Dark Energy and the expanding universe

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L. N. Granda

Departamento de Física
Universidad del Valle

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Outline

- The Cosmological Constant
- Quintesence
- Scalar-tensor model

1. The cosmological constant

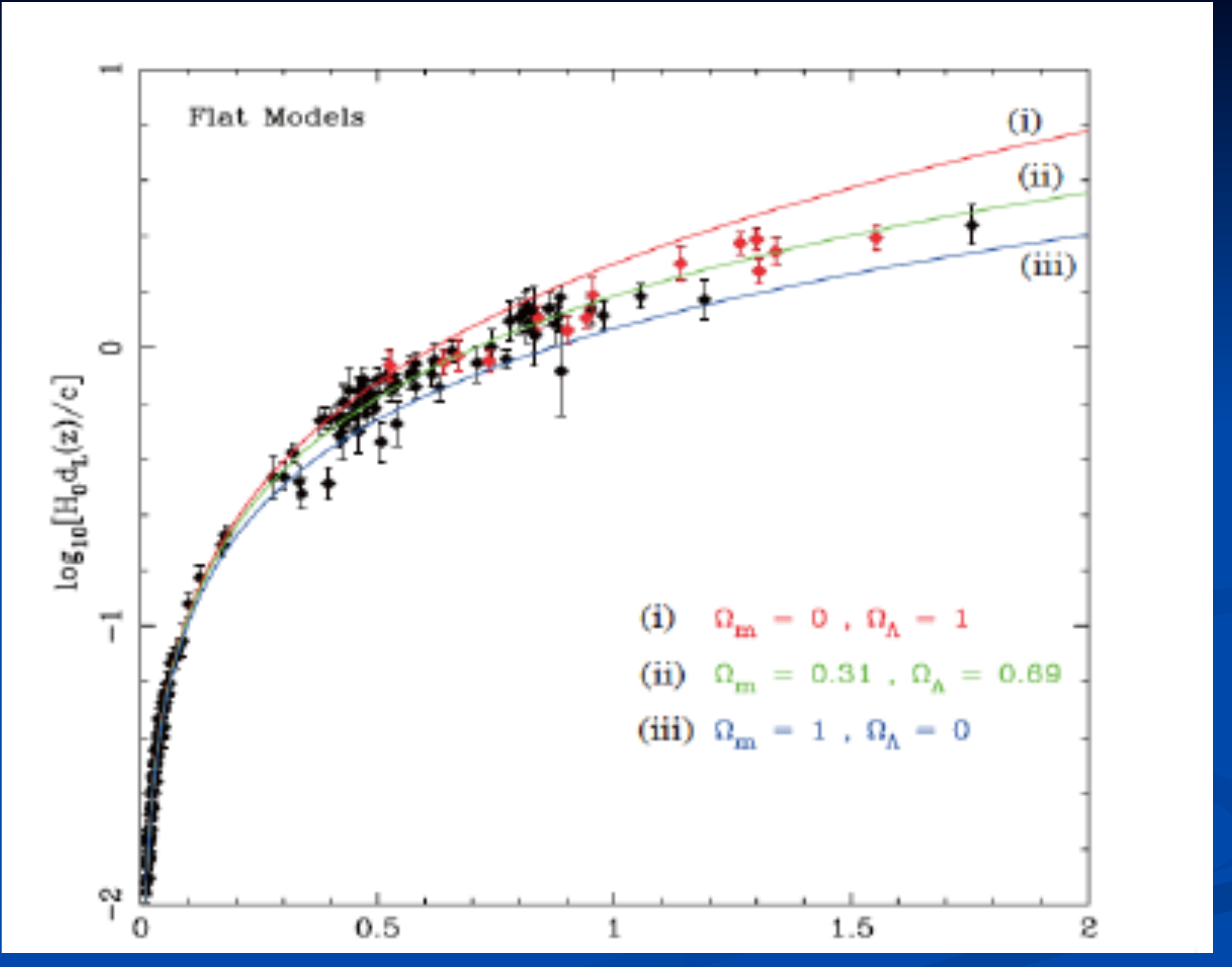
Homogeneity and Isotropy:

The Dynamics and Geometry of the Universe are described by two parameters:

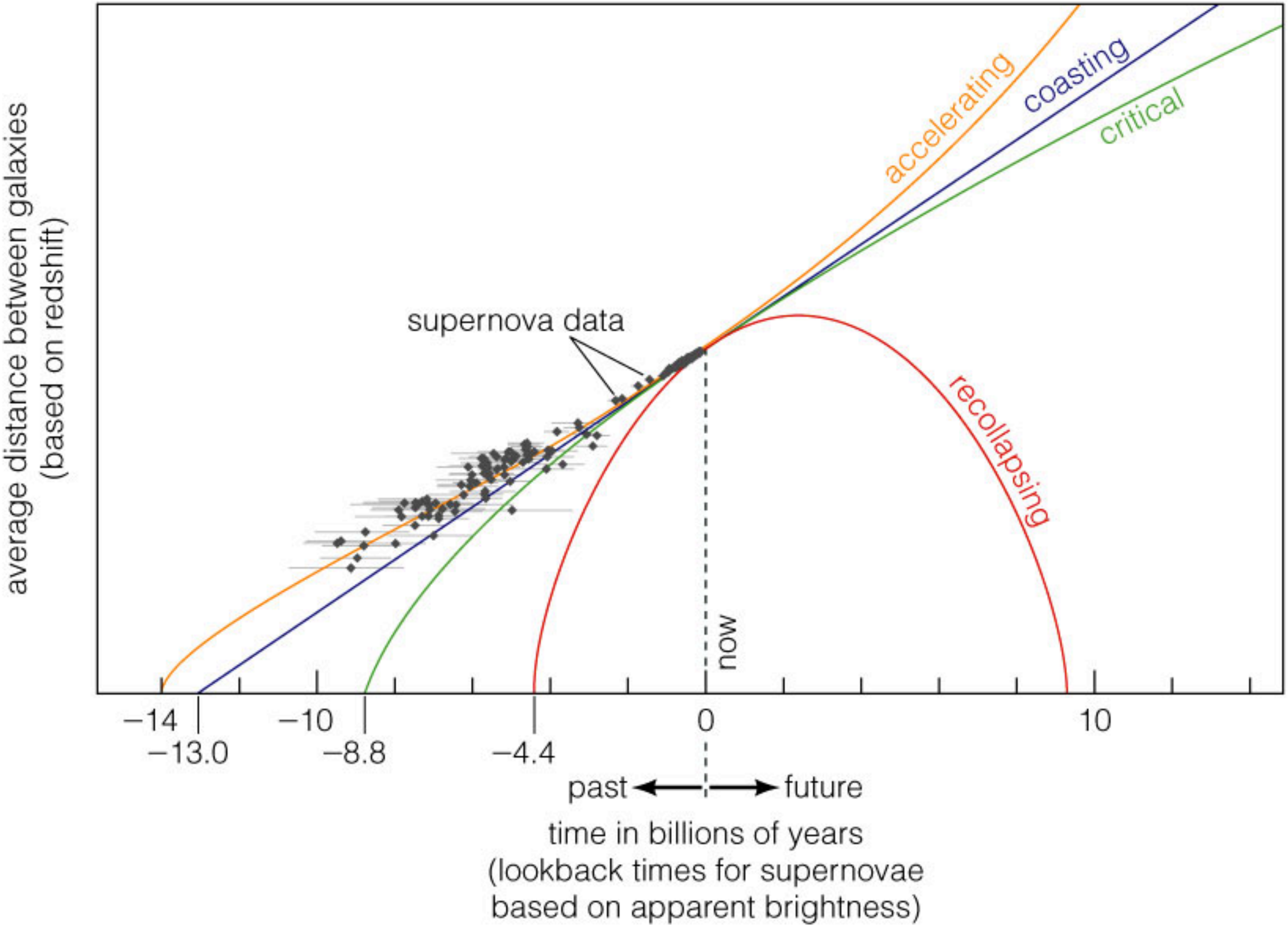
- The scale factor $a(t)$
- The curvature parameter K

