Exact solutions of Einstein and Einstein-Maxwell equations

0.1 Topics

- Exact solutions in General Relativity
- Cosmology and Astrophysics
- Quantum Fields
- Alternative theories

0.2 Participants

0.2.1 ICRANet participants

- G.A.Alekseev
- V.A.Belinski
- T.Damour
- M.Novello
- H.Quevedo

0.2.2 Past collaborators

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0.2.3 Students

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- D.Pugliese (PhD)
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0.3 Research activity

0.3.1 Exact solutions in General Relativity

• The old problem how to generate the exact stationary axisymmetric solutions corresponding to the charged masses with horizons in the framework of Inverse Scattering Method (ISM) was investigated (V.Belinski, G.Alekseev). It was shown that applicability of the ISM in presence of electromagnetic field is not restricted only to the cases with naked singularities (as it have been erroneously stated by some authors). We showed that solutions of Einstein-Maxwell equations with horizon also follows from ISM and they are of the same solitonic character. The mathematical procedure of analytical continuations of the naked-singularity solitonic solutions in the space of their parameters which procedure results in solitonic solutions with horizon has been described [3].

• It was found the new way of derivation of the Kerr solution by adding to the Schwarzchild black hole the solitonic vortex made from the pure gravitational field (V.Belinski et al.). With this method, we can figure out how rotational energy can contribute to the mass of the resulting Kerr black hole. The interpretation of the extremal black hole as a whirl of pure gravity is proposed. Also we suggest a new point of view on the relation between the mass and angular momentum of a Kerr black hole [4].

• The physical effects of the motion of particles in the gravitational field of naked singularities in Einstein and Einstein-Maxwell theories have been investigated (H.Quevedo et al.). The properties of circular orbits of neutral and charged test particles in the equatorial plane of a central charged and rotating mass in General Relativity have been analyzed. For black holes and naked singularities it was explored all the spatial regions where circular orbits can exist and the behavior of the energy and the angular momentum of the corresponding test particles has been studied. In particular, it was found all the radii at which a test particle can have zero angular momentum due to the repulsive gravity effects generated by naked singularities. All the stability zones of circular orbits was classified. It was shown that the geometric structure of the stability zones of black holes is completely different from that of naked singularities. The essential part of these results can be formulated in a plausible manner by using the model of an accretion disk made of stable test particles which are rotating around the central mass. The conclusion is that the main difference between a black hole and a naked singularity consists in the different geometric structure of their accretion disks [16, 17, 18].

• In relation to the Kerr solution the following interesting result has been obtained. It was found (first time in the literature) the explicit representation of this solution in the synchronous (Gaussian) coordinate system. This representation is useful for the description of the Kerr geometry from the point of view of an freely falling observer. It is reasonable to expect that other physical

applications of this representation will follow [13].

0.3.2 Cosmology and Astrophysics

Cosmology

• It was continuation of the work on the book "Cosmological Singularity" by V.Belinski and T.Damour. The by-product of this work is represented by the two publications. The first paper represents a short review of the old results and contemporary development on the problem of cosmological singularity [1]. The other is the much more detailed exposition of the BKL theory including the recent ideas of T.Damour, M.Henneaux and H.Nicolai on the extension of this theory to the string models and their intriguing proposal which is known as Hidden Symmetry Conjecture [2].

• The new results in relation to the BKL theory was obtained by T.Damour and his collaborators and can be described as follows. BKL pioneered the study of the statistical properties of the never-ending oscillatory behavior (among successive Kasner epochs) of the geometry near a space-like singularity. Damour and his collaborators showed how the use of a "cosmological billiard" description allows one to refine and deepen the understanding of these statistical properties. Contrary to previous treatments, the authors do not quotient the dynamics by its discrete symmetry group (of order 6), thereby uncovering new phenomena, such as correlations between the successive billiard corners in which the oscillations take place. Starting from the general integral invariants of Hamiltonian systems, the authors showed how to construct invariant measures for various projections of the cosmological-billiard dynamics. In particular, it was exhibited, for the first time, an invariant measure on the "Kasner circle" which parametrizes the exponents of successive Kasner epochs. Also one of the main results is that there was established the relation between the billiard dynamics that arose in recent studies suggesting the hidden presence of Kac-Moody symmetries in cosmological billiards [9].

