Numerical methods for relativistic plasma physics

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Supernovae, Hypernovae and Binary Driven Hypernovae
ICRANet, Pescara
• **Outline of the physical problem**

• **Strategies followed so far**

• **Implementation of a hydrodinamical code**

• **Perspectives and conclusions**
Outline of the physical problem: The EMBH model for GRBs

Collapsing Black Hole

Creation of an electron-positron plasma by means of the Schwinger Effect

Baryonic remnant
Outline of the physical problem: The EMBH model for GRBs

The (optically thick) plasma expands and accelerates, dragging with it matter from the remnant.
Outline of the physical problem: The EMBH model for GRBs

Transparency is reached, photons escape (proper GRB)

Interaction with interstellar medium, prompt emission, afterglow
Outline of the physical problem: Equations of motion

Spherical symmetry assumption → Reissner-Nordstrom metric:

\[
ds^2 = -g_{tt}(r)dt^2 + g_{rr}(r)dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2,
\]

where \(g_{tt}(r) = \left[1 - \frac{2GM}{c^2r} + \frac{Q^2G}{c^4r^2}\right] \equiv \alpha(r)^2\) and \(g_{rr}(r) = \alpha(r)^{-2}\).

Stress-energy tensor:

\[
T^{\mu\nu} = pg^{\mu\nu} + (p + \rho)U^{\mu}U^{\nu} + \Delta T^{\mu\nu}
\]

(Dissipative effects (heat conduction, viscosity))

Equation of state:

\[
\Gamma(\rho, T) = 1 + \frac{p}{\epsilon}
\]

(For now, \(\Gamma = \text{constant} = 4/3\))
Outline of the physical problem: Equations of motion

Equations of motion (baryons+pairs)

Baryon number conservation:

\[ (n_B U^\mu)_{,\mu} = 0 \]

Energy-momentum conservation:

\[ (T^{\mu\nu})_{,\nu} = 0 \]
Outline of the physical problem: Equations of motion

Equations of motion (baryons+pairs)

Baryon number conservation:
\[ (n_B U^\mu)_{;\mu} = 0 \]

Energy-momentum conservation:
\[ (T^{\mu\nu})_{;\nu} = 0 \]

Some definitions:
\[ \epsilon \equiv \rho - \rho_B \quad \text{Comoving internal energy density} \]
\[ \rho_B \equiv n_B m_B c^2 \quad \text{Comoving baryon mass density} \]
\[ \gamma \equiv \sqrt{1 + U^r U_r} \quad \text{Lorentz gamma factor,} \]
\[ V^r \equiv \frac{U^r}{U^t} \quad \text{radial coordinate velocity} \]
Outline of the physical problem: Equations of motion

Final system of equations

\[ E \equiv \epsilon \gamma, \quad D \equiv \rho_B \gamma \]

**Energy density**  **Mass density**

\[
\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} D V^r \right)
\]

\[
\frac{\partial E}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} E V^r \right) - p \left[ \frac{\partial \gamma}{\partial t} + \frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} \gamma V^r \right) \right]
\]

\[ S_r \equiv \alpha(p + \rho)U^t U_r = (D + \Gamma E)U_r \quad \text{Radial momentum density} \]

\[
\frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} S_r V^r \right) - \alpha \frac{\partial p}{\partial r} - \frac{\alpha}{2}(p + \rho) \left[ \frac{\partial g_{tt}}{\partial r} (U^t)^2 + \frac{\partial g_{rr}}{\partial r} (U^r)^2 \right]
\]
Outline of the physical problem: Equations of motion

Final system of equations

\[ E \equiv \epsilon \gamma, \quad D \equiv \rho_B \gamma \]

\[ \frac{\partial D}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} D V^r \right) \]

\[ \frac{\partial E}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} E V^r \right) - p \left[ \frac{\partial \gamma}{\partial t} + \frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} \gamma V^r \right) \right] \]

\[ S_r \equiv \alpha(p + \rho) U^t U_r = (D + \Gamma E) U_r \]

\[ \frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} S_r V^r \right) - \alpha \frac{\partial p}{\partial r} - \frac{\alpha}{2} (p + \rho) \left[ \frac{\partial g_{tt}}{\partial r} (U^t)^2 + \frac{\partial g_{rr}}{\partial r} (U^r)^2 \right] \]
Outline of the physical problem: Equations of motion