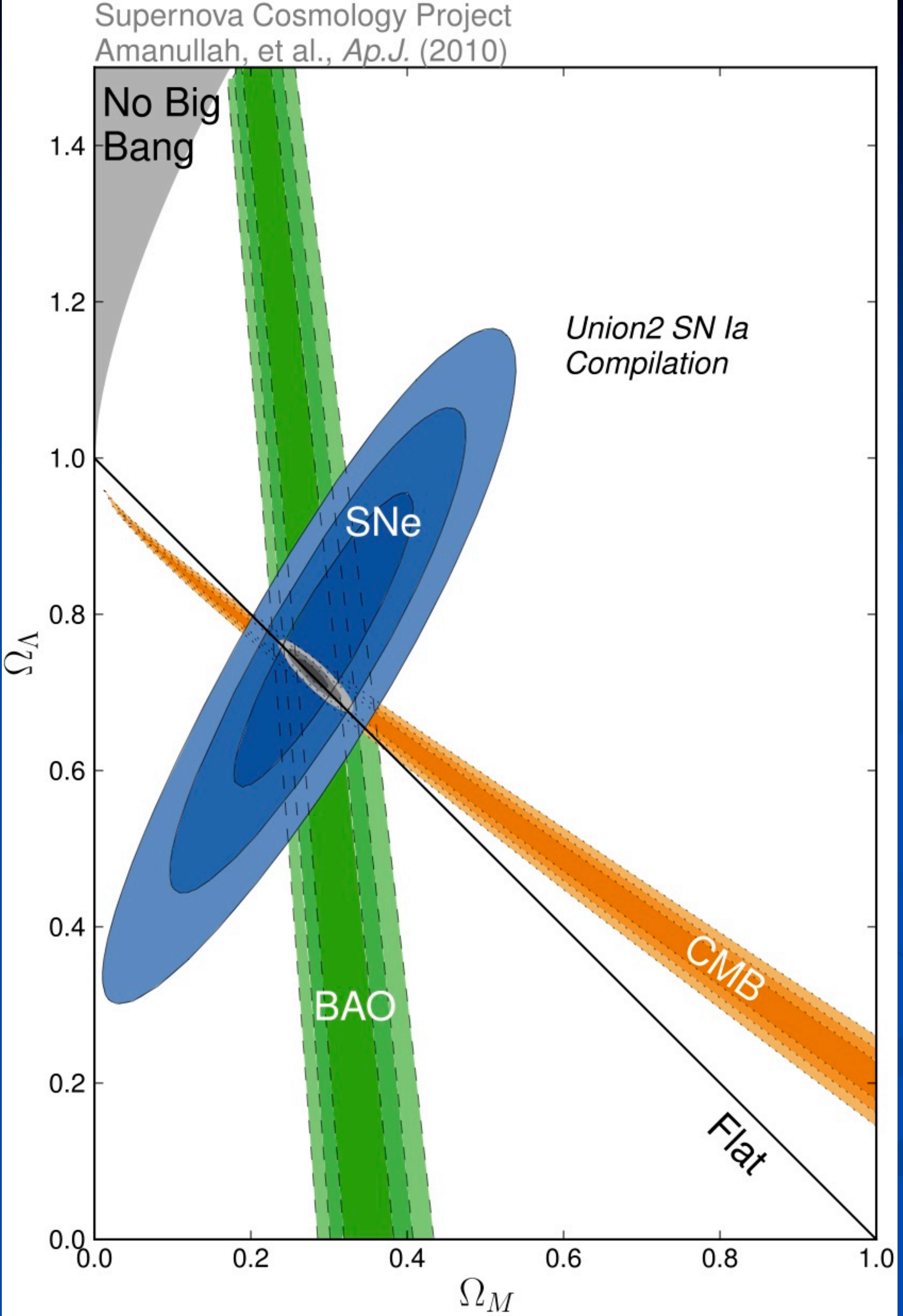
The metric of the space time:

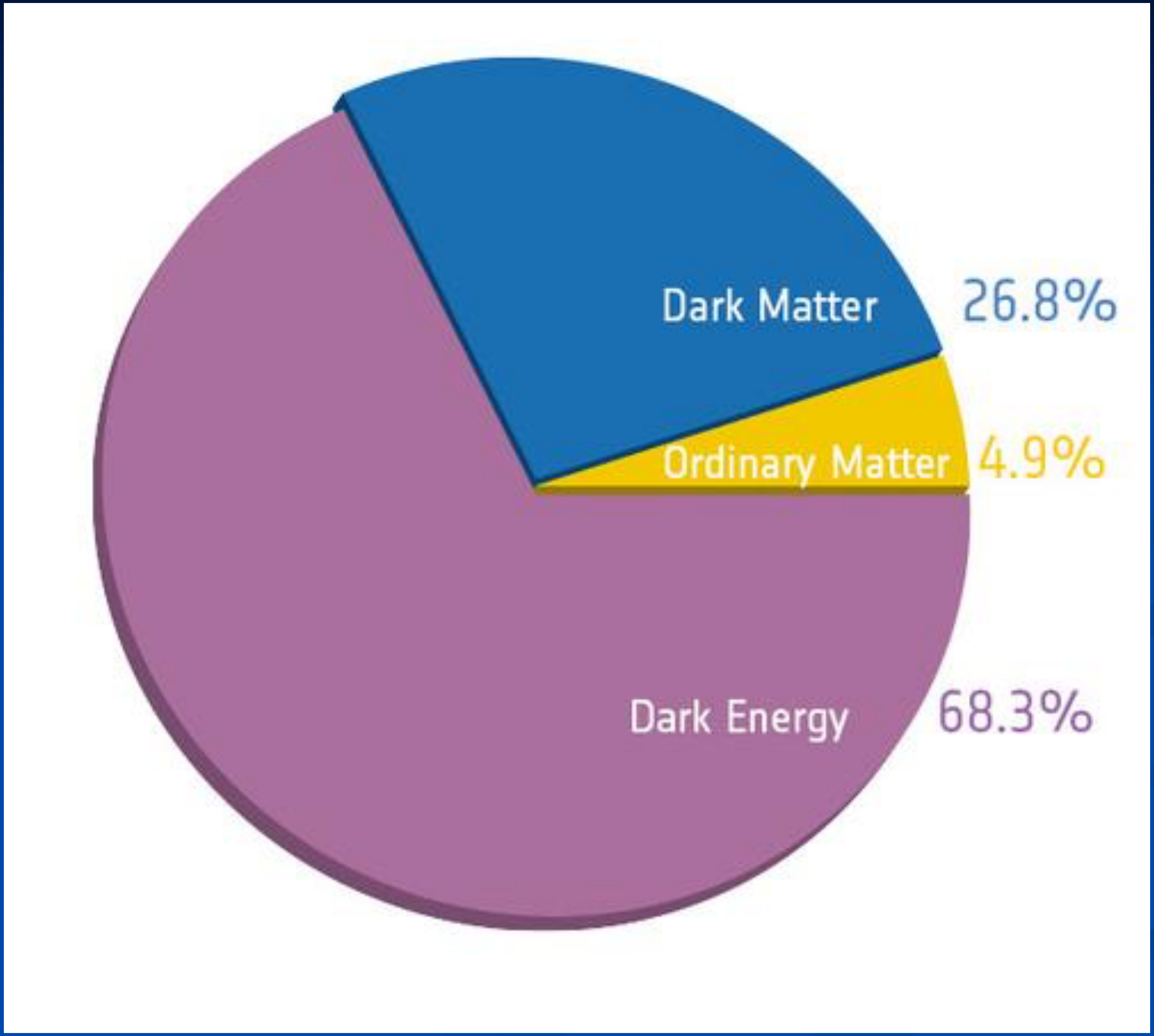
$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2(\theta)d\phi^2)\right)$$



