Orbital Angular Momentum (OAM) of light beams and photons

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The Orbital Angular Momentum

Among the properties of light still poorly exploited in Astronomy, **is the Orbital Angular Momentum (OAM)** and associated **Optical Vorticity (OV),** which is instead already used in Chemistry, Biology, and Quantum and classical Communications.

M. Harwit (2003, The Astrophys. J.) was the first to point out the interest of OAM for Astronomy, but his paper was largely ignored.

We took up and developed some of his ideas, and shall show in the following that OAM can actually be used in Astronomy, e.g. in the ***optical domain*** to overcome to Rayleigh criterium of angular resolution, for coronagraphic applications and hopefully to detect intrinsic OAM in celestial sources.
In the ***radio domain,*** it can be used for interstellar and interplanetary plasma physics diagnostic, for radio interferometry from the Moon, for measuring ***the rotation of Black Holes***.

Total EM field Angular Momentum

Electromagnetic (EM) beams do not only carry energy, power (Poynting flux, linear momentum), and spin angular momentum (SAM, wave polarization), but also *orbital angular momentum* (OAM).

The total angular momentum **J**$^{\mathrm{EM}}$ can be separated into two parts [*van Enk & Nienhuis*, 1992]:

$$\mathbf{J}^{\mathrm{EM}} = \frac{\varepsilon_0}{2i\omega}\left\{\int \mathbf{E}^* \times \mathbf{E}\ \mathrm{d}^3x + \int \sum \mathbf{E}_i^{\ *}\left(\left[\left(\mathbf{x} - \mathbf{x_0}\right)\times\nabla\right]\right)\mathbf{E}_i\hat{\mathbf{e}}_i\right\}\mathrm{d}^3x$$

- the first part is the ***spin angular momentum*** **(SAM)** **S**$^{\mathrm{EM}}$, a.k.a. ***wave polarization*****,**
- the second part is the ***orbital angular momentum*** **(OAM) L**$^{\mathrm{EM}}$.

In general, **both linear momentum P$^{\mathrm{EM}}$,** ***and*** **angular momentum J**$^{\mathrm{EM}}$ = **S**$^{\mathrm{EM}}$ + **L**$^{\mathrm{EM}}$ **are radiated all the way out to the far zone (***see e.g. Jackson, Classical Electrodynamics***).**

SAM vs. OAM

• SAM is tied to the helicity (***polarization***) of the light beam and for a ***single photon*** its value is:

$$S_z = \pm (h/2\pi)$$

• OAM is tied to the ***spatial structure of the wavefront***: the orbital terms are generated by the *gradient of the phase;* it determines the helicoidal shape of the wave front; for a ***single photon*** it assumes the value :

$$L_z = \ell \, (h/2\pi)$$

with $\ell = 0$ for a plane wave with $\boldsymbol{S} \,//\, \boldsymbol{k}$, and $\ell \neq 0$ for a helicoidal wave front because ***S*** precesses around ***k***.

Polarization enables only two photon spin states, but actually photons can exhibit multiple OAM eigenstates, ***allowing single photons to encode much more information*** **(A. Zeilinger and collaborators)**.

Papers on Angular Momentum

It was postulated by Poynting already in 1909, Proc. Roy. Soc. London

The Wave Motion of a Revolving Shaft, and a Suggestion as to the Angular Momentum in a Beam of Circularly Polarised Light.

By J. H. Poynting, Sc.D., F.R.S.

(Received June 2,—Read June 24, 1909.)

The analogy between circularly polarised light and the mechanical model suggests that a similar relation between torque and energy may hold in a beam of such light incident normally on an absorbing surface. If so, a beam of wave-length λ containing energy E per unit volume will give up angular momentum $E\lambda/2\pi$ per second per unit area. But in the case of light waves $E = P$, where P is the pressure exerted. We may therefore put the angular momentum delivered to unit area per second as

$$P\lambda/2\pi.$$

Two more recent papers:

Light with a twist in its tail

Miles Padgett and L. Allen

Contemporary Physics (2000) vol. 41, nr.5, pag. 275-285

Twisted photons

G. Molina-Terriza, J. Torres and L. Torner

nature physics | VOL 3 | MAY 2007 | www.nature.com/naturephysics

We can say that the Orbital Angular Momentum represents ***a fundamentally new optical degree of freedom*** of light .

It arises as a consequence of the ***spatial distribution of the intensity and phase*** of an optical field ***- even down to the single photon limit*** (as was shown by ***A. Zeilinger*** et al.***)***.

Researchers have begun to appreciate its implications for our understanding of the ***many ways in which light and matter can interact,*** for its practical potential for ***quantum information*** applications, and finally for its ***astronomical interest***.

The mathematics of OAM

PHYSICAL REVIEW A VOLUME 45, NUMBER 11 1 JUNE 1992

Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes

L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman
Huygens Laboratory, Leiden University, P.O. Box 9504, 2300 RA Leiden, The Netherlands
(Received 6 January 1992)

$$u_{pl}(r,\phi,z) = \frac{C}{(1+z^2/z_R^2)^{1/2}}\left[\frac{r\sqrt{2}}{w(z)}\right]^l L_p^l\left[\frac{2r^2}{w^2(z)}\right] exp\left[\frac{-r^2}{w^2(z)}\right]\times$$
$$\times\, exp\left[\frac{-ikr^2 z}{2(z^2+z_R^2)}\right] exp(-il\phi) exp\left[i(2p+l+1)tan^{-1}\frac{z}{z_R}\right]$$

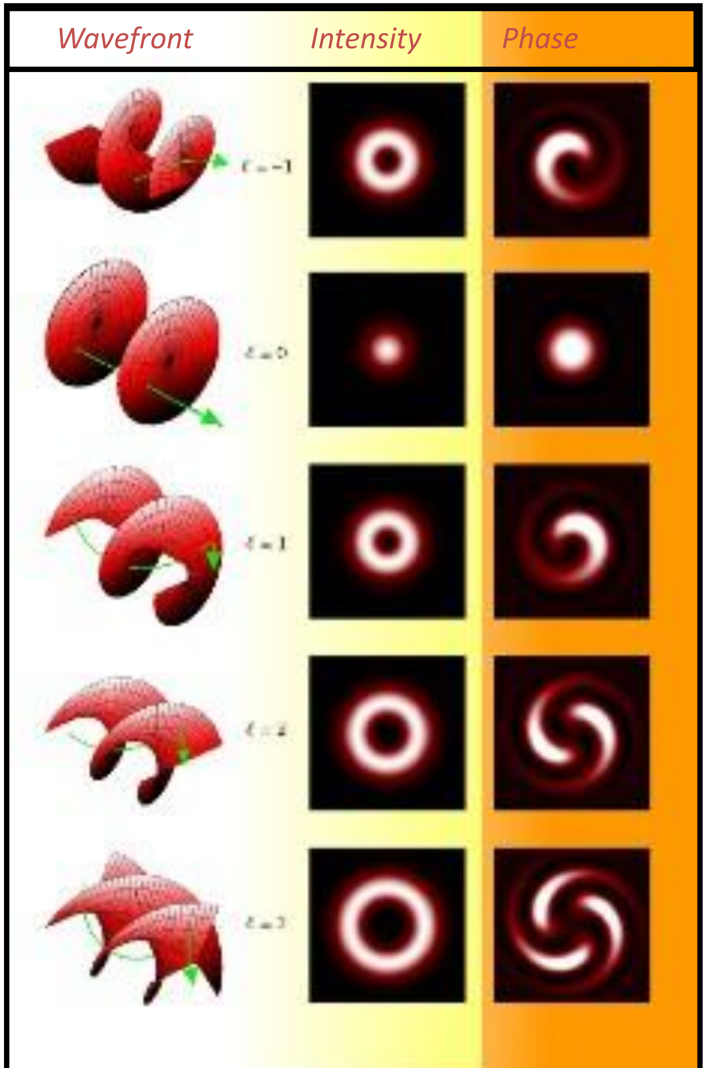
the mathematical representation in terms of Laguerre - Gauss modes contains two integer numbers:

ℓ = nr. of helicoidal twists along a wavelength, ***p*** = nr. of radial nodes

The red ovals underline the general terms applying also to non-laser beams. In the following we concentrate on ℓ.

Graphical representation of L-G modes for p = 0

ℓ = topological charge



The wavefront has a *helical shape* composed by $\boldsymbol{\ell}$ lobes disposed around the propagation axis *z*.

A phase singularity called *Optical Vortex* is nested inside the wavefront, along the axis z.

Another representation of OPTICAL VORTICES OV

helicoidal shape of the wavefront

↓

indetermination of the phase on the axis around which the wavefront twists

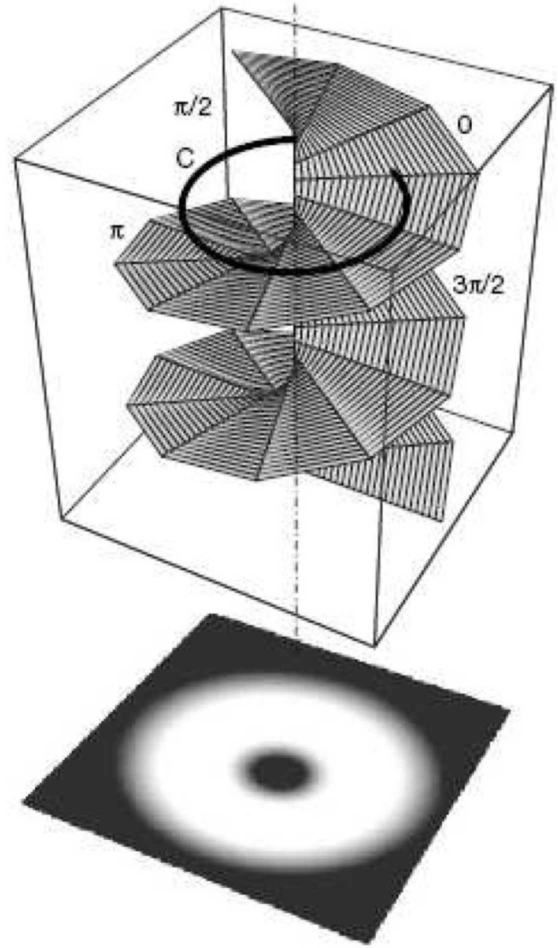
↓

zero intensity of the field on such axis (destructive interference)

↓

Optical Vortex described by the *topological charge:*

$$Q = \frac{1}{2\pi} \oint_c \nabla\chi \cdot d\vec{s} \quad \Longleftrightarrow \quad Q = \ell$$



Example: OAM IN A LASER PARAXIAL BEAM

In a PLANE EM wave:

$E_z = B_z = 0$, ***S*** is parallel to ***k*** → ***J*** = 0

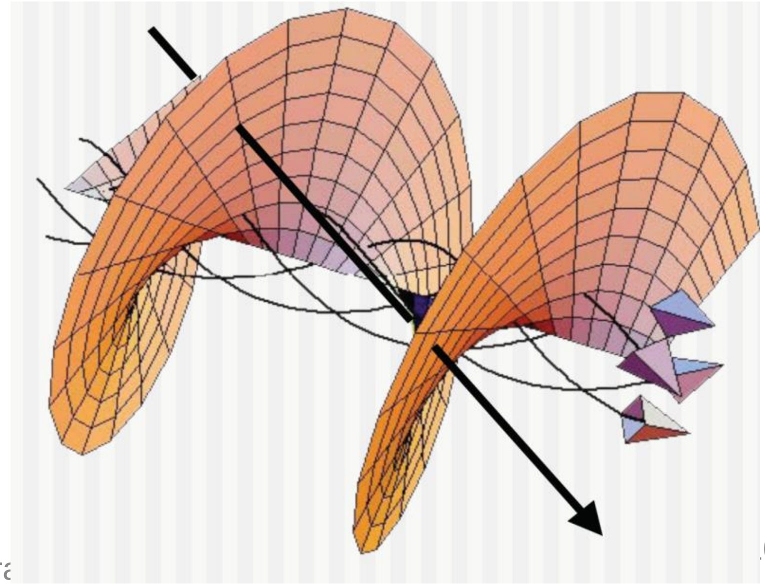
In a LASER generated paraxial beam:

$E_z \neq 0$, $B_z \neq 0$, ***S*** is no longer parallel to ***k***

↳ ***S*** gets a radial + an azimuthal component: → ***J*** = $J_z \neq 0$

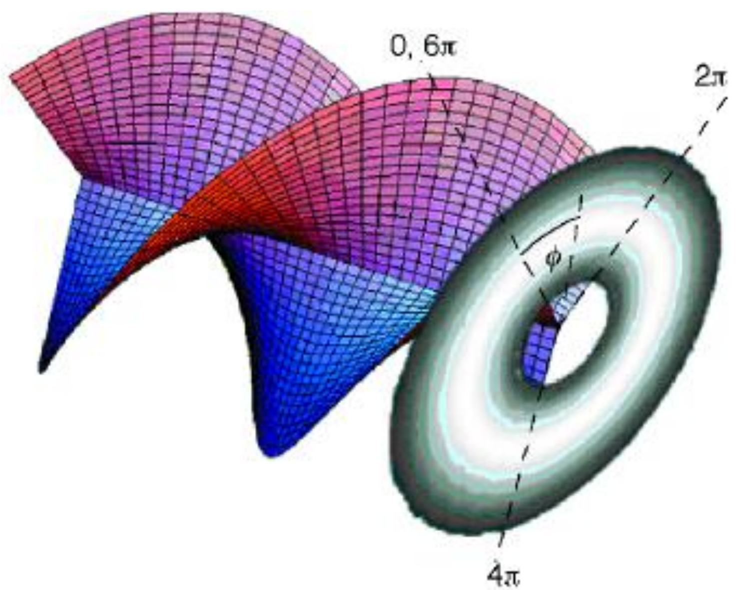
↓

Poynting's vector rotates around the average direction of propagation :



Imparting OAM onto a laser beam

The generation of beams carrying OAM proceeds thanks to the insertion in the optical path of *a phase modifying device* which imprints vorticity on the phase distribution of the incident beam.

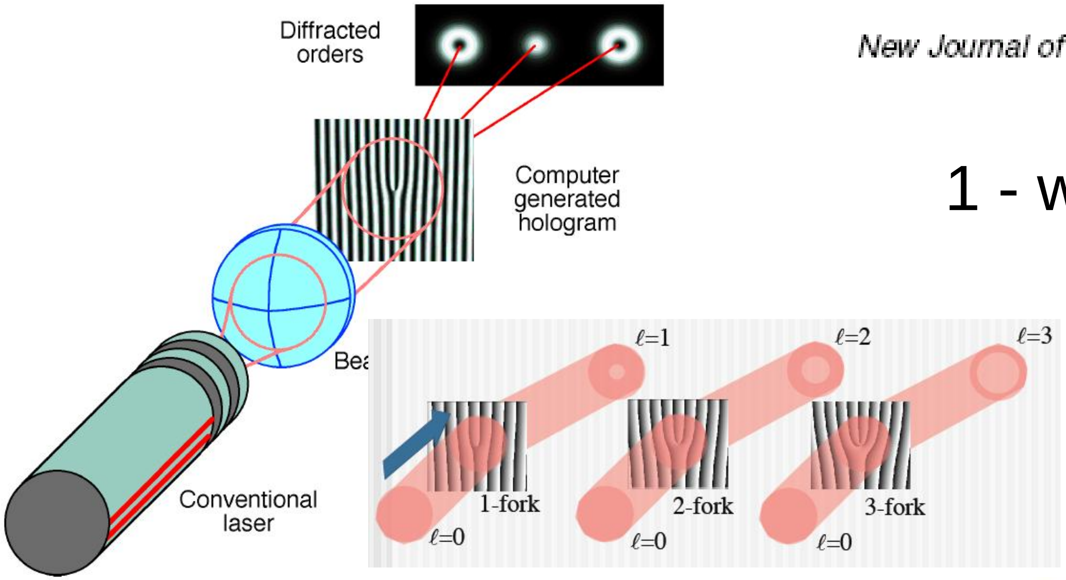


Imparting OAM onto a laser beam

Jonathan Leach, Eric Yao and Miles J Padgett
Department of Physics and Astronomy, Kelvin Bld, University Ave, University of Glasgow, Glasgow G12 8QQ, UK
E-mail: j.leach@physics.gla.ac.uk

New Journal of Physics **6** (2004) 71

1 - with a fork hologram



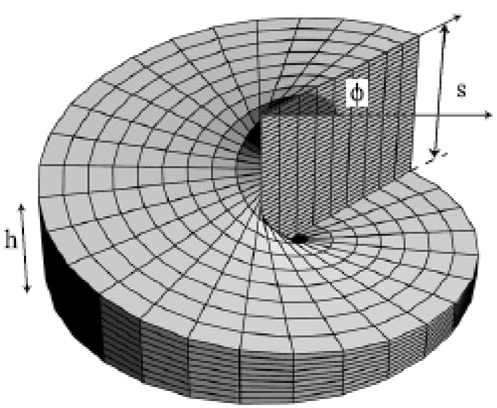
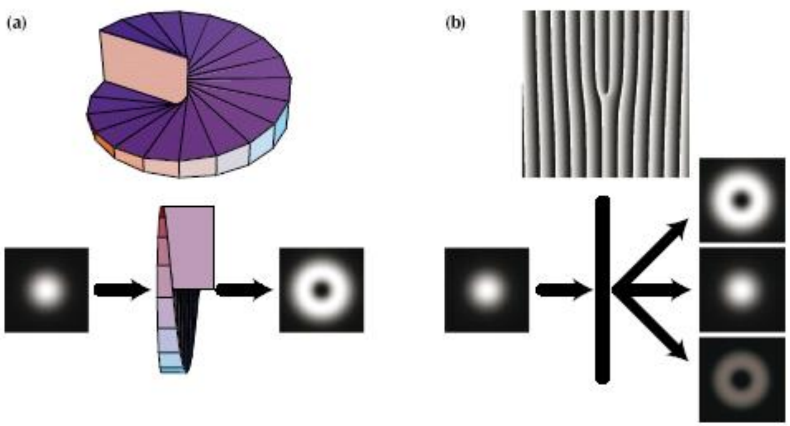


Figure 1. A spiral phase plate of refractive index n. The thickness of the phase plate h is proportional to the azimuthal position given by ϕ.



2 - with a spiral plate

Our results

Our first results with a $\ell = 1$ fork hologram: overcoming the Rayleigh limit in the laboratory

PRL 97, 163903 (2006) PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2006

Overcoming the Rayleigh Criterion Limit with Optical Vortices

F. Tamburini, G. Anzolin, G. Umbriaco, A. Bianchini, and C. Barbieri

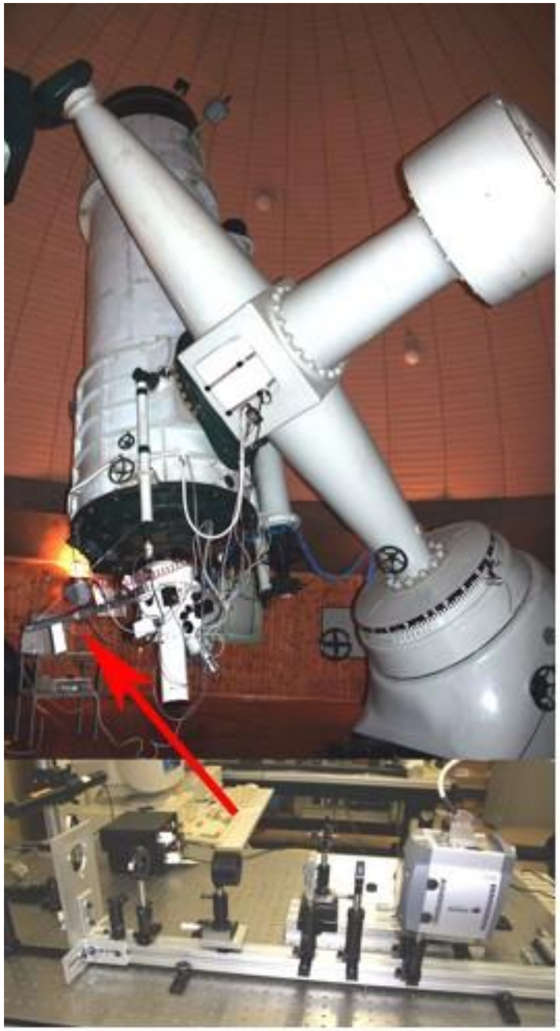
Department of Astronomy, University of Padova, vicolo dell' Osservatorio 2, Padova, Italy

(Received 12 June 2006; published 16 October 2006)

We experimentally and numerically tested the separability of two independent equally luminous monochromatic and white light sources at the diffraction limit, using optical vortices (OV). The diffraction pattern of one of the two sources crosses a fork hologram on its center generating the Laguerre-Gaussian (LG) transform of an Airy disk. The second source, crossing the fork hologram in positions different from the optical center, generates nonsymmetric LG patterns. We formulated a criterion, based on the asymmetric intensity distribution of the superposed LG patterns so created, to resolve the two sources at angular distances much below the Rayleigh criterion. Analogous experiments in white light allow angular resolutions which are still one order of magnitude below the Rayleigh criterion. The use of OVs might offer new applications for stellar separation in future space experiments.

DOI: 10.1103/PhysRevLett.97.163903 PACS numbers: 42.25.−p, 42.40.Eq, 42.40.Jv, 42.87.Bg

Our first results with a ℓ = 1 fork hologram: producing Optical Vortices with starlight



Optical vortices with starlight
G. Anzolin, F. Tamburini, A. Bianchini, G. Umbriaco, and C. Barbieri
(2008, Astron. & Astrophys.)

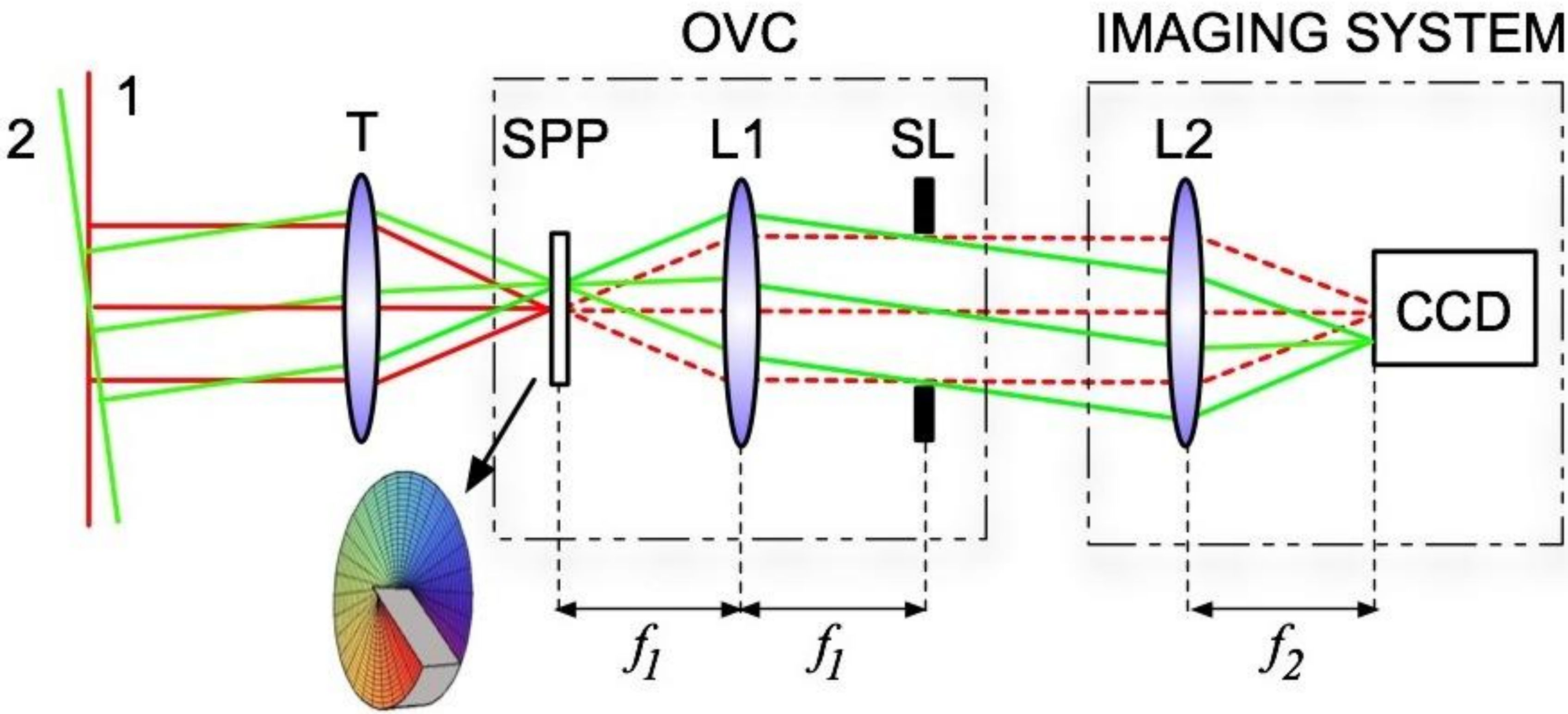
The previously described device with a l = 1 fork hologram was taken to the 122 cm Asiago telescope.

