Loop Quantum Cosmology: Effective Dynamics

Edison Montoya

Escuela de Física, Universidad Industrial de Santander, Bucaramanga, Colombia.

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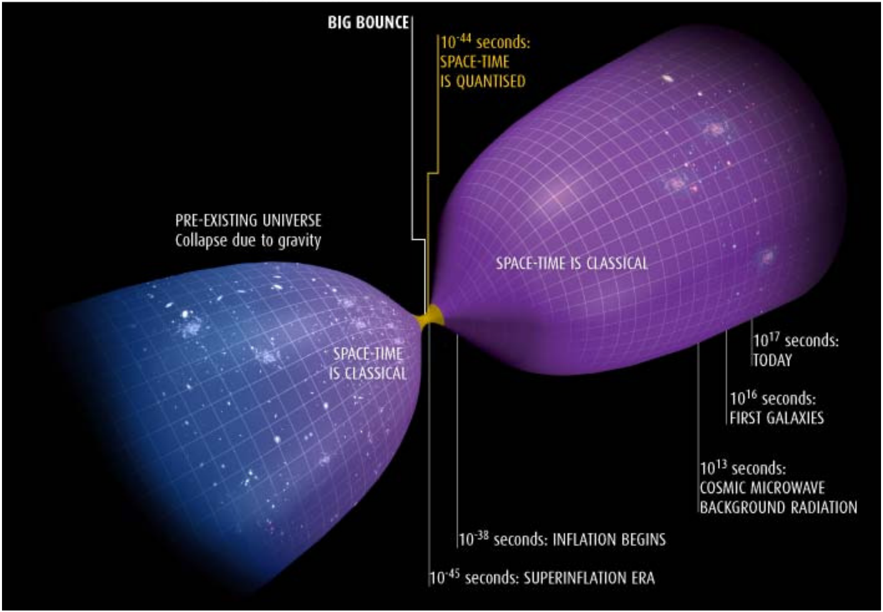

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The implications of the Big Bounce

- The qualitative picture is that non-perturbative quantum geometry corrections create a '*repulsive*' force.
- The universe follows a classical trajectory till the matter density ρ reaches about $\sim 1\%$ of the Planck density $\rho_{\rm Pl}$.
- When the density is near to the Planck density, the universe **bounces**.
- Spacetime fluctuations are severely constrained on both sides of the bounce. At very early times, before the bounce, the universe was as classical as ours.
- The 'horizon problem' disappears.
- LQC calculations provide an *a priori* justification for using classical general relativity during inflation.

Introduction

Loop Quantum Gravity

Is a non-perturbative and background independent quantization of General Relativity. Some important results of the theory are the statistical description of the black holes entropy and the discrete spectrum of the volume and area operators, which gives a picture of a discrete space-time.

Loop Quantum Cosmology

is a reduce model that uses the Loop Quantum Gravity techniques in order to quantize cosmological models. A significant result from this theory is the resolution of the Big Bang singularity, which is replace with a Bounce that take place near to the Planck density.

Loop quantum cosmology has provided a complete description of the quantum dynamics in the case of isotropic cosmological models and singularity resolution has been shown to be generic. The theory has been generalized to anisotropic universes. The idea is to solve the quantum dynamics for these models.

Effective Dynamics

This effective theory comes from the construction of the full quantum theory and we expect that it gives some insights about the quantum dynamics of semiclassical states. The accuracy of the effective equations has been established in the isotropic cases and thus we expect that they should give an excellent approximation of the full quantum evolution for semiclassical states.

- We analyzed the solutions to the effective equations that come from the improved LQC dynamics of the Bianchi I, II and IX models.

These models are of interest to the issue of singularity resolution in the context of the Belinskii, Khalatnikov and Lifshitz (BKL) conjecture.

Belinskii, Khalatnikov and Lifshitz (BKL) Conjecture

Classical universes that have the BKL behavior are universes in which the dynamics near to the big bang singularity is dominated by the time derivatives, which turn been more important than the spatial derivatives. Therefore locally approach the Bianchi IX universe.

Mixmaster

In Bianchi IX when the universe approaches the big bang singularity, there is an oscillatory behavior between Bianchi I solutions with Bianchi II transitions. This is known as the mixmaster behavior, studied first by Misner.

We want to study homogeneous and anisotropic universes, these kind of universes are classified as Bianchi models. We are interested in

- **Bianchi I**, which the spacetime is like $M = \Sigma \times \mathbb{R}$ where Σ is a spatial 3-manifold which have a 3-dimensional group of symmetries generated by three translations.

$$\mathrm{d}s^2 = -N^2 \mathrm{d}t^2 + a_1(t)^2 \mathrm{d}^2 x + a_2(t)^2 \, \mathrm{d}y^2 + a_3(t)^2 \, \mathrm{d}z^2 \qquad (1)$$

- **Bianchi II**, which the spacetime is like $M = \Sigma \times \mathbb{R}$ where Σ is a spatial 3-manifold which have a 3-dimensional group of symmetries generated by two translations and a rotation on a null 2-plane.

$$\mathrm{d}s^2 = -N^2 \mathrm{d}t^2 + a_1(t)^2 \left(\mathrm{d}x - \alpha z \, \mathrm{d}y\right)^2 + a_2(t)^2 \, \mathrm{d}y^2 + a_3(t)^2 \, \mathrm{d}z^2 \qquad (2)$$

where α is a switch (Bianchi I $\alpha = 0$, Bianchi II $\alpha = 1$).

- **Bianchi IX**, which the spacetime is like $M = \Sigma \times \mathbb{R}$ where Σ is a spatial 3-manifold which have a 3-dimensional group of symmetries generated by three rotations that can be identified by the symmetry group $SU(2)$. The Bianchi IX metric can be construct from the fiducial co-triads

$$\begin{aligned} {}^o\omega^1_a &= \sin\beta\sin\gamma(\mathrm{d}\alpha)_a + \cos\gamma(\mathrm{d}\beta)_a, \\ {}^o\omega^2_a &= -\sin\beta\cos\gamma(\mathrm{d}\alpha)_a + \sin\gamma(\mathrm{d}\beta)_a, \\ {}^o\omega^3_a &= \cos\beta(\mathrm{d}\alpha)_a + (\mathrm{d}\gamma)_a, \end{aligned} \tag{3}$$

with physical co-triads $\omega^i_a = a^i(t){}^o\omega^i_a$ and 3-metric $q_{ab} := \omega^i_a\omega_{bi}$. The spacetime metric is

$$g_{\mu\nu} = -n_\mu n_\nu + q_{\mu\nu} \tag{4}$$

We use the fiducial triads and co-triads to introduce a convenient parametrization of the phase space variables, E^a_i, A^i_a given by

$$E^a_i = p_i L_i V_0^{-1} \sqrt{|{}^o q|}\, {}^o e^a_i \qquad \text{and} \qquad A^i_a = c^i L_i^{-1}\, {}^o\omega^i_a. \tag{5}$$