T. R. Choudhury and T. Padmanabhan, Astron. Astrophys. 429, 807 (2005).


average distance between galaxies
(based on redshift)
accelerating
coasting
critical
recollapsing
supernova data
now
−14
−13.0
−10
−8.8
−4.4
0
10
past
future
time in billions of years
(lookback times for supernovae
based on apparent brightness)


Supernova Cosmology Project
Amanullah, et al., *Ap.J.* (2010)
No Big Bang
Union2 SN Ia Compilation
SNe
BAO
CMB
Flat
Ω_Λ
Ω_M
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4



Dark Matter
26.8%
Ordinary Matter
4.9%
Dark Energy
68.3%

$$\overbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}^{Geometry} = \underbrace{8\,\pi\,G\,T_{\mu\nu}}_{Source}$$

- Cosmological constant
- $$T_{\mu\nu} = T^{(m)}_{\mu\nu} + T^{\phi}_{\mu\nu}$$
- $$R \;\Rightarrow\; F(R)$$

The Energy-Momentum tensor

Perfect fluid

$$T_{\mu\nu} = (p + \rho)\, u_\mu u_\nu + p g_{\mu\nu}$$

The Continuity Equation

$$\nabla^\mu T_{\mu\nu} = 0 \;\Rightarrow\; \dot{\rho} + 3\frac{\dot{a}}{a}\,(\rho + P) = 0$$

The Equation of State

$$P = w\rho, \quad w - constant$$

$$\rho \propto a^{-3(1+w)} = \begin{cases} a^{-3}, & \text{for } w = 0\text{, Ordinary and dark Matter} \\ a^{-4}, & \text{for } w = 1/3\text{, Radiation} \end{cases}$$

The Cosmological Equations with Dark Energy

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\left(\frac{\rho_{m0}}{a^3} + \frac{\rho_{r0}}{a^4} + \rho_{DE}\right) - \frac{k}{a^2}$$

$$\dot{a}^2 \propto \frac{\rho_{m0}}{a} + \frac{\rho_{r0}}{a^2} + a^2\rho_{DE} + const$$

$$\rho_{DE} \propto a^p, \quad p > -2$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3P\right)$$

$$P = w\rho \quad \Rightarrow \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(1 + 3w\right)\rho$$

Then, the condition for accelerated expansion:

$$\boxed{w < -\frac{1}{3}}$$

The First Model
The cosmological Constant

What we know about Dark Energy:

1. Smoothly distributed through space
2. Varies very slowly with time
3. Constitutes about 70% of Energy in the Universe

$$\rho \approx \text{constant}, \quad P \approx -\rho$$

$$\rho \propto a^{-3(1+w)}$$

$$\text{if } w = -1 \quad \Rightarrow \quad \rho = const = \rho_\Lambda$$

The cosmological Constant

$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda\, g_{\mu\nu}}_{Cosmological\, Constant} = 8\,\pi\, G\, T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda\, g_{\mu\nu} = \underbrace{8\,\pi\, G\, (T_{\mu\nu} + \rho_{\Lambda}\, g_{\mu\nu})}_{Vacumm\, Energy}$$

$$\Lambda = 8\,\pi\, G\, \rho_{\Lambda}$$

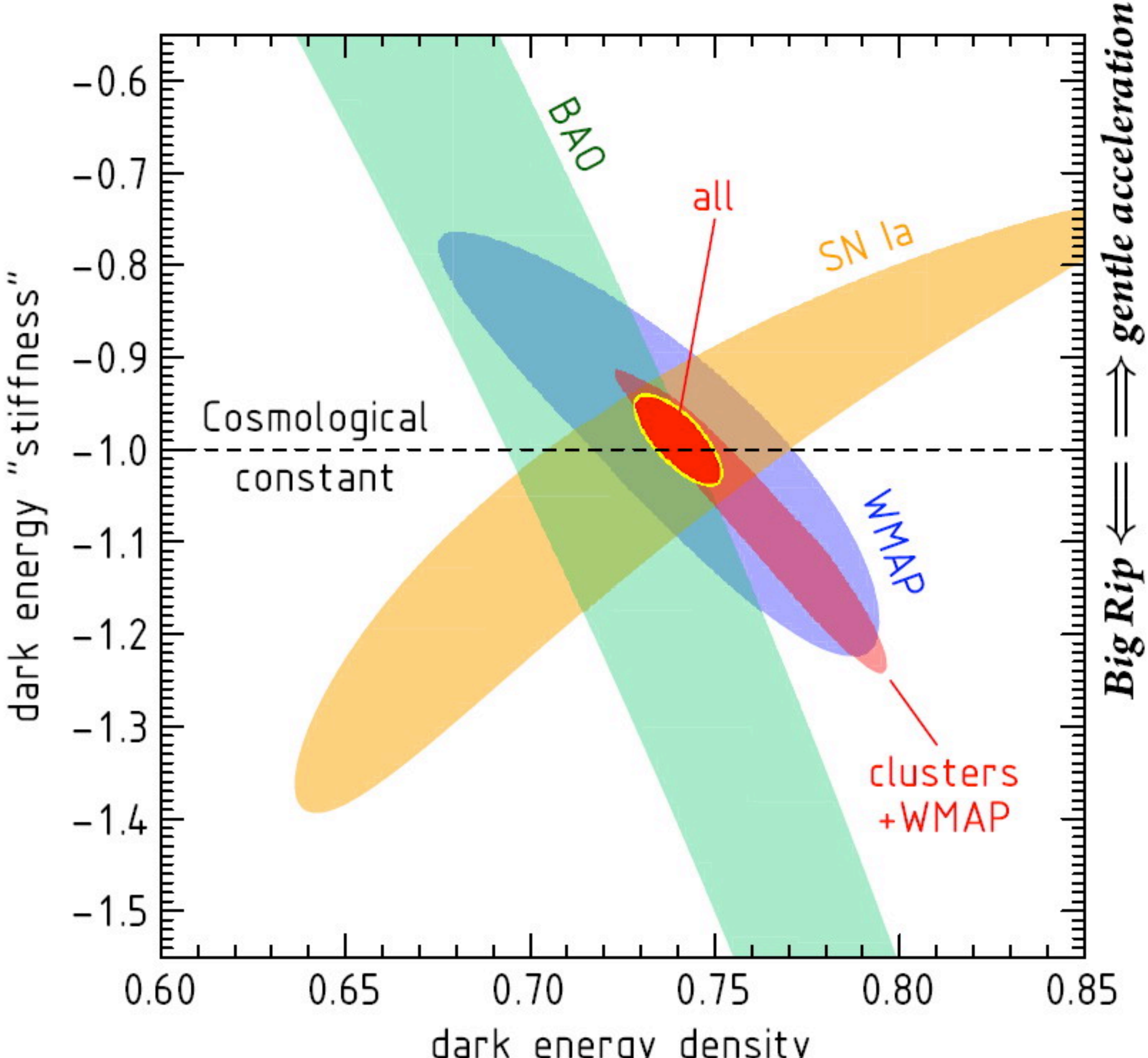
The cosmological Constant

$$\dot{a}^2 \propto \frac{\rho_{m0}}{a} + \frac{\rho_{r0}}{a^2} + a^2\rho_\Lambda + const$$

