Numerical relativity: from vacuum to matter spacetimes

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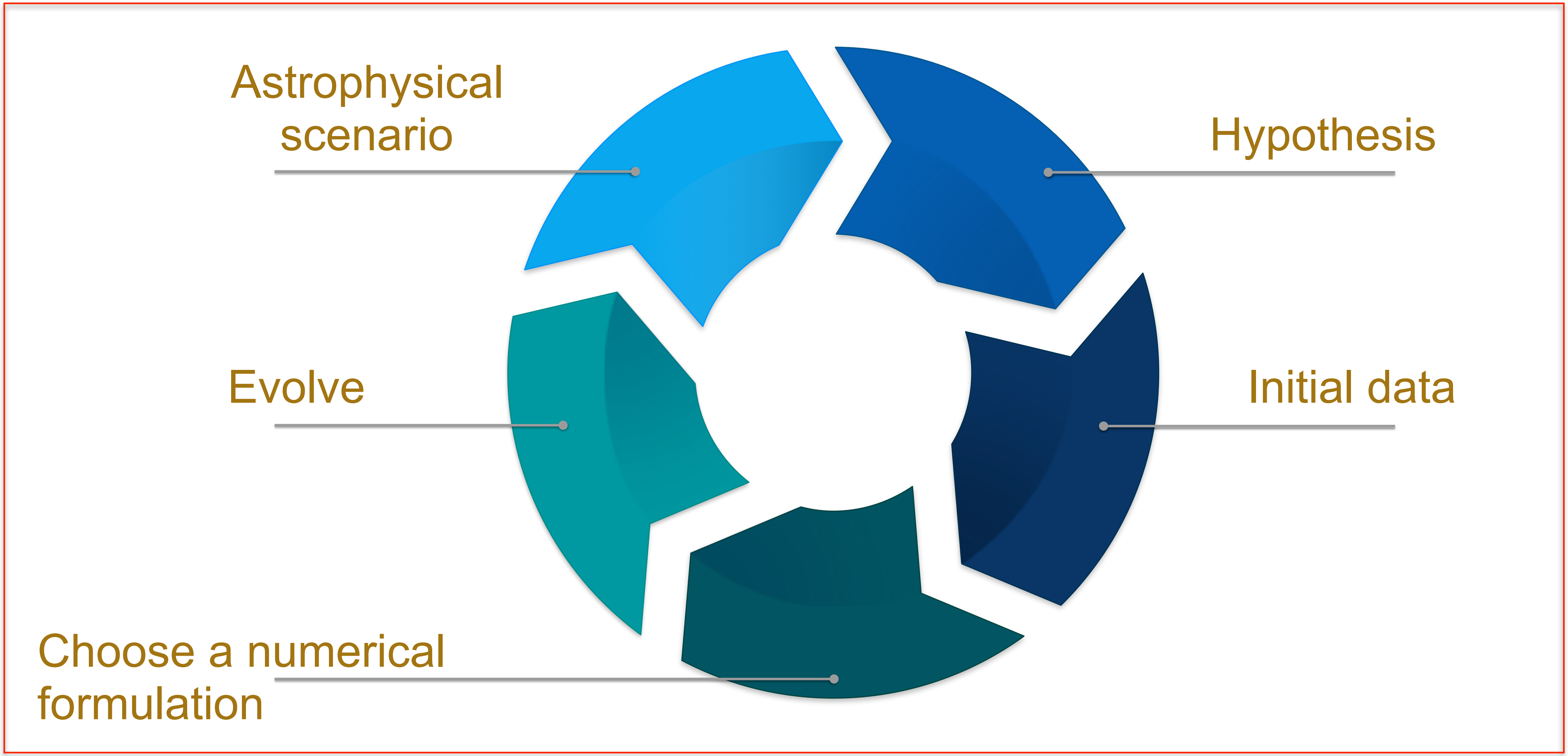
Take go home

Numerical relativity has reached a stage that allows to study realistic astrophysical scenarios

Roadmap



Astrophysical scenario

Hypothesis

Initial data

Choose a numerical formulation

Evolve

To be on the same page …

Formulation, gauge conditions and numerical code:

BSSN: Local one in town

Gauge: Punture gauge conditions:

$$\frac{d}{dt}\alpha = -\alpha^2 f(\alpha)\, K$$

Control size of volume elements

$$\partial_t^2 \beta^i = \alpha^2 \xi\, \partial_t \tilde{\Gamma}^i - \eta\, \partial_t \beta^i$$

Control distortion of volume elements

Numerical code: AMR GRMHD Illinois code embedded in Cactus

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What astrophysical problems can be addressed with NR?

The simplest one: Vacuum

$$R_{\mu\nu} = 8\pi\left(\cancelto{0}{T_{\mu\nu}} - \frac{1}{2}g_{\mu\nu}T\right)$$

One-body problem:

Schwarzschild (1916):

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \left(1-\frac{2M}{r}\right)^{-1}dr^2 + r^2 d\Omega^2$$

Kerr (1963):

$$\begin{aligned} ds^2 = &-\left(\frac{\Delta - a^2\sin^2\theta}{\rho^2}\right)dt^2 - \frac{2a\sin^2\theta\,(r^2+a^2-\Delta)}{\rho^2}\,dt\,d\phi \\ &+ \left(\frac{\left(r^2+a^2\right)^2 - \Delta\, a^2\sin^2\theta}{\rho^2}\right)\sin^2\theta\, d\phi^2 + \frac{\rho^2}{\Delta}\,dr^2 + \rho^2\, d\theta^2 \end{aligned}$$

☑ From theoretical point of view: Excellent

☐ From astrophysical point of view: Crude approx

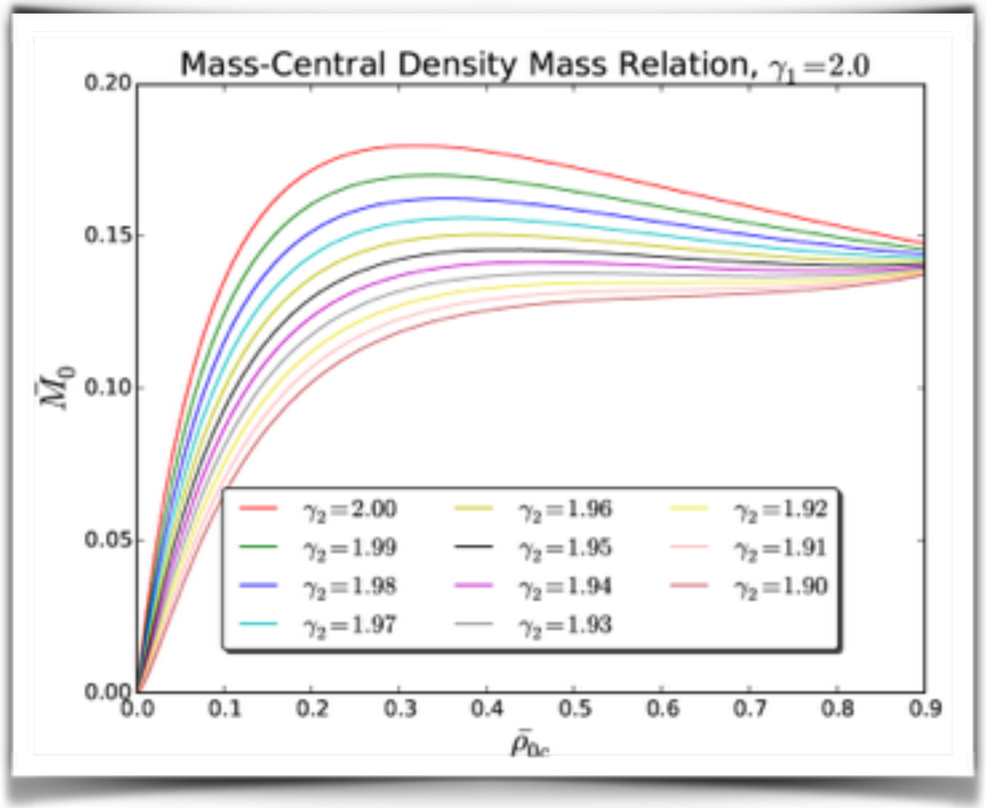
What astrophysical problems can be addressed with NR?

A simple one: Single star

$$R_{\mu\nu} = 8\pi \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right)$$

☑ From astrophysical point of view: EoS, NS stability, ..