Real star images were fed to the optical train.

Our first results with a $\ell = 2$ phase plate:

OVs for astronomical coronagraphy

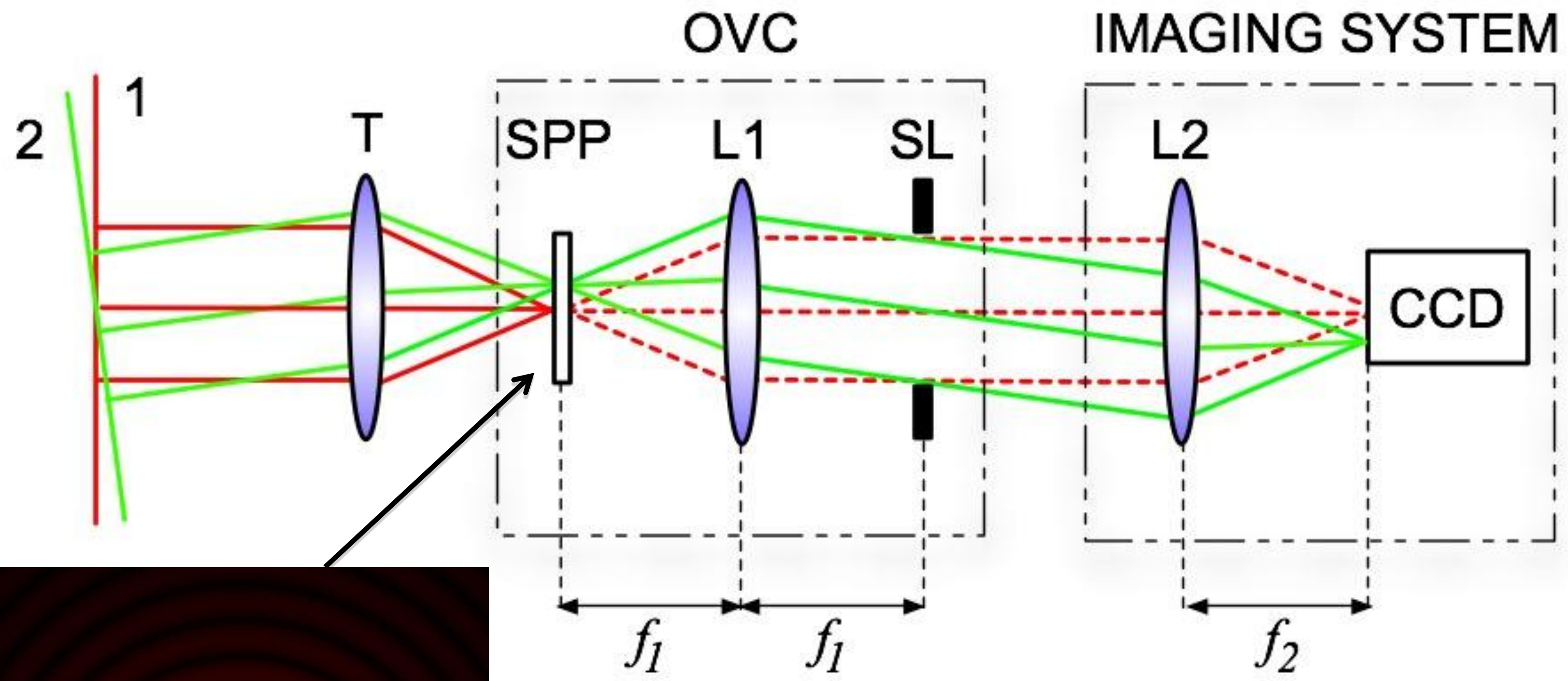
OVs for Coronagraphy

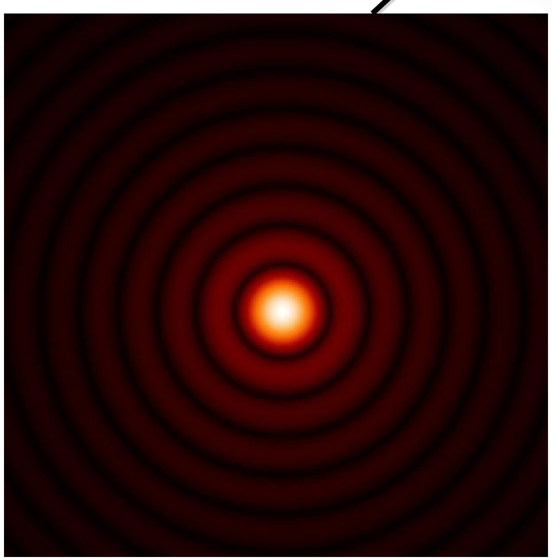


Phase mask placed in the telescope focal plane. It generates a $\ell = 2$ (more generally, an even topological charge) OV.

Consider two stars in a close binary system (1 on axis, 2 off-axis): the off-axis secondary star will pass through the Lyot mask, while the ring of the primary is blocked.

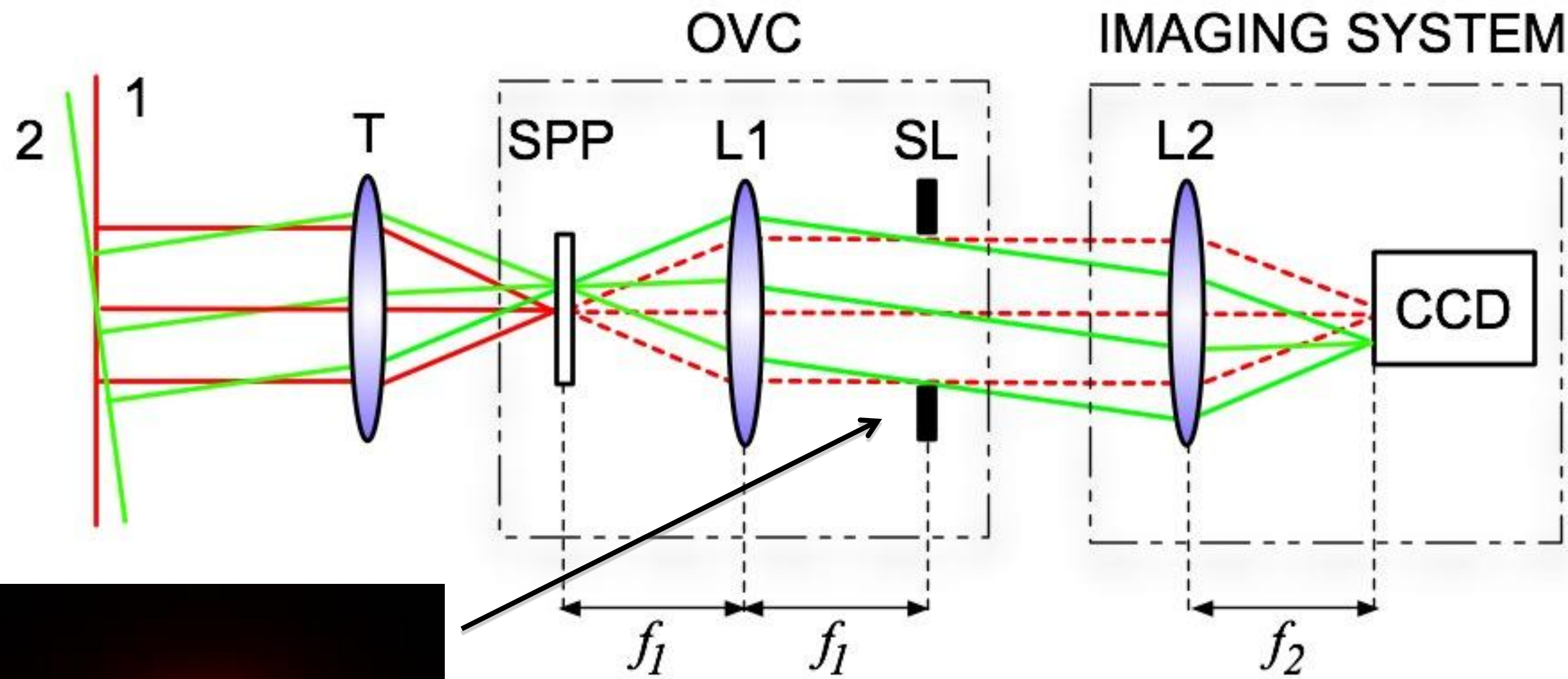
OVs for Coronagraphy



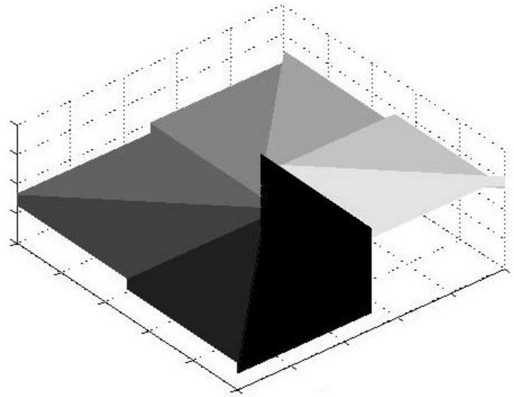


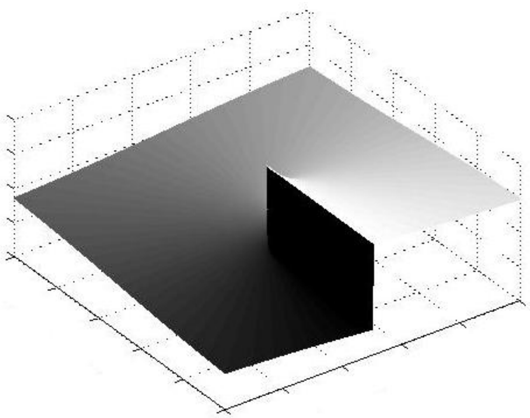
Incident Airy diffraction pattern that crosses the optical singularity of the SPP

OVs for Coronagraphy



Optical vortex then blocked by a circular aperture (Lyot stop)





N steps	Fainting	λ_{min}
8	$2.27 \cdot 10^{-4}$	624
16	$6.70 \cdot 10^{-5}$	585
32	$1.75 \cdot 10^{-5}$	567
64	$4.42 \cdot 10^{-6}$	558
128	$1.09 \cdot 10^{-6}$	554
512	$7.21 \cdot 10^{-8}$	551
∞	$5.63 \cdot 10^{-10}$	550

With an ideal spiral phase mask, the achieved contrast is sufficient for the direct detection of extra-solar planets.