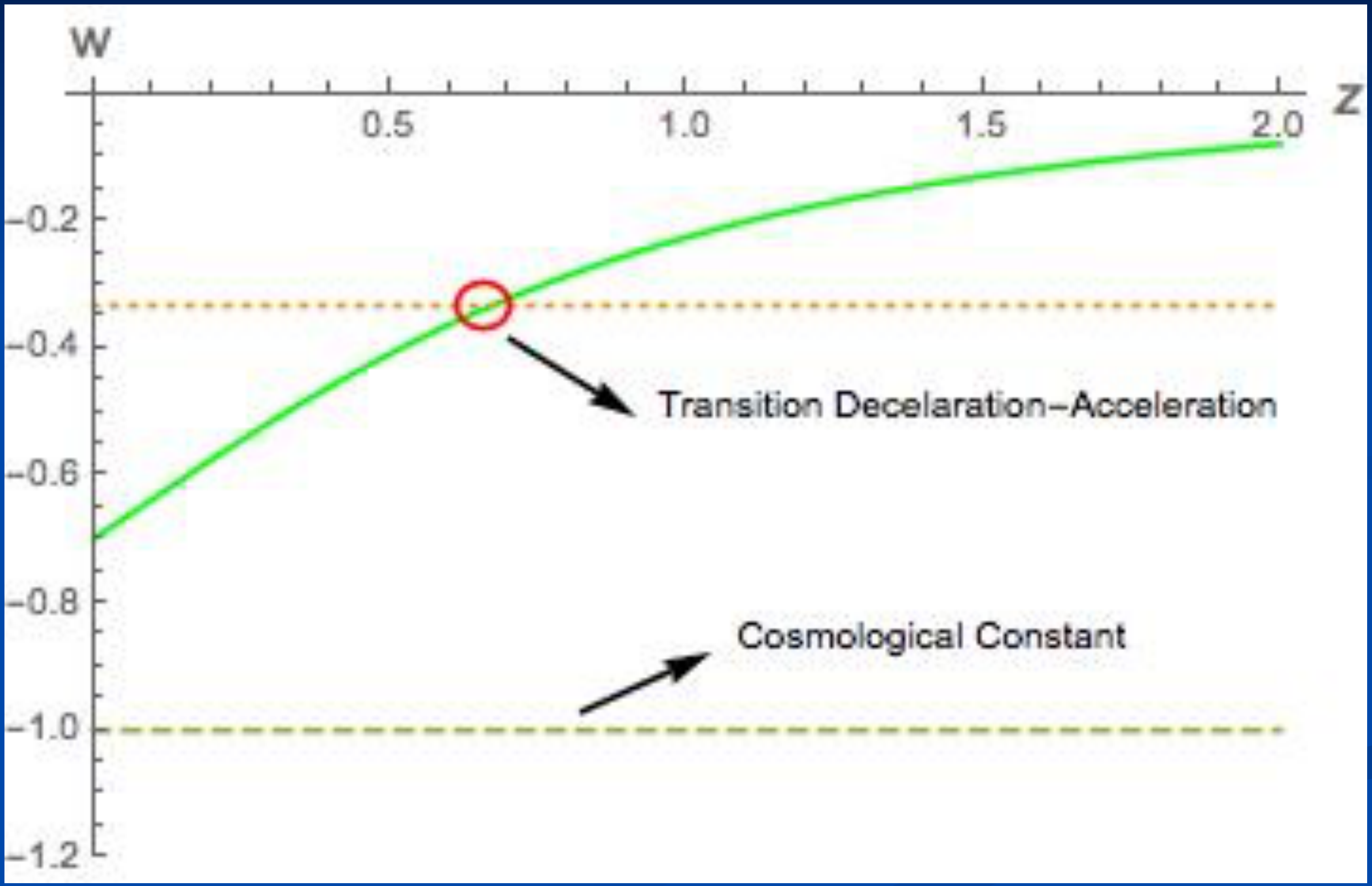
As $a \to \infty$
$\rho_\Lambda a^2$ dominates, and $\ddot{a} > 0$

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_\Lambda$$

$$a = a_0 e^{H_0 t}, \quad H_0 = \left(\frac{8\pi G}{3}\rho_\Lambda\right)^{1/2}$$

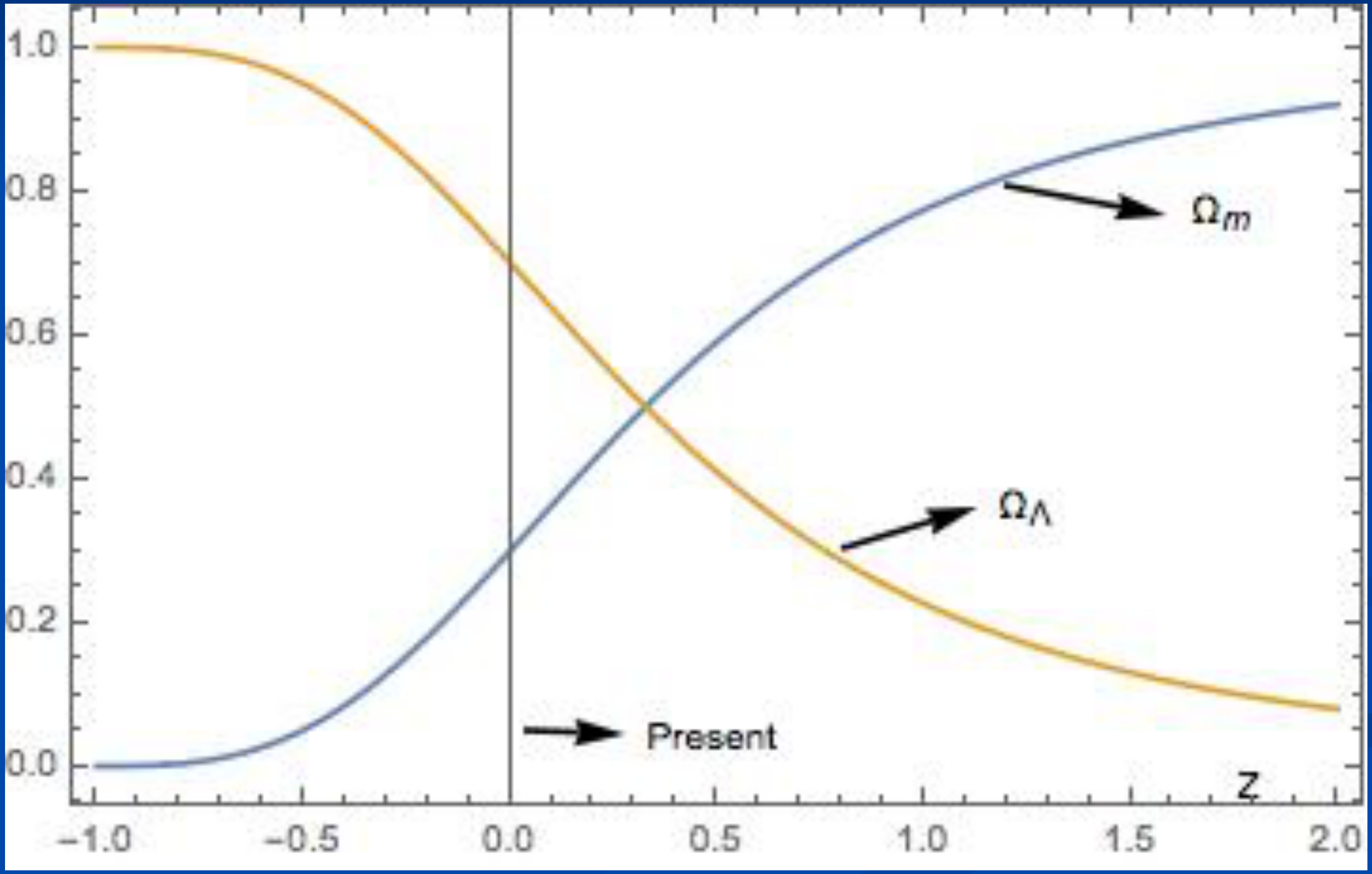

BAO
all
SN Ia
Cosmological
constant
WMAP
clusters
+WMAP
dark energy "stiffness"
dark energy density
−0.6
−0.7
−0.8
−0.9
−1.0
−1.1
−1.2
−1.3
−1.4
−1.5
0.60
0.65
0.70
0.75
0.80
0.85
Big Rip ⟸ ⟹ gentle acceleration