Final system of equations

\[ E \equiv \epsilon \gamma, \quad D \equiv \rho_B \gamma \]

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\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} D V^r \right)
\]

\[
\frac{\partial E}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} E V^r \right) - p \left[ \frac{\partial \gamma}{\partial t} + \frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} \gamma V^r \right) \right]
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\[ S_r \equiv \alpha(p + \rho)U^t U_r = (D + \Gamma E)U_r \]

\[
\frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} S_r V^r \right) - \frac{\partial p}{\partial r} - \frac{\alpha}{2}(p + \rho) \left[ \frac{\partial g_{tt}}{\partial r} (U^t)^2 + \frac{\partial g_{rr}}{\partial r} (U^r)^2 \right]
\]

Transport
Expansion work \((PdV)\)
Outline of the physical problem: Equations of motion

Final system of equations

\[ E \equiv \epsilon \gamma, \quad D \equiv \rho_B \gamma \]

\[
\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} DV^r \right)
\]

\[
\frac{\partial E}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} EV^r \right) - p \left[ \frac{\partial \gamma}{\partial t} + \frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} \gamma V^r \right) \right]
\]

\[ S_r \equiv \alpha(p + \rho)U^tU_r = (D + \Gamma E)U_r \]

\[
\frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} S_r V^r \right) - \frac{\alpha}{\gamma} \frac{\partial p}{\partial r} - \frac{\alpha}{2}(p + \rho) \left[ \frac{\partial g_{tt}}{\partial r} (U^t)^2 + \frac{\partial g_{rr}}{\partial r} (U^r)^2 \right]
\]
Outline of the physical problem: Equations of motion

Final system of equations

\[ E \equiv \epsilon \gamma, \quad D \equiv \rho_B \gamma \]

\[
\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} DV^r \right)
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\[ S_r \equiv \alpha (p + \rho) U^t U_r = (D + \Gamma E) U_r \]

\[
\frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} S_r V^r \right) - \frac{\partial p}{\partial r} - \frac{\alpha}{2} (p + \rho) \left[ \frac{\partial g_{tt}}{\partial r} (U^t)^2 + \frac{\partial g_{rr}}{\partial r} (U^r)^2 \right]
\]
Strategies followed

- **Direct numerical solving of the GRHD equations,**
  Wilson and Salmonson, Lawrence Livermore National Laboratory, University of California (1999).

- **Approximate code using information from the Livermore code,**
  Ruffini, Xue, Bianco, ICRANet (1999-present):
  
  - No gravitational interaction, special relativity.
  
  - **Pulse-like structure (from the Livermore code) of constant width in the coordinate frame** and uniform velocity.
  
  - Integration done until transparency is reached.
Strategies followed: approximate code

$$U_r(r) = U \frac{r}{R}$$

**Fig. 3.** Lorentz gamma factor $\gamma$ as a function of radius. Three models for the expansion pattern of the PEM-pulse are compared with the results of the one dimensional hydrodynamic code for a $1000 M_\odot$ black hole with charge to mass ratio $\xi = 0.1$. The 1-D code has an expansion pattern that strongly resembles that of a shell with constant coordinate thickness.
**Strategies followed**: approximate code, *interaction with baryons*

**Assumptions:**
- the PEM pulse does not change its geometry during the interaction;
- the collision between the PEM pulse and the baryonic matter is assumed to be inelastic,
- the baryonic matter reaches thermal equilibrium with the photons and pairs of the PEM pulse.

\[
B = \frac{M_{\text{Baryons}}}{E_{\text{Pulse}}} \leq 10^{-2}
\]

**Fig. 7.** Here we see a comparison of Lorentz factor $\gamma$ for the one-dimensional (1-D) hydrodynamic calculations and slab calculations ($M_{\text{BH}} = 10^3 M_\odot, \xi = 0.1$ EMBH and $B \simeq 1.3 \cdot 10^{-4}$). The calculations show good agreement.
Strategies followed: Livermore code, operator splitting

\[
\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} DV^r \right)
\]

\[
\frac{\partial E}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} EV^r \right) - p \left[ \frac{\partial \gamma}{\partial t} + \frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} \gamma V^r \right) \right]
\]

\[S_r \equiv \alpha(p + \rho)U^t U_r = (D + \Gamma E)U_r\]

\[
\frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} S_r V^r \right) - \alpha \frac{\partial p}{\partial r} - \frac{\alpha}{2}(p + \rho) \left[ \frac{\partial g_{tt}}{\partial r} (U^t)^2 + \frac{\partial g_{rr}}{\partial r} (U^r)^2 \right]
\]
Strategies followed: Livermore code, operator splitting

\[
\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} DV^r \right)
\]

\[
\frac{\partial E}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} EV^r \right) - p \left[ \frac{\partial \gamma}{\partial t} + \frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} \gamma V^r \right) \right]
\]

\[S_r \equiv \alpha(p + \rho)U^tU_r = (D + \Gamma E)U_r\]