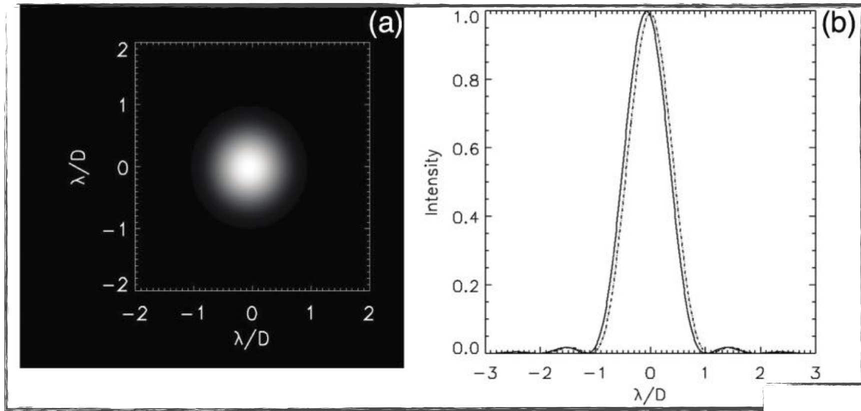
Subrayleigh l=2 Coronagraphy

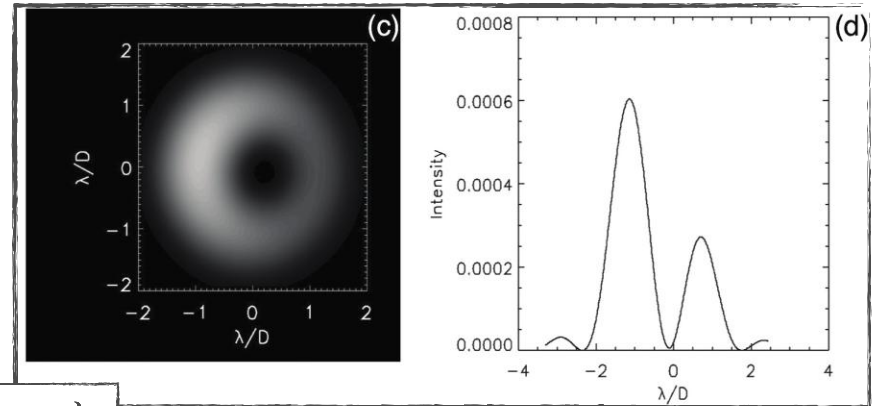
Sub-Rayleigh optical vortex coronagraphy.

Mari E, Tamburini F, Swartzlander GA Jr, Bianchini A, Barbieri C, Romanato F, Thidé B.

Opt Express. 2012 Jan 30;20(3):2445-51. doi: 10.1364/OE.20.002445.

We have investigated numerically the super-resolution capabilities of an optical vortex coronagraph (OVC), equipped with an N-step spiral phase plate in its optical path, when the separation of the two sources is below the Rayleigh separability criterion.
Our numerical calculations show that a fraction of the light from the secondary source can be detected yielding a sub-Rayleigh resolution of at least 0.1 λ/D.



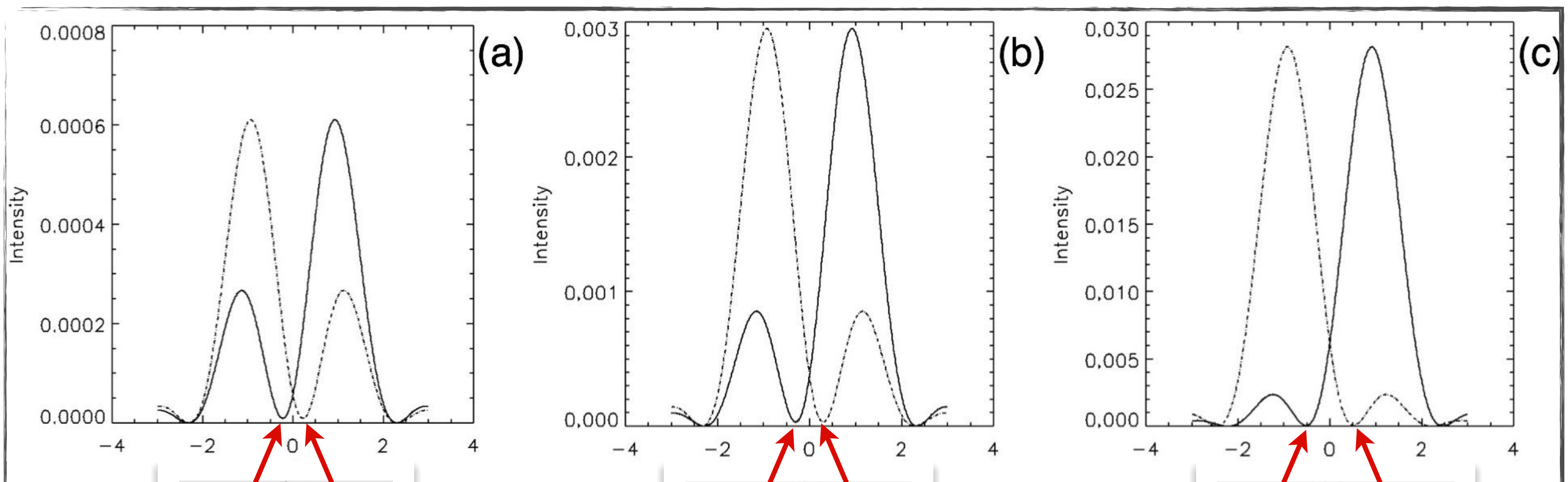


$$\Delta\theta = 0.1\frac{\lambda}{D}$$

UNRESOLVED

On-axis source is extinguished by OVC. Off-axis source assumes an asymmetric doughnut pattern and so can be detected

Examples:

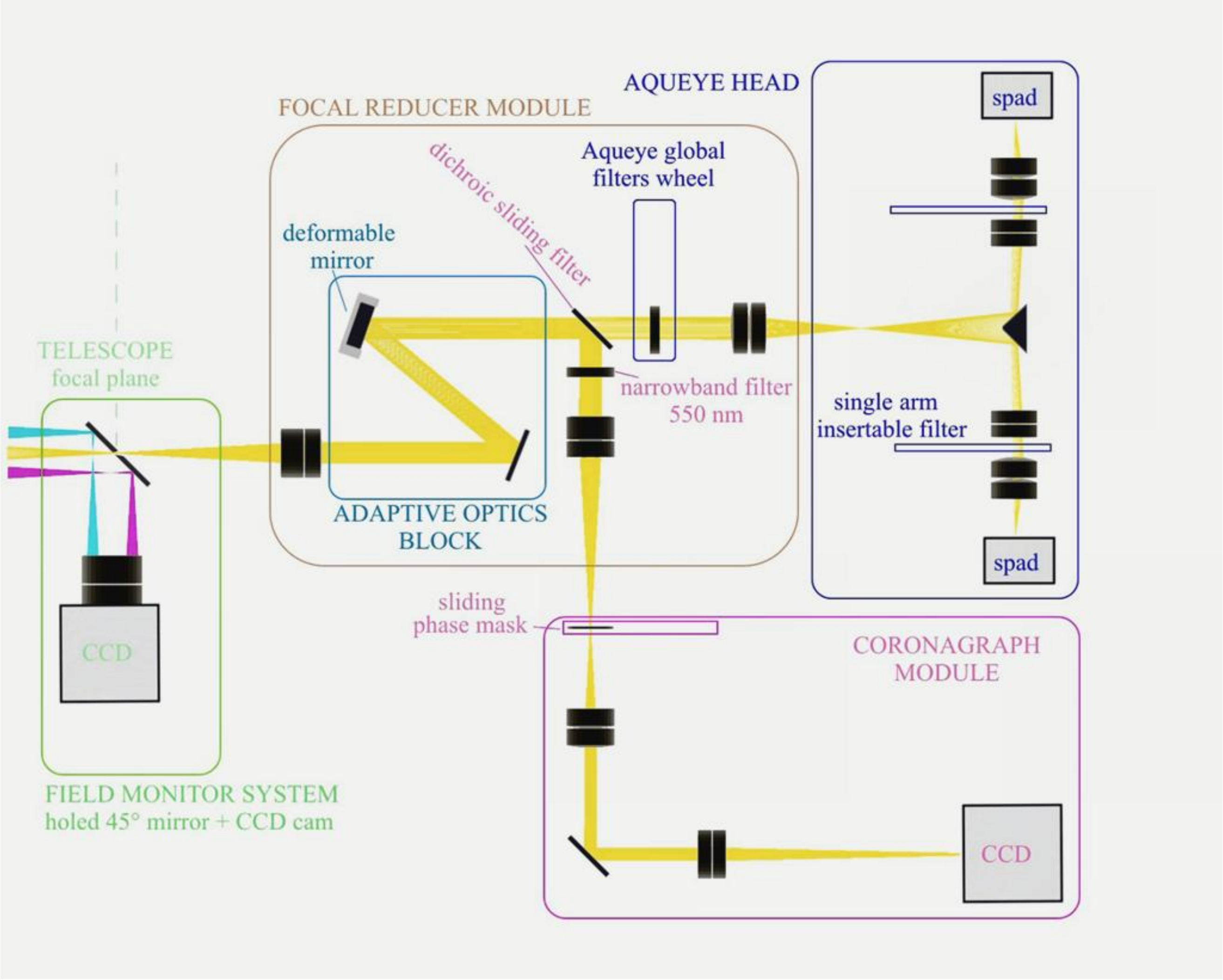


$\Delta\theta = 0.1\lambda/D$ $\Delta\theta = 0.2\lambda/D$ $\Delta\theta = 0.5\lambda/D$

Two important caveats

- Phase plates are chromatic devices, in a simple implementation of the technique a ***very narrow filter*** is necessary for optimal attenuation (e.g. 1 nanometer or so).
- Seeing is a terrible nuisance, a very good ***adaptive optics device*** is required to restore as much as possible the Airy figure (e.g. to achieve a Strehl ratio of 0.5 or better).
- We are working on this latter aspect in order to go to the 1.8m telescope at Cima Ekar in the near future.

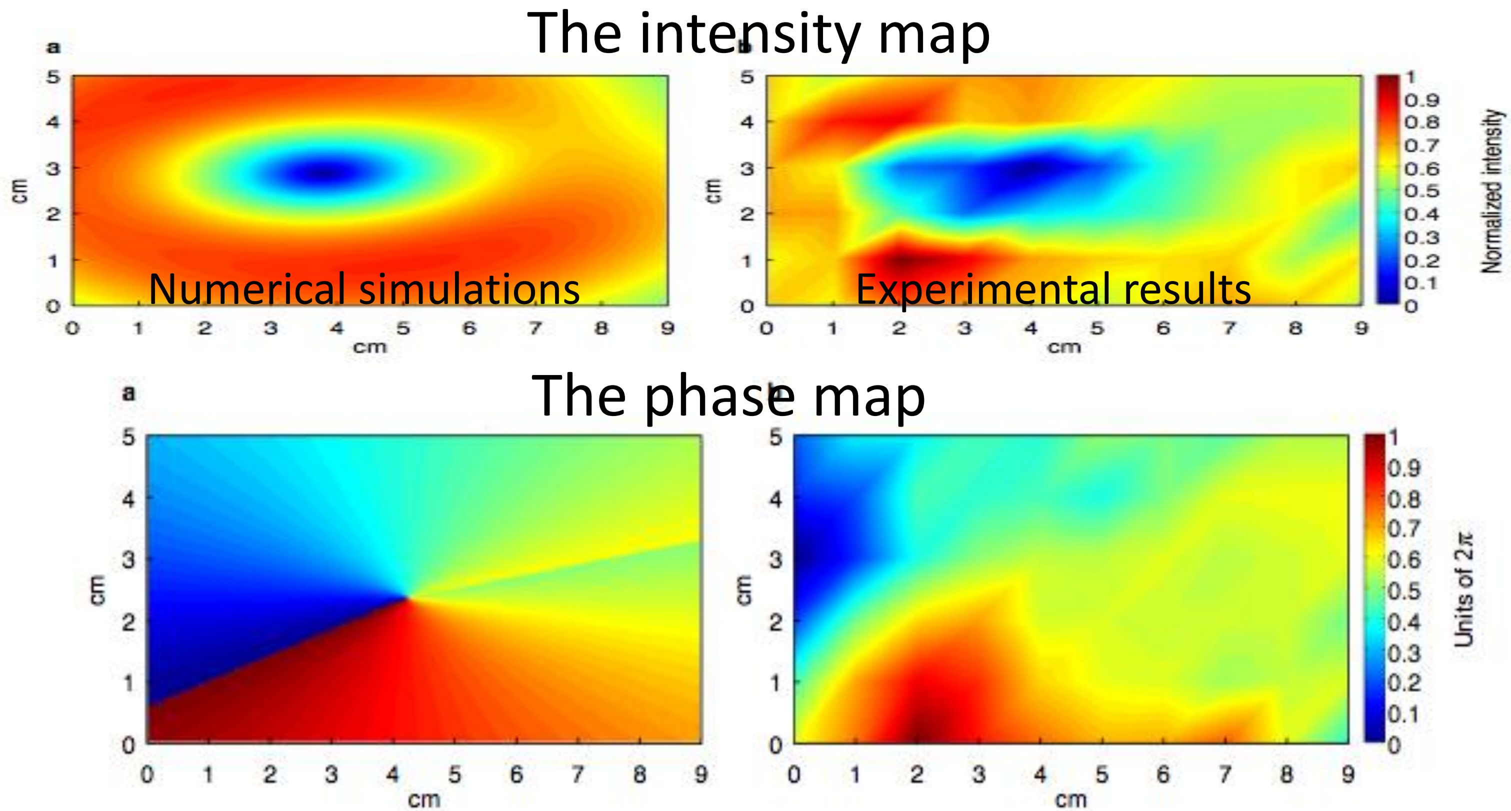
The preliminary drawing of the new OAM coronagraph for the Asiago 1.8m telescope



4 - Radio Applications

We applied OAM to the radio domain. We have shown (in the anechoic chamber of Uppsala University) how OAM and vorticity can be readily imparted onto ***radio beams***. The frequency was 1.4 GHz.