Tolman-Oppenheimer–Volkoff



$$\frac{dm}{dr} = 4\pi r^2 \rho,$$

$$\frac{dP}{dr} = -\frac{\rho m}{r^2} \left(1 + \frac{P}{\rho}\right) \left(1 + \frac{4\pi P r^3}{m}\right) \left(1 - \frac{2m}{r}\right)^{-1}$$

Lora-Clavijo et al.: in preparation

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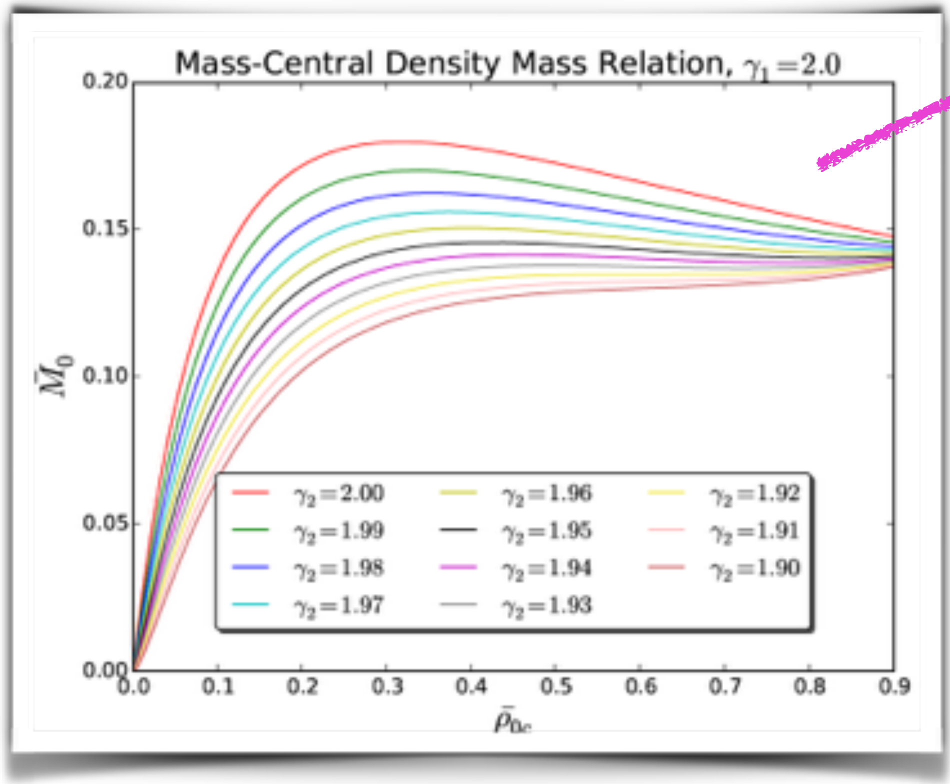
Tolma

What about B field?



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Lora-Clavijo et al.: in preparation

What astrophysical problems can be addressed with NR?

A bit more complicated: Pulsars in full GR

- Regime:
  - Flat-spacetime:

1. Dipole magnetic B-field (Maxwell in vacuum)
2. Pulsar magnetosphere is well-described by force-free proposed by Goldreich & Julian ’69 justification by Philippov et al. ’13

$$L = (1 \pm 0.05)\,\mu^2\,\Omega^4$$

3. Back-reaction of B-field onto the matter is ignored
4. Deviations from sphericity are ignored (e.g. due to rotation)

(Komissarov '02, McKinney '06, Spitkovsky ’06)

What astrophysical problems can be addressed with NR?

A bit more complicated: Pulsars in full GR

- Regime:
  - Curve-spacetime:

1. Frame dragging induces an enhanced E field that modifies the structure of the magnetosphere

2. A GRMHD simulation shows a possible deviation in the pulsar spin down luminosity from flat spacetime (Palenzuela '12)

What can we say about it?

What astrophysical problems can be addressed with NR?

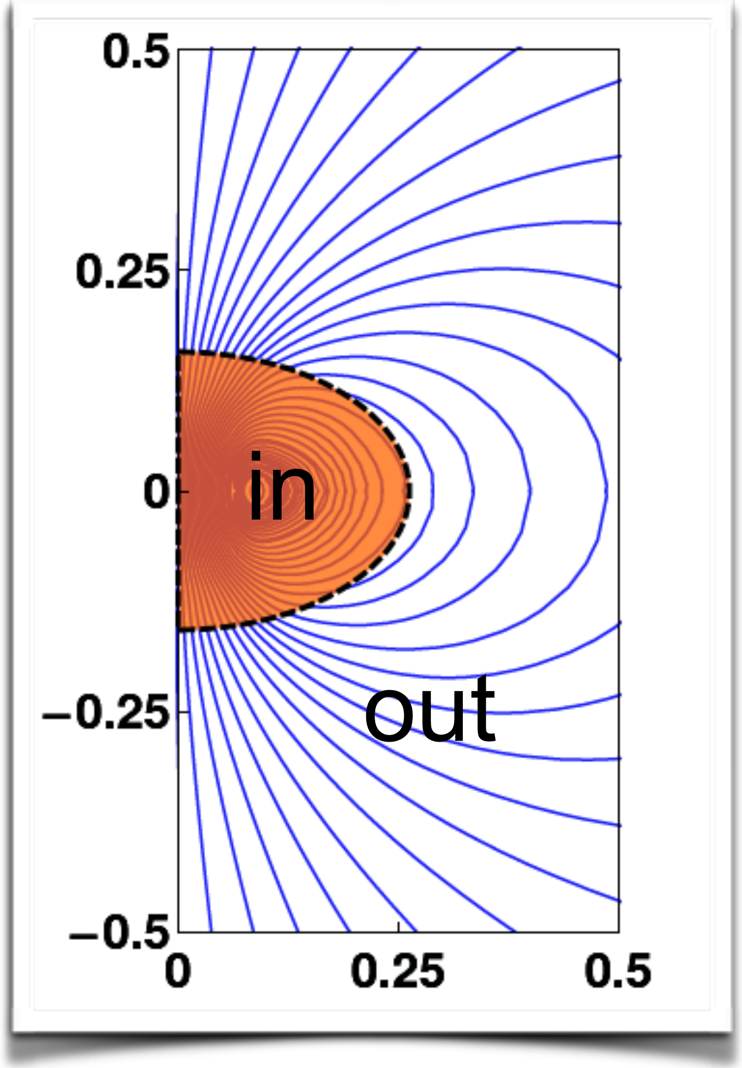
A bit more complicated: Pulsars in full GR

- Regime:
  - In: Ideal MHD (frozen-in B field)
  - Out: Force-free electrodynamics

$$T^{ab} = T^{ab}_{\rm fluid} + T^{ab}_{\rm EM} \simeq T^{ab}_{\rm EM}$$

  - What about B field back-reaction?