The Equation of State for the ΛCDM



The Density Parameter (k=0)

$$\Omega = \Omega_m + \Omega_\Lambda = 1$$



Problems

1. Fine tuning

$$\rho_{vac} \approx M_p^4 \approx (10^{27}\, eV)^4 \approx 10^{108}\, eV^4$$
$$\rho_{vac}^{(obs)} = \rho_\Lambda \approx (10^{-3}\, eV)^4 \approx 10^{-12}\, eV^4$$
$$\rho_{vac} \approx 10^{120} \rho_{vac}^{(obs)}$$

2. Coincidence problem

Why $\Omega_\Lambda \approx \Omega_m$

Why is the vacuum energy so important now?

2. Quintessence

Dynamical Dark Energy Scalar field models - Quintessence

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G\left(T^{(m)}_{\mu\nu} + T^{(\phi)}_{\mu\nu}\right)$$

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - V(\phi)\right)$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$H^2 = \frac{8\,\pi\,G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\frac{\ddot{a}}{a} = -\frac{8\,\pi\,G}{3}\left[\dot{\phi}^2 - V(\phi)\right]$$

The Universe accelerates if $\dot{\phi}^2 < V(\phi)$

Dynamical Dark Energy (Quintessence)

Dark energy doesn't vary quickly, but maybe slowly.

$$w_\phi = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} \geq -1$$

$$\rho = \rho_0\, exp\left[-\int 3(1+w_\phi)\,\frac{d\,a}{a}\right]$$

- As $\dot{\phi} \to 0, \quad w_\phi \to -1$ and $\rho \to const$
- As $V(\phi) \to 0, \quad w_\phi \to 1$ and $\rho \to a^{-6}$

$$\rho \propto a^{-m}, \;\; 0 < m < 6$$

Power-Law Solution

$$a(t) \propto t^p$$

$$a \propto t^p = \begin{cases} t^{2/3}, & \text{for Ordinary and dark Matter} \\ t^{1/2}, & \text{for Radiation} \\ p > 1, \ \ddot{a} > 0 \ \Rightarrow & \text{accelerated expansion} \end{cases}$$

$$\phi \propto ln\, t \quad V(\phi) = V_0\, exp\Big(- \sqrt{\frac{16\,\pi}{p}} \frac{\phi}{m_{pl}} \Big)$$

$$m_\phi \approx 10^{-33}\, eV \approx 10^{60}\, M_p$$

3. Scalar-Tensor Model

Scalar with non-minimal coupling to curvature and to Gauss-Bonnet

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{2}\xi\phi^2 R - \eta(\phi)G + \mathcal{L}_m\right]$$

$$H^2 = \frac{\kappa^2}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + 3\xi H\left(2\phi\dot{\phi} + H\phi^2\right) + 24H^3\frac{d\eta}{d\phi}\dot{\phi} + \rho_m\right]$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} + 6\xi\left(2H^2 + \dot{H}\right)\phi + 24H^2\left(H^2 + \dot{H}\right)\frac{d\eta}{d\phi} = 0$$

Exact Solutions with V=0

x=ln a

$$H^2 = \frac{\kappa^2}{3}\left[\frac{1}{2}H^2\left(\frac{d\phi}{dx}\right)^2 + 3\xi H^2\left(2\phi\frac{d\phi}{dx} + \phi^2\right) + 24H^4\frac{d\eta}{dx}\right]$$

$$\frac{d}{dx}\left[H^2\left(\frac{d\phi}{dx}\right)^2\right] + 6H^2\left(\frac{d\phi}{dx}\right)^2 + 6\xi\left(4H^2 + \frac{dH^2}{dx}\right)\phi\frac{d\phi}{dx} + 24H^2\left(2H^2 + \frac{dH^2}{dx}\right)\frac{d\eta}{dx} = 0$$

$$\frac{d\eta}{dx} = \frac{1}{H^2}\left(g_1 + g_2\phi^2\right)$$

The Friedmann Equation

$$\left(\frac{d\phi}{dx}\right)^2 + 12\xi\phi\frac{d\phi}{dx} + 6\left(\xi + 8g_2\right)\phi^2 + 48g_1 - \frac{6}{\kappa^2} = 0$$

The Equation of Motion

$$\left[\frac{d}{dx}\left(\frac{d\phi}{dx}\right)^2 + 6\left(\frac{d\phi}{dx}\right)^2 + 24\xi\phi\frac{d\phi}{dx} + 48\left(g_1 + g_2\phi^2\right)\right]H^2 + \left[\left(\frac{d\phi}{dx}\right)^2 + 6\xi\phi\frac{d\phi}{dx} + 24\left(g_1 + g_2\phi^2\right)\right]\frac{dH^2}{dx} = 0$$

$$g_1 = 1/(8\kappa^2)$$

$$\frac{d\phi}{dx} = \left(-6\xi \pm \sqrt{6\xi(6\xi - 1) - 48g_2}\right)\phi$$

$$\phi(x) = \phi_0 e^{-\lambda x}, \qquad \lambda = 6\xi \pm \sqrt{6\xi(6\xi - 1) - 48 g_2}$$

$$\left(\lambda^2 - 6\xi\lambda + 24 g_2 + \frac{3}{\kappa^2 \phi_0^2} e^{2\lambda x}\right) \frac{dH^2}{dx} +$$
$$2\left(-\lambda^3 + 3\lambda^2 - 12\xi\lambda + 24 g_2 + \frac{3}{\kappa^2 \phi_0^2} e^{2\lambda x}\right) H^2 = 0$$

$$w_{DE} = -1 - \frac{1}{3H^2}\frac{dH^2}{dx}$$

1. $(x >> 1)$ if $\lambda > 0$ $\qquad$ $H^2 = H_0^2 e^{-2x}$

$$w = -1/3$$

2. $\lambda < 0$

$$H^2 = H_0^2 \exp\left(\frac{2\lambda^3 - 6\lambda^2 + 24\xi\lambda - 48g_2}{\lambda^2 - 6\xi\lambda + 24g_2} x\right)$$

$$w = -1 - \frac{1}{3}\left(\frac{2\lambda^3 - 6\lambda^2 + 24\xi\lambda - 48g_2}{\lambda^2 - 6\xi\lambda + 24g_2}\right)$$