Experimental verification of photon angular momentum and vorticity with radio techniques
F. Tamburini, E. Mari, B. Thidé, C. Barbieri, and F. Romanato: Appl. Phys. Lett. **99, 204102 (2011);**



5 - The Radio Experiment in Venice - 1

Encoding many channels on the same frequency through radio vorticity: first experimental test

F. Tamburini, E. Mari, A. Sponselli, B. Thidé, A. Bianchini, F. Romanato

New Journal of Physics ***14 (2012) 033001*** 1367-2630/12/033001+17$33.00

A radio broadcast of OAM at 2.4 GHz over a distance of 442 meters was performed in June 2011 from a lighthouse on the island of San Giorgio Maggiore to the Doge's Palace.

It was shown experimentally that it is possible to use two beams of incoherent radio waves, transmitted on the same frequency but ***encoded in two different orbital angular momentum states, to simultaneously transmit two independent radio channels***.

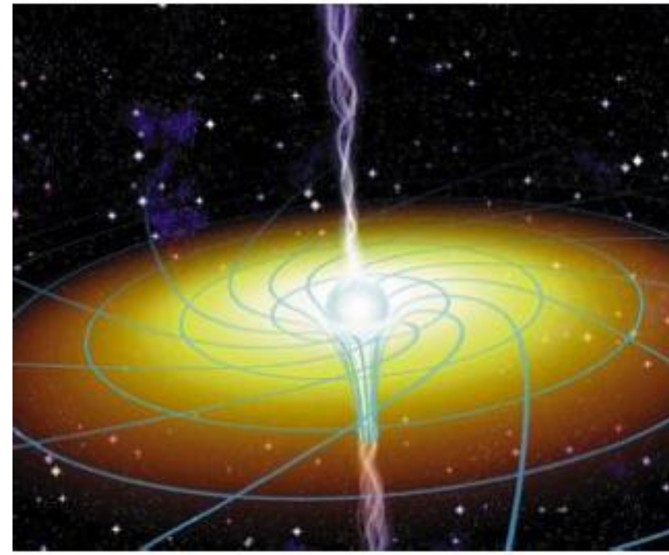
This novel radio technique allows the implementation of, in principle, an ***infinite number of channels in a given, fixed bandwidth***, even without using polarization, multiport or dense coding techniques. This paves the way for innovative techniques in radio science and entirely new paradigms in radio communication protocols that might offer a solution to the problem of radio-band congestion.

5 - The Radio Experiment in Venice - 2

Public announcement of the reception and tuning of the twisted signal 'segnale ricevuto' (signal received).

Detecting rotating black holes

Twisting of light around rotating black holes
F. Tamburini et al.
Nature Physics. Vol. 7, March 2011, p. 197



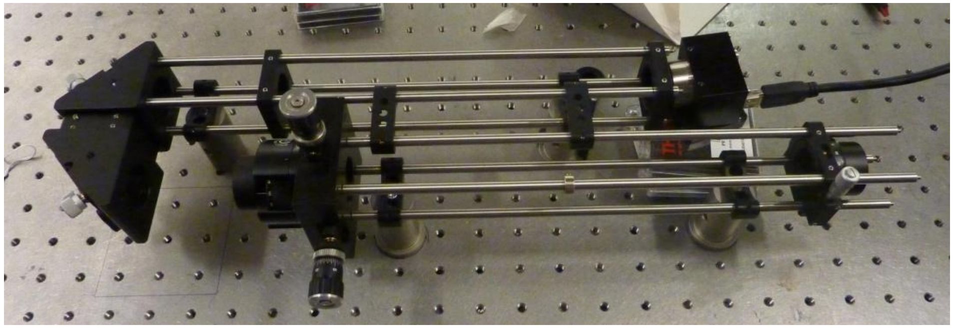
Spacetime around a black hole can twist the light emitted as a result of matter falling onto the object's accretion disc. The twisty space-time would have the same effect as a lens, and that effect should be observable via the Orbital Angular Momentum.

Computer simulations revealed that the light twists will depend on how fast the black hole is rotating, giving a precise means of measuring that ***rate of rotation*** via the amount of OAM.

Those measurements, in turn, could shed light on how black holes form, and help to detect ***Hawking radiation***, emitted by black holes as they evaporate over time (the bigger they are, the more slowly they evaporate). This radiation was predicted by Stephen Hawking in 1974, but has not yet been directly observed.

A new avenue: OAM sorter

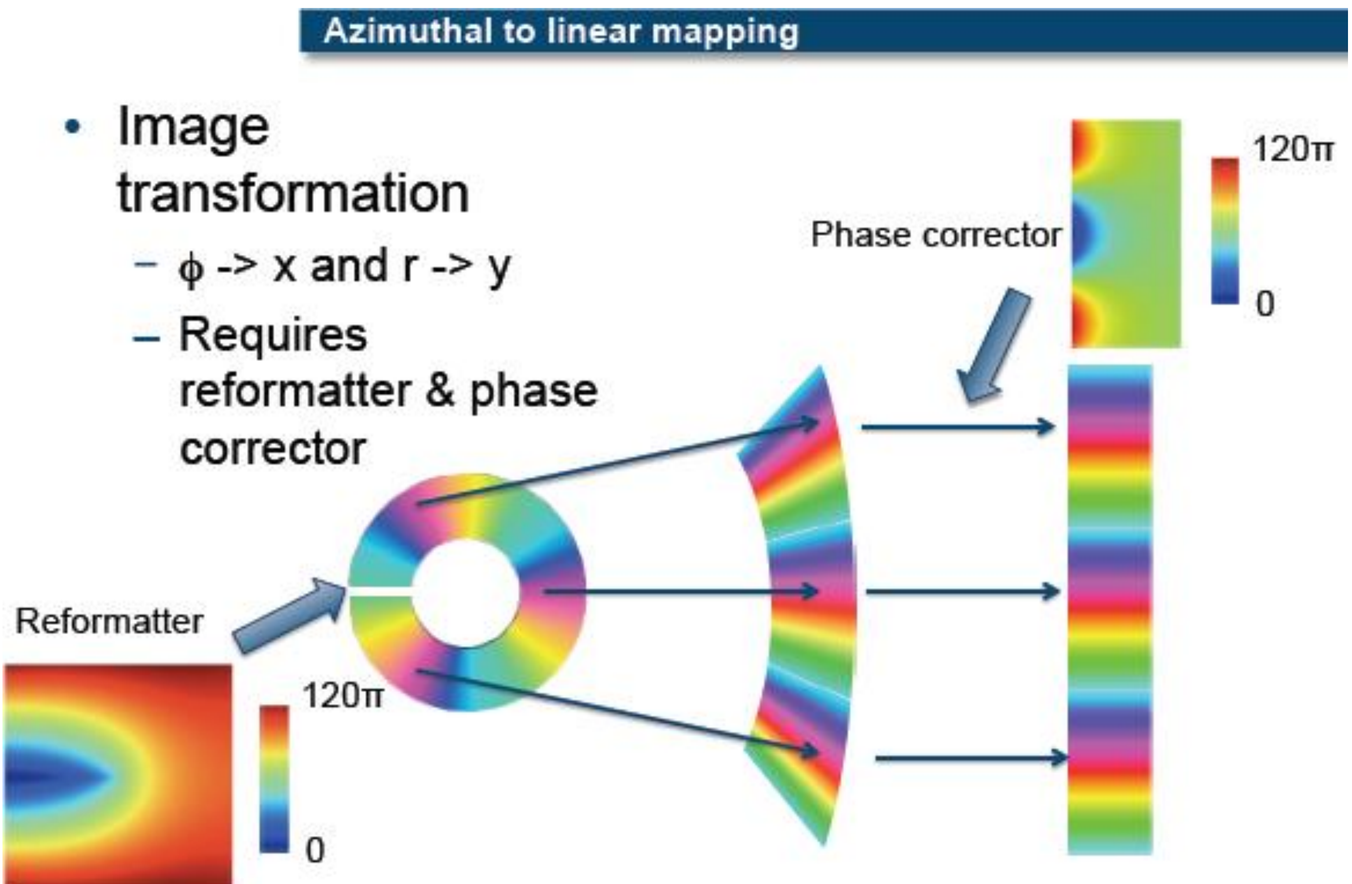
OAM sorter: an optical device to perform the OAM spectrum of incident light



G. C. G. Berkhout, M. P. J. Lavery, J. Courtial, M.W. Beijersbergen and Miles J. Padgett, *Efficient Sorting of Orbital Angular Momentum States of Light*, PRL 105, 153601 (2010):

"We present a method to efficiently sort OAM states of light using two static optical elements. The optical elements perform a Cartesian to log-polar coordinate transformation, converting the helically phased light beam corresponding to OAM states into a beam with transverse phase gradient. A subsequent lens then focuses each input OAM state to a different lateral position."

“We present a method to efficiently sort OAM states of light using two static optical elements. ***The optical elements perform a Cartesian to log-polar coordinate transformation, converting the helically phased light beam corresponding to OAM states into a beam with transverse phase gradient.*** *A subsequent lens then focuses each input OAM state to a different lateral position.”*



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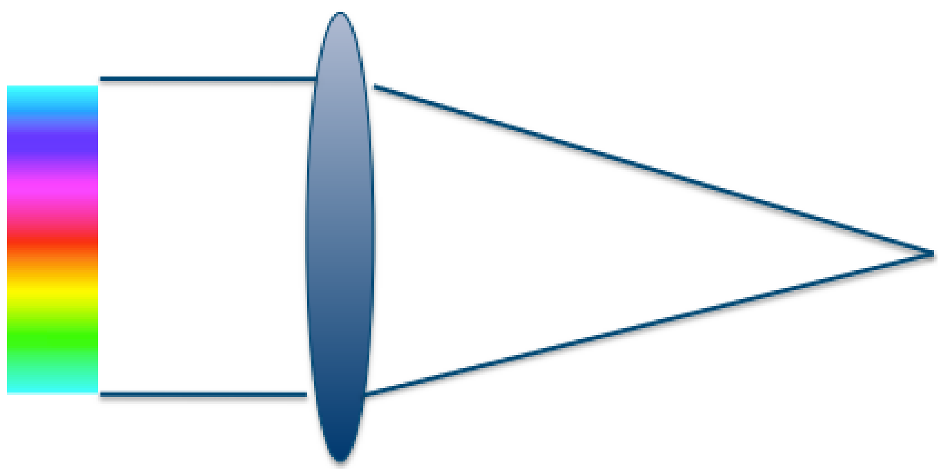
- A "plane-wave" is focused by a lens
- A phase ramp of 2π displaces the spot