Astrophysics

A ground-based network of interferometric gravitational wave (GW) detectors is currently being upgraded and is expected, thanks to an improved sensitivity, to detect, within a few years, the GW signals emitted during the inspiral and merger of compact binaries. The realization of this exciting observational prospect depends, however, on our theoretical ability to accurately compute, within Einstein's theory of general relativity, the motion of compact binaries and its associated GW emission. Recent developments have made it clear that the most efficient way to theoretically understand the late stages of the dynamics of compact binaries is to combine the knowledge coming from analytical relativity techniques, such as traditional post-Newtonian

expansions, or the newer effective-one-body (EOB) formalism (first developed by T.Damour and his students around 1999-2001), with the knowledge coming from numerical relativity simulations. In EOB formalism one maps the general relativistic two-body problem onto that of a test particle moving in an effective external metric. This effective-one-body approach defines, in a non-perturbative manner, the late dynamical evolution of a coalescing binary system of compact objects. During the last year analyzing the number of concrete physical models of black hole and neutron star binaries it was managed to show that the prediction of the effective one-body formalism, based purely on known analytical results, agrees strikingly well with the numerical results. Both by numerical calculations and analytical approach the number of relations for the physical parameters was established for the binaries (e.g. the gauge invariant relation between the binding energy and the angular momentum and the expected waveforms essentially up to the merger). All these results show that effective-one-body approach can be safely applied for the calculation of the gravitational radiation from the binary systems [6, 7, 10].

0.3.3 Quantum Fields

• During the last twelve years a series of papers has appeared trying to introduce particle creation by the Schwarzschild black hole as a quasi-classical tunneling process through an infinitesimal neighborhood of the black hole horizon. We showed that this line of research is erroneous. The correct approach shows that there is no way for particle creation to occur by quantum tunneling through an infinitesimal neighborhood of the black hole horizon. This result is the straightforward consequence of the regularity of the horizon, the equivalence principle and the general covariance of the relativistic theory of gravity. Moreover, we also confirm the less evident statement that no particle creation by quantum tunneling through the black hole horizon is possible independent of the size of the presupposed tunneling domain [5].

• T. Damour (and collaborators) study the mini–superspace quantization of spatially homogeneous (Bianchi) cosmological universes sourced by a Dirac spinor field. The quantization of the homogeneous spinor leads to a finite-dimensional fermionic Hilbert space and thereby to a multi-component Wheeler-DeWitt equation whose main features are the presence of spin-dependent Morse-type potentials and the appearance of a q-number squared-mass term. The authors gave the exact quantum solution of the Bianchi type-II system (which contains both scattering states and bound states), and discuss the main qualitative features of the quantum dynamics of the (classically chaotic in the BKL sense) Bianchi type-IX system. The comparison of the exact quantum dynamics of fermionic cosmological billiards to previous works that described the spinor field as being either classical or Grassmann-valued has been presented [8].

0.3.4 Alternative theories

In spite of the fact that the main research in ICRANet follow the well-established Einstein General Relativity and conventional Quantum Theory, the non-standard ways for the attempts to resolve the contemporary problems in physics also are welcomed. This kind of activity is represented by our brazilian collaborators under the leadership of prof. M.Novello. The 2011 activity in this fields can be outlined as follows.

• In relation to the LHC search for the Higgs boson it is reasonable to investigate also the alternative ways for understanding the origin of the mass. It was analyzed the recent proposal according to which gravity is what is really responsible for the generation of mass of all bodies [11].

• Up until recently, the idea of using scalar or vector fields to mimic gravitational processes was limited to kinematic issues. This meant that the propagation processes of these fields (if they satisfy the non-linear equations) may be described by a change in the geometry of space-time in terms of the effective metrics. This used to be done only in issues referring to kinematics rather than dynamics. However, now we have been able to show that there are exceptional dynamics where the very dynamics is represented as function of the effective metrics [12].

• It was shown how a static and spherically symmetric geometry can be obtained as a solution to the equations of the Spinorial Theory of Gravitation (STG), proposed earlier by M.Novello. For the weak field this theory is the same as the General Relativity, because this new solution coincides with Schwarzschild solution. However, in the subsequent orders, both theories differ. So, STG includes predictions which may be tested in the future [14].

• The new kind of duality has been found, namely, that Maxwell electromagnetism can be mapped into the Born-Infeld theory in a curved spacetime. This map includes all possible solutions of Maxwell equations [15].

0.4 Participation in the conferences

• 12 Italian-Korean Symposium on Relativistic Astrophysics, 4-8 July, 2011, Pescara (Italy). The talk: V. Belinski and H. W. Lee “Kerr rotation as solitonic whirl around Schwarzschild black hole.”

• 3-nd Galileo-XuGuangqi Meeting, 11-15 October, 2011, Beijing (China). Chairman (V.Belinski) of the parallel Session “Early Universe”. The talk: V.Belinski “Basic facts on the Cosmological Singularity”.

• Conference ”Classical and Quantum Integrable Systems (CQIS-2011)”, Institute for High Energy Physics (Protvino, Russia) , January 24-27, 2011. The talk: G.A. Alekseev ”Integrability of symmetry reduced bosonic dynamics and solutions with rational monodromy data in heterotic string effective theory”.

• International Conference on Symmetries in Physics (Zacatecas, Mexico, February 2011), H.Quevedo.

• 9th International Workshop on Applied Category Theory Graph-Logic (San Antonio, Texas, USA, March 2011), H.Quevedo.

0.5 Teaching activity

V.Belinski "On the tunneling through the black hole horizon", the course of 4 lectures for Erazmus Mundus Joint Doctorate Program, Nice University "Sophia Antipolis", Nice (France), 11-17 September, 2011.

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