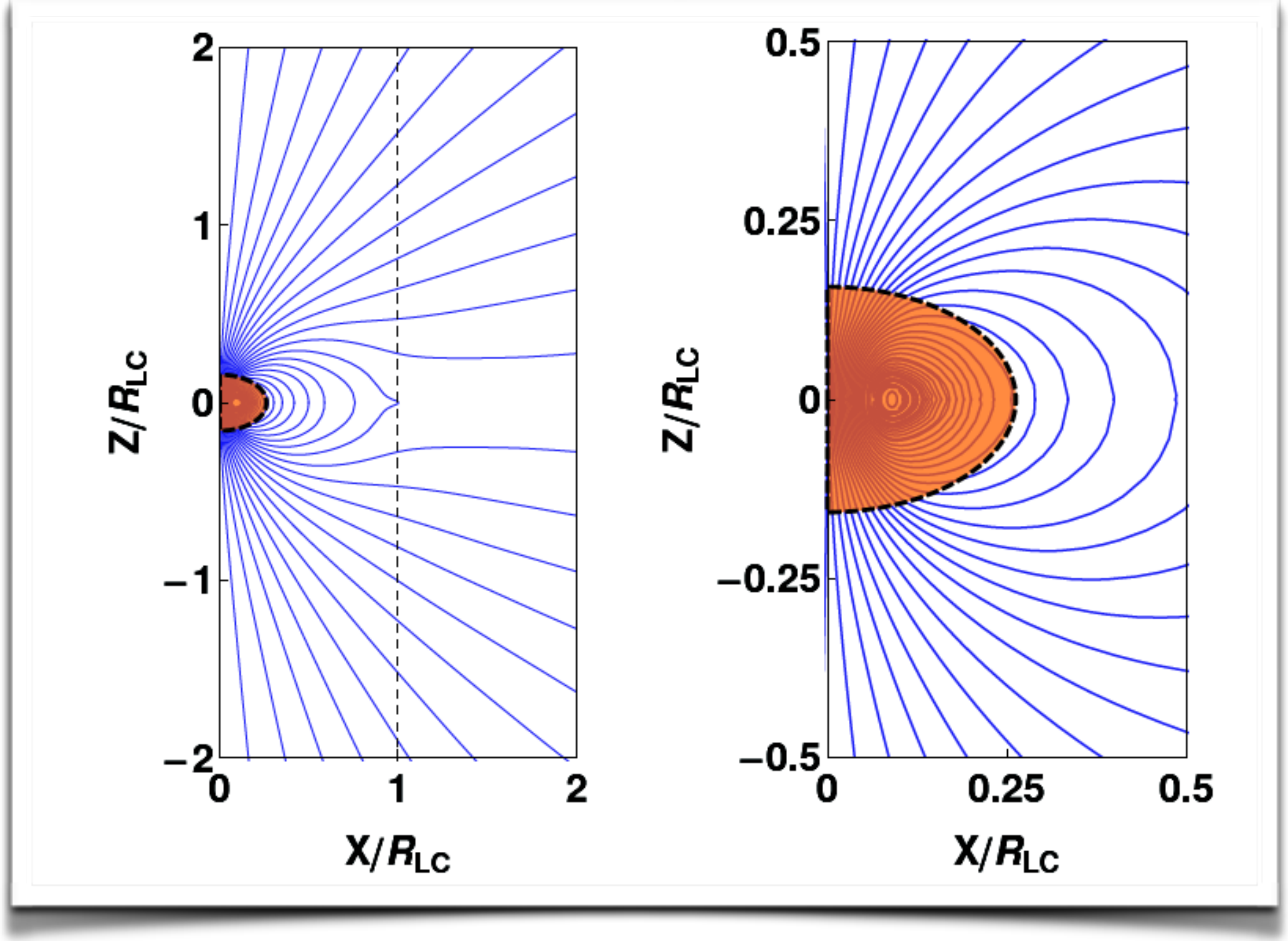
$$\mathcal{M}/|W| \ll 1$$



Ruiz et al. 2014

What astrophysical problems can be addressed with NR?

Pulsars in full GR



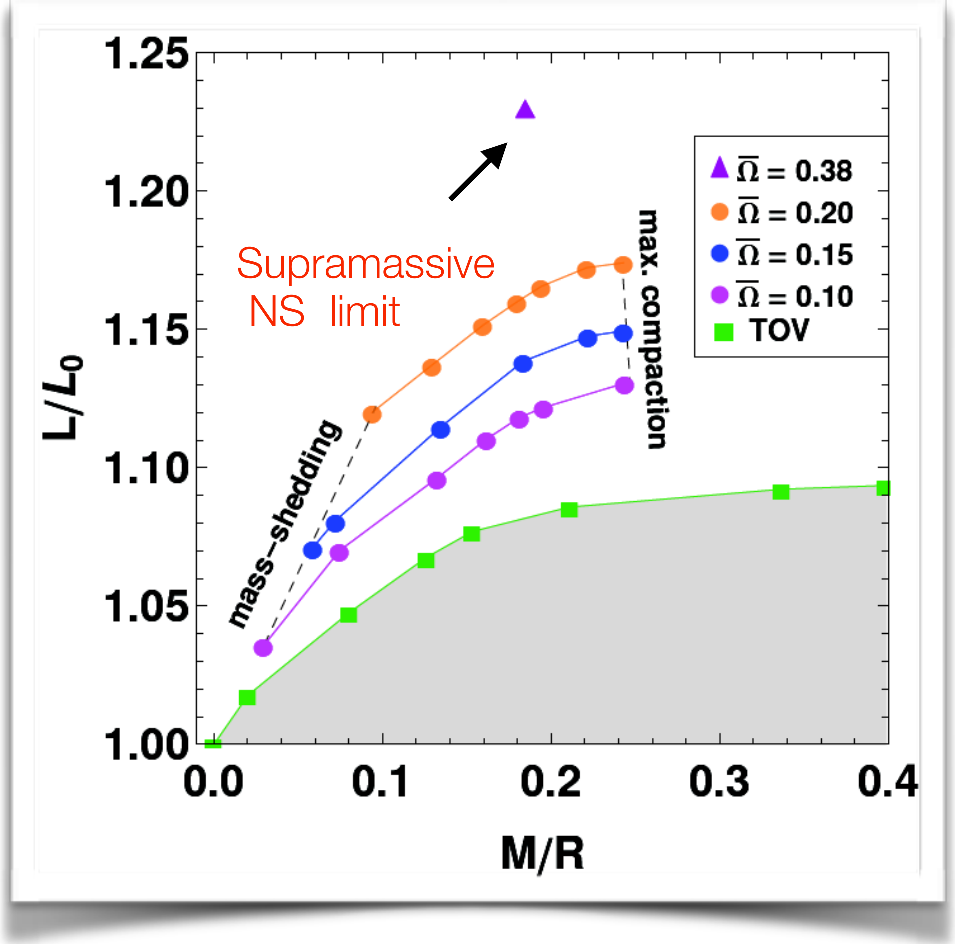
Oblateness alone does NOT matter!

$$L_0 = 1.02\,\mu^2\,\Omega^4$$

$$\simeq 10^{43}\left(\frac{B}{10^{12}\,G}\right)^2\left(\frac{R}{10\,Km}\right)^6\left(\frac{P}{ms}\right)^{-4}\frac{\text{erg}}{\text{sec}}$$

What astrophysical problems can be addressed with NR?

Pulsars in full GR



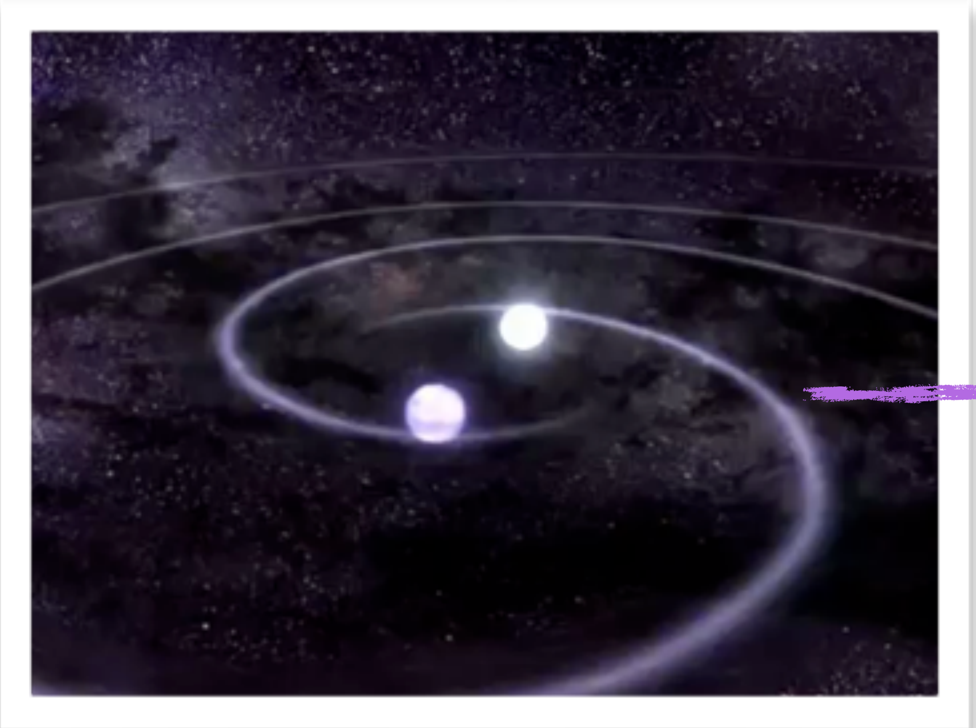
Larger compaction → higher enhancement

Faster rotation → higher enhancement

What about the two-body problem?

The simplest one: Vacuum

$$R_{\mu\nu} = 8\pi \cancelto{0}{\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right)}$$



Credit: NASA/Tod Strohmayer (GSFC)
Dana Berry (Chandra X-Ray Observatory)

GWs carry out energy and momenta:
The orbit shrinks and then the system collapses

Let’s try a numerical evolution:

But the singularity?

$$ds^2 = -\left(\frac{1 - M/(2r)}{1 + M/(2r)}\right)^2 dt^2 + \boxed{\left(1 + \frac{M}{2r}\right)^4} (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2)$$