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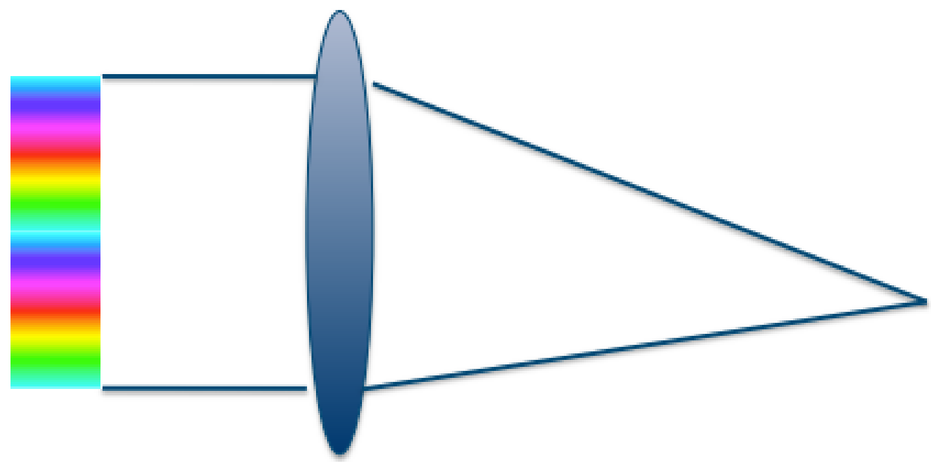

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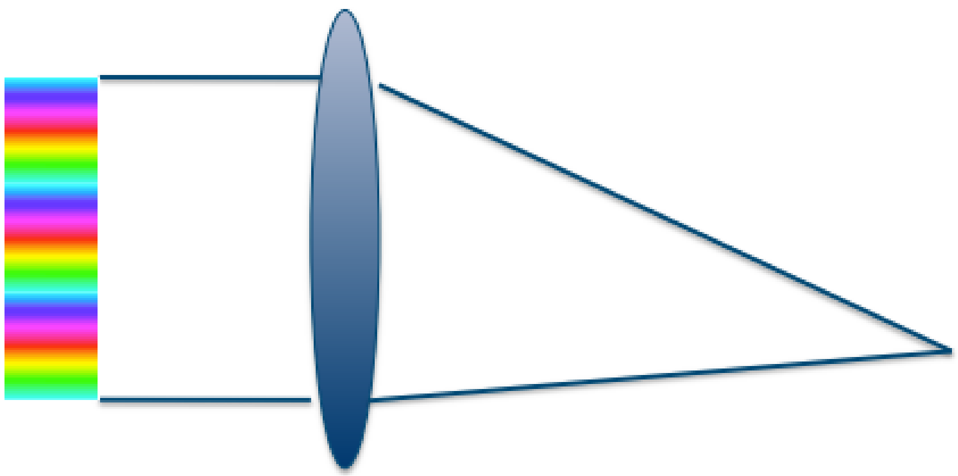

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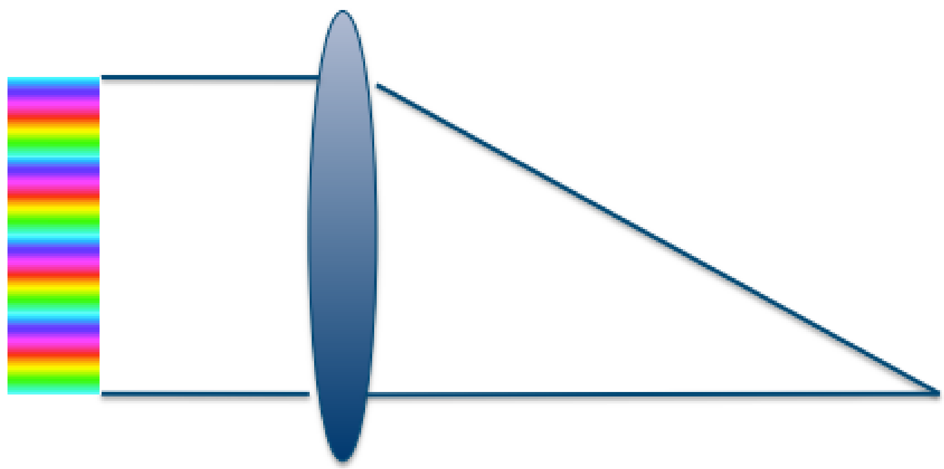
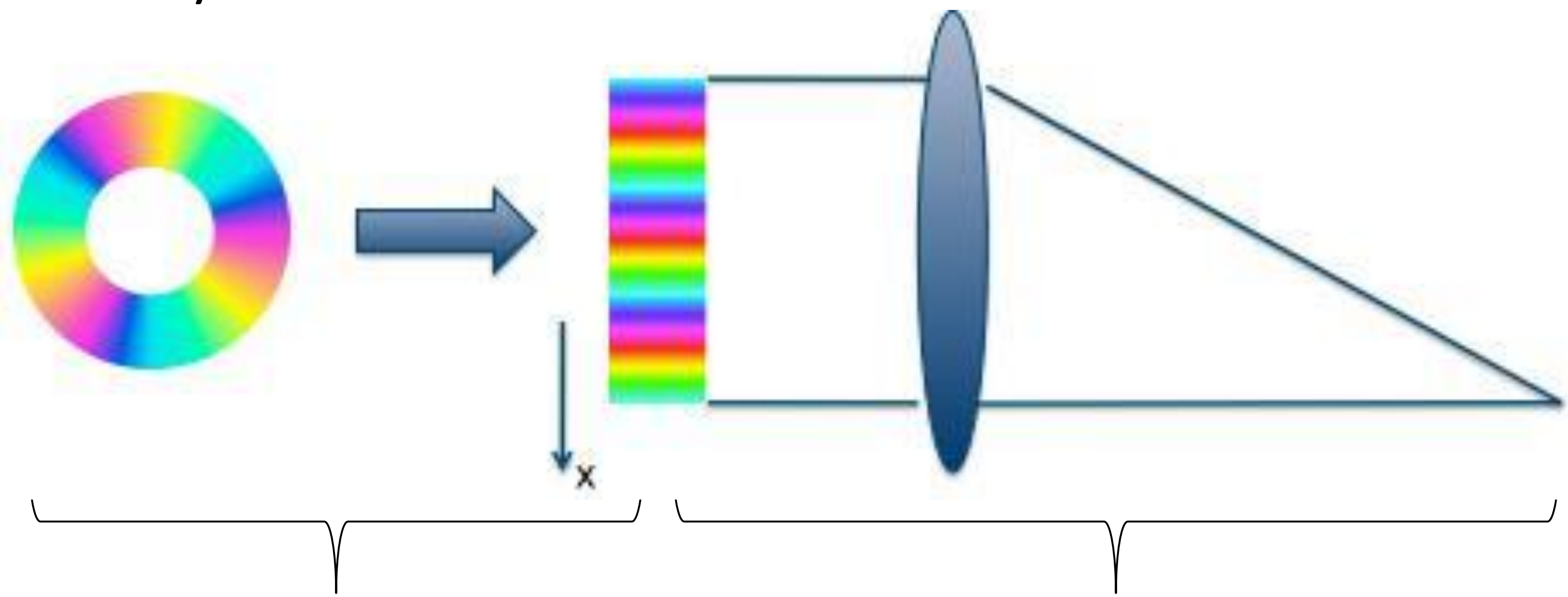


Image transformation:

$\Phi \longrightarrow x$

$r \longrightarrow y$

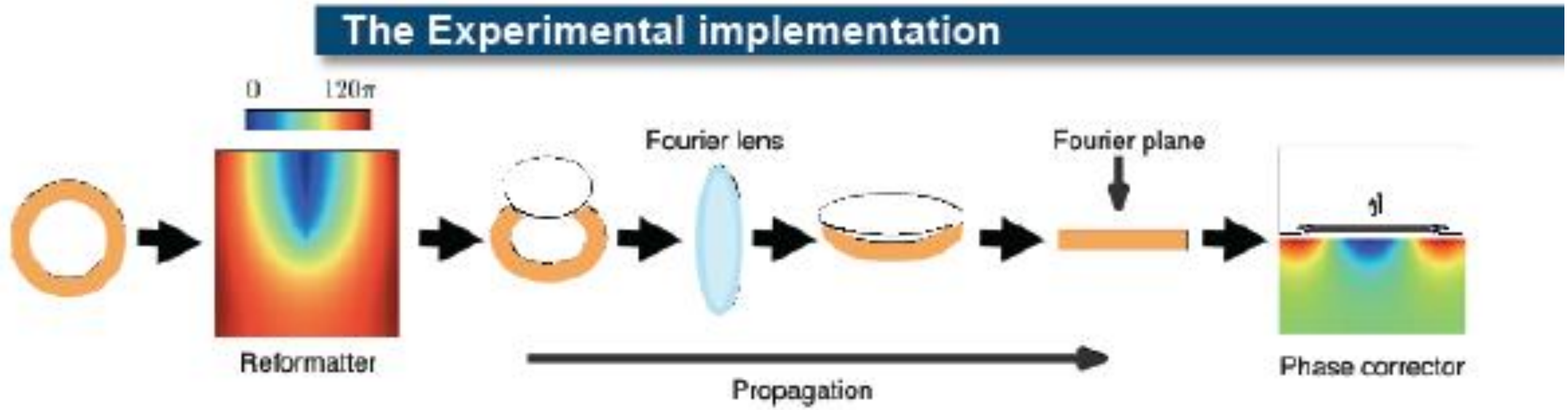


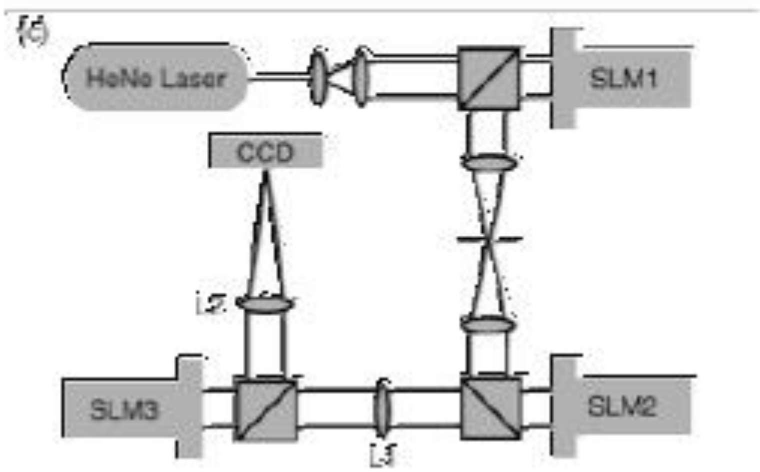
we convert helical phase
to linear phase

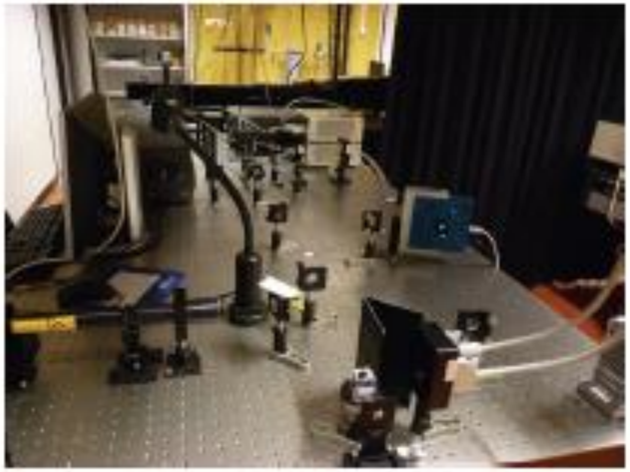
each phase profile corresponds
to a different x-displacement

OAM spectrum!