Thus, a point in the phase space is now coordinatized by six real numbers (p_i, c^i) with Poisson brackets given by

$$\{c^i,\, p_j\} \;=\; 8\pi G\gamma\, \delta^i_j\,. \tag{6}$$

Additionally to the matter degrees of freedom, e.g., the scalar field ϕ and its momenta p_ϕ, with Poisson bracket $\{\phi, p_\phi\} = 1$.
The relation between the new and the old variables is

$$p_i = a_j a_k L_j L_k \ ,$$

with $i \neq j \neq k \neq i$, L_i fiducial lengths, $V_0 = L_1 L_2 L_3$.

The choice of the physical triads and connections has fixed the Gauss and vector constraint and only remain the Hamiltonian constraint

$$\mathcal{C}_H = \int_V \left[\frac{N E^a_i E^b_j}{16\pi G \sqrt{|q|}} \left(\epsilon^{ij}{}_k F_{ab}{}^k - 2(1+\gamma^2) K^i_{[a} K^j_{b]} \right) + N \mathcal{H}_{\mathrm{matt}} \right] d^3x \qquad (7)$$

with $F_{ab}{}^k = 2\partial_{[a} A^k_{b]} + \epsilon_{ij}{}^k A^i_a A^j_b$ and $\mathcal{H}_{\mathrm{matt}}$ Hamiltonian of matter.
At classical level the Hamiltonian constraint for **Bianchi I and II** is given by

$$\begin{aligned} \mathcal{C}_{H_{BII}} &= -\frac{1}{8\pi G \gamma^2} \Bigg[p_1 c_1 p_2 c_2 + p_1 c_1 p_3 c_3 + p_2 c_2 p_3 c_3 + \alpha p_2 p_3 c_1 \\ &\qquad - (1+\gamma^2) \left(\frac{\alpha p_2 p_3}{2 p_1} \right)^2 \Bigg] + \frac{p_\phi^2}{2} \approx 0 \end{aligned} \qquad (8)$$

with lapse $N = V = \sqrt{p_1 p_2 p_3}$ and $\mathcal{H}_{\mathrm{matt}} = \frac{p_\phi^2}{2V}$.

Bianchi Models

- Directional scale factors, $a_i = L_i^{-1}\sqrt{\frac{p_j p_k}{p_i}}$.
- Hubble parameters, $H_i = \frac{a_i'}{a_i} = \frac{1}{2}\left(\frac{p_j'}{p_j} + \frac{p_k'}{p_k} - \frac{p_i'}{p_i}\right)$.
- Expansion, $\theta = \frac{V'}{V} = H_1 + H_2 + H_3$.
- Matter density, $\rho = \frac{p_\phi^2}{2V^2} = \frac{p_\phi^2}{2p_1 p_2 p_3}$.
- Density parameter, $\Omega = \frac{24\pi G\rho}{\theta^2}$.
- Shear, $\sigma^2 = \frac{1}{3}[(H_1 - H_2)^2 + (H_1 - H_3)^2 + (H_2 - H_3)^2]$.
- Shear parameter, $\Sigma^2 = \frac{3\sigma^2}{2\theta^2}$
- Curvature parameter, $K = -\frac{3\,{}^{(3)}R}{2\theta^2}$, where $\Omega + \Sigma^2 + K = 1$ for the classical Bianchi.
- Ricci scalar R.

Bianchi Models

Intrinsic curvature, one feature of Bianchi II and IX models is that the spatial curvature is different from zero. The intrinsic spatial curvature is given by

$$ {}^{(3)}R = -\frac{1}{2}\left[x_1^2 + x_2^2 + x_3^2 - 2(x_1 x_2 + x_1 x_3 + x_2 x_3)\right], \tag{9} $$

where

$$ x_1 = \alpha_1 \sqrt{\frac{p_2 p_3}{p_1^3}}, \quad x_2 = \alpha_2 \sqrt{\frac{p_1 p_3}{p_2^3}}, \quad x_3 = \alpha_3 \sqrt{\frac{p_1 p_2}{p_3^3}}. \tag{10} $$

The values of α_i are:

- Bianchi I, $\alpha_1 = \alpha_2 = \alpha_3 = 0$.
- Bianchi II, $\alpha_1 = 1, \alpha_2 = 0, \alpha_3 = 0$ or permutations.
- Bianchi IX, $\alpha_1 = \alpha_2 = \alpha_3 = l_0^2$, with $l_0 = V_0^{1/3}$.
- The isotropic FLRW model with $k = 1$, is obtained from the Bianchi IX model by setting $x_1 = x_2 = x_3$.

- The elementary functions on the classical phase space are the momenta p_i and the holonomies of the gravitational connection A_a^i .
- The elementary functions are promoted to operators on $\mathcal{H}_{\mathrm{kin}}^{\mathrm{grav}}$.
- The Hamiltonian constraint is written in terms of the elementary variables and promoted to operator.

In the last step there are some subtleties, namely

- The curvature is calculated from the connection and not from the holonomies, $F_{ab}{}^k = 2\partial_{[a}A_{b]}^k + \epsilon_{ij}{}^k A_a^i A_b^j$.
- The connection is calculated from the holonomies

$$A_a = \lim_{l_k \to 2\bar{\mu}_k} \sum_k \frac{1}{2 l_k L_k}\left(h_k^{(l_k)} - (h_k^{(l_k)})^{-1}\right) \quad \Rightarrow \quad A_a^k = \frac{\sin(\bar{\mu}_k c_k)}{\bar{\mu}_k L_k}{}^o\omega_a^k$$

with

$$\bar{\mu}_1 = \lambda\sqrt{\frac{p_1}{p_2 p_3}}, \qquad \bar{\mu}_2 = \lambda\sqrt{\frac{p_2}{p_1 p_3}}, \qquad \bar{\mu}_3 = \lambda\sqrt{\frac{p_3}{p_1 p_2}}.$$