3. The restriction

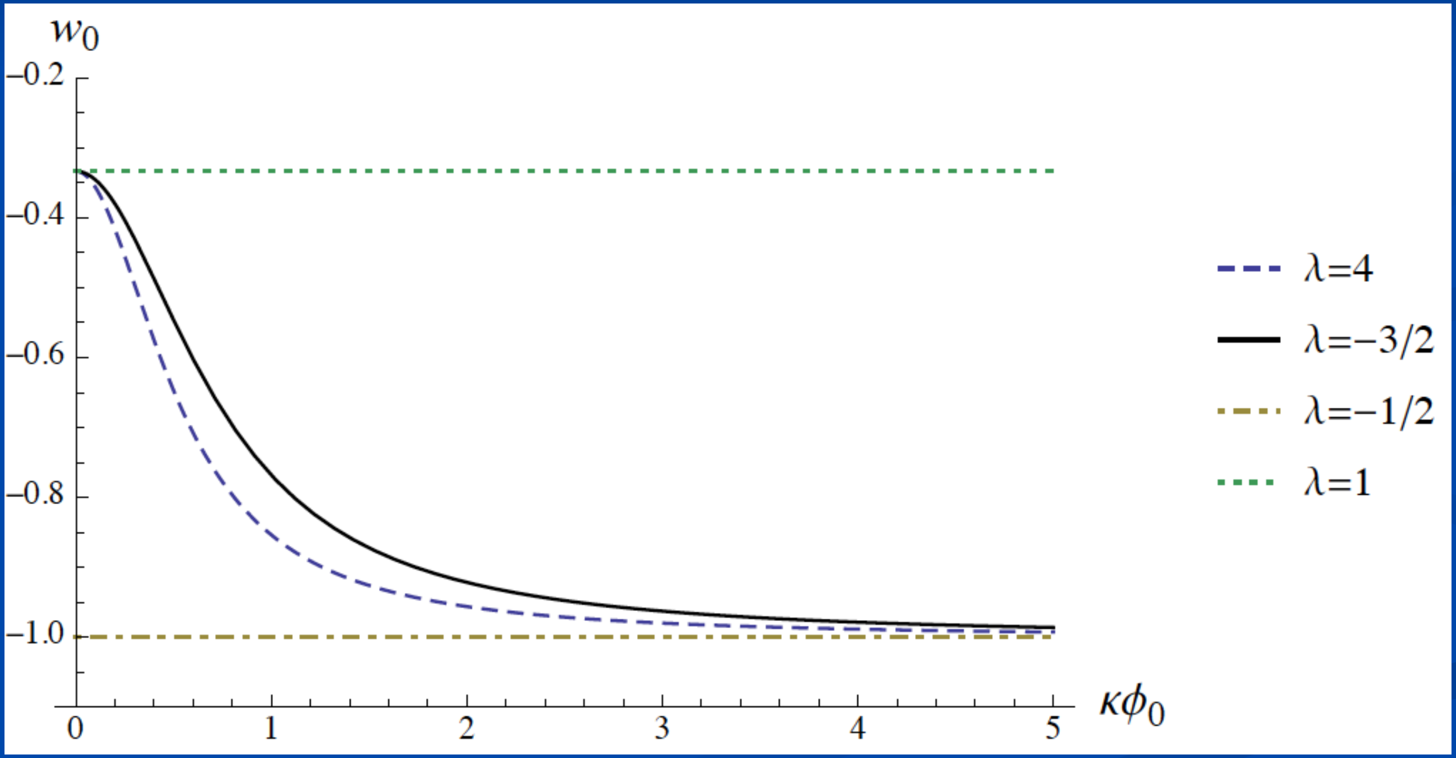
$$2\lambda^3 - 6\lambda^2 + 24\xi\lambda - 48g_2 = 0$$

$$H^2 = h_0^2\left[\kappa^2\phi_0^2(\lambda^2 - 6\xi\lambda + 24g_2) + 3e^{2\lambda x}\right]^{-1/\lambda}$$

$$w = -1 + \frac{2e^{2\lambda x}}{\kappa^2\phi_0^2(\lambda^2 - 6\xi\lambda + 24g_2) + 3e^{2\lambda x}}$$

$$w(x=0)=w_0$$

$$\frac{2\lambda^3-2\lambda^2}{2\lambda+1}=-\frac{1}{(\kappa\phi_0)^2}\frac{3w_0+1}{w_0+1}$$



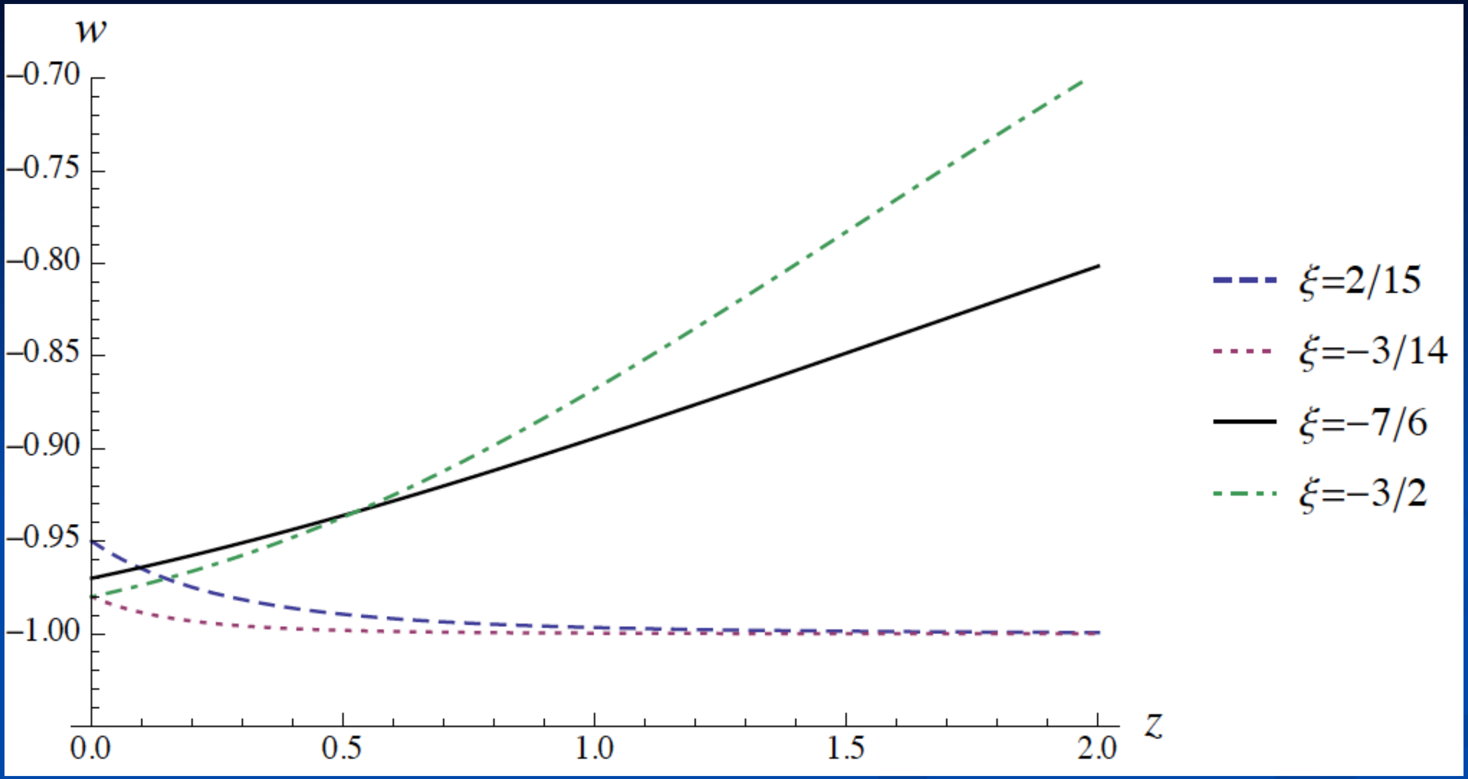


Fig. 2 *The evolutions of the DE EOS parameter w as function of z for $\xi = (2/15, -3/14, -7/6, -3/2)$ which give $(\lambda = 2, 3, -1, -3/2)$ and $w_0 \approx -0, 95, -0.98, -0.97, -0.98$ respectively, for $\kappa\phi_0 = 4$. Though the current values are very similar, the DE evolves to different scenarios depending ξ (which defines the sign of λ).*