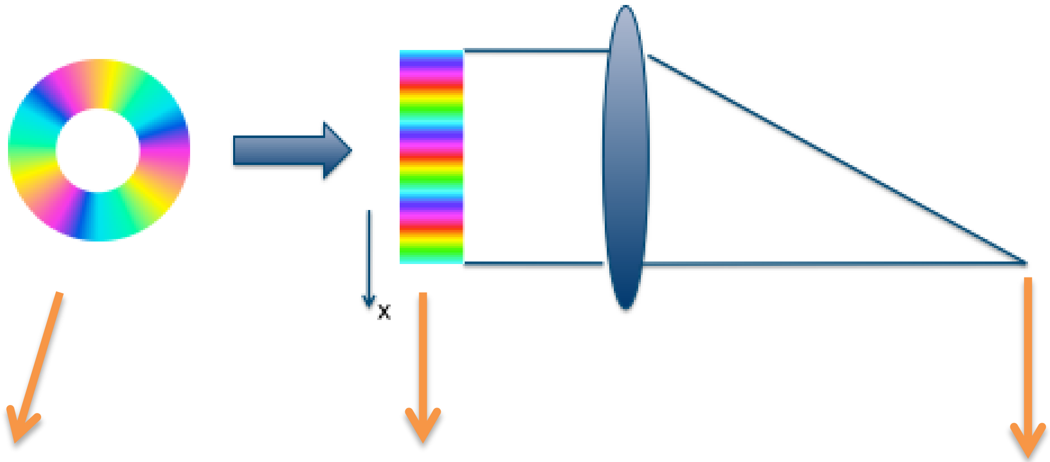
Implementation in the laboratory



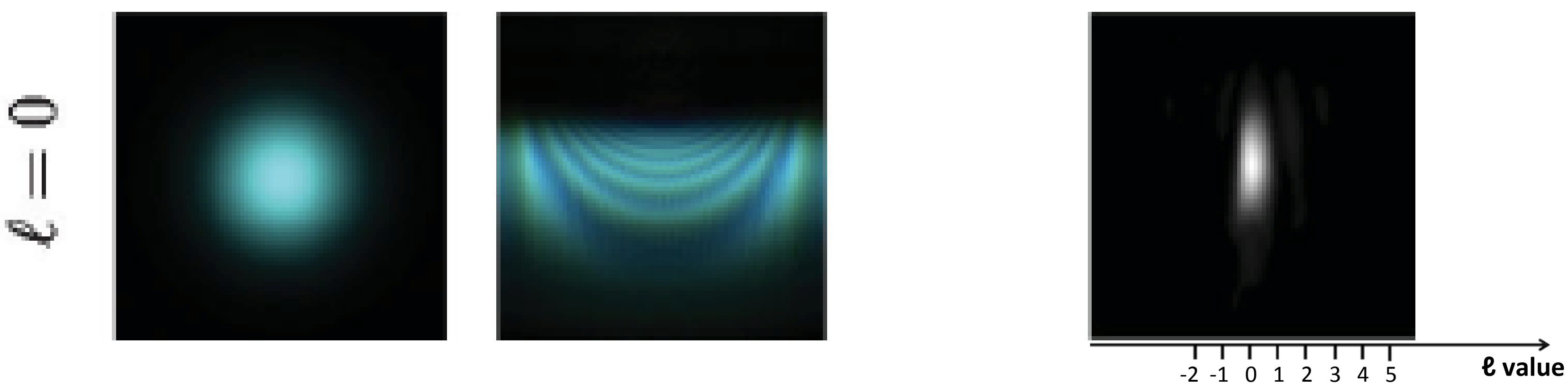




The results

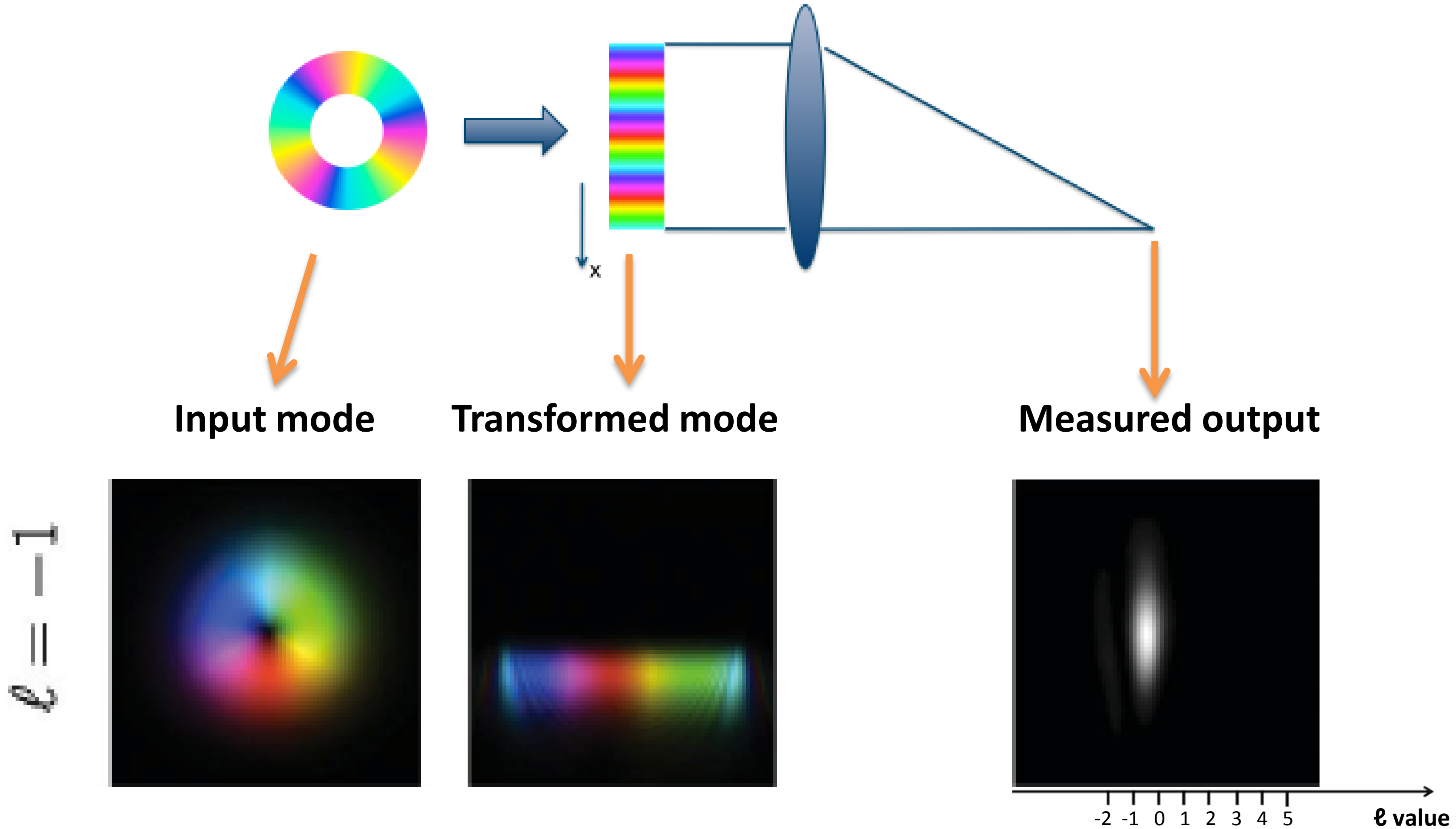


Input mode **Transformed mode** **Measured output**



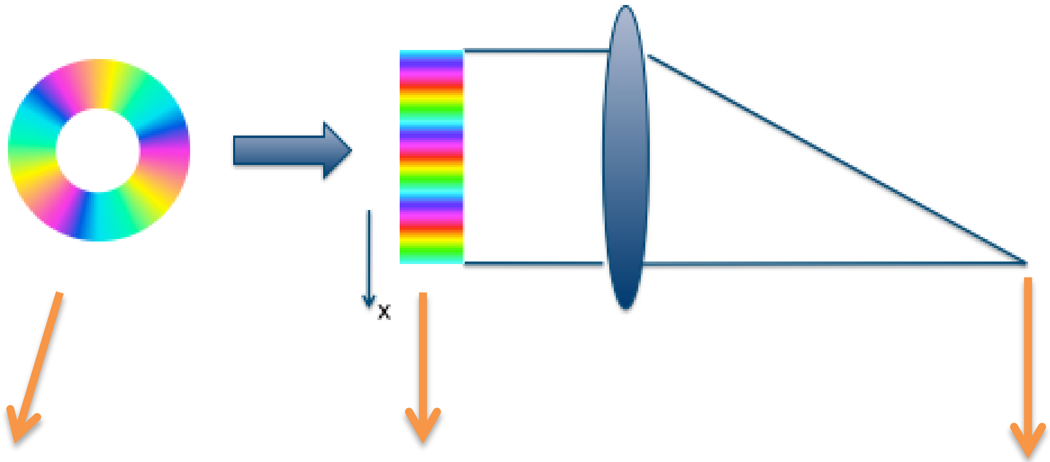
(lateral position) α (OAM state)

The results

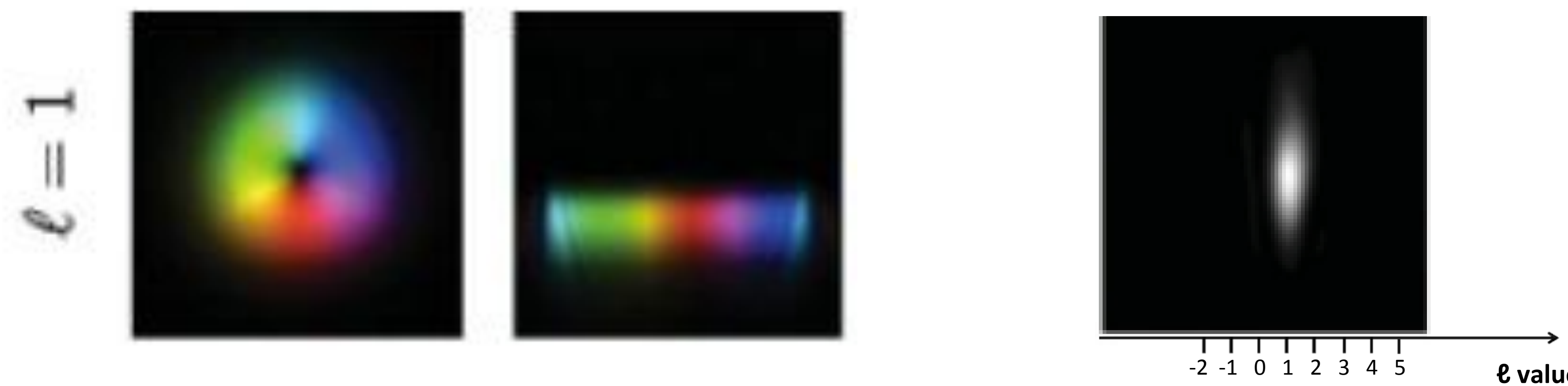


(lateral position) α (OAM state)

The results

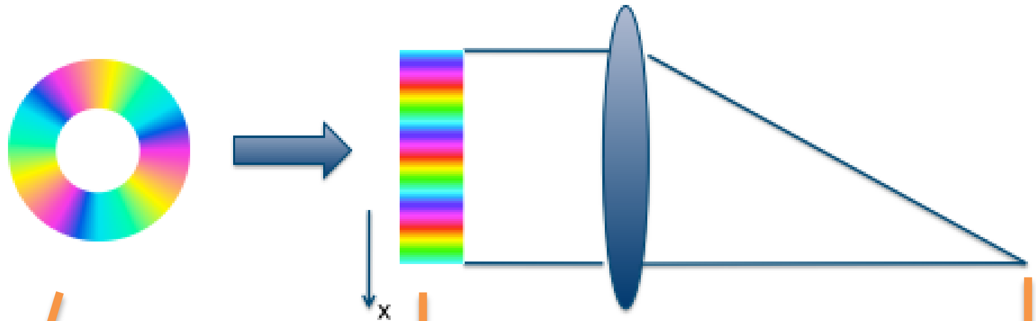


Input mode **Transformed mode** **Measured output**



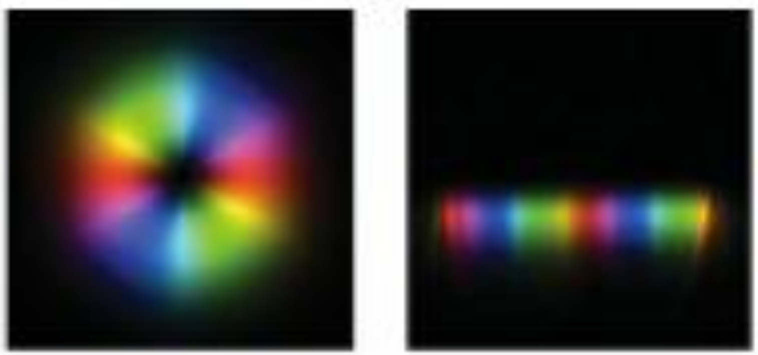
(lateral position) α (OAM state)