The value of λ is chosen such that $\lambda^2 = 4\sqrt{3}\pi\gamma\ell_{\mathrm{Pl}}^2$.

$$
\begin{aligned}
\mathcal{H}_{BII} = & \frac{p_1 p_2 p_3}{8\pi G \gamma^2 \lambda^2} \Big[\sin \bar{\mu}_1 c_1 \sin \bar{\mu}_2 c_2 + \sin \bar{\mu}_2 c_2 \sin \bar{\mu}_3 c_3 + \sin \bar{\mu}_3 c_3 \sin \bar{\mu}_1 c_1 \Big] \\
& + \frac{1}{8\pi G \gamma^2} \left[\frac{\alpha (p_2 p_3)^{3/2}}{\lambda \sqrt{p_1}} \sin \bar{\mu}_1 c_1 - (1+\gamma^2) \left(\frac{\alpha p_2 p_3}{2 p_1} \right)^2 \right] - \frac{p_\phi^2}{2} \approx 0
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{H}_{BIX} = & - \frac{p_1 p_2 p_3}{8\pi G \gamma^2 \lambda^2} \big(\sin \bar{\mu}_1 c_1 \sin \bar{\mu}_2 c_2 + \sin \bar{\mu}_2 c_2 \sin \bar{\mu}_3 c_3 + \sin \bar{\mu}_3 c_3 \sin \bar{\mu}_1 c_1 \big) \\
& - \frac{\vartheta}{8\pi G \gamma^2 \lambda} \left(\frac{(p_1 p_2)^{3/2}}{\sqrt{p_3}} \sin \bar{\mu}_3 c_3 + \frac{(p_2 p_3)^{3/2}}{\sqrt{p_1}} \sin \bar{\mu}_1 c_1 + \frac{(p_3 p_1)^{3/2}}{\sqrt{p_2}} \sin \bar{\mu}_2 c_2 \right) \\
& - \frac{\vartheta^2 (1+\gamma^2)}{32\pi G \gamma^2} \left[2(p_1^2 + p_2^2 + p_3^2) - \left(\frac{p_1 p_2}{p_3} \right)^2 - \left(\frac{p_2 p_3}{p_1} \right)^2 - \left(\frac{p_3 p_1}{p_2} \right)^2 \right] \\
& + \frac{p_\phi^2}{2} \approx 0
\end{aligned}
$$

with lapse $N = V = \sqrt{p_1 p_2 p_3}$.

Given that **Bianchi IX** is spatially compact, the inverse triad corrections are important. The terms related to the curvatures, F_{ab}^{k} and K_a^i, contain some negative powers of p_i which are not well defined operators. To solve this problem is used the same idea as Thiemann's strategy.

$$|p_i|^{(\ell-1)/2} = -\frac{\sqrt{|p_i|}L_i}{4\pi G\gamma j(j+1)\bar{\mu}_i\ell}\mathrm{Tr}(\tau_i h_i^{(\bar{\mu}_i)}\{h_i^{(\bar{\mu}_i)-1}, |p_i|^{\ell/2}\})\,, \tag{11}$$

The eigenvalues for the operator $\widehat{|p_i|^{-1/4}}$ are given by

$$J_i(V, p_1, p_2, p_3) = \frac{h(V)}{V_c}\prod_{j\neq i} p_j^{1/4}\,, \tag{12}$$

with

$$h(V) = \sqrt{V+V_c} - \sqrt{|V-V_c|}, \quad \text{and} \quad V_c = 2\pi\gamma\lambda\ell_{\mathrm{Pl}}^2. \tag{13}$$

The correction term which comes from the operator $\epsilon_k^{ij}E_i^aE_j^b/\sqrt{|q|}$ is

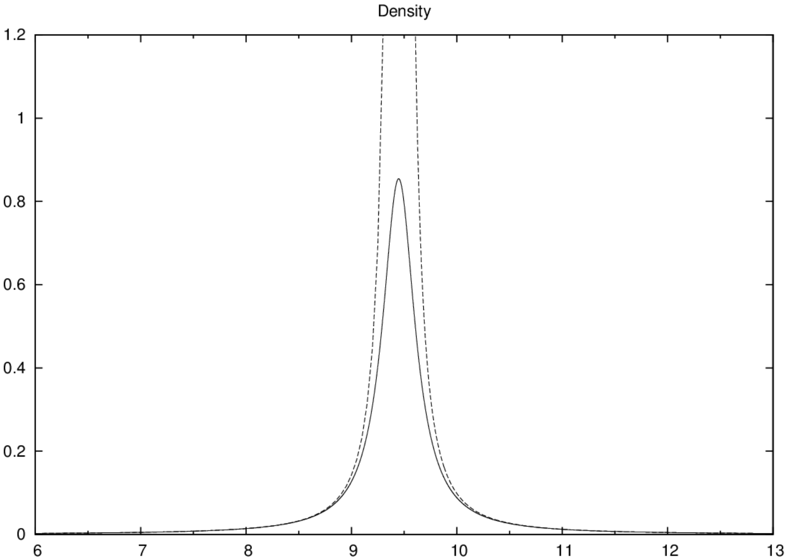
$$A(V) = \frac{1}{2V_c}(V+V_c-|V-V_c|) \tag{14}$$

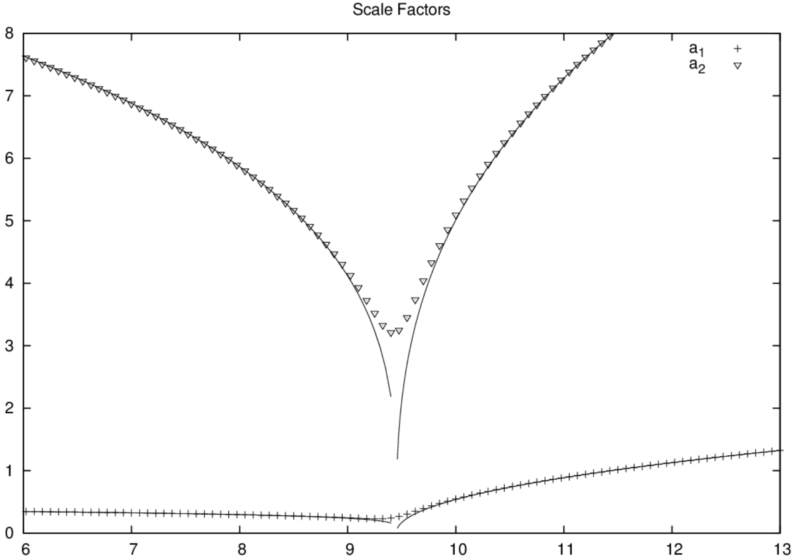
Effective Bianchi I and II

Matter Source: Massless scalar field.