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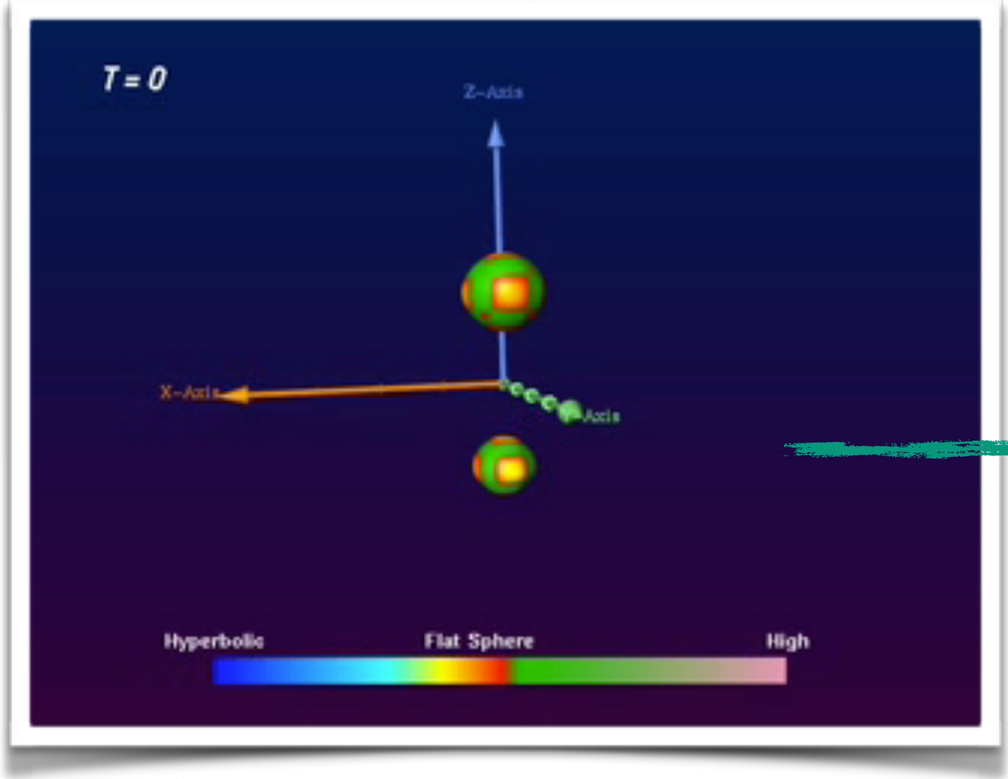


Courtesy: M. Alcubierre

Let's try a numerical evolution:

Factor out the singular term & evolve regular term

(Brandt-Bruegmann '97)

$$ds^2 = -\left(\frac{1 - M/(2r)}{1 + M/(2r)}\right)^2 dt^2 + \boxed{\left(1 + \frac{M}{2r}\right)^4} (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2)$$

What about the two-body problem?

Ingredients for a successful evolution:

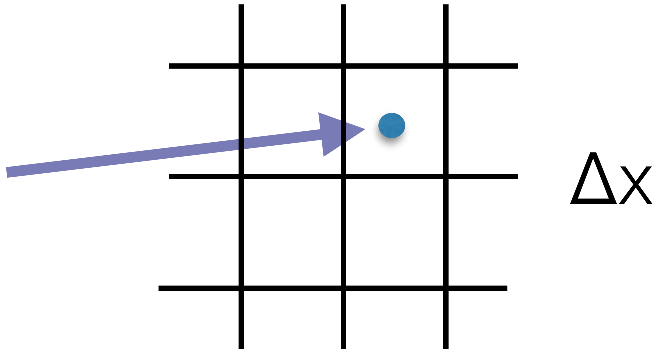
Campanelli et al. ’06
Baker et al. ‘06

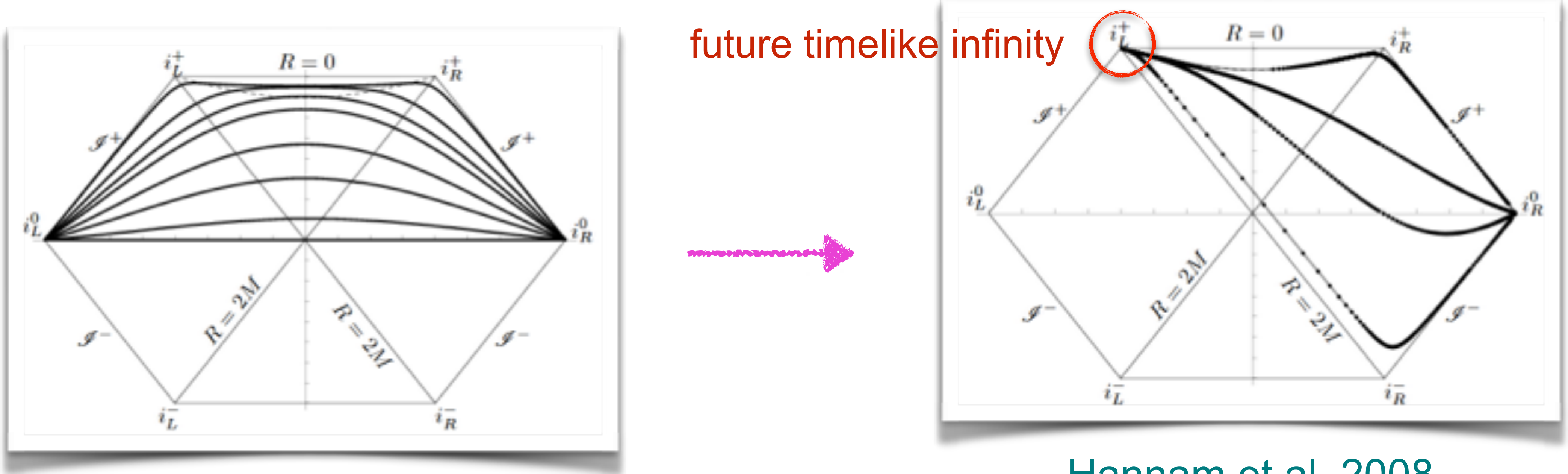
Puncture gauge:

$$\frac{d}{dt}\alpha = -\alpha^2 f(\alpha)\,K$$

$$\partial_t^2 \beta^i = \alpha^2 \xi\, \partial_t \tilde{\Gamma}^i - \eta\, \partial_t \beta^i$$

Singular term:





future timelike infinity

Hannam et al. 2008

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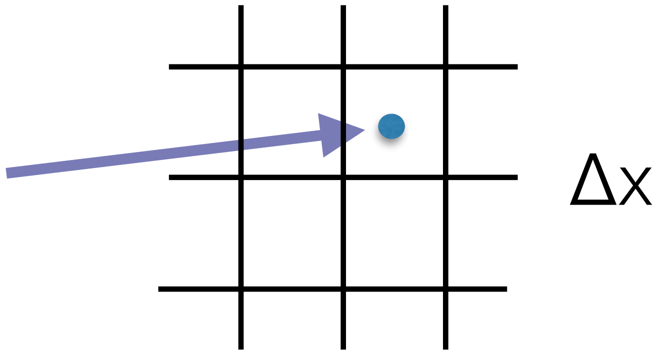