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\]
Strategies followed: Livermore code, operator splitting

\[ \frac{\partial D}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} DV^r \right) \]

\[ \frac{\partial E}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} EV^r \right) - p \left[ \frac{\partial \gamma}{\partial t} + \frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} \gamma V^r \right) \right] \]

\[ S_r \equiv \alpha(p + \rho)U^t U_r = (D + \Gamma E)U_r \]

\[ \frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} S_r V^r \right) - \alpha \frac{\partial p}{\partial r} - \frac{\alpha}{2}(p + \rho) \left[ \frac{\partial g_{tt}}{\partial r} (U^t)^2 + \frac{\partial g_{rr}}{\partial r} (U^r)^2 \right] \]
Strategies followed: Livermore code, *operator splitting*  

\[
\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} D V^r \right)
\]

\[
\frac{\partial E}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} E V^r \right) - p \left[ \frac{\partial \gamma}{\partial t} + \frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} \gamma V^r \right) \right]
\]

\[
S_r \equiv \alpha (p + \rho) U^t U_r = (D + \Gamma E) U_r
\]

\[
\frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} S_r V^r \right) - \alpha \frac{\partial p}{\partial r} - \frac{\alpha}{2} (p + \rho) \left[ \frac{\partial g_{tt}}{\partial r} (U^t)^2 + \frac{\partial g_{rr}}{\partial r} (U^r)^2 \right]
\]
Strategies followed: Livermore code, operator splitting

\[
\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} DV^r \right)
\]

\[
\frac{\partial E}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} EV^r \right) - p \left[ \frac{\partial \gamma}{\partial t} + \frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} \gamma V^r \right) \right]
\]

\[
S_r \equiv \alpha(p + \rho)U^t U_r = (D + \Gamma E)U_r
\]

\[
\frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} S_r V^r \right) - \frac{\partial p}{\partial r} - \frac{\alpha}{2}(p + \rho) \left[ \frac{\partial g_{tt}}{\partial r} (U^t)^2 + \frac{\partial g_{rr}}{\partial r} (U^r)^2 \right]
\]
Strategies followed: Livermore code, grid implementation

Leap-frog method
Strategies followed: Livermore code, grid velocity

\[
\dot{D} + D \frac{\dot{\gamma}}{\gamma} + \frac{1}{\gamma} \frac{\partial}{\partial x^i} \left( \gamma D (V^i - V_g^i) \right) + \frac{D}{\gamma} \frac{\partial}{\partial x^i} \left( \gamma V_g^i \right) = 0,
\]

\[
\dot{S}_i + S_i \frac{\dot{\gamma}}{\gamma} - \frac{1}{\gamma} \frac{\partial}{\partial x^j} \left( S_i (V^j - V_g^i) \right) + S_i \frac{\partial}{\partial x^i} \left( \gamma V_g^i \right) + \alpha \frac{\partial P}{\partial x^i} - S_j \frac{\partial \beta^i}{\partial x^j} + (D + \Gamma E) \left( W \frac{\partial \alpha}{\partial x^i} + \frac{U_k U_j}{2W} \frac{\partial \gamma^k}{\partial x^i} \right) = 0,
\]

\[
\dot{E} + \Gamma E \frac{\dot{\gamma}}{\gamma} + \frac{1}{\gamma} \frac{\partial}{\partial x^i} \left( E (V^i - V_g^i) \right) + \frac{\Gamma E}{\gamma} \frac{\partial}{\partial x^i} \left( \gamma V_g^i \right) + (\Gamma - 1) E \left[ \frac{\dot{W}}{W} + \frac{1}{\gamma W} \frac{\partial}{\partial x^i} \left( W (V^i - V_g^i) \right) \right] = 0.
\]

\[
\text{det}(g_{\alpha\beta}) = -\alpha^2 \gamma^2
\]