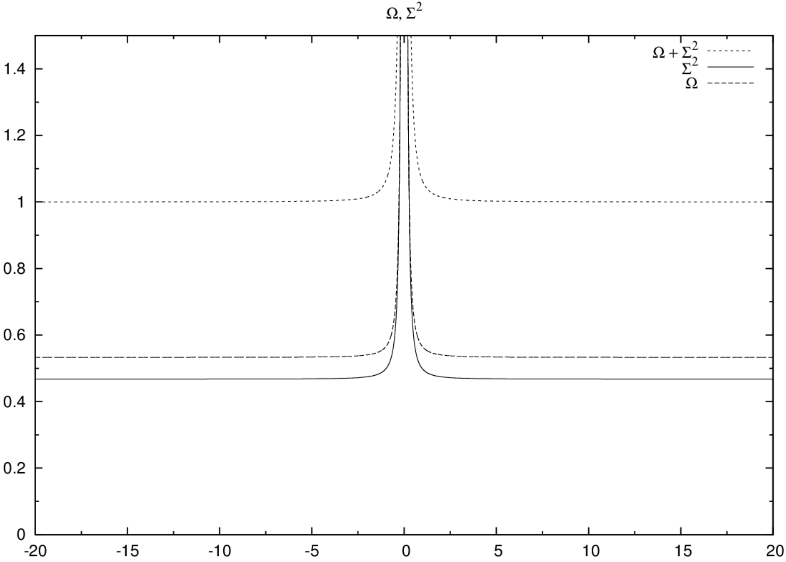
Bianchi II

Classical Limit





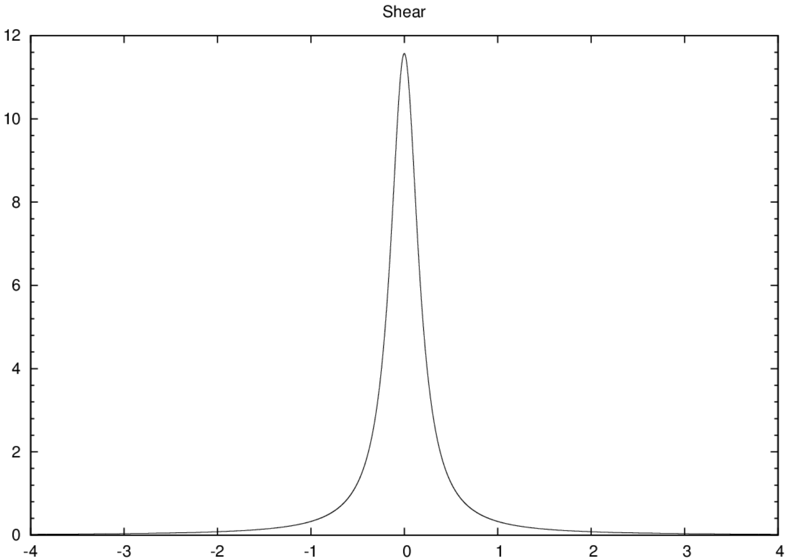
Scale Factors
8
7
6
5
4
3
2
1
0
6
7
8
9
10
11
12
13
a_1 +
a_2 ▽



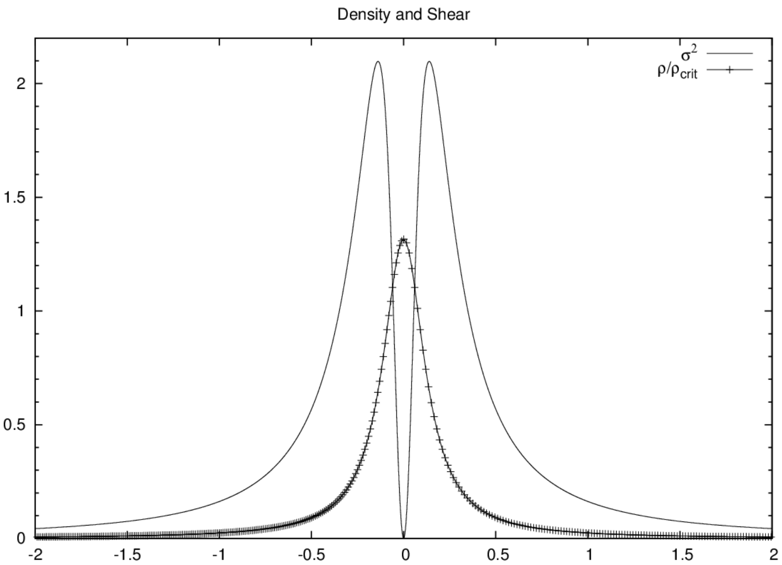
Ω, Σ^2
$\Omega + \Sigma^2$
Σ^2
Ω
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-20
-15
-10
-5
0
5
10
15
20