4. The general case

$$H^2 = h_0^2 e^{-\frac{\gamma}{\beta}x}\left[\kappa^2\phi_0^2\beta + 3e^{2\lambda x}\right]^{-\frac{2\beta-\gamma}{2\beta\lambda}}$$

$$\beta = \lambda^2 - 6\xi\lambda + 24g_2, \quad \gamma = -2\lambda^3 + 6\lambda^2 - 24\xi\lambda + 48g_2$$

$$w = -1 + \frac{\gamma(\kappa\phi_0)^2 + 6e^{2\lambda x}}{3\beta(\kappa\phi)^2 + 9e^{2\lambda x}}$$

$$\lim_{x\to-\infty} w = -1 + \frac{\gamma}{3\beta} \quad \text{and} \quad \lim_{x\to\infty} w = -\frac{1}{3} \quad \text{for} \quad \lambda > 0$$

$$\lim_{x\to-\infty} w = -\frac{1}{3} \quad \text{and} \quad \lim_{x\to\infty} w = -1 + \frac{\gamma}{3\beta} \quad \text{for} \quad \lambda < 0$$

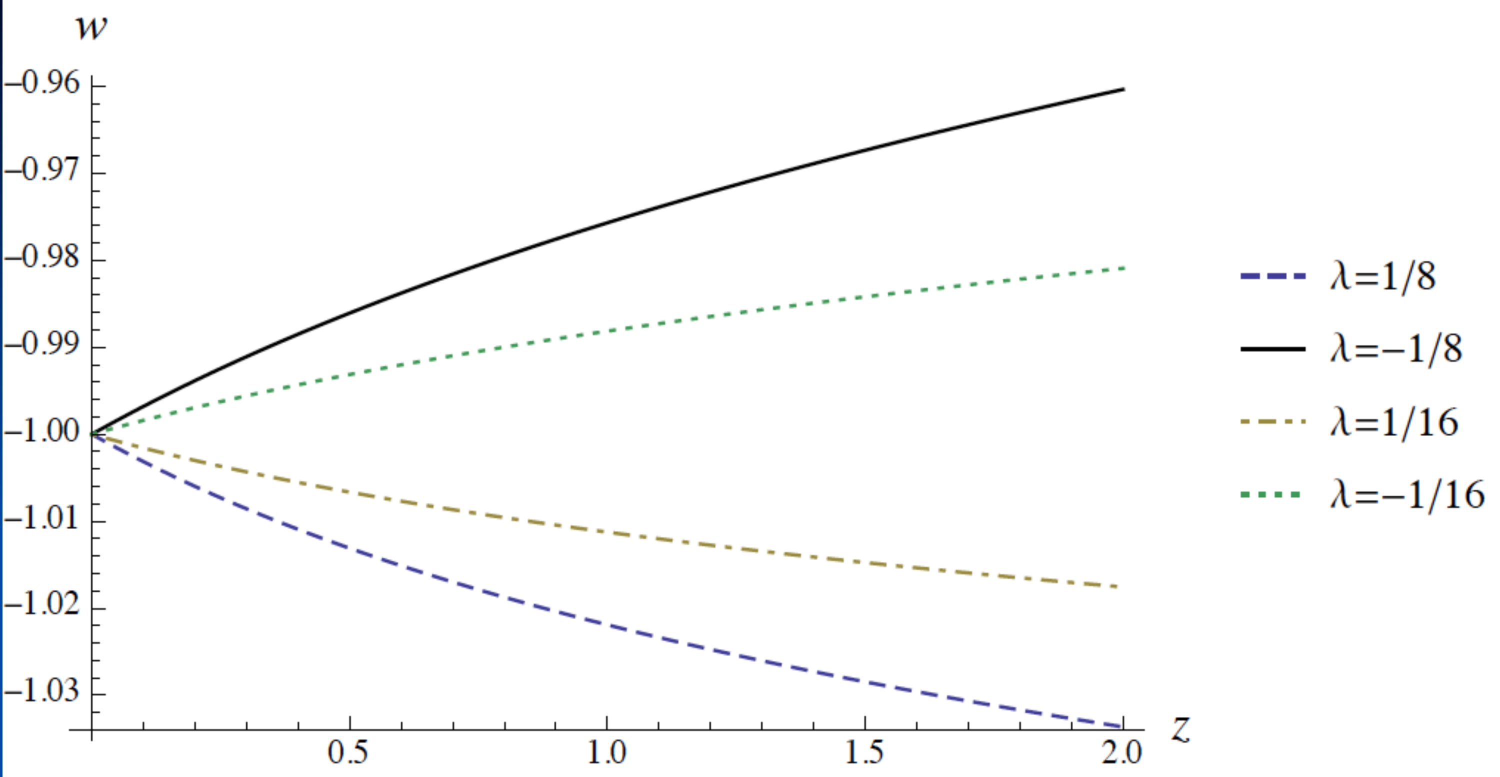


Fig. 3 *The behavior of the DE EOS w as function of the redshift. The initial value of the scalar field is $\kappa\phi_0 = 2$ and the current value is $w_0 = -1$. For $\lambda = (1/8, -1/8, 1/16, -1/16)$ the corresponding coupling parameters take the values $\xi = (5, -5, 10, -10)$ and $g_2 = 0.28$ for all cases.*

