + k \). If instead we bend the mica so that the image is formed in the focus, the length will become \( h \). The intensities of the two images will obviously be approximately in the inverse ratio of their lengths. Their ratio is then

\[ \frac{2l + k}{h} \]

If, for instance, \( h = 1 \) cm, \( l = 4 \) cm the ratio is 9. Then the intensity is almost tenfold increased.

I shall now describe the way in which I have actually succeeded in obtaining these images.

The source of the rays consisted of a tube of the shape and size approximately indicated in Figure 1.

I created the vacuum with a Cacciari rotational pump, model Gaede. The cathode \( K \) was concave, with a radius of 6 or 7 cm when it was desired to concentrate the rays on the anticathode as much as possible; if instead one wanted the whole surface of the anticathode be hit by the rays, the cathode was made with a smaller radius. The anticathode was generally of iron and sometimes was cut almost perpendicularly to the cathode rays, in order to do without the slit. Instead, in other experiments it was cut as the spout of a flute, in order to present a large surface to the detecting instruments. Since the radiations typical of iron are largely absorbed by the glass of the bulb, I thought it useful to equip the tube with little window of aluminum \( R \). During the work the tube was kept attached to the pump, so that after a short time, it assumed a regular enough running. The tube was driven by a big induction coil with a Wehnelt switch; in ordinary conditions the equivalent spark was 10 or 12 cm long.

The tube was contained in a small wooden box sheathed by lead 6 mm thick on the side of the instruments and 3 mm thick on the other sides.

To obtain fairly precise images it was necessary that the reflecting plate of mica be as regular as possible. Therefore, it was carefully chosen among many samples; nevertheless I never succeeded in finding plates that, in reflecting the light, were
more regular than an ordinary windowpane. This is the cause of the irregularities and smudges we can observe in the reported images. The mica was bent by binding it fast on a turned brass cylinder. Then a layer of sealing wax (little more than half a centimeter thick) was spread on the convex part. When the sealing wax had cooled one could remove the fastenings and detach the mirror from the cylinder. In this way I succeeded in obtaining cylindrical mirrors relatively precise given the limit imposed by the natural irregularity of the plates used. They had mostly dimensions of $4 \times 6$ cm but usually their aperture was reduced to $4 \times 2$ cm for making use of the less irregular parts, which were judged by trying the mirrors by the reflection of the ordinary light. The mirror was mounted on a graduated circle in order to be able to position it correctly. (The angle by which it was turned for the study of the third order of the $K_\alpha$ of the iron was of $16^\circ 50'$). The detection of the rays was performed photographically.

I carried out first a few experiments of orientation with planar crystals to verify the nature of the anticathode and the intensities of the reflections of the various orders. It resulted that the pair $K_\alpha$ $K_\alpha'$ ($\lambda = 1.932; 1.928$), scarcely resolvable in the experimental conditions in which I found myself, and the $K_\beta$ ($\lambda = 1.748$) were emitted. The $K_\gamma$ was scarcely visible due to the low intensity. The most intense orders were the first and the third. I preferred to work in the third in order not to be obliged to use incidence angles too close to 90°.

Then I experimented with the indicated method to obtain images first on the anticathode which was also working as an aperture.

The distances anticathode-crystal and crystal-image varied from 18 to 22 cm. The exposure lasted about ten minutes.

I could immediately ascertain the very strong increase in intensity which can be obtained in this way. A rough idea of this is given by Figs. 2, 1, and 2, 2 which represent two photographs of the 3rd order of iron $K_\alpha$ obtained approximately in the same conditions of exposure and operation of the tube, the first one with flat mica and the second one with curved mica. The increase in intensity was indeed such that, particularly using mirrors of 6 cm of aperture, accustoming my eyes a few minutes to the darkness of the room, it was possible to see clearly the images on a screen of barium platinum cyanide. From Fig. 2, 2 it is clearly visible that the emission intensity of the central part of the anticathode, where the cathode rays were concentrated, is considerably greater than that of the side parts. It is possible to see this because the method of images allows one to observe the slit “alla Lockyer”, that is, to observe point by point what happens in the slit. To put this more clearly in evidence I made the following experience: I placed before the window of aluminum a lead thread of about 1 mm in diameter and shifted the photographic plate to carry it to the point where the image of the aluminum window was forming.

Fig. 2, 3 gives the result of this experiment; in the figure the gap in the image produced by the lead thread is clearly visible.
Finally Fig. 2, 4 represents an attempt to obtain an image of an object in two dimensions. The anticathode of iron was therefore cut as the spout of a flute and two cross shaped furrows were cut into it and inside them two copper wires were driven in order to form a sort of X. In Fig. 2, 4, one can see the image of this X, obviously together with several irregularities due to the irregularity of the reflector.

This work was carried out at the Institute of Physics of the University of Pisa in Winter 1922.
Figure 1.

Figure 2.