Bianchi II

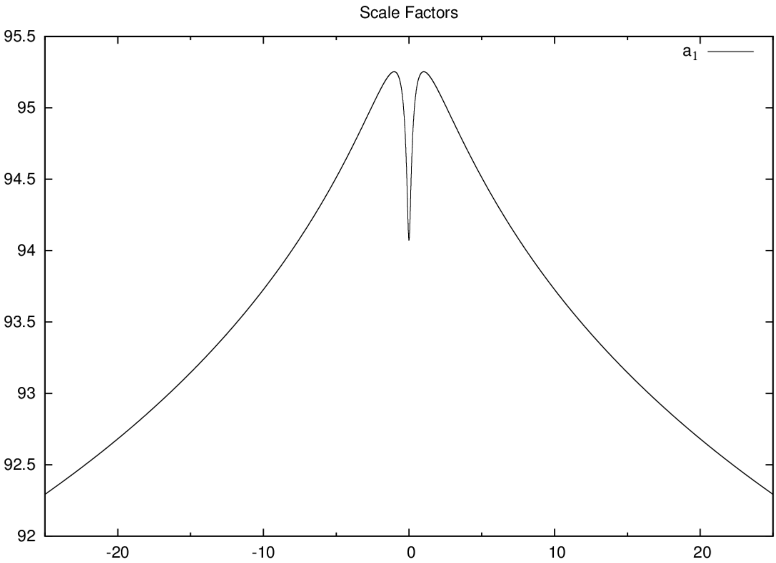
Bianchi I Limit



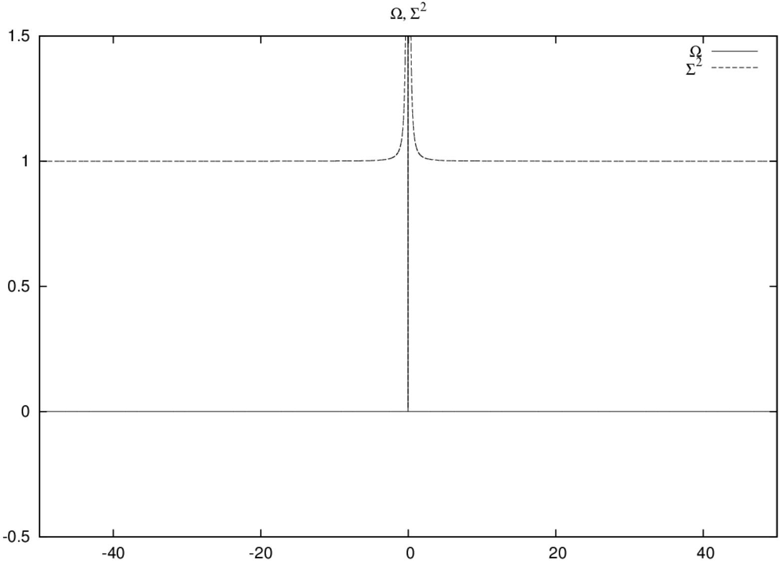
LRS Bianchi II



LRS Bianchi II

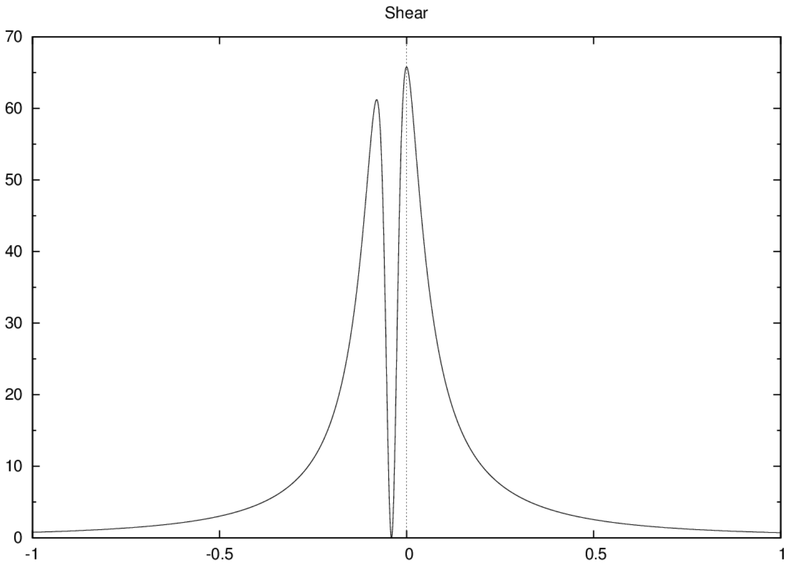


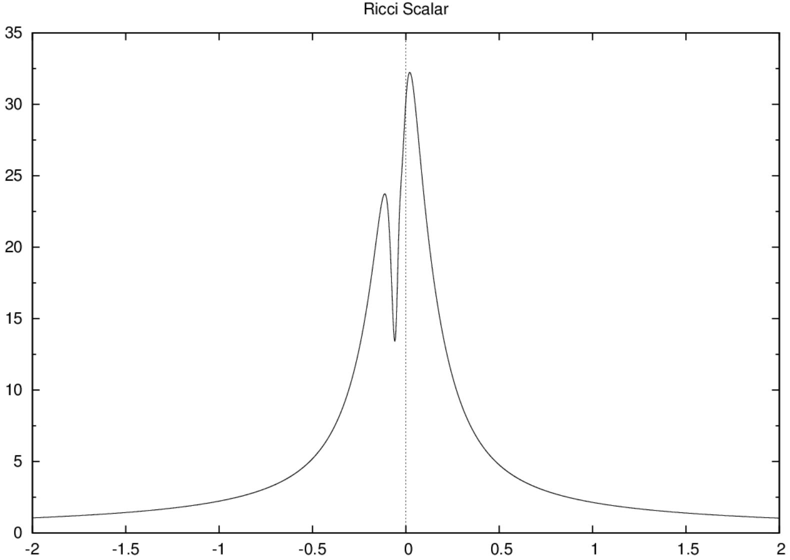
Bianchi II
Vacuum Limit



Bianchi II

Vacuum Limit



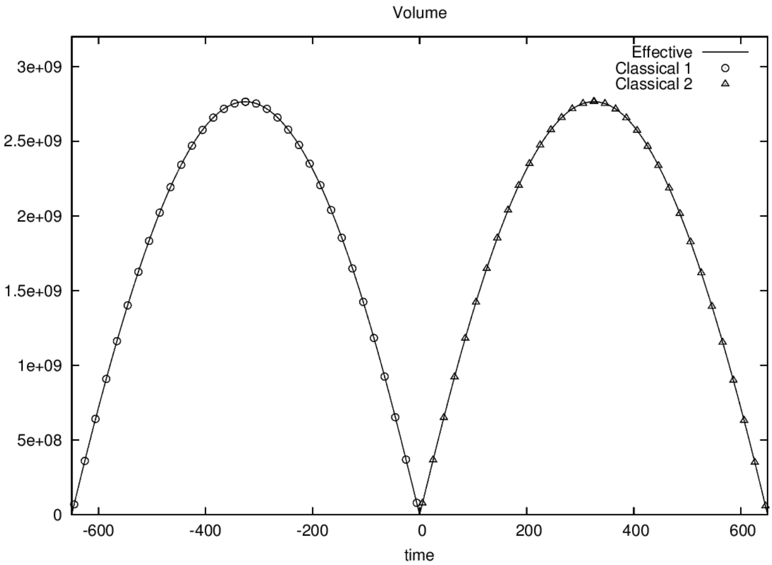

Ricci Scalar
35
30
25
20
15
10
5
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