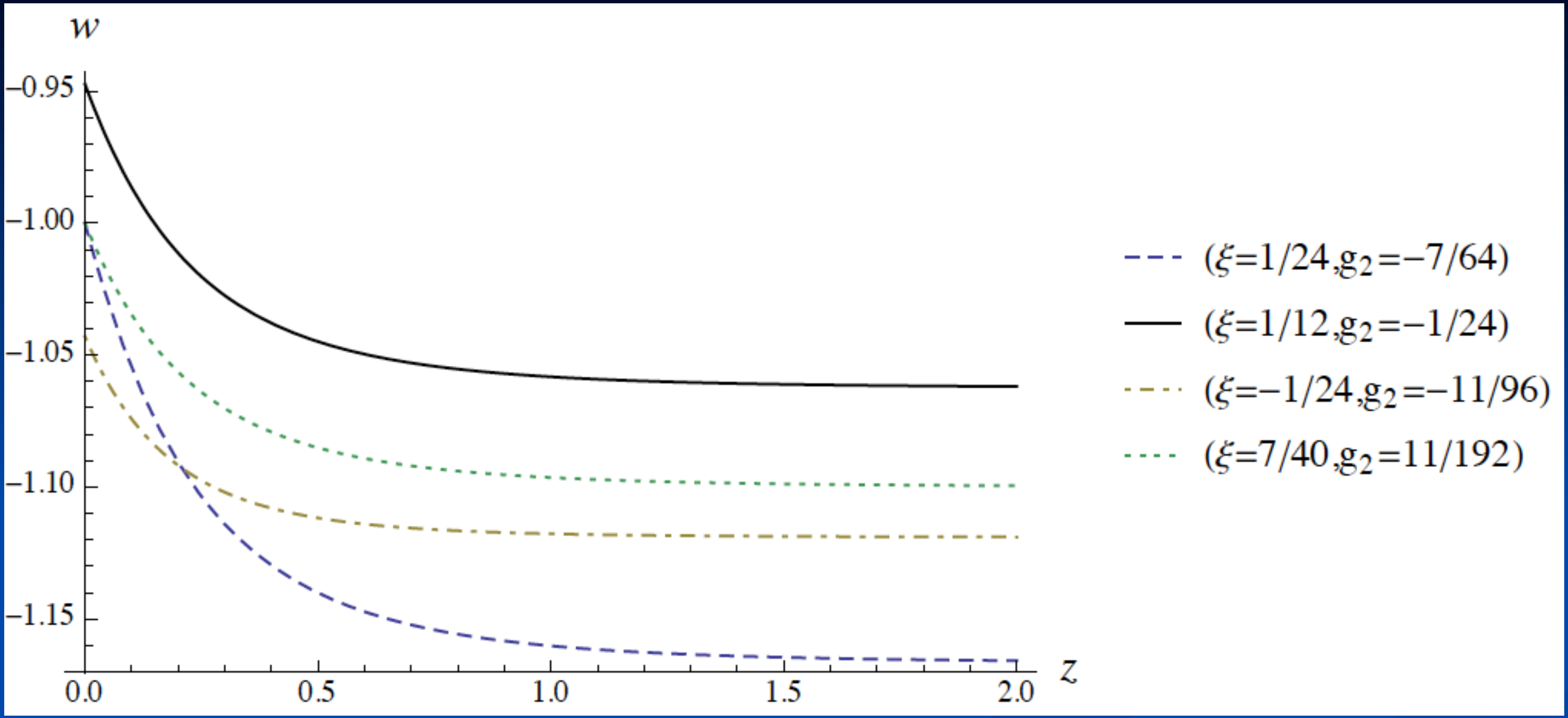


Fig. 4 *The DE EOS w as function of the redshift for small parameters. In all cases $\lambda > 0$ and the initial value of the scalar field is $\kappa\phi_0 = 2$. The curves show different current EOS w_0, but all of them are close to -1, and present transitions from phantom to quintessence phase in the past ($w_0 > -1$), the present ($w_0 = -1$) and the future ($w_0 < -1$).*

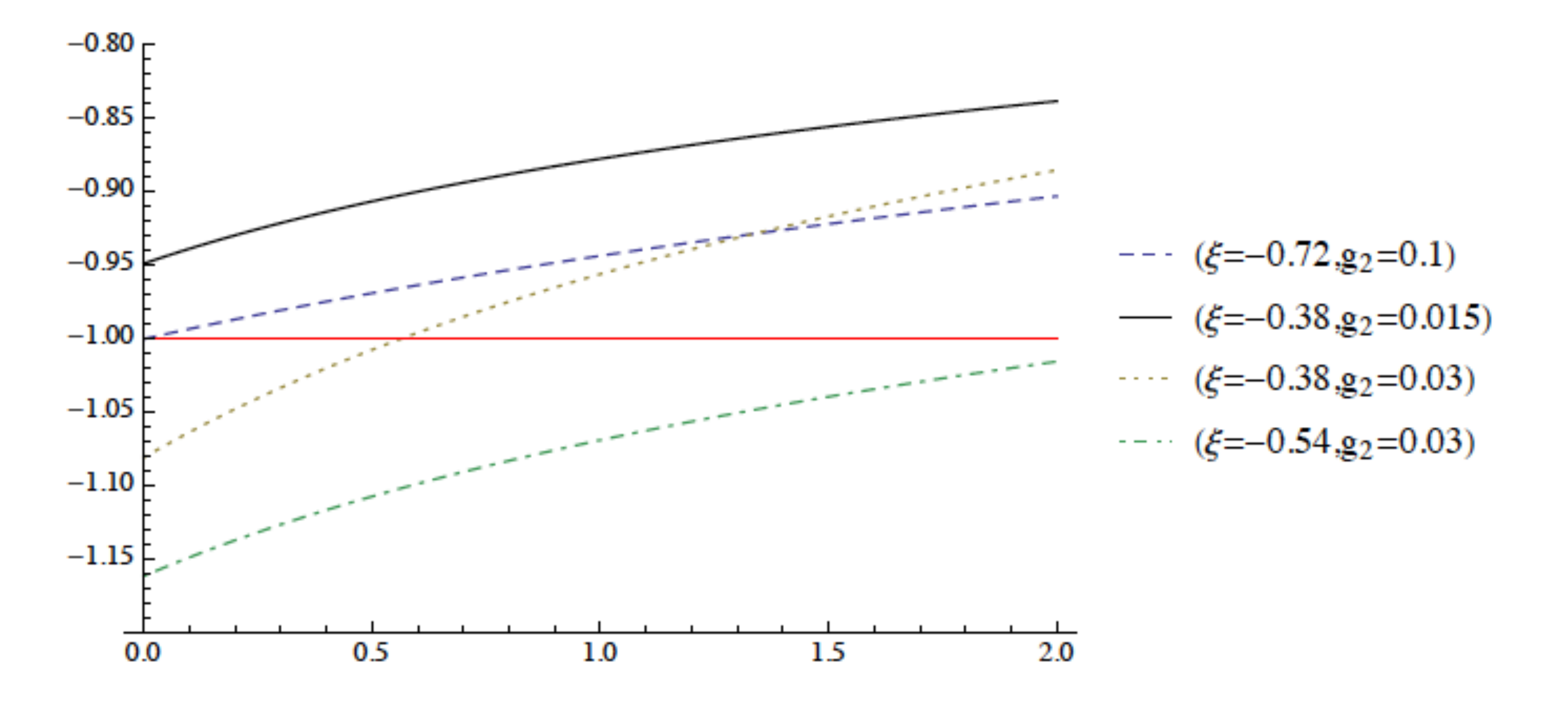


Fig. 5 *The DE EOS w as function of the redshift for $\lambda < 0$. In all cases the initial value of the scalar field is $\kappa\phi_0 = 4$. The curves cover different behaviors of the EOS, but all of them in an acceptable range of values according to the data. Note the transitions from quintessence to phantom phase in the past ($w_0 < -1$), the present ($w_0 = -1$) and the future ($w_0 > -1$). The horizontal line corresponds to the cosmological constant.*

Phantom power-law

$$w < -1 \text{ (or } \dot{H} > 0)$$

$$H = \frac{p}{t_s - t}, \quad \phi = \phi_0 \ln\left(\frac{t_s - t}{t_0}\right)$$

$$\eta(\phi) = \left[c_0'(p,\xi) + c_1'(p,\xi)\phi + c_2'(p,\xi)\phi^2\right] e^{2\phi/\phi_0}$$

$$V(\phi) = \left[d_0'(p,\xi) + d_1'(p,\xi)\phi + d_2'(p,\xi)\phi^2\right] e^{-2\phi/\phi_0}$$

Big-Rip Singularity

$$a \to \infty, \quad p \to \infty, \quad \rho \to \infty$$

At finite time

Little-Rip solutions

$$w < -1 \ (\text{or } \dot{H} > 0)$$

$$H = H_0 e^{ht} \qquad \phi = \phi_0 e^{ht}$$

$$\eta(\phi) = \frac{{\phi_0}^2 \left(h^2\kappa^2(1-4\xi){\phi_0}^2 + 2hH_0\kappa^2(1-4\xi)\phi\phi_0 + 2{H_0}^2\right)}{16{H_0}^4\kappa^2\phi^2}$$

$$V(\phi) = m_1\phi + m_2\phi^2 + m_3\phi^3 + m_4\phi^4$$

Solutions with quintessence and phantom phase

$$H = p\left(\frac{1}{t} + \frac{1}{t_s - t}\right) \qquad \phi = \frac{\phi_0}{t_0}t$$

$t < t_s/2$ Quintessence phase

$t > t_s/2$ Phantom phase

Conclusions

- There is not a definite theoretical model to explain the observations.

- The nature of dark energy is a fundamental problem for cosmology and particle physics. We need more SN1a results complemented by observational constraints on CMB.

- The equation of state plays a central role: If $w=-1$, the DE is a cosmological constant with it's fine-tuning and coincidence problems. If $w>-1$ quintessence models will give the answer.

- If $w<-1$, we will face new theoretical challenges.