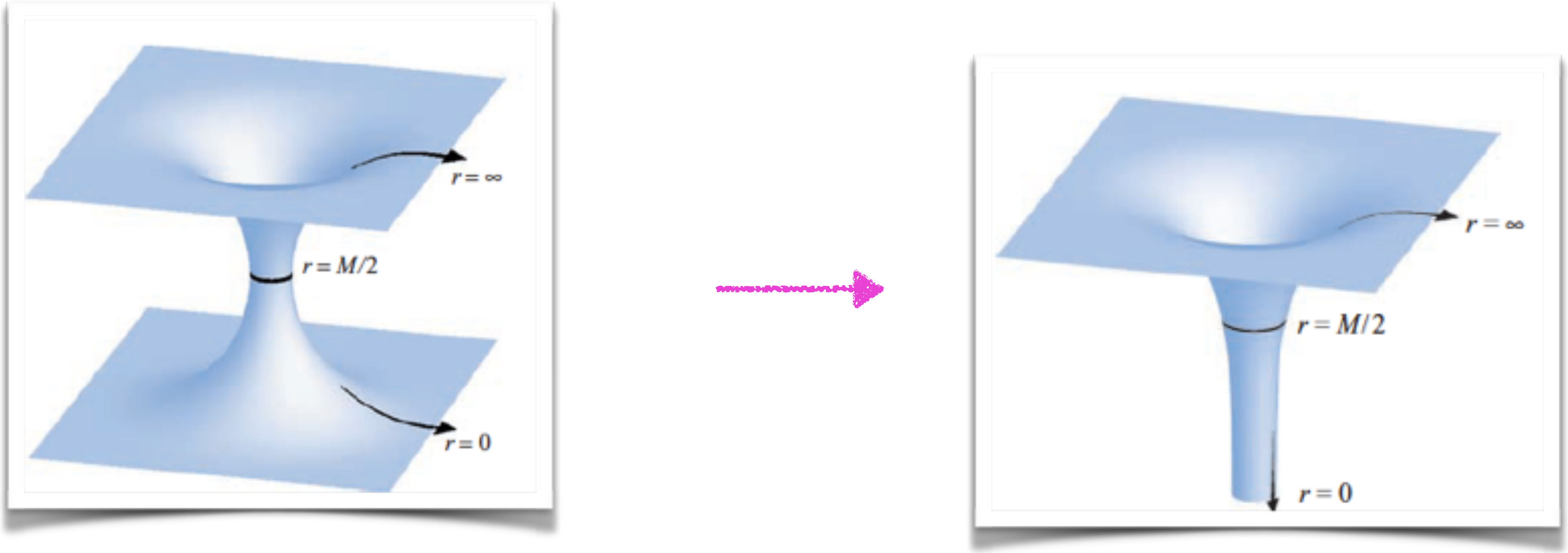

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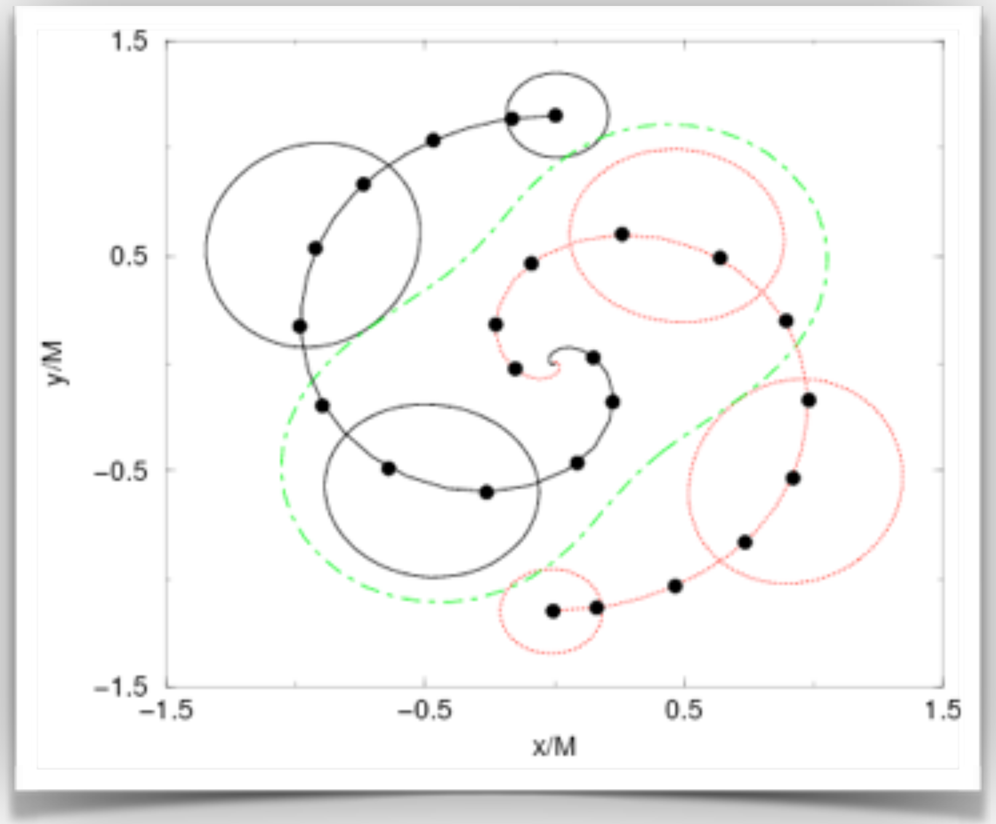
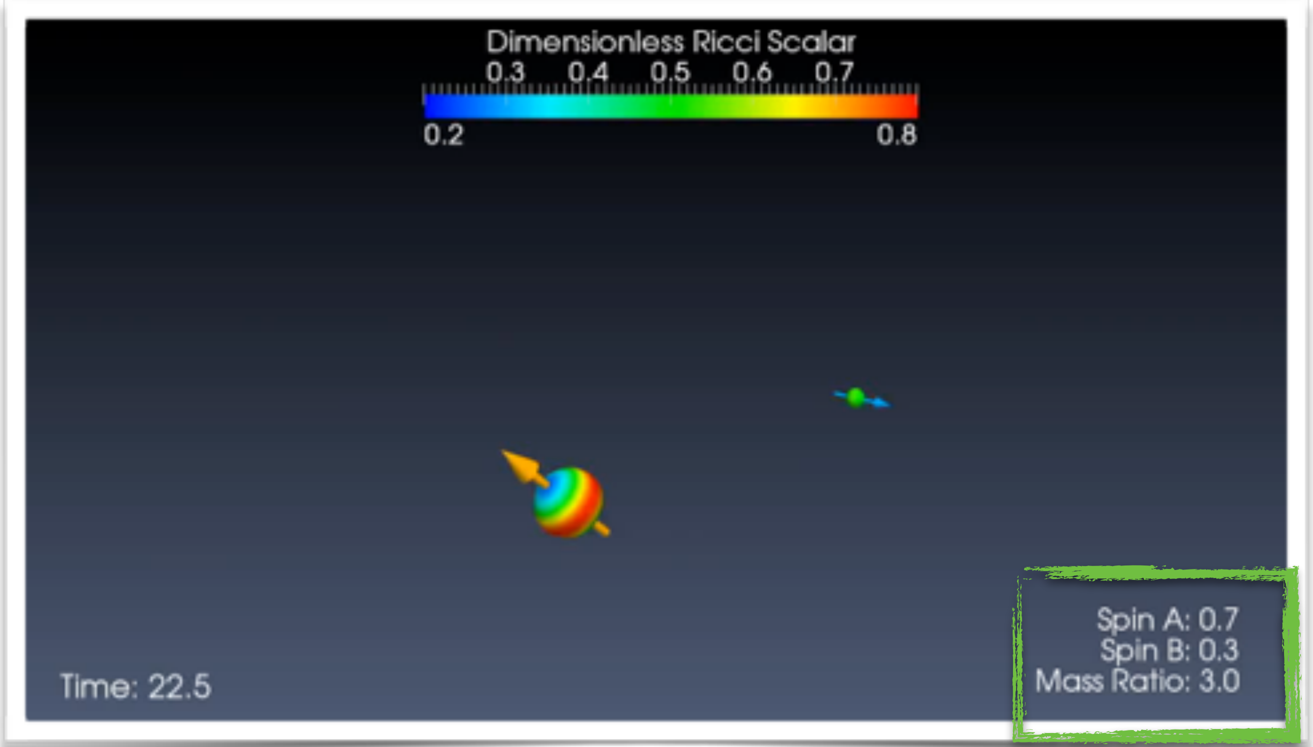
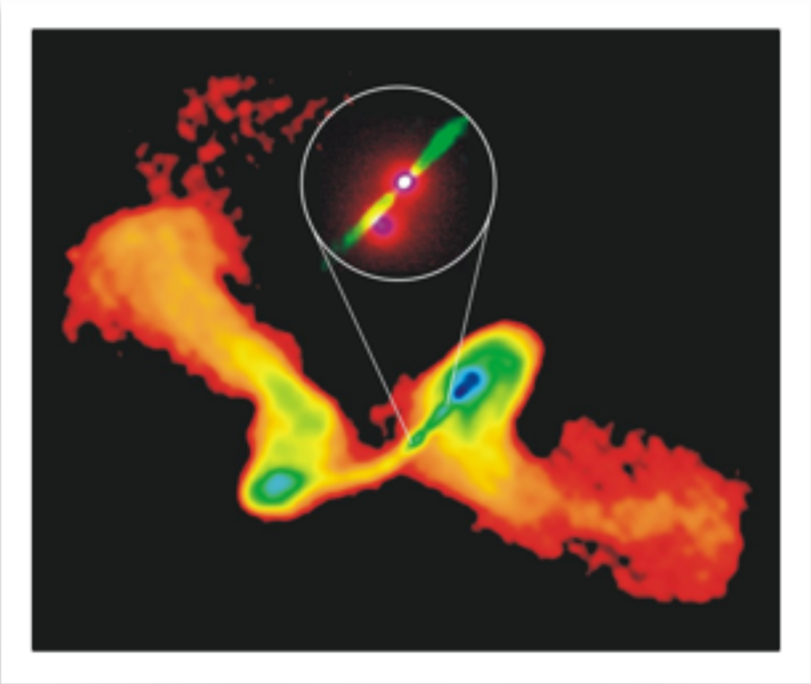




Courtesy: Caltech N. R group



First ingredient to explain, for example, X-shaped galaxies

Radio image of the galaxy NGC 326

What about the two-body problem?

A bit more complicated: matter spacetimes

$$R_{\mu\nu} = 8\pi\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)$$

- ☐ Stellar Evolution: The birth, life, and death of a star (Jorge’s talk)

- ☑ Binary system immerse in a magnetized environment
- ☑ Binary systems: BH-BH, BH-NS and NS-NS

What about the two-body problem?

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Binary BHs and magnetized accretion disk: Near decoupling regime

Gold et al. 2014

☑ Initial data:

- Accretion disk model around a single BH
- Γ = 4/3 radiation pressure dominated, optically thick disk

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Binary BHs and magnetized accretion disk: Postdecoupling regime

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Gold et al. 2014

What about gravitational waves?

A bit more complicated: matter spacetimes

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Binary BHs and magnetized accretion disk: Postdecoupling regime

☑ Waveforms (h+) are plotted in the region of r/M ≥ 40

Gold et al. 2014

What about the two-body problem?