Bianchi II Conclusions

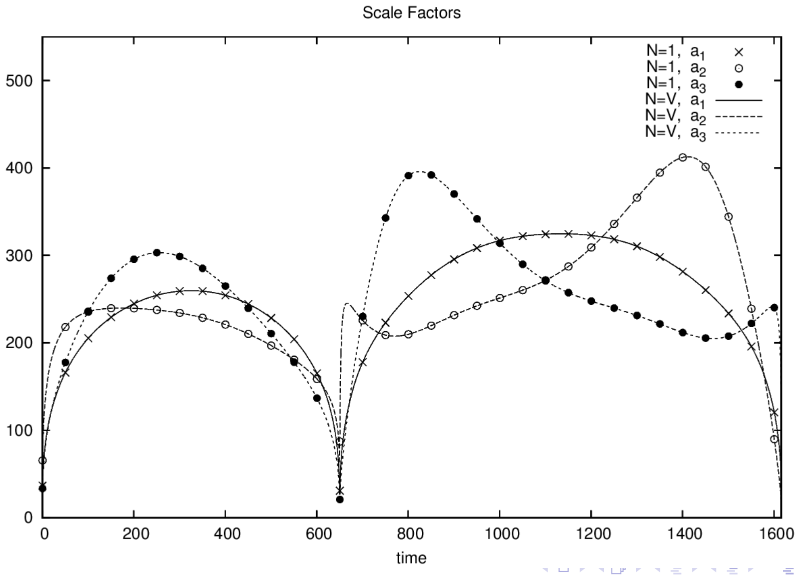
- The classical singularities are resolved, namely, the geodesics are inextensible, i.e., scale factor non zero (or infinite) in a finite time.
- Bianchi I limit is recover and presents all its known facts.
- Shear can be zero at the bounce and non zero in the evolution.
- Some important solutions are LRS, like the one with maximal density and the one with maximal shear.
- In the vacuum limit ($\Omega \approx 0$) is possible to have solutions where all the dynamical contribution comes from the anisotropies ($\Sigma^2 \approx 1$).
- There is one global bounce $\theta = 0$.
- The effective solutions connect anisotropic solutions even when the shear is zero at the bounce.

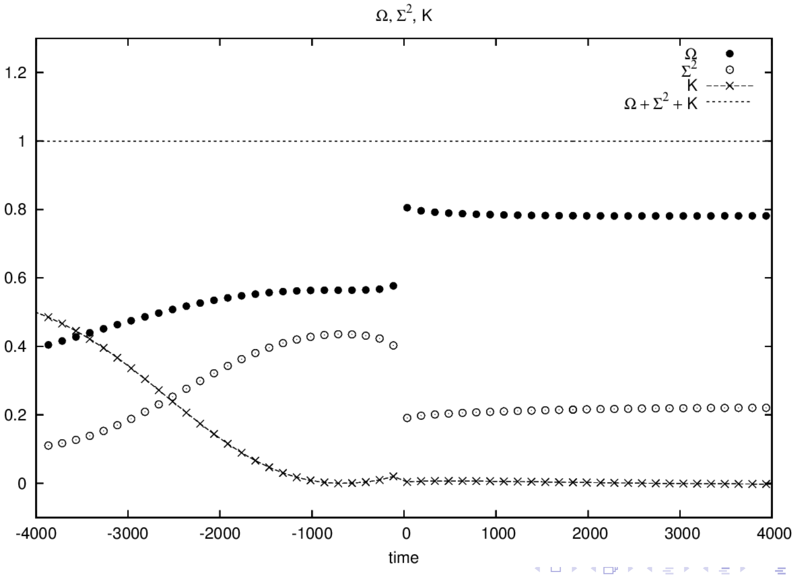
Effective Bianchi IX

Matter Source: Massless scalar field.

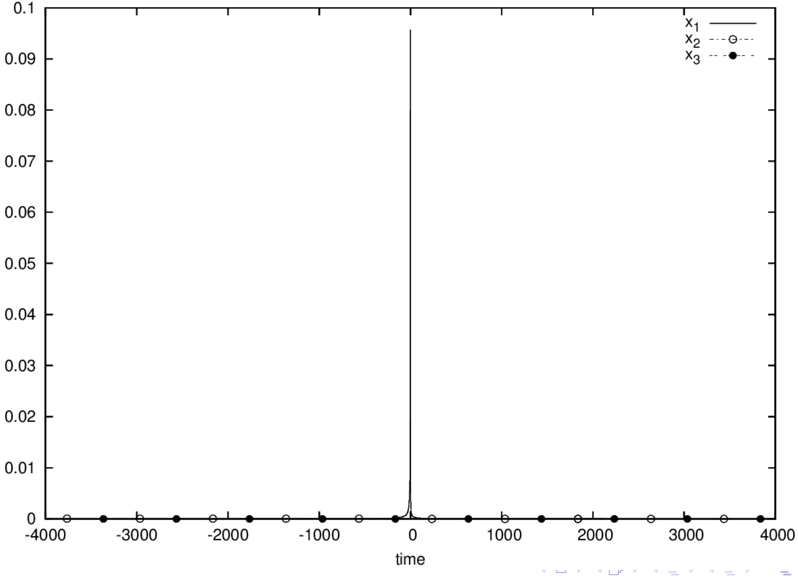


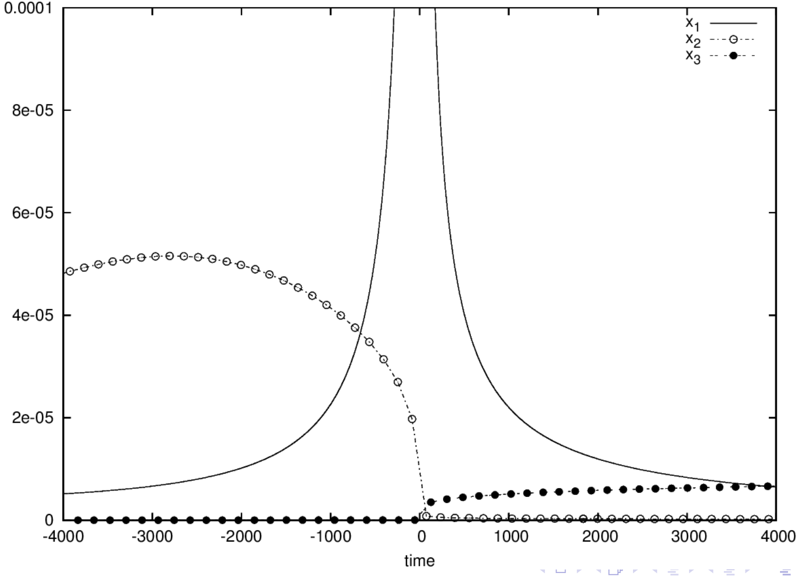
Volume
Effective
Classical 1
Classical 2
3e+09
2.5e+09
2e+09
1.5e+09
1e+09
5e+08
0
-600
-400
-200
0
200
400
600
time