\[
\gamma = \frac{r^2}{\alpha(r)}
\]

\[
W = \text{Lorentz gamma}
\]
Strategies followed: Livermore code, the advection part

\[
\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} DV^r \right)
\]

\[
\frac{\partial E}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} EV^r \right) - p \left[ \frac{\partial \gamma}{\partial t} + \frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} \gamma V^r \right) \right]
\]

\[
\frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} S_r V^r \right) - \frac{\partial p}{\partial r} - \frac{\alpha}{2} (p + \rho) \left[ \frac{\partial g_{tt}}{\partial r} (U^t)^2 + \frac{\partial g_{rr}}{\partial r} (U^r)^2 \right]
\]
Strategies followed: Livermore code, the advection part

\[
\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} DV^r \right)
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\frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} S_r V^r \right) - \alpha \frac{\partial p}{\partial r} - \frac{\alpha}{2} (p + \rho) \left[ \frac{\partial g_{tt}}{\partial r} (U^t)^2 + \frac{\partial g_{rr}}{\partial r} (U^r)^2 \right]
\]

\[
\hat{D} = \gamma D, \quad \hat{E} = \gamma E, \quad \hat{S} = \gamma S.
\]

\[
\gamma = \frac{r^2}{\alpha(r)}
\]

\[W = \text{Lorentz gamma}\]
Strategies followed: Livermore code, the advection part

\[
\frac{\partial D}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} D V^r \right)
\]

\[
\frac{\partial E}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} E V^r \right) - p \left[ \frac{\partial \gamma}{\partial t} + \alpha \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} \gamma V^r \right) \right]
\]

\[
\frac{\partial S_r}{\partial t} = -\frac{\alpha}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2}{\alpha} S_r V^r \right) - \alpha \frac{\partial p}{\partial r} - \frac{\alpha}{2} (p + \rho) \left[ \frac{\partial g_{tt}}{\partial r} (U^t)^2 + \frac{\partial g_{rr}}{\partial r} (U^r)^2 \right]
\]

\[
\hat{D} = \gamma D, \quad \hat{E} = \gamma E, \quad \hat{S} = \gamma S.
\]

\[
\gamma = \frac{r^2}{\alpha(r)}
\]

\[W = \text{Lorentz gamma}\]

\[
\frac{\partial \hat{D}}{\partial t} + \frac{\partial}{\partial x} \left( \hat{D} (V - V_g) \right) = 0,
\]

\[
\frac{\partial \hat{E}}{\partial t} + \frac{\partial}{\partial x} \left( \hat{E} (V - V_g) \right) = 0,
\]

\[
\frac{\partial \hat{S}}{\partial t} + \frac{\partial}{\partial x} \left( \hat{S} (V - V_g) \right) = 0.
\]

\[
\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{v}) = 0
\]

1-D conservation equations
Strategies followed: Livermore code, the advection part

Conservative scheme for advection:

\[ D^i(t + \delta t) = D^i(t) - \frac{(\Delta M_{D}^{i+1} - \Delta M_{D}^{i})}{Vol_b^i} \]

\[ \Delta M_{D}^{i} = \bar{D}_f^i A_a^i (V^i - V_g^i) dt \]
**Strategies followed: Livermore code, the advection part**

**Conservative scheme for advection:**

\[
D^i(t + \delta t) = D^i(t) - \frac{\Delta M^{i+1}_D - \Delta M^i_D}{Vol_i^i}
\]

\[
\Delta M^i_D = \bar{D}^i_f A^i_a (V^i - V_g^i) dt
\]

- if \((V^i - V_g) > 0\), \(\bar{D}^i_f = D^{i-1} + \frac{1}{2} \nabla \tilde{D}^{i-1}[dx_b^{i-1} - (V^i - V_g^i)dt]\)
- if \((V^i - V_g) < 0\), \(\bar{D}^i_f = D^i - \frac{1}{2} \nabla \tilde{D}^i[dx_b^i + (V^i - V_g^i)dt]\)
Strategies followed: Livermore code, some usual problems

Numerical dissipation

- Appropriate choice of interpolated boundary density for advection (second order).
- Inclusion of a grid velocity.

Unphysical oscillations

- The same, plus artificial viscosity.