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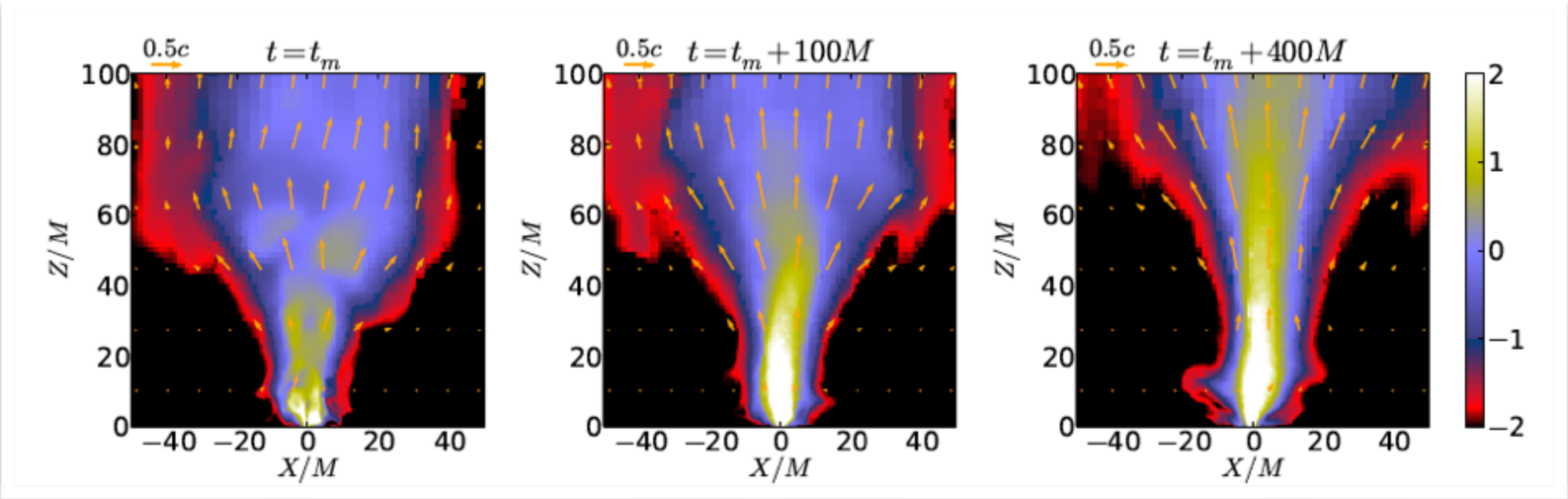
Why magnetized accretion disk are important?



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Why magnetized accretion disk are important?

Multi-messenger astronomy:

Gravitational radiation: Coupled to the dynamics of the source

Electromagnetic Radiation: Interaction of charged particles with matter and/or radiation around the source

What about the two-body problem?

Binary BH-NS and NS-NS: Precursor of Short Gamma ray burst?

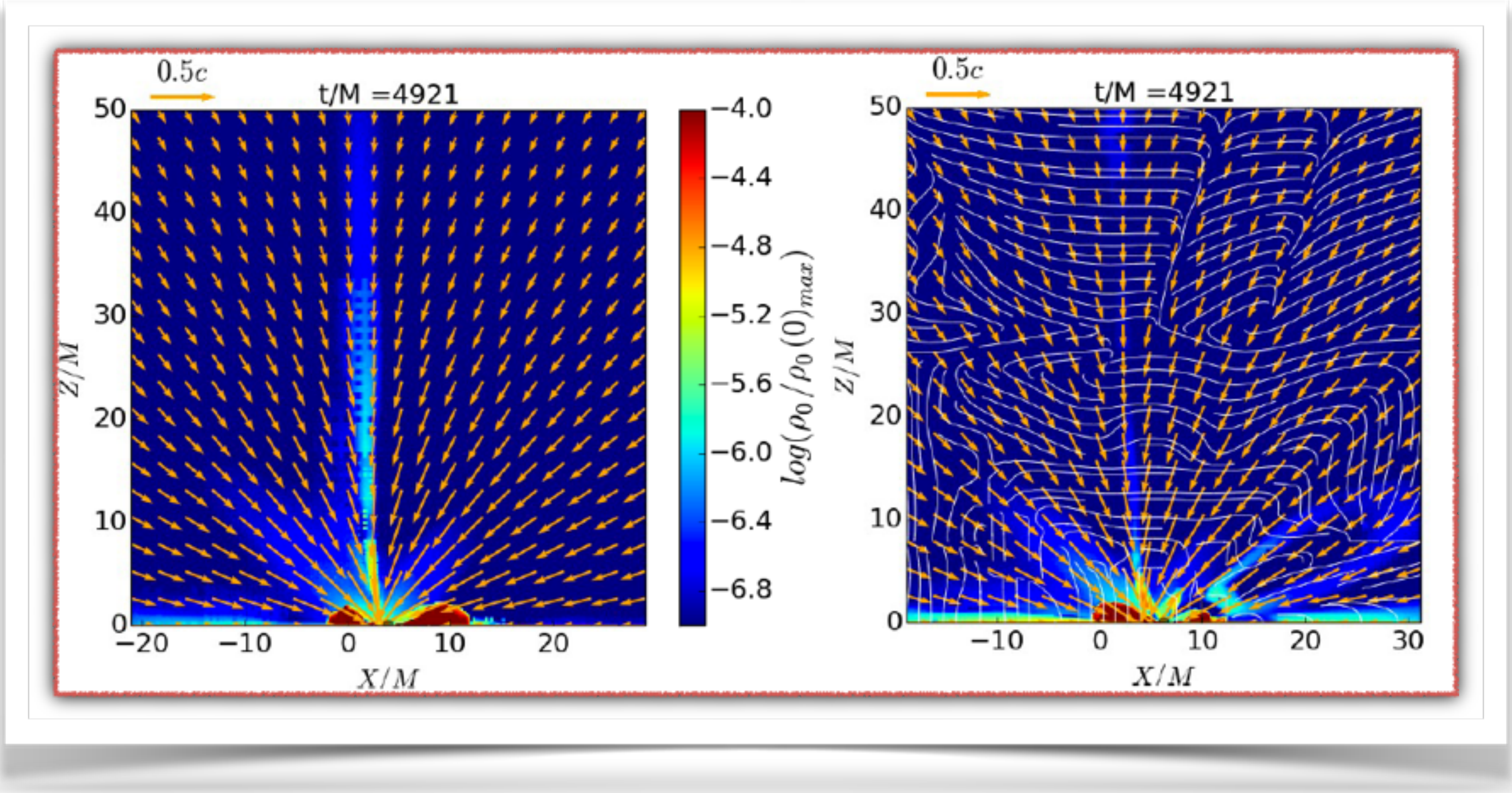
GRB: Flashes of gamma rays associated with extremely energetic explosions

- ☑ Long gamma-ray bursts (> 2s): Associated with rapid star formation, core-collapse supernova (Jorge's talk)
- ☑ Short gamma-ray bursts (< 2s): Associated with BH-NS and NS-NS merger (hypothesis)

Can we prove it numerically?

What about the two-body problem?

Let's start with BH-NS binaries:



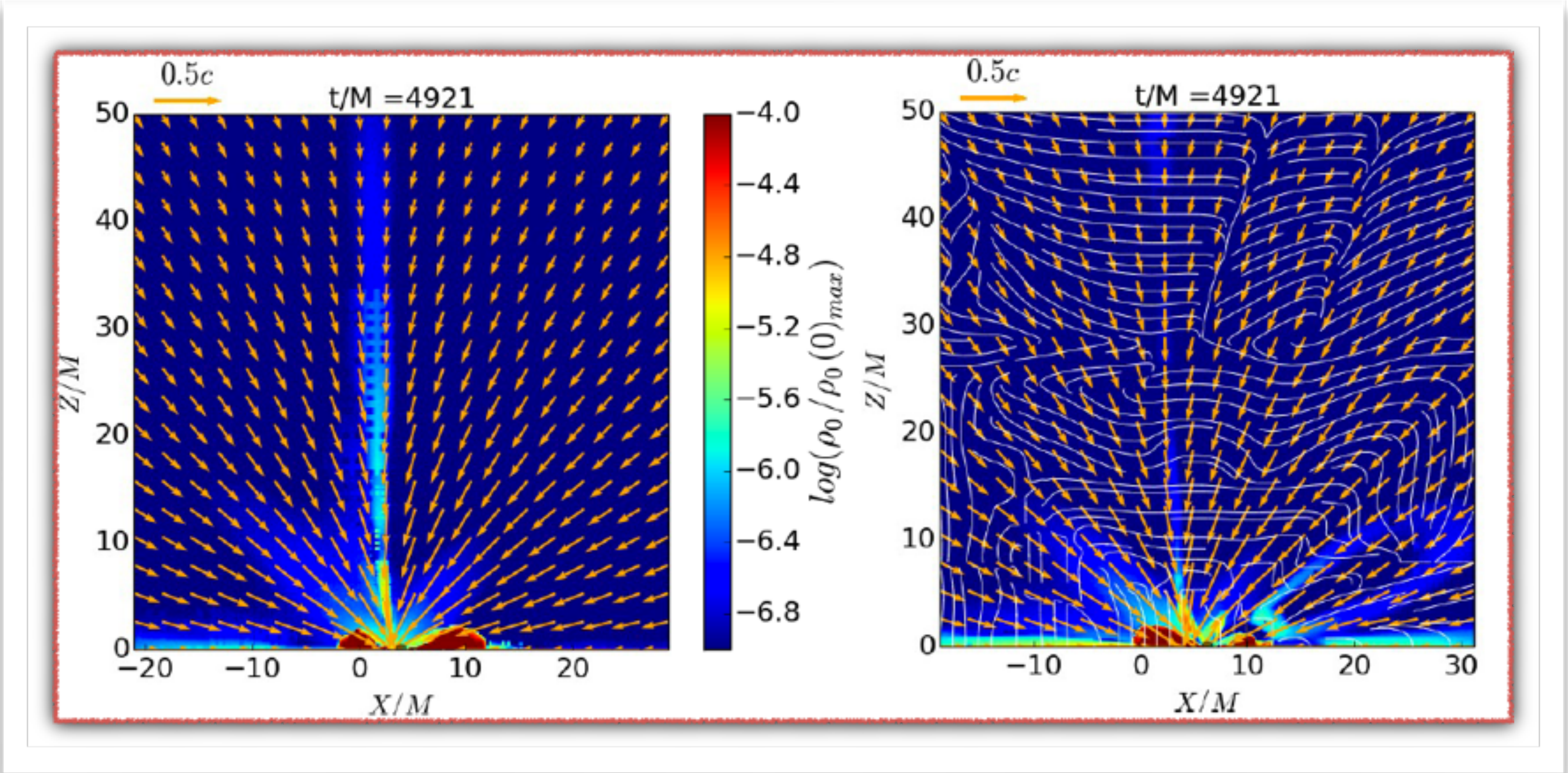
B=0

B=Interior B-field

No purely hydrodynamic simulations of an accretion disk onto BH have shown a jet. A missing ingredient?

What about the two-body problem?

Let’s start with BH-NS binaries:



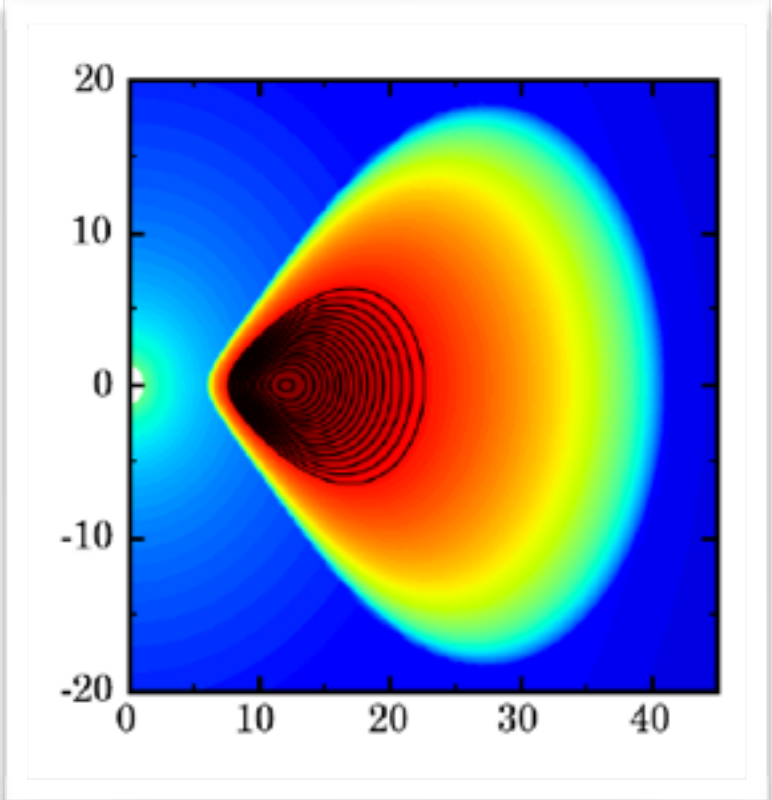
B=0

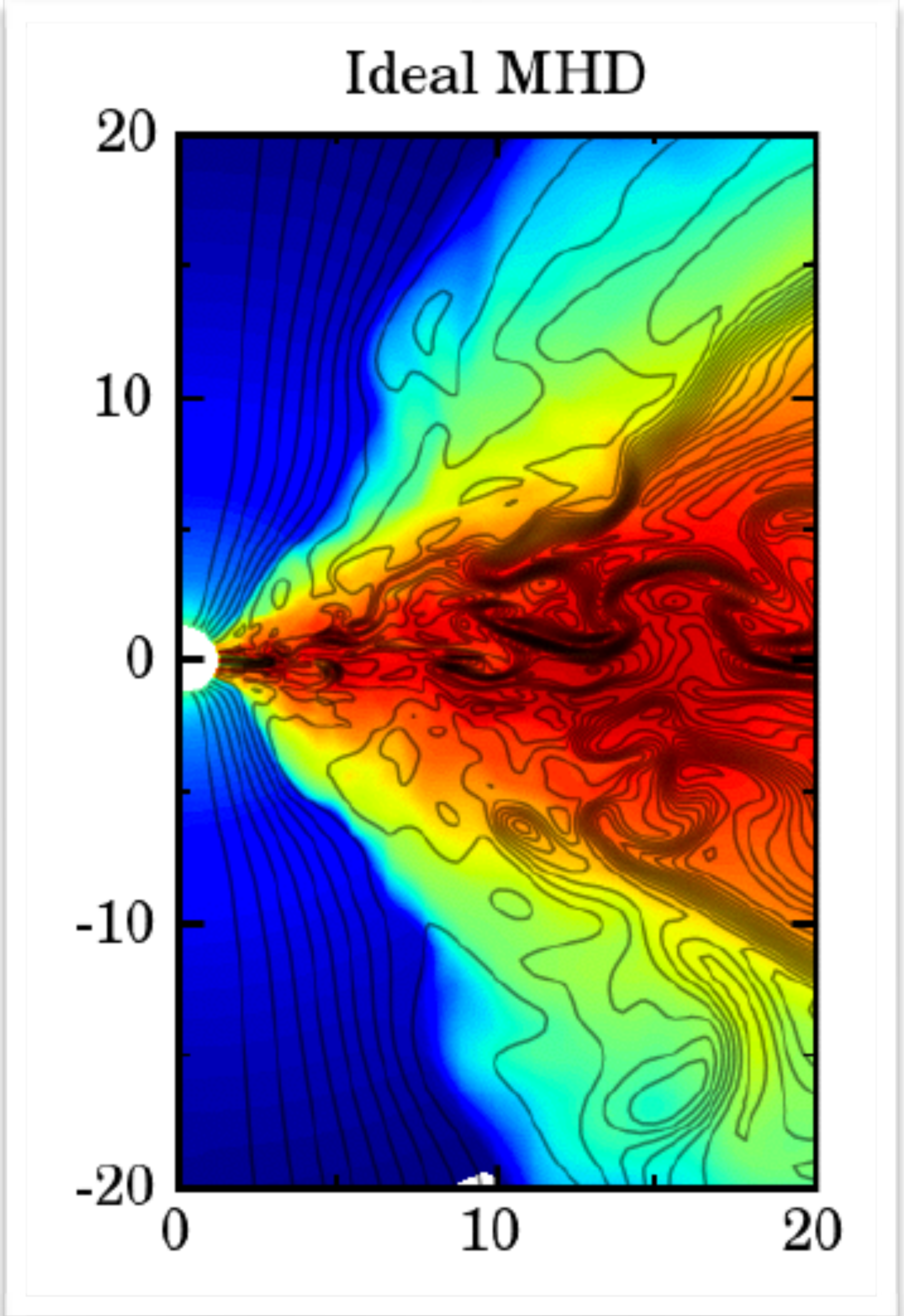
B=Interior B-field

No GR-MHD BHNS simulations with initial B field confined in the NS interior have shown a jet. WAIT!

What about the two-body problem?

We know that GRMHD studies of magnetized accretion disk onto a BH have shown a jet (eg. McKinney et al. '12)





What about the two-body problem?

MHD- BHNS simulation: Jet if the disk is artificially seeded with a purely poloidal B field then the system launches a jet (Etienne et al. '12)

What is the issue?