Scale Factors
N=1, a_1
N=1, a_2
N=1, a_3
N=V, a_1
N=V, a_2
N=V, a_3
0
100
200
300
400
500
0
200
400
600
800
1000
1200
1400
1600
time



Ω, Σ^2, K
Ω
Σ^2
K
$\Omega + \Sigma^2 + K$
1.2
1
0.8
0.6
0.4
0.2
0
-4000
-3000
-2000
-1000
0
1000
2000
3000
4000
time


0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-4000
-3000
-2000
-1000
0
1000
2000
3000
4000
time
x_1
x_2
x_3



0.0001
8e-05
6e-05
4e-05
2e-05
0
-4000
-3000
-2000
-1000
0
1000
2000
3000
4000
time
x_1
x_2
x_3

- All solutions have a bounce. In other words, singularities are resolved. In Bianchi IX, there is an infinite number of bounces and recollapses.
- Both effective theories in the large volume limit (compared to the Planck volume) describe the same dynamics.
- Bianchi I and the isotropic case $k = 0, 1$ are limiting cases of Bianchi IX.
- A set of quantities that are very useful are the functions x_i associated to the intrinsic curvature ${}^{(3)}R$, because they can be used to determine which kind of solution is obtained.

Qualitative Bianchi IX

Independent of the matter content.

In order to study the qualitative behavior of the Bianchi IX theories.

- We first consider the maximal density for each effective theory. This maximal density is reach when the sine functions are equal to one in each theory. This let the maximal density as a function only of the triads (p_1, p_2, p_3).
- Second we do a change of variables to the Misner varibles (V, β_+, β_-),

$$a_1 = V^{1/3} e^{\beta_+ + \sqrt{3}\beta_-}, \quad a_2 = V^{1/3} e^{\beta_+ - \sqrt{3}\beta_-}, \quad a_3 = V^{1/3} e^{-2\beta_+}. \tag{15}$$

The variables used to write the effective theories are the triads (p_1, p_2, p_3), which in the new variables (V, β_+, β_-) are

$$p_1 = V^{2/3} e^{-\beta_+ - \sqrt{3}\beta_-}, \quad p_2 = V^{2/3} e^{-\beta_+ + \sqrt{3}\beta_-}, \quad p_3 = V^{2/3} e^{2\beta_+} \tag{16}$$

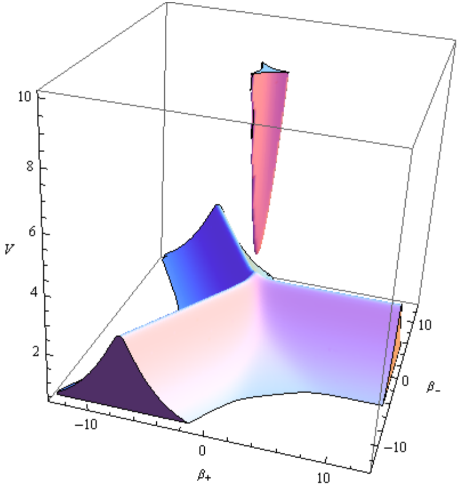


Figure : Density region ($\rho \geq 0$) with $N = 1$.

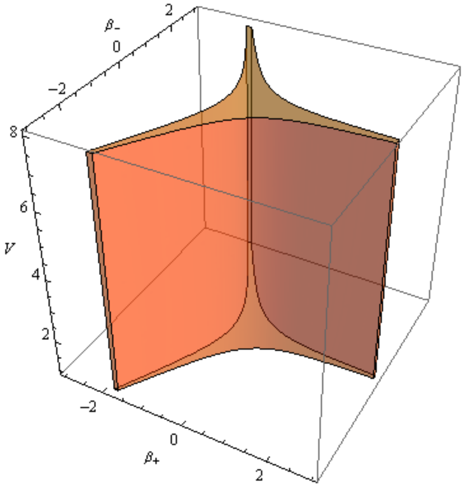


Figure : Density region ($\rho \geq 0$) with $N = V$.

Bianchi IX

Density Region

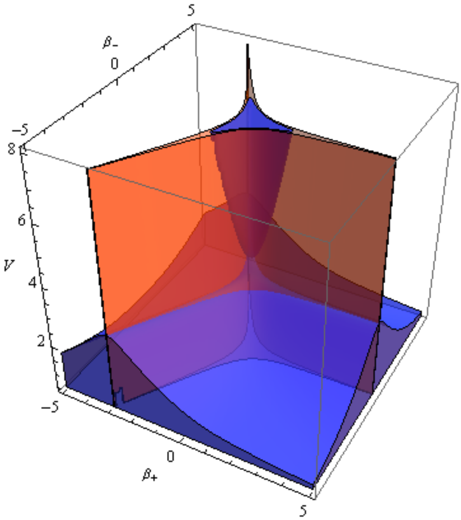


Figure : Comparison of the regions for ρ with $N = V$ and $N = 1$.

Bianchi IX

Density Region

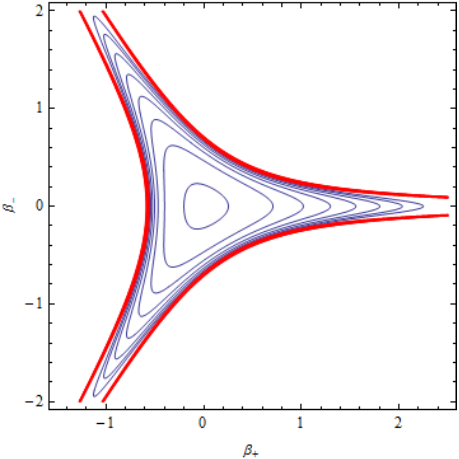


Figure : Comparison of the region boundaries for ρ with $N = V$ and $N = 1$, for different volumes.