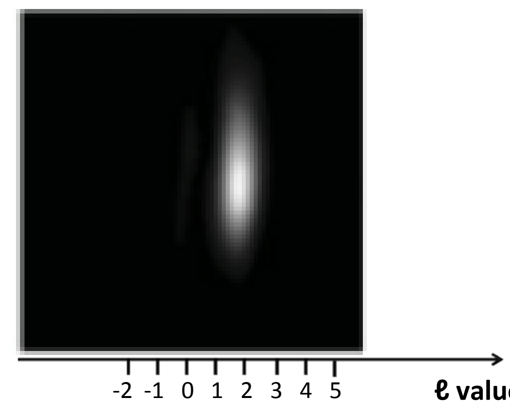
The results



Input mode **Transformed mode** **Measured output**

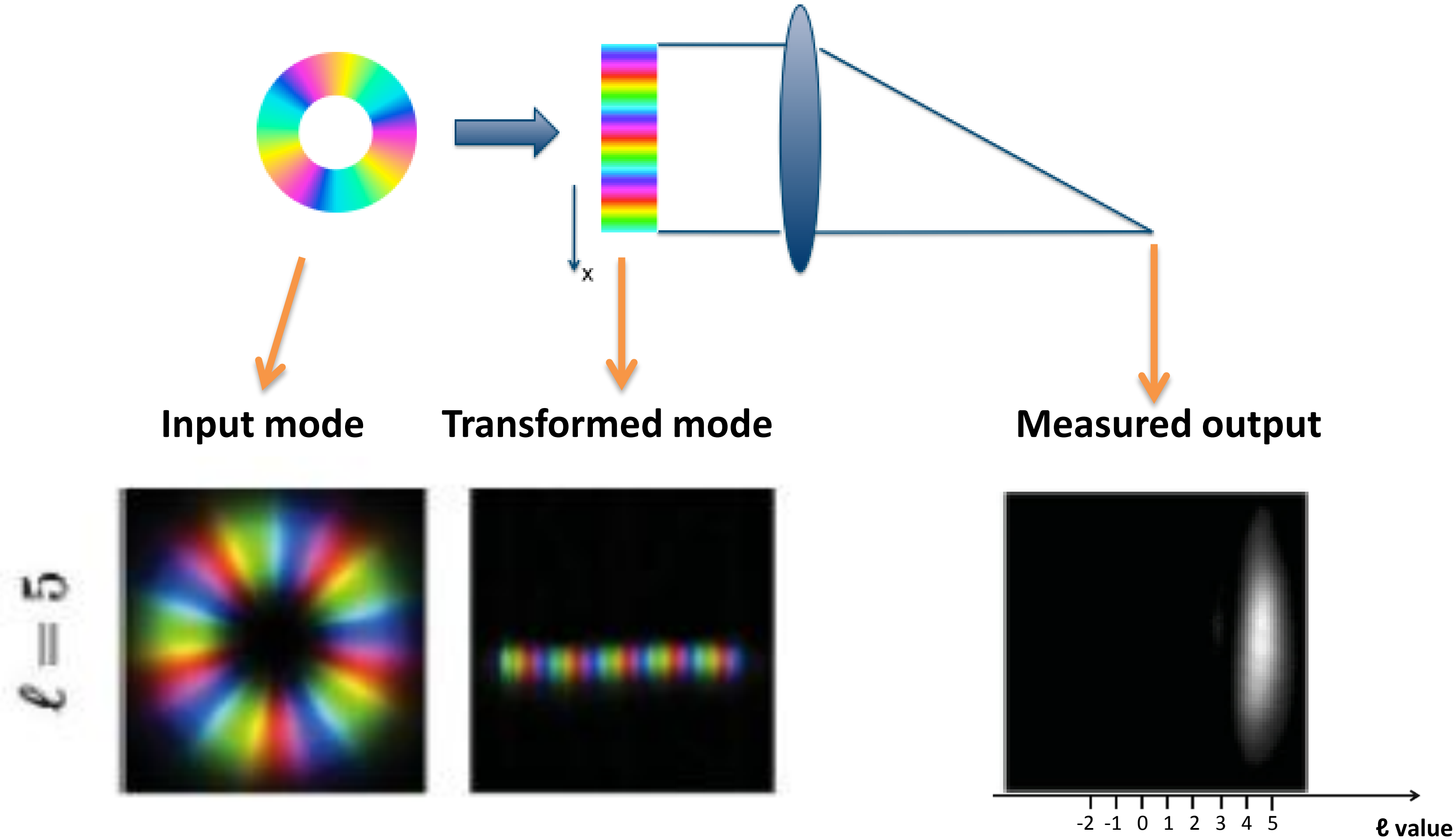
$\ell = 2$



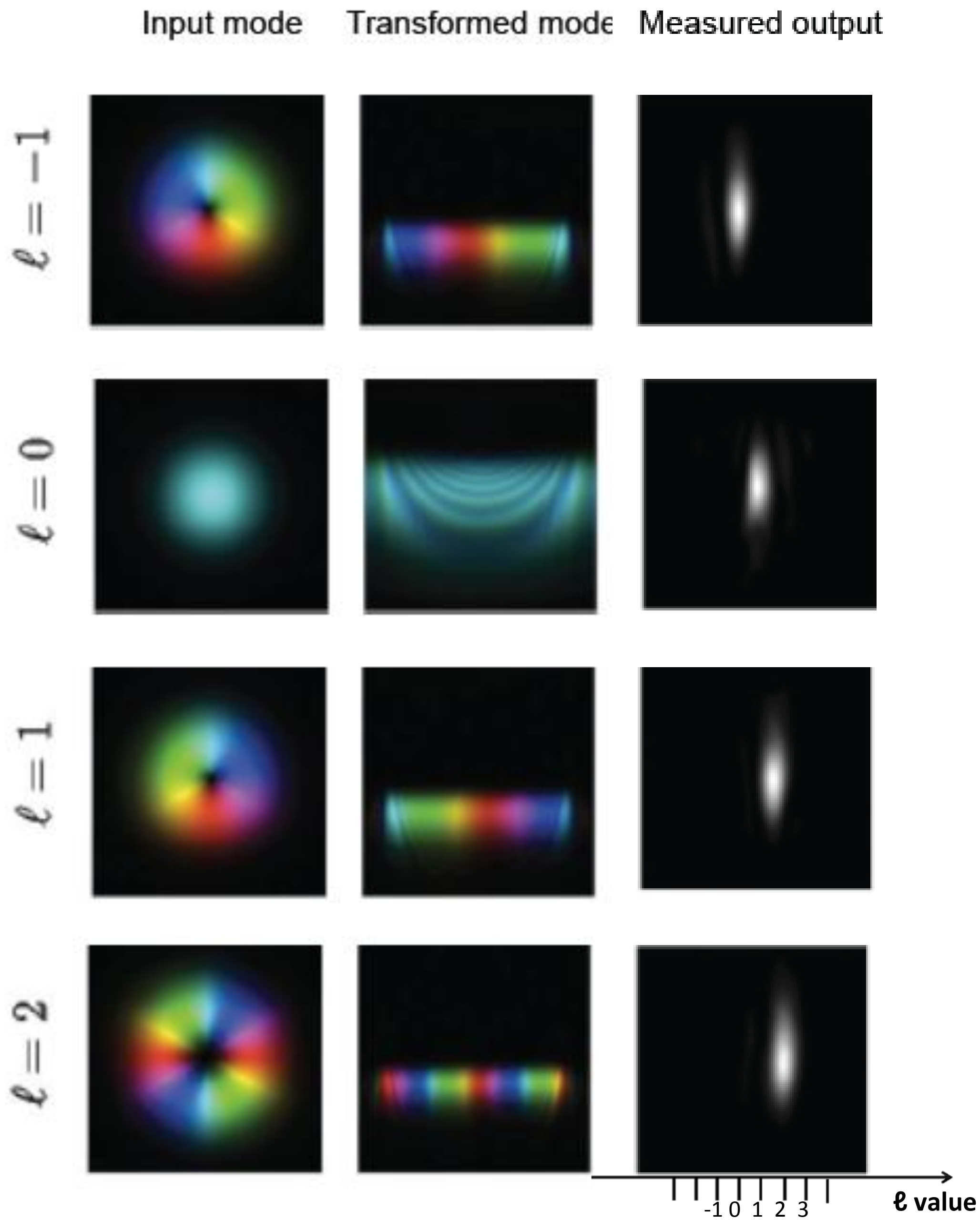


(lateral position) α (OAM state)

The results



(lateral position) α (OAM state)



Results in the laboratory

OAM sorter in Astrophysics

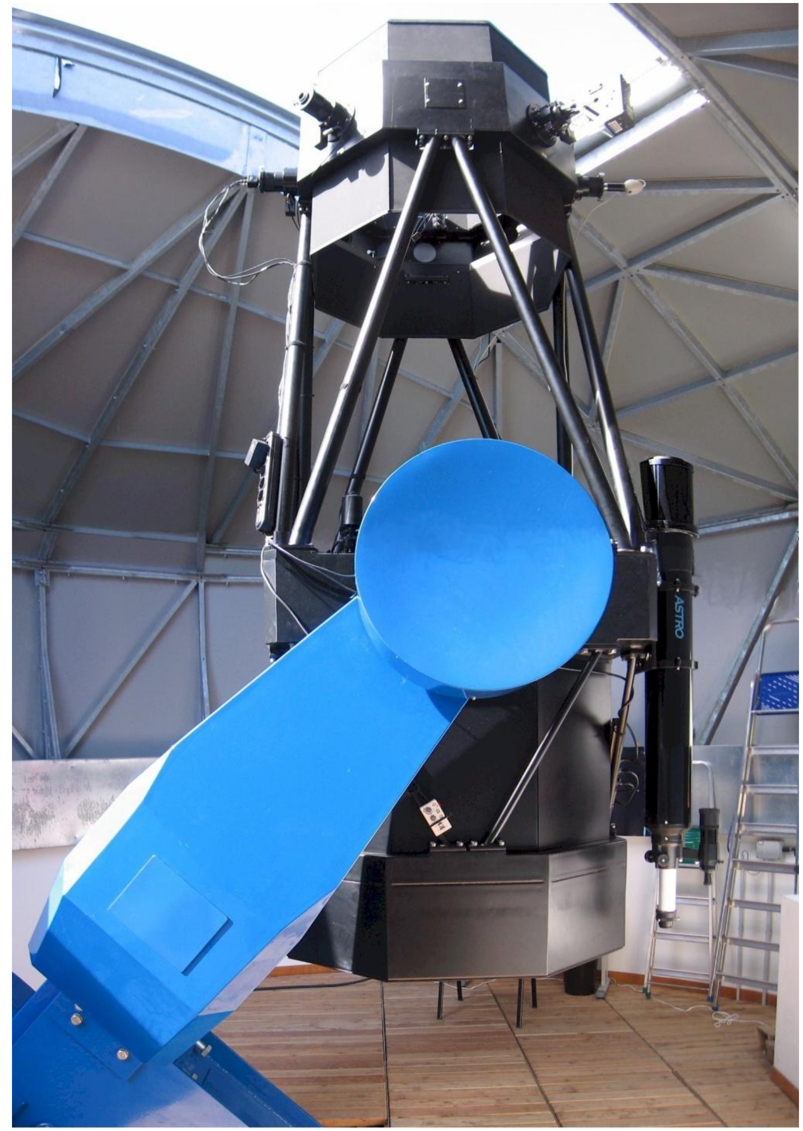
Up to now: such a sorting technique has been used in fields of optical communications and quantum optics

Our goal: detection of OAM of light coming from astrophysical sources

Test to be carried out soon in an 80-cm amateur telescope (Celado, Italy): for the first time we will try to measure an OAM spectrum from an astrophysical object.

As before, crucial problems with misalignments and seeing.

The 80-cm Celado Telescope



The OAM sorter in the laboratory