The fluid motion, after tidal disruption, becomes strongly toroidal, dragging the B field into a toroidal configuration. Beckwith, Hawley & Krolik 2008: We need a strong poloidal component

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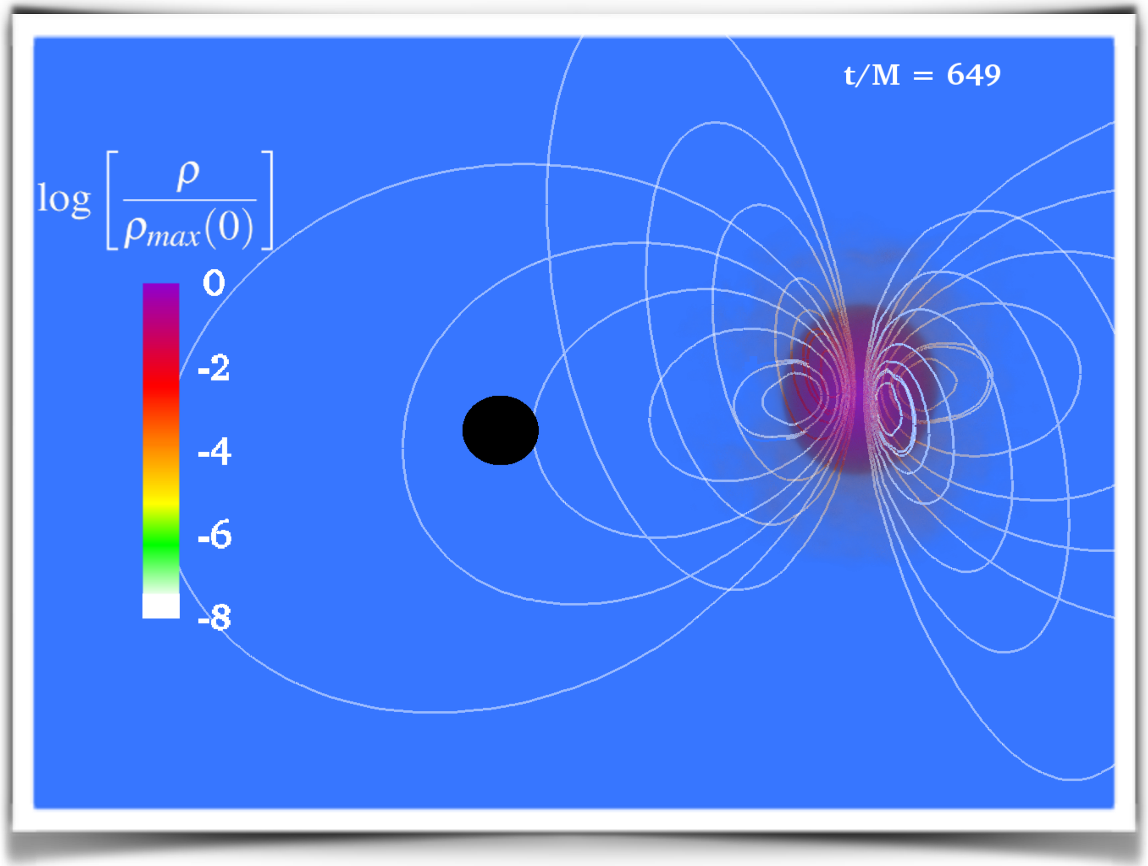
What if we use Pulsars?



New Features:

1) B field lines attached to the fluid thread the BH before tidal disruption.

2) After tidal disruption, the exterior and interior fluid elements in the disk are linked via the B field: Strong poloidal B field.

BH-NS simulations: Setup

CTS initial data: NS is an irrotational, unmagnetized polytrope n=1.
BH: a/m = 0.75 and mass ratio 3:1

BH (NS) resolved by 60 (80) points

Two orbits prior to tidal disruption, we insert the dipole B field generated by a current loop such that (Frozen in-condition):

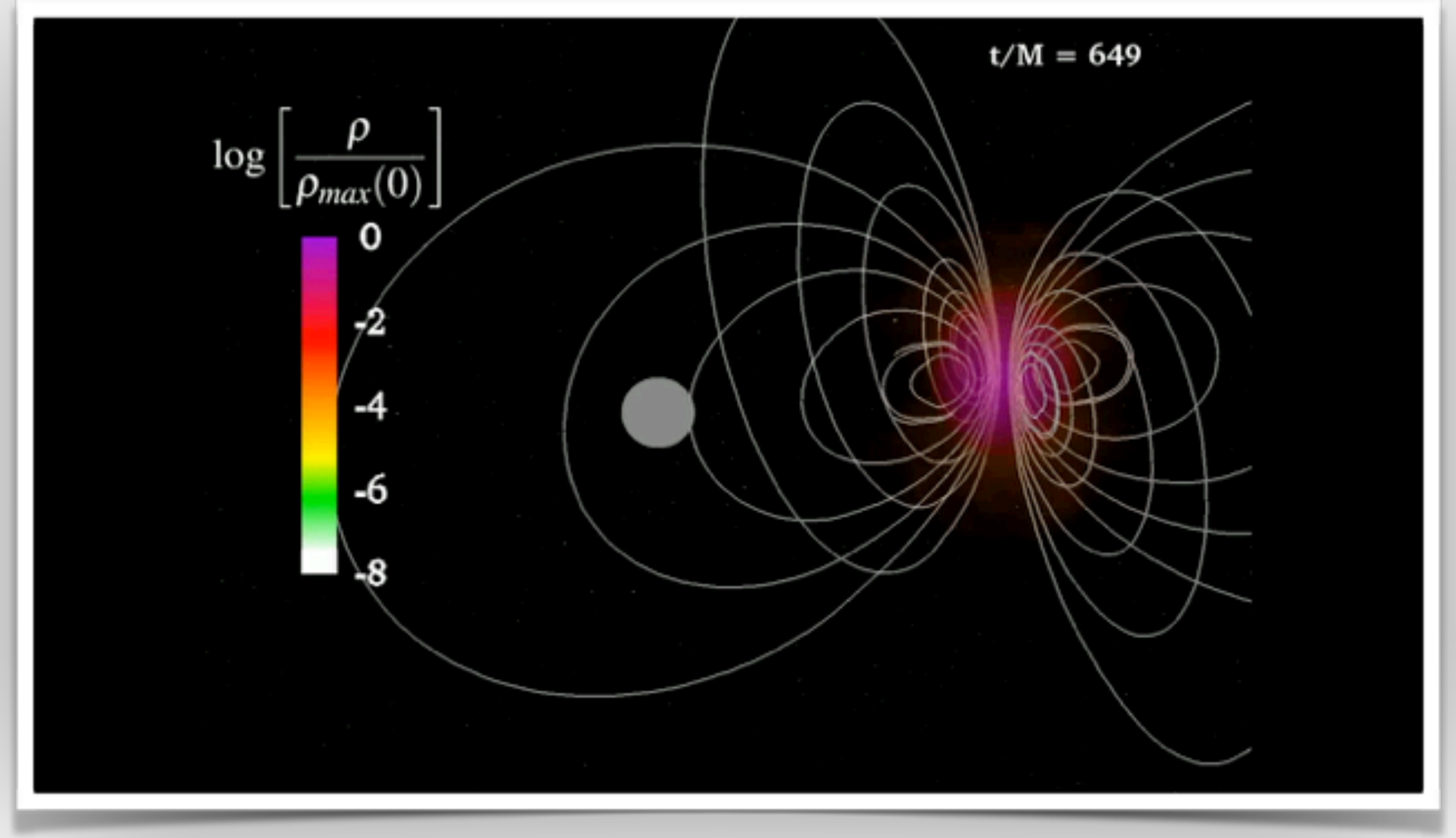
$$\beta \equiv P_{\rm gas}/P_{\rm mag}$$

1. NS Interior: the ratio of the gas to magnetic pressure is $\beta \sim 20$. The B field is dynamically week.

2. NS Exterior: $\beta > 0.01$. The exterior is magnetic pressure dominated. We set a variable atmosphere at B-insertion time such that our MHD code can handle it.

BH-NS simulation

Paschadilis et al. 2015



Disk life-time:

$$\Delta t \sim M_{\rm disk}/\dot{M} \sim 0.5(M_{\rm NS}/1.4M_\odot){\rm s}$$

Consistent with typical sGRB T_{90}

$$L_{EM} \sim 10^{51}\,(a/m)^2\,(m/5.6M_\odot)^2\,(B/10^{15}\,G)^2{\rm erg\ \ s^{-1}}$$

What about NS-NS?

☐ aLIGO/Virgo: the best-bet rate for detection BH–NS mergers is ~10/yr

☑ aLIGO/Virgo: the best-bet rate for detection NS–NS mergers is ~40/yr

Our best chance: NS-NS

✤ Rumor of a gravitational wave detection at LIGO detector from a NS-NS binary system

NS-NS simulation

Preliminary results

Lang et al. in preparation

NS-NS simulation

Preliminary results

Lang et al. in preparation

$$\boxed{L_{EM} \sim 10^{51}\,(a/m)^2\,(m/5.6M_\odot)^2\,(B/10^{15}\,G)^2 \mathrm{erg\ \ s}^{-1}}$$

Final Comments

Numerical relativity is the current tool to tackle unresolved problems in theoretical astrophysics and GR

BH-NS and NS-NS binaries are viable sGRB engines

Multi-messenger astronomy: New observational window!