Positivity: $S_r \equiv \alpha (p + \rho) U^t U_r = (D + \Gamma E) U_r$
Strategies followed: Livermore code, ordering prescription

1. pressure acceleration
2. viscosity
3. velocities $U$, $V$ and $W$
4. pressure $PdV$ work on fluid
5. advection of state variables
6. velocities again
7. pressure $PdV$ work again
8. time step $dt$ calculation
9. grid update
10. output and post processing when appropriate.
Implementation: shock evolution
Implementation: shock evolution

Pulse 0.24s - 2.24s, M=10xMs, Q=0.1xM

Constant thickness in lab frame
Implementation: comparison with the approximate code

M=10 Ms, Q=0.1 M
Implementation: shock profile
Implementation: shock profile
**Implementation**: interaction with baryons

- Very different physical behaviour depending on whether we include a grid velocity or not.
- Artificial viscosity needed to prevent instabilities.
- Instabilities generated anyway, depending on the chosen initial conditions.

So, is our scheme reliable?
Implementation: tests, Riemann shock tube (1D)

Fig. 2.4. Various regions in the shock tube problem. They are: (1) the undisturbed high density fluid; (2) the rarefaction wave; (3) a region of constant velocity and pressure which features a contact discontinuity separating regions of different density; (4) the shock itself; and (5) the undisturbed low density fluid.
**Implementation:** tests, *Riemann shock tube (1D)*

First attempt (Wilson's ordering prescription) not entirely successful:

Anninos & Fragile (LLNL, 2003): optimal AV scheme, different ordering.

**Artificial viscosity**

The scalar viscosity $Q_i$ is computed as a local quantity in a dimensionally split fashion, and active only in convergent flows for which $\nabla_i V^i < 0$

$$Q_i = (D + E + PW)\Delta l(\nabla_i V^i) [k_{q2} \Delta l(\nabla_i V^i)(1 - \phi^2) - k_{q3} C_s]$$
Implementation: tests, Riemann shock tube, moderate boost

Maximum boost factor = 1.49

Best choice of AV parameters, k1 = 0.32, k2 = 0.000005
Implementation: *tests, Riemann shock tube, moderate boost*

10% change in $k_1$

$k_1 = 1.1 \times 0.32, k_2 = 0.000005$
Implementation: tests, Riemann shock tube, moderate boost

30% change in $k_1$

$k_1 = 1.3 \times 0.32, k_2 = 0.000005$
Implementation: tests, Riemann shock tube, high boost

Maximum boost factor = 3.59

Best choice in AV parameters, $k_1=0.05$, $k_2=1.2$
Implementation: tests, Riemann shock tube

Anninos & Fragile also need to change the AV parameters for high boosts. They obtain:

These AV schemes seem to fail dramatically for \textbf{gamma}>3!!
**Perspectives**

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<td>√&lt;sup&gt;e&lt;/sup&gt;</td>
<td>√&lt;sup&gt;f&lt;/sup&gt;</td>
<td>×&lt;sup&gt;g&lt;/sup&gt;</td>
</tr>
<tr>
<td>rGlimm</td>
<td>√</td>
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<td>×</td>
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<td>Sym-HRSC</td>
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<td>√</td>
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<td>van Putten</td>
<td>√&lt;sup&gt;i&lt;/sup&gt;</td>
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<td>√&lt;sup&gt;j&lt;/sup&gt;</td>
<td>×&lt;sup&gt;k&lt;/sup&gt;</td>
</tr>
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</table>

<sup>a</sup>D: excessive dissipation; O: oscillations; SE: systematic errors.

<sup>b</sup>All finite difference methods are extended by directional splitting.

Martí, Müller (2003)
Conclusions

• We reproduced a hydrodynamical code similar to the one developed by Wilson and Salmonson (1999). In the absence of baryonic matter, the thickness of the PEM pulse remains constant during its evolution, which is in agreement with Wilson's results. Besides, the gamma vs. r curve coincides with that obtained using the constant thickness approximation.

• When the fluid velocities are high, this code leads to excessive numerical dissipation and does not reproduce shocks if a grid velocity is not included.

• When applied to the interaction of the plasma with a baryonic remnant, the code produces results that depend on the implementation of the grid velocity, and that may develop instabilities depending on that, the initial conditions, and the AV scheme.

• The Riemann Shock Tube test verifies that AV schemes become unreliable for high fluid velocities (gamma>3), and that therefore our current case (gamma>100) should be treated using a different Eulerian scheme.
Thank you