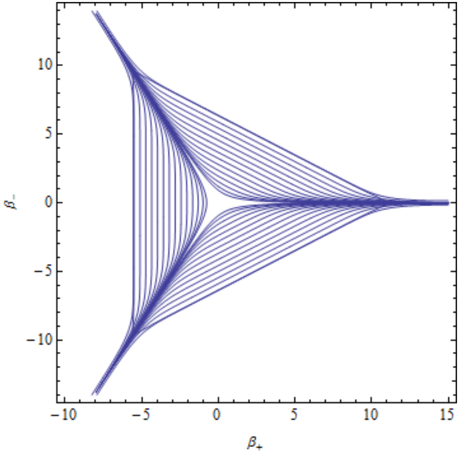


Figure : Comparison of the region boundaries for ρ with $N = 1$, for different volumes.

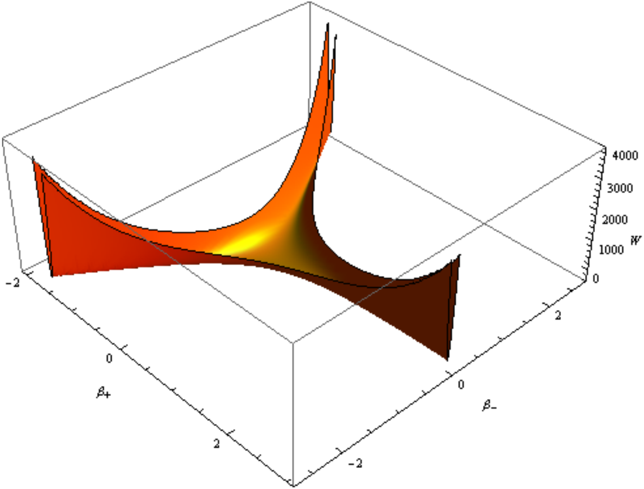


Figure : Potential at $V = 10\,V_{\rm pl}$, in the region $\rho \geq 0$, with $N = V$.

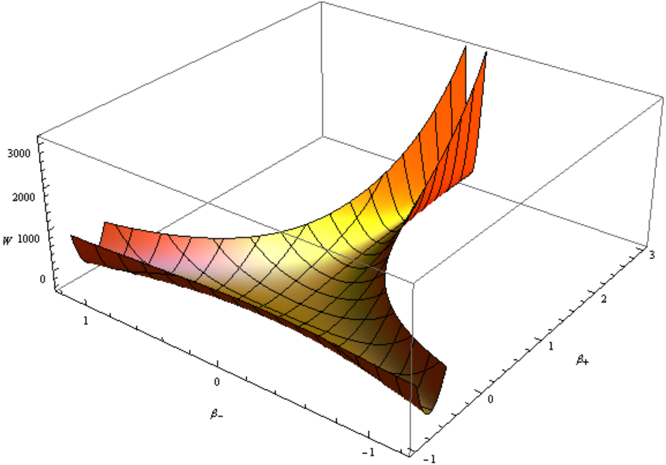


Figure : Potential at $V = 10\,V_{\rm pl}$, in the region $\rho \geq 0$, with $N = V$.

Bianchi IX

Potential

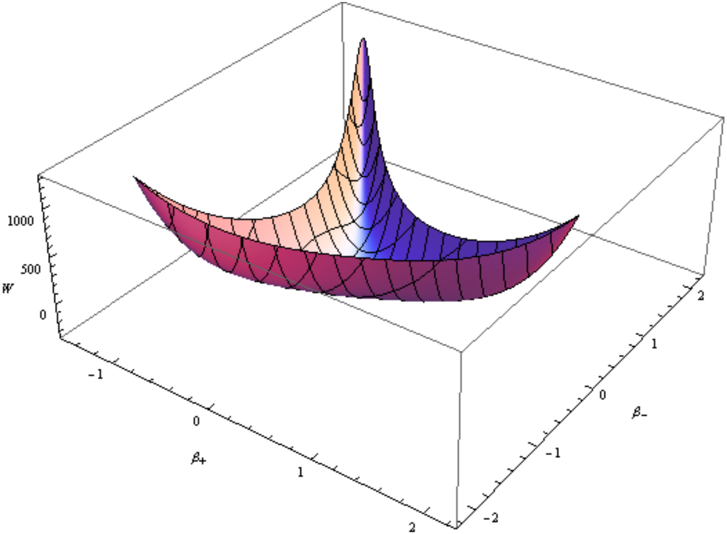


Figure : Potential at $V = 10V_{\mathrm{pl}}$, in the region $\rho \geq 0$, with $N = 1$.

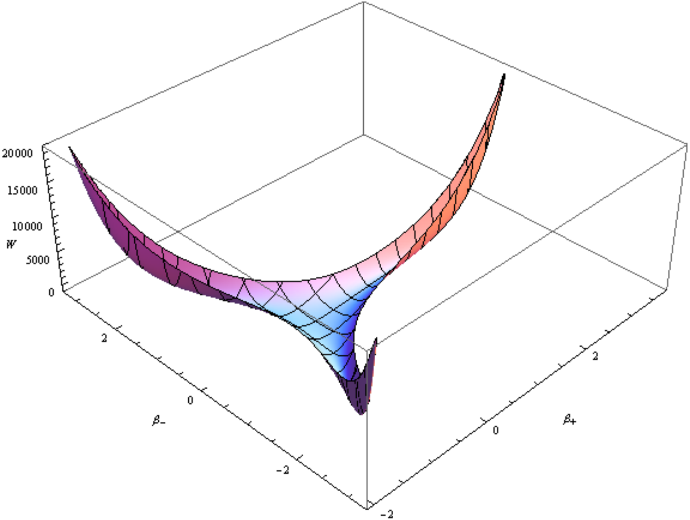


Figure : Potential at $V = 20\,V_{\text{pl}}$, in the region $\rho \geq 0$, with $N = 1$.

- There is an isotropization process.
- Some classical solutions are discarded.
- The triangular symmetry of the classical theory (associated with the Misner variables) is still present, but the qualitative behavior in the anisotropic directions changes drastically due to the quantum effects.
- The potential walls are modify such that there is **not chaotic behavior**.
- These results remain true even when they are consider the quantization ambiguities or different inverse triad corrections or ambiguities in the definition of the matter density.

Acknowledgments

Thank You!!!

Questions

- Is the bouncing non-singular behavior generic for inhomogeneous configurations?
- Are we a step forward toward generic quantum singularity resolution?
- How to understand inhomogeneities in quantum theory?
- What are the observable predictions?
- What can we say about primordial gravitational waves?